Structured Sparse Signal Recovery Algorithm for Finite Alphabet Constellation Symbol Decoding in Communication Systems

Ashok Bandi

SPC Lab, Dept. ECE, IISc

Dec 3, 2016

Outline

- Introduction
- Symbol Based Modulation (SBM) and Index Based Modulation (IBM)
  - System Model
  - Structured Sparsity in SBS/IBS
  - Problem formulation to leverage the structure
  - Proposed IESR SSR algorithm for SBS/IBS
  - Simulation results
- Future work

Introduction

General communication system can be represented as

$$\mathbf{y} = \Phi\mathbf{x} + \mathbf{w}, \quad (1)$$

where $\mathbf{y} \in \mathbb{C}^{Mx1}$ is the observation vector,

$\Phi \in \mathbb{C}^{MxN}$ is the measurement matrix,

$\mathbf{x} \in \mathbb{C}^{Nx1}$ is the unknown vector,

$\mathbf{w} \sim \mathcal{N}(0, \sigma^2\mathbf{I}_N)$ is the AWGN noise vector

In most of the applications, entries of $\mathbf{x}$ come from a finite alphabet set or constellation like *M-PSK*, *M-QAM* etc. In this work, we are interested in recovering such $\mathbf{x}$ in the following cases.

- Symbol Based Modulation Schemes (SBM)
  - MIMO
  - GSM
- Index Based Modulation Schemes (IBM)
  - GSSK-MU-MIMO
  - MBM
  - GSSK-MBM

SBM and/or IBM

- In SBM information is conveyed through a symbol from a known finite alphabet set like BPSK, QAM, etc.
- In IBM, unlike SBM, information is conveyed through antenna indices (i.e., one of the possible channel states) like Space shift keying (SSK) modulation.
- In IBM, Symbol decoding can be interpreted as decoding a symbol from $\{0, 1\}$ constellation, and decoded symbol vector is mapped to get actual binary data. Here 0 or 1 indicates the usage of that particular channel realization for transmission.
- Advantages of IBM:
  - Spectral efficiency
  - power efficiency
  - improvement in performance
- Disadvantages of IBM:
  - Increase in number of antennas for higher data rate and independent channel states.
- The combination of SBM and IBM, like GSM, offers advantages of both techniques minimizes the impact of disadvantages.

MIMO

System model

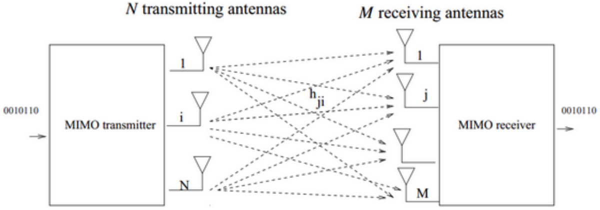


Figure : *MxN* MIMO system

In all the subsequent slides

- $\Phi \in \mathbb{C}^{MxN}$ represents channel matrix between transmitter and receiver.
- $\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$ is unknown transmitted vector with entries x_is coming from known finite length alphabet $\mathcal{A}$.
- $\mathbf{y} \in \mathbb{C}^{Mx1}$ is the received vector.

- Let $\mathcal{A} \triangleq \{a_1, \ldots, a_L\}$ be a known finite alphabet set of length L and i^{th} entry of $\mathbf{x}$ i.e., $x_i \in \mathcal{A}$.
- Now $\mathbf{x}$ can be written as

$$\mathbf{x} = \mathbf{G}\mathbf{a}, \text{where } \mathbf{a} \triangleq \begin{pmatrix} a_1 \\ \vdots \\ a_L \end{pmatrix} \text{ and } \mathbf{G} \in \{0,1\}^{NxL} \text{ is unknown binary matrix}$$

- Let $\mathbf{g_i} \in \{0,1\}^{1xL}$ is i^{th} row of $\mathbf{G}$, and $x_i = \mathbf{g_i}\mathbf{a}$.Then $\mathbf{x}$ can be written as

$$\mathbf{x} = (I_{NxN} \otimes \mathbf{a}^T) \begin{pmatrix} \mathbf{g_1}^T \\ \vdots \\ \mathbf{g_N}^T \end{pmatrix}$$

- An element x_i in $\mathbf{x}$ represents a structured sparse vector $\mathbf{g_i}$ in the transformed domain with structure being one non-zero element in $\mathbf{g_i}$
- Let $\mathbf{B} = I_{NxN} \otimes \mathbf{a}^T$ with size $NxNL$, and $\mathbf{g} = \begin{pmatrix} \mathbf{g_1}^T \\ \vdots \\ \mathbf{g_N}^T \end{pmatrix}$, then (1) can be written as

$$\mathbf{y} = \Psi\mathbf{g} + \mathbf{w}, \text{where } \Psi = \Phi\mathbf{B}$$

Breakthrough

Problem of recovering a non-sparse complex valued vector $\mathbf{x}$ becomes the problem of recovering structured binary sparse block vector $\mathbf{g}$ with structure being one active element in each block i.e. $\mathbf{g_i}$

- The problem of recovering $\mathbf{g}$ in MIMO case can be formulated as follows

$$
\begin{aligned}
\min_{\mathbf{g}} \quad & f_1(\mathbf{g}) \triangleq \frac{\|\mathbf{y} - \Psi \mathbf{g}\|_2^2}{2\sigma^2} \qquad (2) \\
\text{subject to} \quad & \\
C_1 : \quad & g_{ij} \in \{0, 1\}, \; i = 1, \ldots, N, \; j = 1, \ldots, L \\
C_2 : \quad & \sum_{j=1}^{L} g_{ij} = 1, \; i = 1, \ldots, N \\
C_3 : \quad & \sum_{i=1}^{N} \sum_{j=1}^{L} g_{ij} = N
\end{aligned}
$$

- The problem in (2) is combinatorial in nature, hence difficult to solve directly. Equivalent convex problem with linear constraints is as follows.

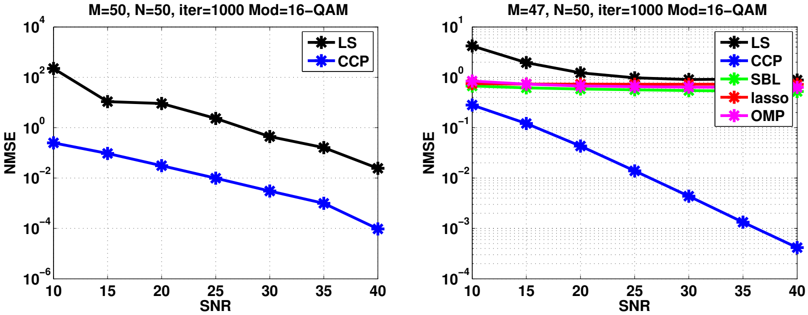
$$
\min_{\mathbf{g}} f(\mathbf{g}) \triangleq \frac{\|\mathbf{y} - \Psi \mathbf{g}\|_2^2}{2\sigma^2} + \lambda \left(\sum_{i=1}^{N} \sum_{j=1}^{L} g_{ij} - N \right)^2 + \mu \sum_{i=1}^{N} \left(\sum_{j=1}^{L} g_{ij} - 1 \right)^2 \qquad (3)
$$

$$
\textbf{subject to } C1 : 0 \leq g_{ij} \leq 1, \forall \; i = 1, \ldots, N, \; j = 1, \ldots, L
$$

- $f(\mathbf{g})$ in (3) is convex in $\mathbf{g}$ and can be solved using CVX

MIMO

Simulation results



Advantages

- Decoding of binary vector gives improvement in bit error rate over regular SBM schemes.
- Dimension of constellation increases with increase in alphabet size and number transmit antennas. Hence improvement in performance.

Disadvantage

- Increase in dimension of the vector with alphabet size and number of antennas.

MIMO

Summary

- Problem of recovering complex valued vector, whose elements come from a known finite length alphabet, can be modeled as problem of inclusion-exclusion sparse recovery problem (an active element precludes the other entries being active in each block, we refer this kind of structure as inclusion-exclusion sparsity (IES)).
- Simulation results show that in a MxN MIMO system, the transmitter vector of length N can be recovered successfully using $M < N$ receive antennas. This is against the existing fact that in a MxN MIMO system M should be $\geq N$ for successful decoding.
- By exploiting the structure in the signal, we can recover k sparse signal with $M < k$ measurements in single measurement vector case.

Generalized Spatial Modulation (GSM)

Problem formulation

- In GSM, only a subset of antennas are active at a given time. In other words, only a subset of entries are non-zero in the transmitted vector $\mathbf{x}$.
- In GSM, information is conveyed through active antenna indices and modulation symbols.
- Consider a GSM system with N transmit antennas and at a given time only N_a antennas are active, and modulation symbols from $\mathcal{A}$.
- An element x_i in $\mathbf{x}$ represents a structured sparse vector $\mathbf{g_i}$ in the transformed domain. Since only N_a entries are non-zero in $\mathbf{x}$, so the number of active blocks in $\mathbf{g}$ i.e. N_a.
- The problem of recovering $\mathbf{g}$ in GSM case can be formulated as follows

$$\min_{\mathbf{g}} \quad f(\mathbf{g}) \triangleq \frac{\|\mathbf{y} - \Psi\mathbf{g}\|_2^2}{2\sigma^2} \tag{4}$$

subject to

$$C_1 : \quad g_{ij} \in \{0,1\},\ i = 1,\dots,N,\ j = 1,\dots,L$$

$$C_2 : \quad \sum_{j=1}^{L} g_{ij} \in \{0,1\},\ i = 1,\dots,N$$

$$C_3 : \quad \sum_{i=1}^{N}\sum_{j=1}^{L} g_{ij} = N_a$$

- Constraint C2 ensures the sparsity within the blocks and C3 ensures the sparsity of overall vector $\mathbf{g}$.
- The convex problem with combinatorial constraints in (4) can be converted to non-convex problem with linear constraints as follows

$$\min_{\mathbf{x}} \quad f(\mathbf{g}) \triangleq \underbrace{\frac{\|\mathbf{y}-\Psi\mathbf{g}\|_2^2}{2\sigma^2}}_{f_1(\mathbf{g})} + \underbrace{\lambda\left(\sum_{i=1}^{N}\sum_{j=1}^{L} g_{ij} - N_a\right)^2}_{f_2(\mathbf{g})} \underbrace{-\mu\sum_{i=1}^{N}\left(\sum_{j=1}^{L} g_{ij}\right)\left(\sum_{j=1}^{L} g_{ij} - 1\right)}_{f_3(\mathbf{g})} \tag{5}$$

subject to

$$C1: \quad 0 \leq g_{ij} \leq 1, \forall\ i = 1, \ldots, N,\ j = 1, \ldots, L$$

$$C_2: \quad \sum_{j=1}^{L} g_{ij} \leq 1,\ \ i = 1, \ldots, N$$

- Function $f(\mathbf{g})$ in (5) is difference of convex functions, so the convex-concave procedure (CCP) can be applied.
- CCP is a majorization-minimization procedure, where the concave part is replaced with affine upper bound at current iterate and minimizes the surrogate function.
- The optimization problem at $\mathbf{g}^k$ is as following

$$\min_{\mathbf{x}} \quad f(\mathbf{g}) \triangleq \underbrace{\frac{\|\mathbf{y}-\Psi\mathbf{g}\|_2^2}{2\sigma^2}}_{f_1(\mathbf{g})} + \underbrace{\lambda\left(\sum_{i=1}^{N}\sum_{j=1}^{L} g_{ij} - N_a\right)^2}_{f_2(\mathbf{g})} - \mu \nabla f_3(\mathbf{g^k})^T \mathbf{g} \tag{6}$$

subject to $\quad$ **C1:** $0 \leq g_{ij} \leq 1$, **C2:** $\sum_{j=1}^{L} g_{ij} \leq 1,\ \forall\ i = 1, \ldots, N,\ j = 1, \ldots, L$

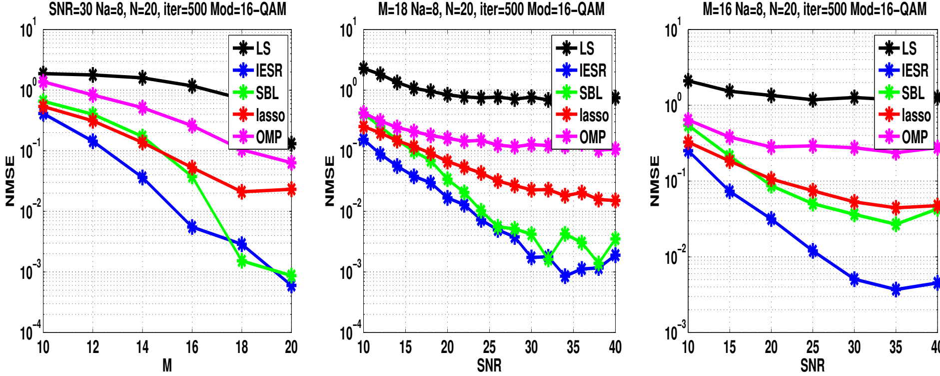


Figure : NMSE of the proposed algorithm (a) versus M and (b) versus SNR compared against popular LS and SSR algorithms, with $N = 20$, $N_a = 8$, and $L = 16$

- Consider a uplink MU-MIMO scenario with N users. Assume each user is equipped with L antennas and communicates with base station (BS) using GSSK modulation with $K \leq L$ antennas. In other words each user communicate with BS by transmitting tones through a subset of $K \leq L$ antennas. And at any time only $N_a \leq N$ users allowed to communicate with BS

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) \triangleq \underbrace{\frac{\|\mathbf{y}-\Phi\mathbf{x}\|_2^2}{2\sigma^2}}_{f_1(\mathbf{x})} \underbrace{+\lambda\left(\sum_{i=1}^{N}\sum_{j=1}^{L} x_{ij} - N_a K\right)^2}_{f_2(\mathbf{x})} \underbrace{-\mu\sum_{i=1}^{N}\left(\sum_{j=1}^{L} x_{ij}\right)\left(\sum_{j=1}^{L} x_{ij} - K\right)}_{f_3(\mathbf{x})} \qquad (7)$$

subject to

$$C1: \quad 0 \leq x_{ij} \leq 1, \forall\; i = 1, \ldots, N, \; j = 1, \ldots, L$$

$$C_2: \quad \sum_{j=1}^{L} x_{ij} \leq k, \; i = 1, \ldots, N$$

- Here Φ is channel matrix between all the users and BS, $\mathbf{x}$ concatenated transmitted vector from all the users and $\mathbf{y}$ is received vector at the BS.
- Above problem, (7), is in the form as (5) hence CCP can be applied.

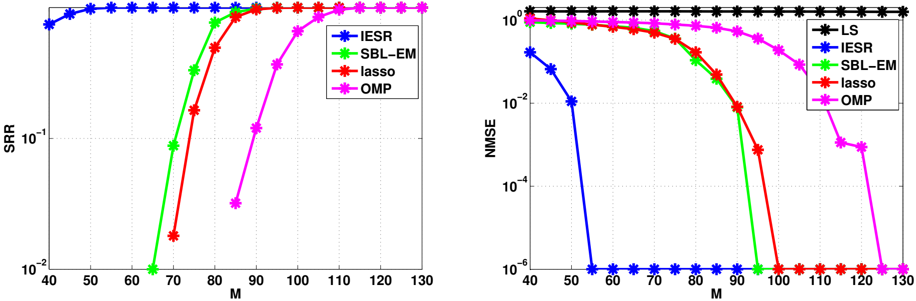


Figure : (a) SRR and (b) NMSE of the proposed algorithm for the recovery of inclusion-exclusion sparse vectors compared against popular SSR algorithms, with $N = 16$, $N_a = 8$, $L = 16$, $k = 6$, and SNR$= 40$ dB.

Block Sparse Recovery

Problem formulation

- Binary block sparse recovery vector can also be modeled as IES recovery problem.
- Consider a block sparse vector $\mathbf{x}$ with N blocks and each block of length L. Assume only N_a blocks are active. Then $\mathbf{x}$ can be recovered by solving following optimization problem.

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) \triangleq \underbrace{\frac{\|\mathbf{y}-\Phi\mathbf{x}\|_2^2}{2\sigma^2}}_{f_1(\mathbf{x})} + \lambda \underbrace{\left(\sum_{i=1}^{N}\sum_{j=1}^{L} x_{ij} - N_a L\right)^2}_{f_2(\mathbf{x})} - \mu \underbrace{\sum_{i=1}^{N}\left(\sum_{j=1}^{L} x_{ij}\right)\left(\sum_{j=1}^{L} x_{ij} - L\right)}_{f_3(\mathbf{x})} \tag{8}$$

subject to

$$C1: \quad 0 \le x_{ij} \le 1, \forall\ i = 1, \ldots, N,\ j = 1, \ldots, L$$

$$C_2: \quad \sum_{j=1}^{L} x_{ij} \le L,\ \ i = 1, \ldots, N$$

- The problem in (8) is in the same form as in (5), hence CCP can be applied.

Block Sparse Recovery

Simulation results

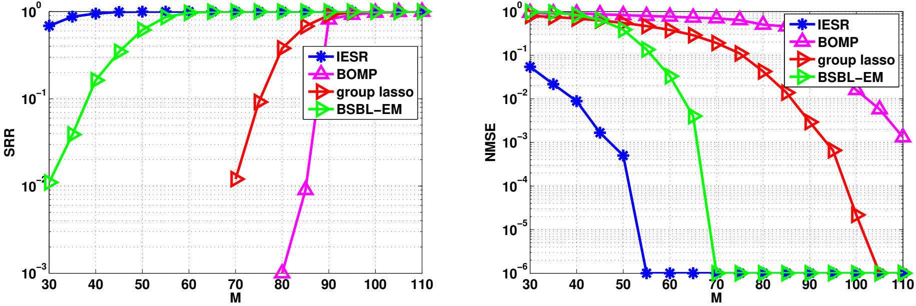
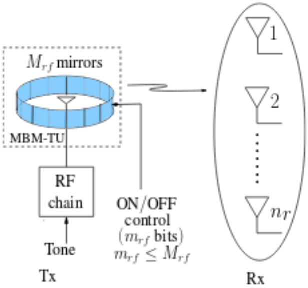


Figure : (a) SRR and (b) NMSE of the proposed algorithm compared against popular SSR algorithms in the block-sparse setting, with $N = 24$, $N_a = 12$, $L = 8$, and $k = 8$ and SNR$= 30$ dB.

Media Based Modulation

- Varying the end to end channel based on the input is called Media Based Modulation (MBM).
- Carrier is modulated after leaving the transmitter by changing RF properties of the medium.
- All others traditional modulations are referred as Source Based Modulations (SBM).
- Small perturbation near the tx in a rich scattering environment results an independent end-to-end channel. RF mirrors are used for creating perturbations.



- If r_s bits are used for SBM and r_m bit for MBM, total $r_s + r_m$ can be transmitted by combining SBM and MBM, and receiver will receive one of the points from constellation of $2^{(r_s+r_m)}$ points.

Advantages of MBM

- Increasing the spectral efficiency without increasing energy unlike SBM, where increasing r_s results exponential increase in energy.
- Deep fades do not have persisting effect because of Constellation diversity. As constellation size increases, this converts static multi-path fading channel into non-fading AWGN.
- In a $1xD$ SIMO-MBM system received vector spans in D receive dimension unlike SIMO-SBM which spans in single complex dimension, which is equivalent to SIMO-SBM with D times bandwidth.
- Possibility of choosing subset of channel similar to multi user diversity gain in scheduling.

Disadvantages of MBM

- Random arrangements of constellation points and all points are used with equal probability. While in SBM constellation can be used with non uniform probability to realize shaping gain.
- MBM is Linear Time variant, can trouble the traditional channel equalization techniques
- Signal in single dimension at the input is spread across the multiple dimension at output.

Flavors of MBM:

- MIMO-MBM: MBM discussed in the above single antenna case can be extended to MIMO. This is referred as MIMO-MBM.
- GSM-MIMO: MBM combined with multiple antenna case using GSM is referred as GSM-MBM.
- GSSK-MIMO: MBM combined with multiple antenna case using GSSK is referred as GSSK-MBM.

Data Decoding in GSSK-MBM:

- Consider a BS with M received antennas and a user with N antennas, each equipped with N_{rf} mirrors. Also assume user wants communicate with BS using GSSK modulation using $N_a \leq N$ antennas
- Now the decoding at BS can be formulated as follows

$$\min_{\mathbf{x}} \quad f(\mathbf{g}) \triangleq \underbrace{\frac{\|\mathbf{y}-\Phi\mathbf{x}\|_2^2}{2\sigma^2}}_{f_1(\mathbf{x})} + \lambda \underbrace{\left(\sum_{i=1}^{N}\sum_{j=1}^{L} x_{ij} - N_a\right)^2}_{f_2(\mathbf{x})} - \mu \underbrace{\sum_{i=1}^{N}\left(\sum_{j=1}^{L} x_{ij}\right)\left(\sum_{j=1}^{L} x_{ij} - 1\right)}_{f_3(\mathbf{x})} \qquad (9)$$

subject to

$$C1: \quad 0 \leq x_{ij} \leq 1, \forall\ i = 1,\ldots,N,\ j = 1,\ldots,L$$

$$C_2: \quad \sum_{j=1}^{L} x_{ij} \leq 1,\ i = 1,\ldots,N$$

- Here Φ is concatenated channel matrix between BS and all possible mirror patterns at all antennas. $L = 2^{N_{rf}}$ is number of possible mirror activation patterns at each antenna.
- The problem in (9) is in the same form as in (5), hence CCP can be applied

Simulation results: GSSK-MBM

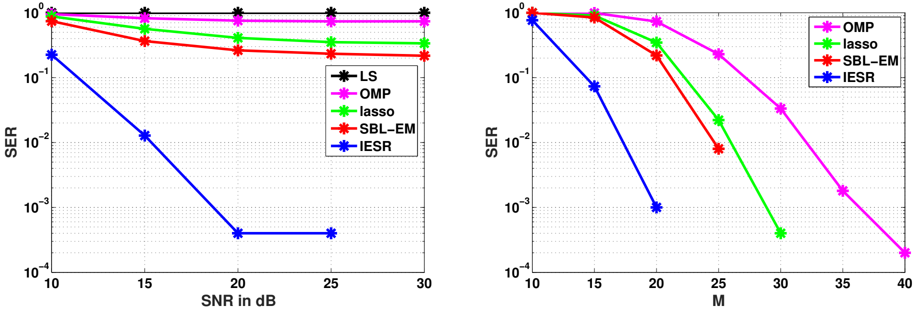


Figure : SER of the proposed algorithm for decoding data with MBM, compared against popular SSR algorithms, with $N = 10$, $N_a = 8$, $N_{rf} = 4$, and $L = 16$.

Future work

- Coming up with an efficient algorithm instead of using CVX.
- Bounds or exact expression for the number of measurements needed given the sparsity and length of the vector
- Extending this work to recover the vector when maximum number of non-zeros is known instead of exact number of non-zeros.