Limited Feedback Hybrid Precoding for Multi-User Millimeter Wave Systems IEEE TWC, Vol. 14, No. 11, November 2015

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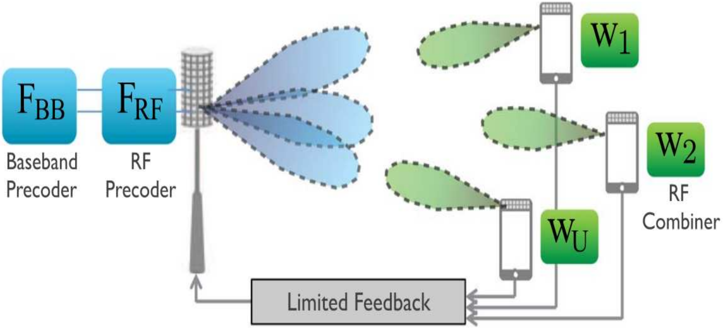
Motivation for Hybrid Precoding in mmWave Systems

- Large bandwidths available in the mmWave spectrum make mmWave communication desirable for WLAN and future cellular systems.
- High quality communication links in mmWave systems requires large antenna arrays at both the AP or BS and the MS.
- In conventional lower frequency systems, precoding was done in the baseband to have a better control over the entries in the precoding matrix.
- High cost and power consumption of mixed signal components make fully digital baseband precoding unlikely with current semiconductor technologies.
- Complete CSI is infeasible in mmWave systems, which necessitates the use of limited feedback systems.

Contributions

- Developing a hybrid precoding/combining algorithm for DL MU mmWave systems. Assumption is that mobile stations employ analog-only combining while the base station performs hybrid analog/digital precoding.
- Analyzing the performance of the proposed algorithm in special cases:
  - when the channels are single-path and
  - when the number of transmit and receive antennas are very large, which are relevant for mmWave systems.
- Characterizing the average rate loss due to joint analog and digital codebook quantization.

System Model



System Model contd.

- Consider a multi-user mmWave system, in which one BS communicates with U MSs simultaneously.
- Number of antennas in BS and each MS are N_{BS} and N_{MS} respectively. Number of RF chains in BS is N_{RF}.
- Number of streams per user $= 1$. Total number of streams $= U$.
- Baseband precoder: $\mathbf{F}_{BB} = \left[\mathbf{f}_1^{BB}, \mathbf{f}_2^{BB}, ..., \mathbf{f}_U^{BB}\right]$.
- RF precoder: $\mathbf{F}_{RF} = \left[\mathbf{f}_1^{RF}, \mathbf{f}_2^{RF}, ..., \mathbf{f}_U^{RF}\right]$.

System Model contd.

- Sampled transmitted signal: $\mathbf{x} = \mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s}$, where $\mathbf{s} = [s_1, s_2, ..., s_U]^T$ is the $U \times 1$ vector of transmitted symbols such that $\mathbb{E}[\mathbf{s}\mathbf{s}^*] = \frac{P}{U}\mathbf{I}_U$ and P is the average total transmitted power.
- Assumptions:
  - $U \leq N_{RF}$ and BS will use U out of the N_{RF} RF chains.
  - Equal power allocation among different users' streams.
  - Entries of $\mathbf{F}_{RF}$ are of constant modulus. $|[\mathbf{F}_{RF}]_{m,n}|^2 = N_{BS}^{-1}$.
  - Angles of the analog phase shifters are quantized and have a finite set of possible values.
  - Narrowband block-fading channel model.

System Model contd.

- Received signal of the u^{th} MS is

$$\mathbf{r}_u = \mathbf{H}_u \sum_{n=1}^{U} \mathbf{F}_{RF} \mathbf{f}_n^{BB} s_n + \mathbf{n}_u \tag{1}$$

- $\mathbf{H}_u$ is the $N_{MS} \times N_{BS}$ channel matrix between the BS and the u^{th} MS, $\mathbf{n}_u$ ~$\mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$.
- At the u^{th} MS, the RF combiner is used to process the received signal $\mathbf{r}_u$:

$$y_u = \mathbf{w}_u^* \mathbf{H}_u \sum_{n=1}^{U} \mathbf{F}_{RF} \mathbf{f}_n^{BB} s_n + \mathbf{w}_u^* \mathbf{n}_u \tag{2}$$

- Channel Model:
  - mmWave channels have limited scattering.
  - Geometric channel model:

$$\mathbf{H}_u = \sqrt{\frac{N_{BS}N_{MS}}{L_u}} \sum_{l=1}^{L_u} \alpha_{u,l} \mathbf{a}_{MS}\left(\theta_{u,l}\right) \mathbf{a}_{BS}^{*}\left(\phi_{u,l}\right) \tag{3}$$

- $\alpha_{u,l}$ is the complex gain of the l^{th} path, $\mathbb{E}[|\alpha_{u,l}|^2] = \bar{\alpha}$
- $\theta_{u,l}, \phi_{u,l} \in [0, 2\pi]$ are the l^{th} path's angles of arrival and departure respectively.
- $\mathbf{a}_{BS}(\phi_{u,l})$ and $\mathbf{a}_{MS}(\theta_{u,l})$ are the antenna array response vectors of the BS and u^{th} MS respectively.

System Model contd.

- Uniform planar arrays (UPA) and uniform linear arrays (ULA) are assumed in this paper.

$$\mathbf{a}_{BS}\left(\phi\right) = \frac{1}{\sqrt{N_{BS}}}\left[1, e^{j\frac{2\pi}{\lambda}dsin(\phi)}, \ldots, e^{j(N_{BS}-1)\frac{2\pi}{\lambda}dsin(\phi)}\right] \quad (4)$$

- λ is the signal wavelength.
- d is the distance between antenna elements.
- Array response vector at the MS $\mathbf{a}_{MS}(\theta_{u,l})$ can be written in a similar fashion.

Problem Formulation

- Objective: To efficiently design the analog (RF) and digital (baseband) precoders at the BS and the analog combiners at the MS's to maximize the sum-rate of the system.
- Achievable rate of user u is

$$R_u = \log_2 \left(1 + \frac{\frac{P}{U} \left| \mathbf{w}_u^* \mathbf{H}_u \mathbf{F}_{RF} \mathbf{f}_u^{BB} \right|^2}{\frac{P}{U} \sum_{n \neq u} \left| \mathbf{w}_u^* \mathbf{H}_u \mathbf{F}_{RF} \mathbf{f}_n^{BB} \right|^2 + \sigma^2} \right) \tag{5}$$

- Sum rate of the system is

$$R_{sum} = \sum_{u=1}^{U} R_u \tag{6}$$

Problem Formulation contd.

- Availability of only quantized angles for the RF phase shifters due to constraints on the RF hardware.
- Models for RF beamforming codebooks:
  - General quantized beamforming codebooks.
  - Beamsteering codebooks (This paper considers beamsteering codebooks for analog beamforming).

Problem Formulation contd.

- Precoding Design Problem:

$$\left\{\mathbf{F}_{RF}^{*}, \left\{\mathbf{f}_u^{*BB}\right\}_{u=1}^{U}, \left\{\mathbf{w}_u^{*}\right\}_{u=1}^{U}\right\}$$
$$= \operatorname{argmax} \sum_{u=1}^{U} \log_2 \left(1 + \frac{\frac{P}{U}\left|\mathbf{w}_u^{*}\mathbf{H}_u\mathbf{F}_{RF}\mathbf{f}_u^{BB}\right|^2}{\frac{P}{U}\sum_{n\neq u}\left|\mathbf{w}_u^{*}\mathbf{H}_u\mathbf{F}_{RF}\mathbf{f}_n^{BB}\right|^2 + \sigma^2}\right)$$
$$\text{s.t} \quad [\mathbf{F}_{RF}]_{:,u} \in \mathcal{F}, u = 1, 2, ..., U,$$
$$\mathbf{w}_u \in \mathcal{W}, u = 1, 2, ..., U,$$
$$\left\|\mathbf{F}_{RF}\left[\mathbf{f}_1^{BB}, \mathbf{f}_2^{BB}, ..., \mathbf{f}_U^{BB}\right]\right\| = U.$$

Problem Formulation contd.

- Mixed integer programming problem which requires a search over the entire $\mathcal{F}^U \times \mathcal{W}^U$ space of all possible $\mathbf{F}_{RF}$ and $\{\mathbf{w}_u\}_{u=1}^U$ combinations.
- Joint design of baseband precoder $\mathbf{F}_{BB}$ is needed which requires the feedback of the complete CSI.
- Large training and feedback overhead.
- Main directions of designing the precoders (in the existing literature for traditional MU-MIMO systems):
  - Iterative coordinated beamforming designs.
  - Non-iterative designs with CSI at the transmitter.
  - Non-iterative designs with CSI at the receiver.

Two-Stage Multi-User Hybrid Precoding

- Design of precoder done in two stages.
- First Stage:
  - BS RF precoder and MS RF combiner jointly designed to maximize the desired signal power of each user, neglecting the interference among users.
  - For each MS u,

$$\{\mathbf{g}_u^\star, \mathbf{v}_u^\star\} = \underset{\substack{\forall \mathbf{g}_u \in \mathcal{W} \\ \forall \mathbf{v}_u \in \mathcal{F}}}{\operatorname{argmax}} \left\| \mathbf{g}_u^* \mathbf{H}_u \mathbf{v}_u \right\| \tag{7}$$

  - MS u sets $\mathbf{w}_u = \mathbf{g}_u^\star$, BS sets $\mathbf{F}_{RF} = [\mathbf{v}_1^\star, \mathbf{v}_2^\star, ..., \mathbf{v}_U^\star]$.

Two-Stage Multi-User Hybrid Precoding contd.

- Second Stage: Multi-user digital precoding design.
  - Each MS u estimates its effective channel $\bar{\mathbf{h}}_u^* = \mathbf{w}_u^* \mathbf{H}_u \mathbf{F}_{RF}$.
  - MS quantizes $\bar{\mathbf{h}}_u$ using a codebook $\mathcal{H}$ of size $2^{B_{BB}}$ and feeds back $\hat{\mathbf{h}}_u$ where

$$\hat{\mathbf{h}}_u = \underset{\hat{\mathbf{h}}_u \in \mathcal{H}}{\operatorname{argmax}} \|\bar{\mathbf{h}}_u^* \hat{\mathbf{h}}_u\| \tag{8}$$

  - BS designs $\mathbf{F}_{BB} = \hat{\mathbf{H}}^* \left(\hat{\mathbf{H}}\hat{\mathbf{H}}^*\right)^{-1}$, $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, ..., \hat{\mathbf{h}}_U]^*$.
  - BS normalizes $\mathbf{f}_u^{BB} = \frac{\mathbf{f}_u^{BB}}{\|\mathbf{F}_{RF}\mathbf{f}_u^{BB}\|_F}$, $u = 1, 2, ..., U$.

Performance Analysis with Infinite-Resolution Codebooks

- Perfomance analysis done for two cases: Single path channels and large number of antennas.
- Analysis assuming perfect effective channel knowledge and continuous angles for the RF beamsteering vectors.
- Each MS u knows its channel $\mathbf{H}_u$ and BS perfectly knows the effective channels $\bar{\mathbf{h}}_u, u = 1, 2, ..., U$.
- Case 1: Single-Path Channel

Theorem

Define the $N_{BS} \times U$ matrix $\mathbf{A}_{BS}$ to gather the BS array response vectors associated with the U AoDs, i.e., $\mathbf{A}_{BS} = [\mathbf{a}_{BS}(\phi_1), \mathbf{a}_{BS}(\phi_2), ..., \mathbf{a}_{BS}(\phi_U)]$, with maximum and minimum singular values $\sigma_{max}(\mathbf{A}_{BS})$ and $\sigma_{min}(\mathbf{A}_{BS})$ respectively. Then the achievable rate of user u is lower bounded by

$$R_u \geq \log_2 \left(1 + \frac{SNR}{U} N_{BS} N_{MS} |\alpha_u|^2 G\left(\{\phi_u\}_{u=1}^{U}\right)\right) \tag{9}$$

where $G\left(\{\phi_u\}_{u=1}^{U}\right) = 4\left(\frac{\sigma^2_{max}(\mathbf{A}_{BS})}{\sigma^2_{min}(\mathbf{A}_{BS})} + \frac{\sigma^2_{min}(\mathbf{A}_{BS})}{\sigma^2_{max}(\mathbf{A}_{BS})} + 2\right)^{-1}$, $SNR = \frac{P}{\sigma^2}$.

Performance Analysis with Infinite-Resolution Codebooks contd.

- The lower bound separates the dependence on the channel gains and the AoDs, which can be used to claim the optimality of the proposed algorithm in some cases and to give useful insights into the gain of the proposed algorithm over analog-only beamsteering solutions.

Proposition

Denote the single-user rate as $\mathring{R}_u = \log_2\left(1 + \frac{SNR}{U} N_{BS} N_{MS} |\alpha_u|^2\right)$. *When Algorithm 1 is used to design the hybrid precoders and RF combiners described above, the relation between the achievable rate by any user u, and the single-user rate,* $\mathring{R}_u$ *satisfies*

1. $\mathbb{E}\left[\mathring{R}_u - R_u\right] \leq K(N_{BS}, U)$.
2. $\lim_{N_{BS}\to\infty} R_u = \mathring{R}_u$ *almost surely.*

where $K(N_{BS}, U)$ *is a constant whose value depends only on* N_{BS} *and* U.

Performance Analysis with Infinite-Resolution Codebooks contd.

Corollary

Let R_{BS} denote the rate achieved by user u when the BS employs analog-only beamsteering designed according to the algorithm 1. Then, the relation between the average achievable rate using algorithm 1 R_u and the average rate of analog-only beamsteering solution when the number of MS antennas goes to infinity satisfies: $\lim_{N_{MS}\to\infty} \mathbb{E}[|R_u - R_{BS}|] = \infty$.

Rate Loss with Limited Feedback

- Analyze the rate loss due to the joint RF/baseband quantization.
- Two cases: Single-path channels and large-dimensional regimes.
- Case 1: Single-Path Channel

Theorem

Let R_u^Q denote the rate acheived by user u when algorithm 1 is used to design the hybrid precoders and RF combiners under single-path assumptions mentioned before. Then the average rate loss per user, $\overline{\Delta R}_u = \mathbb{E}[R_u - R_u^Q$, is upper bounded by

$$\overline{\Delta R}_u \leq \log_2 \left(\frac{1 + \frac{SNR}{U} N_{BS} N_{MS} \overline{\alpha} \left(1 + \frac{U-1}{N_{BS}}\right) 2^{-\frac{B_{BB}}{U-1}}}{|\overline{\mu}_{BS}|^2 |\overline{\mu}_{MS}|^2} \right), \tag{10}$$

where

$$|\overline{\mu}_{BS}| = \min_{\mathbf{f}_u \in \mathcal{F}} \max_{\mathbf{f}_n \in \mathcal{F}} |\mathbf{f}_u^* \mathbf{f}_n|$$

$$|\overline{\mu}_{MS}| = \min_{\mathbf{w}_u \in \mathcal{W}} \max_{\mathbf{w}_n \in \mathcal{W}} |\mathbf{w}_u^* \mathbf{w}_n|.$$

Rate Loss with Limited Feedback contd.

- The above theorem can be used to determine how the number of baseband and RF quantization bits should scale with the different system and channel parameters to be within a constant gap of the optimal rate.

Corollary

To maintain a rate loss of $\log_2(b)$ *bps/Hz per user, the number of baseband quantization bits should satisfy*

$$\begin{aligned} B_{BB} =& \frac{U-1}{3} SNR_{dB} \\ &+ (U-1)\log_2\left(\frac{N_{BS}N_{MS}\overline{\alpha}}{U}\left(1-\frac{U-1}{N_{BS}}\right)\right) \\ &- (U-1)\log_2\left(|\overline{\mu}_{BS}|^2|\overline{\mu}_{MS}|^2 b - 1\right) \end{aligned}$$

Conclusions

- Proposed a low-complexity hybrid analog/digital precoding algorithm for DL MU mmWave systems.
- Performance analysis of the proposed algorithm when the channels are single-path and when the system dimensions are very large (not considered in this presentation).
- Results indicate that interference management in multi-user mmWave systems is required even when the number of antennas is large.
- Average rate loss due to limited feedback is characterized.
- Future work of the authors: To develop efficient mmWave precoding and channel estimation algorithms for multi-user cellular systems taking into consideration the out-of-cell interference.