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SNR and Noise Variance Estimation for MIMO Systems

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# SNR and Noise Variance Estimation for MIMO Systems

**System Model:** $\mathbf{y}_i = \mathbf{H}\mathbf{x}_i + \mathbf{n}_i, \quad i = 1...N$

$\mathbf{y}_i : M_R x 1, \quad \mathbf{H} : M_R x M_T, \quad \mathbf{x}_i : M_T x 1$

$\mathbf{X} = [\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_N}]$

**SNR definition:**

$\rho_{xy} = \frac{\|h_{xy}\|^2}{\sigma^2}$

$\rho = \frac{\|\mathbf{H}\|_\mathbf{F}^\mathbf{2}}{\sigma^2}$

**CRLB for SNR estimation: Data aided(DA)**

$\mathbf{\Theta} = [(vec(\mathbf{H}))^T (vec(\mathbf{H}^*))^T \sigma^2]^T$

# SNR and Noise Variance Estimation for MIMO Systems

CRLB of $\rho_{xy} = 2\alpha\rho_{xy} + \frac{\rho_{xy}^2}{NM_R}$

CRLB of $\rho = \frac{2\rho}{N} + \frac{\rho^2}{2N}$

**Non-data aided model(NDA)**

**-No closed form expression**

**-Numerical evaluation**

**CRLB of Noise Variance: DA**

$CRLB[\sigma^2] = \frac{\sigma^4}{NM_R}$

$CRLB[\sigma_k^2] = \frac{\sigma_k^4}{N}$

**Mixed DA and NDA model**

$N = N_p + N_d$

$\frac{1}{CRLB[N]_{DA\_NDA}} = \frac{1}{CRLB[N_p]_{DA}} + \frac{1}{CRLB[N_d]_{NDS}}$

# SNR and Noise Variance Estimation for MIMO Systems

*ML Estimators:DA*

$\widehat{SNR} = \frac{\|\hat{\mathbf{H}}\|^2}{\hat{\sigma^2}}$

1) Noise Variance

$$\hat{\sigma}^2_{DA\_ML} = \frac{1}{NM_R} \sum_{i=1}^{N} (\mathbf{y_i} - \mathbf{Hx_i})^H (\mathbf{y_i} - \mathbf{Hx_i})$$

2)Channel Coefficients

ML estimate is the one which maximizes $-\sum_{i=1}^{N} \|\mathbf{y_i} - \mathbf{Hx_i}\|^2$

*ML Estimators:NDA*

-No closed form solution

-Approximation in the low SNR and high SNR regimes

-EM algorithm for iteratively attaining ML estimate

Nondata-Aided Joint Channel Estimation and Equalization for OFDM Systems in Very Rapidly Varying Mobile Channels

Habib Senol,Erdal Panayirci, *Kadir Has University, Turkey*, H. Vincent Poor, *Princeton University*

The received signal model is $\mathbf{y}(m) = \mathbf{S}(m)\mathbf{h}(m) + \mathbf{w}(m)$

Estimate $\mathbf{h} = [\mathbf{h}^T(0), \mathbf{h}^T(1), ..., \mathbf{h}^T(M-1)]^T \in \mathcal{C}^{MNL}$ from $\mathbf{y} = [\mathbf{y}^T(0), \mathbf{y}^T(1), ..., \mathbf{y}^T(M-1)]^T \in \mathcal{C}^{MN}$

Basis Expansion: $\tilde{\mathbf{h}}_l = \mathbf{\Psi}\mathbf{c}_l$ and $\mathbf{c}_l = \mathbf{\Psi}^{\dagger}\tilde{\mathbf{h}}_l$
where $\mathbf{\Psi}$ is the orthonormal discrete Legendre polynomial basis (DLP-BEM)

- Time domain implementation
- Exploit time diversity of the channel
- CRLB for the estimator of channel coefficient vector

SAGE-MAP

1. Expectation step: Compute an estimate of the log-likelihood function of 'hidden data' given $\mathbf{y}$ and a current estimate of $\mathbf{c}$
2. Maximization step: $c$ is updated as the value that maximizes this estimate of the log likelihood function

The initial channel estimate $\mathbf{c}^{(0)}$ can be determined with the aid of pilot symbols

Also computes

1. Bayesian MSE lower bound
2. Convergence rate
3. Complexity

Sparse Bayesian Methods for Low-Rank Matrix Estimation

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Martin Leussi, *Harvard Medical School*,
Rafael Molina, *Universidad de Granada*,
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$\mathbf{X} \in \mathbb{R}^{mxn}$ with rank $r \ll min(m, n)$

1. Matrix Completion: $\mathbf{Y} = f(\mathbf{X})$ such that $Y_{ij} = X_{ij} : (i, j) \in \Omega$
2. Robust PCA: $\mathbf{Y} = \mathbf{X} + \mathbf{E}$

$\mathbf{X} = \mathbf{AB}^T$

1. The goal is to achieve column sparsity in A and B.
2. Approximate Bayesian Inference: Compute posterior distribution approximations by minimizing the KL divergence for each latent variable

1. Estimate the effective rank
2. Sparsity of the estimate of E
3. Computational complexity

Block-Sparse Recovery via Convex Optimization

Ehsan Elhamifar, Rene Vidal
*The Johns Hopkins University, Baltimore*

$\mathbf{y} = \mathbf{Bc}, \quad \mathbf{B} \in \mathbb{R}^{DXN}, N > D$

- Block-Sparse recovery using convex programs
  $P_{l_q/l_1} : min \sum_{i=1}^{n} \|c[i]\|_q$ s.t. $\quad \mathbf{y} = \mathbf{Bc}$
  and
  $P'_{l_q/l_1} min \sum_{i=1}^{n} \|B[i]c[i]\|_q$ s.t. $\quad \mathbf{y} = \mathbf{Bc}$
  for $q \geq 1$
- Does not restrict the blocks of a dictionary to have linearly independent columns
- No restriction on the length of the blocks
- Uniqueness of blocksparse representation