Main Presentation

Hybrid Codebook Construction and Sum Rate Maximization in mmWave Multiuser Systems

Vaibhav Baranwal

Communication & Networks
M.Tech 2016-18

Department of Electrical Communication Engineering
Indian Institute of Science

23^{rd} Jun, 2018

Introduction

- Millimeter wave frequencies are a promising candidate for 5G communications due to large available bandwidth

Introduction

- Millimeter wave frequencies are a promising candidate for 5G communications due to large available bandwidth
- The mmWave frequency ranges from 30GHz to 300GHz

Introduction

- Millimeter wave frequencies are a promising candidate for 5G communications due to large available bandwidth
- The mmWave frequency ranges from 30GHz to 300GHz
- Due to large frequency, the channel gets degraded due to path loss, penetration loss, fading etc and hence it faces challenge in improving system capacity and providing good quality of service (QoS) to the users

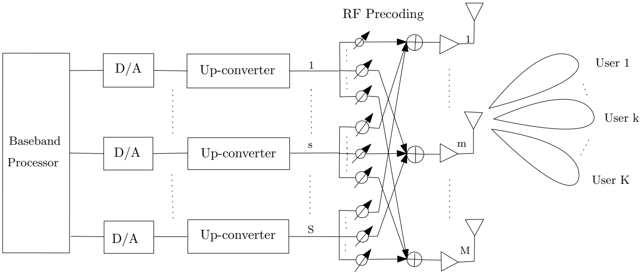
Introduction

- Millimeter wave frequencies are a promising candidate for 5G communications due to large available bandwidth
- The mmWave frequency ranges from 30GHz to 300GHz
- Due to large frequency, the channel gets degraded due to path loss, penetration loss, fading etc and hence it faces challenge in improving system capacity and providing good quality of service (QoS) to the users
- To reduce the losses we use beamforming techniques to generate beamforming vectors

Introduction

- Millimeter wave frequencies are a promising candidate for 5G communications due to large available bandwidth
- The mmWave frequency ranges from 30GHz to 300GHz
- Due to large frequency, the channel gets degraded due to path loss, penetration loss, fading etc and hence it faces challenge in improving system capacity and providing good quality of service (QoS) to the users
- To reduce the losses we use beamforming techniques to generate beamforming vectors
- We have simulated a hybrid precoding method from the literature to generate the RF precoder and baseband precoder to select the beamformers[1]

Consider a multiple input single output (MISO) system,



The system is consists of a baseband processor, digital-to-analog converter (DAC), upconverter, S RF chains, M transmit antennas, and K number of users.

Notations

The system model is expressed as

$$\mathbf{y} = \mathbf{H}\underline{\mathbf{F}}\underline{\mathbf{G}}\mathbf{s} + \mathbf{n}$$

Here,
$\mathbf{H}$: channel matrix of the system of size $K \times M$,
$\underline{\mathbf{F}}$: RF precoder of size $M \times S$,
$\underline{\mathbf{G}}$: baseband precoder of size $S \times K$,
$\mathbf{s}$: transmitted vector of size $K \times 1$,
$\mathbf{n}$: noise vector of size $K \times 1$,
$\mathbf{y}$: received vector of size $K \times 1$.

Here, $M \geq S \geq K$

Channel Model

Here, $\mathbf{s}^T = [s_1, \ldots, s_K]$ with $s_k \sim \mathcal{CN}(0,1), \forall\, k \in \mathcal{K} \triangleq \{1, 2, \ldots, K\}$

$\mathbf{H}^H = [\mathbf{h}_1, \ldots, \mathbf{h}_K]$. Here, $\mathbf{h}_k$ is given by

$$\mathbf{h}_k = \sqrt{\frac{M}{N_{cl} N_{ray}}} \sum_{m_p=1}^{N_{cl}} \sum_{n_p=1}^{N_{ray}} \alpha_{m_p,n_p} \mathbf{a}\left(\phi_{m_p,n_p}\right)$$

$$\text{where, } \mathbf{a}_{ULA}(\phi) = \sqrt{\frac{1}{M}} \left[1, e^{j\frac{2\pi}{\lambda} d\, sin(\phi)}, \ldots, e^{j(M-1)\frac{2\pi}{\lambda} d\, sin(\phi)}\right]$$

λ is the signal wavelength
d is the antenna spacing
ϕ is the angle of departure (AoD)

$\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I_K})$ is the AWGN noise vector.
$\alpha_{m_p,n_p} \sim \mathcal{CN}(0, \sigma_{m_p}^2)$

Sum Rate Expression

Problem of maximizing system sum rate is given as

$$\max_{\underline{F},\underline{G}} \sum_{k=1}^{K} \underline{R}_k$$

$$\text{s.t.} \quad \underline{R}_k = \log\left(1 + \underline{\text{SINR}}_k\right) \geq \gamma_k, \; \forall k \in \mathcal{K}$$

$$\underline{\mathbf{F}} \in \mathcal{F}_{RF}, \; \|\underline{\mathbf{FG}}\|_{\mathcal{F}}^2 \leq P$$

$$\underline{\text{SINR}}_k = \frac{|\mathbf{h_k^H}\underline{\mathbf{F}}\underline{\mathbf{g_k}}|^2}{\sum_{l=1,l\neq k}^{K} |\mathbf{h}_k^H \underline{\mathbf{F}}\underline{\mathbf{g}}_l|^2 + \sigma^2}$$

RF Codebook Design

Some examples of the RF codebooks are

- q-bit quantizer Codebook

$$\mathbf{F}(m,n) = \frac{1}{\sqrt{M}} e^{j\frac{\pi(4(m-1)(n-1)-2N)}{2^{q+1}}}, \ \forall m \in \mathcal{M}, \forall n \in \mathcal{N}$$

- IEEE 802.15.3c Codebook

$$\mathbf{F}(m,n) = \frac{1}{\sqrt{M}} e^{j\frac{\pi}{2}\text{floor}\left(\frac{4(m-1)\left(\text{mod}\left((n-1)+\frac{N}{4},N\right)\right)}{N}\right)}, \ \forall m \in \mathcal{M}, \forall n \in \mathcal{N}$$

- DFT Codebook

$$\mathbf{F}(m,n) = \frac{1}{\sqrt{M}} e^{-\frac{j2\pi(m-1)(n-1)}{M}}, \ \forall m \in \mathcal{M}, \forall n \in \mathcal{N}$$

Here, $\mathcal{M} \in \{1,\ldots,M\}$, $\mathcal{N} \in \{1,\ldots,N\}$

- DFT-based Multilevel Codebook [2]

$$\mathcal{F}_m = \left\{ f_1^{(m)}, f_2^{(m)}, \dots, f_{M/N}^{(m)} \right\}$$
$$= \left\{ \frac{1}{\sqrt{N}} \sum_{p=1}^{N} \mathbf{u}_t(p) e^{j\omega_m p}, \frac{1}{\sqrt{N}} \sum_{p=N+1}^{2N} \mathbf{u}_t(p) e^{j\omega_m p}, \dots, \right.$$
$$\left. \frac{1}{\sqrt{N}} \sum_{p=M-N+1}^{M} \mathbf{u}_t(p) e^{j\omega_m p} \right\}$$

$$\mathbf{u}_t(n) = \frac{1}{\sqrt{M}} \left[1, e^{-j\frac{2\pi}{M}\omega_m\left(n-\frac{M+1}{2}\right)}, e^{-j\frac{2\pi}{M}\omega_m 2\left(n-\frac{M+1}{2}\right)}, \dots, e^{-j\frac{2\pi}{M}\omega_m (M-1)\left(n-\frac{M+1}{2}\right)} \right]$$

$\omega_m \in \left[-\frac{\pi}{M}, \pi\left(1-\frac{1}{M}\right)\right]$ and is selected by minimizing *var* $\left(\left|\mathbf{u}_t^H(n)\mathbf{f}_k^{(m)}\right|\right)$.

- Random Codebook:- obtained using QR-decomposition method.
- Deterministic Precoder

$$F = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & e^{j\pi \sin \frac{2\pi}{S}} & e^{j\pi \sin \frac{4\pi}{S}} & \dots & e^{j\pi \sin \frac{2\pi(S-1)}{S}} \\ 1 & e^{j2\pi \sin \frac{2\pi}{S}} & e^{j2\pi \sin \frac{4\pi}{S}} & \dots & e^{j2\pi \sin \frac{2\pi(S-1)}{S}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j(M-1)\pi \sin \frac{2\pi}{S}} & e^{j(M-1)\pi \sin \frac{4\pi}{S}} & \dots & e^{j(M-1)\pi \sin \frac{2\pi(S-1)}{S}} \end{bmatrix}$$

- Robustness

$$\mathbf{F}(m,n) = \frac{1}{\sqrt{M}} e^{-\frac{j2\pi\left((m-1)(n-1)+\frac{r}{c}\right)}{M}}, \forall m \in \mathcal{M}, \forall n \in \mathcal{N}$$

Beam Sweep Procedure

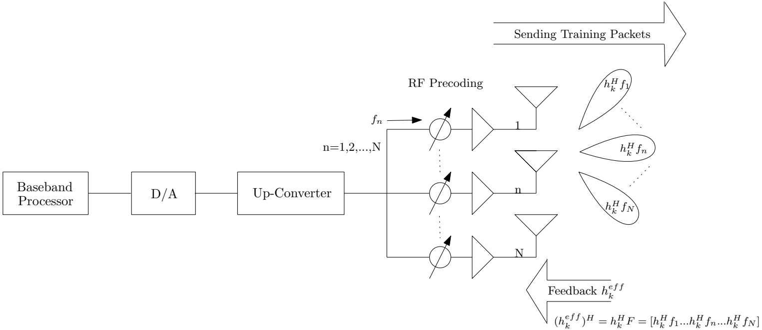


Figure: Beam-sweep Procedure

Virtual Communication System for CSIT

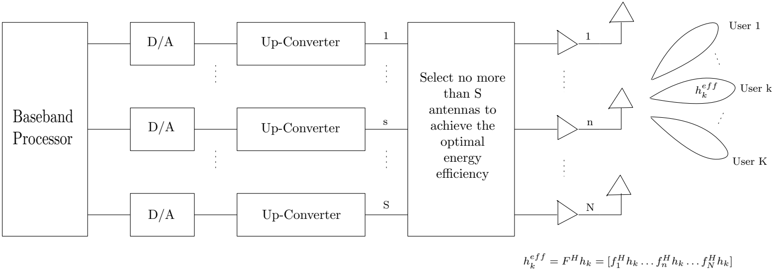


Figure: Virtual Multiuser Communication System

Sum rate maximization problem is expressed as

$$\max_{\mathbf{G}} \sum_{k=1}^{K} R_k,$$
$$\text{s.t. } R_k = \log\left(1 + \text{SINR}_k\right),$$
$$\sum_{k=1}^{K} \|\mathbf{F}\mathbf{g}_k\|_2^2 \leq P, \ \|\ddot{\mathbf{g}}\|_0 \leq S,$$

where, $\ddot{\mathbf{g}} = [\|\tilde{\mathbf{g}}_1\|_2, \ldots, \|\tilde{\mathbf{g}}_N\|_2]^T$

$$\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{F}\mathbf{g}_k|^2}{\sum_{l=1,l\neq k}^{K} |\mathbf{h}_k^H \mathbf{F}\mathbf{g}_l|^2 + \sigma^2}$$

Contd.

We will introduce some variables $a_k, b_k, \forall k \in \mathcal{K}$, So, the sum rate maximization problem can be rewritten as

$$\min_{\{\mathbf{g}_k, a_k, b_k\}} \; -\sum_{k=1}^{K} b_k,$$
$$\text{s.t. } 1 + a_k \geq e^{b_k}, \; \forall k \in \mathcal{K},$$
$$\text{SINR}_k \geq a_k, \; \text{SINR}_k \geq \bar{\gamma}_k, \; \forall k \in \mathcal{K},$$
$$\sum_{k=1}^{K} \|\mathbf{F}\mathbf{g}_k\|_2^2 \leq P, \; \|\ddot{\mathbf{g}}\|_0 \leq S,$$

where $\bar{\gamma}_k = e^{\gamma_k} - 1$

Rearranging the problem

To overcome nonconvex difficulties

$$\min_{\{\mathbf{g}_k, a_k, b_k\}} -\sum_{k=1}^{K} b_k + \lambda \|\ddot{\mathbf{g}}\|_0,$$

$$\text{s.t. } 1 + a_k \geq e^{b_k}, \forall k \in \mathcal{K}, \sum_{k=1}^{K} \|\mathbf{F}\mathbf{g}_k\|_2^2 \leq P,$$

$$\text{SINR}_k \geq a_k, \text{SINR}_k \geq \bar{\gamma}_k, \forall k \in \mathcal{K},$$

Here, λ control the sparsity of solution, i.e., the larger λ the solution is more sparse.

Approximate Problem

Problem can be further approximated as

$$\min_{\{\mathbf{g}_k, a_k, b_k\}} -\sum_{k=1}^{K} b_k + \lambda \|\mathbf{G}\|_{1,\infty}^2,$$

$$\text{s.t. } 1 + a_k \geq e^{b_k}, \forall k \in \mathcal{K}, \sum_{k=1}^{K} \|\mathbf{F}\mathbf{g}_k\|_2^2 \leq P,$$

$$\text{SINR}_k \geq a_k, \text{SINR}_k \geq \bar{\gamma}_k, \forall k \in \mathcal{K},$$

where $\|\mathbf{G}\|_{1,\infty} = \sum_{n=1}^{N} \max_k |\mathbf{g}_k(n)|$ is the $l_{1,\infty}$-norm of matrix $\mathbf{G}$.

Contd.

Here, $\|\mathbf{G}\|_{1,\infty}^2$ can be rewritten as

$$\|\mathbf{G}\|_{1,\infty}^2 = \left(\sum_{n=1}^{N} \max_k |\mathbf{g}_k(n)|\right)^2,$$
$$= \sum_{n=1}^{N}\sum_{m=1}^{N} \left(\left(\max_k |\mathbf{g}_k(n)|\right)\left(\max_k |\mathbf{g}_k(m)|\right)\right),$$
$$= \sum_{n=1}^{N}\sum_{m=1}^{N} \max_{i,j\in\{1,\ldots,K\}} |\mathbf{X}_{i,j}(n,m)|,$$

where $\mathbf{X}_{i,j} = \mathbf{g}_i\mathbf{g}_j^H$, $\forall i,j$. So, $\mathbf{X}_{i,i} = \mathbf{g}_i\mathbf{g}_i^H$, therefore $\mathbf{X}_{i,i} \succeq 0$ and $\text{rank}(\mathbf{X}_{i,i}) = 1$, $\forall i$.

Approximate Problem

Problem can be further approximated as

$$\min_{\{\mathbf{X}_{i,j}, a_k, b_k\}} -\sum_{k=1}^{K} b_k + \lambda \|\mathbf{G}\|_{1,\infty}^2,$$

$$\text{s.t. } 1 + a_k \geq e^{b_k}, \forall k \in \mathcal{K}, \sum_{k=1}^{K} \text{tr}\left(\tilde{\mathbf{F}}\mathbf{X}_{k,k}\right) \leq P,$$

$$\text{SINR}_k \geq a_k, \text{SINR}_k \geq \bar{\gamma}_k, \forall k \in \mathcal{K}, \mathbf{X}_{k,k} \succeq \mathbf{0}, \forall k \in \mathcal{K},$$

$$\text{rank}\left(\mathbf{X}_{i,j}\right) = 1, \forall i, j,$$

where $\tilde{\mathbf{F}} = \mathbf{F}^H \mathbf{F}$, and

$$\text{SINR}_k = \frac{\text{tr}\left(\mathbf{H}_k \mathbf{X}_{k,k}\right)}{\sum_{l=1, l \neq k}^{K} \text{tr}\left(\mathbf{H}_k \mathbf{X}_{l,l}\right) + \sigma^2}$$

where $\mathbf{H}_k = \mathbf{F}^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{F}, \forall k \in \mathcal{K}$.

Further Approximation

Let $\mathbf{X}_k = \mathbf{X}_{k,k}, \forall k \in \mathcal{K}$ and $\mathbf{Z}(n,m) = \max_{k \in \mathcal{K}} |\mathbf{X}_k(n,m)|, \forall k \in \mathcal{K}$.

We will drop the nonconvex constraints $\text{rank}(\mathbf{X}_k) = 1$.

$$\min_{\{\mathbf{X}_k, a_k, b_k\}, \mathbf{Z}} -\sum_{k=1}^{K} b_k + \lambda \text{tr}\left(\mathbf{1}_{N \times N}\mathbf{Z}\right)$$

$$\text{s.t. } 1 + a_k \geq e^{b_k}, \forall k \in \mathcal{K}, \sum_{k=1}^{K} \text{tr}\left(\tilde{\mathbf{F}}\mathbf{X}_k\right) \leq P,$$

$$\text{SINR}_k \geq a_k, \text{SINR}_k \geq \bar{\gamma}_k, \forall k \in \mathcal{K},$$

$$\mathbf{X}_k \succeq \mathbf{0}, \mathbf{Z} \geq |\mathbf{X}_k|, \forall k \in \mathcal{K},$$

Contd.

We will introduce some new variables $\psi_k.\phi_k$, $\forall k \in \mathcal{K}$. The problem can be rewritten as

$$\min_{\{\mathbf{X}_k, a_k, b_k, \psi_k, \phi_k\}, \mathbf{Z}} -\sum_{k=1}^{K} b_k + \lambda \mathrm{tr}\left(\mathbf{1}_{N\times N}\mathbf{Z}\right),$$

$$\text{s.t. } \psi_k^2 \leq \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_k\right), \ \mathbf{X}_k \succeq \mathbf{0}, 1 + a_k \geq e^{b_k}, \forall k \in \mathcal{K}$$

$$\sum_{l=1, l\neq k}^{K} \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_{l,l}\right) + \sigma^2 \leq \phi_k, \forall k \in \mathcal{K}, \sum_{k=1}^{K} \mathrm{tr}\left(\tilde{\mathbf{F}}\mathbf{X}_k\right) \leq P,$$

$$\sum_{l=1, l\neq k}^{K} \bar{\gamma}_k \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_{l,l}\right) + \bar{\gamma}_k\sigma^2 \leq \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_k\right), \ \frac{\psi_k^2}{\phi_k} \geq a_k, \forall K \in \mathcal{K},$$

$$\begin{bmatrix} \mathbf{Z}(n,m) - \Re(\mathbf{X}_k(n,m) & \Im(\mathbf{X}_k(n,m)) \\ \Im(\mathbf{X}_k(n,m)) & \mathbf{Z}(n,m) + \Re(\mathbf{X}_k(n,m)) \end{bmatrix} \succeq \mathbf{0}, \forall k \in \mathcal{K}, m, n.$$

Final Convex Problem

The constraint $\frac{\psi_k^2}{\phi_k} \geq a_k$ is still nonconvex. So, the constraint can be approximated as

$$\frac{\psi_k^2}{\phi_k} \geq \Phi_k^{(l)}(\psi_k, \phi_k) \triangleq 2\frac{\psi_k^{(l)}}{\phi_k^{(l)}}\psi_k - \left(\frac{\psi_k^{(l)}}{\phi_k^{(l)}}\right)^2 \phi_k, \forall k \in \mathcal{K},$$

Now, we will solve the series of convex optimization problem. Here, l denotes the lth iteration. $(\psi_k^{(l)}, \phi_k^{(l)}) \leftarrow (\psi_k, \phi_k)$ at the lth iteration. $\Phi_k^{(l)}(\psi_k, \phi_k)$ is determined at the lth iteration.

Contd.

So, the approximate value of convex problem at the $(l+1)$th iteration can be

$$\min_{\{\mathbf{X}_k, a_k, b_k, \psi_k, \phi_k\}\mathbf{Z}} -\sum_{k=1}^{K} b_k + \lambda \mathrm{tr}(\mathbf{1}_{N\times N}\mathbf{Z}),$$

$$\text{s.t. } \psi_k^2 \le \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_k\right), \ \mathbf{X}_k \succeq \mathbf{0}, 1 + a_k \ge e^{b_k}, \ \forall k \in \mathcal{K},$$

$$\sum_{l=1, l\neq k}^{K} \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_l\right) + \sigma^2 \le \phi_k, \ \forall k \in \mathcal{K}, \ \sum_{k=1}^{K} \mathrm{tr}\left(\tilde{\mathbf{F}}\mathbf{X}_k\right) \le P,$$

$$\sum_{l=1, l\neq k}^{K} \bar{\gamma}_k \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_l\right) + \bar{\gamma}_k\sigma^2 \le \mathrm{tr}\left(\mathbf{H}_k\mathbf{X}_k\right),$$

$$\Phi_k^{(l)}(\psi_k, \phi_k) \ge a_k, \ \forall k \in \mathcal{K},$$

$$\begin{bmatrix} \mathbf{Z}(n,m) - \Re(\mathbf{X}_k(n,m) & \Im(\mathbf{X}_k(n,m)) \\ \Im(\mathbf{X}_k(n,m)) & \mathbf{Z}(n,m) + \Re(\mathbf{X}_k(n,m) \end{bmatrix} \succeq \mathbf{0}, \ \forall k \in \mathcal{K}, m, n.$$

Solution of the Convex Problem

Algorithm for optimal solution

Fix a value of λ. Let the value of our objective function is τ and $\tau^{(l)}$is the value of τ at the lth iteration.

1: Let $l = 0$, take some initial points as $\Gamma^{(l)}$ and get $\tau^{(l)}$.
2: Solve the convex problem with $\Gamma^{(l)}$, and obtain new values of the Γ and τ.
3: If $|\tau - \tau^{(l)}| \leq \zeta$, then τ, Γ will be our optimal solution, otherwise $\Gamma^{(l)} \leftarrow \Gamma, \tau^{(l)} \leftarrow \tau$ and go to step 2.

Selection of λ

Let L^{λ} be the number of nonzero diagonal entries in $\mathbf{Z}$.

Algorithm for choosing λ

1: Generate initial points λ_L, λ_U and compute $\tilde{\tau}^T = \sum_{k=1}^{K} b_k$ and denote Ξ^T as the temoprary solution of the convex problem. Let *flag* $= 1$

2: **while** flag **do**

3: Let $\lambda = \frac{\lambda_L + \lambda_U}{2}$.

4: Solve the convex problem with λ, then obtain the solution of it after iteration and $\tilde{\tau}^{\lambda}$

5: If $L^{\lambda} > S$, let $\lambda_L = \lambda$, otherwise, let $\lambda_U = \lambda$.

6: If $|\tilde{\tau}^{\lambda} - \tilde{\tau}^T| \leq \zeta$ and $L^{\lambda} \leq S$, then let *flag*$=0$ and output the solution of convex problem. Otherwise, $\Xi^T \leftarrow \Xi^{\lambda}, \tilde{\tau}^T \leftarrow \tilde{\tau}^{\lambda}$

7: **end while**

Refining Solution

$\hat{\mathbf{F}}$ is obtained by choosing L^{λ} codewords from RF codebook.
$\bar{\mathbf{h}}_k = \hat{\mathbf{F}}^H \mathbf{h}_k$ is our effective channel.

Now we will refine our solution

$$\max_{\{\bar{g_k}\}} \sum_{k=1}^{K} \bar{R}_k,$$
$$\text{s.t. } \overline{\text{SINR}}_k \geq \bar{\gamma}_k, \forall k \in \mathcal{K},$$
$$\sum_{k=1}^{K} \|\hat{F}\bar{g}_k\|_2^2 \leq P,$$

where $\bar{R}_k = \log\left(1 + \overline{\text{SINR}}_k\right)$, and $\overline{\text{SINR}}_k$ is given by

$$\overline{\text{SINR}}_k \triangleq \frac{\|\bar{\mathbf{h}}_k^H \bar{\mathbf{g}}_k\|_2^2}{\sum_{l=1, l\neq K}^{K} \|\bar{\mathbf{h}}_k^H \bar{\mathbf{g}}_l\|_2^2 + \sigma^2}.$$

Rearranging the Problem

We introduce variable $\bar{a}_k, \bar{b}_k, \bar{\phi}_k$, the problem can be reformulated as

$$\max_{\{\bar{\mathbf{g}}_k, \bar{a}_k, \bar{b}_k, \bar{\phi}_k\}} \sum_{k=1}^{K} \bar{b}_k,$$

$$\text{s.t. } 1 + \bar{a}_k \geq e^{\bar{b}_k}, \forall k \in \mathcal{K}, \sum_{k=1}^{K} \|\hat{F}\bar{g}_k\|_2^2 \leq P$$

$$\frac{\|\bar{\mathbf{h}}_k^H \bar{\mathbf{g}}_k\|_2^2}{\bar{\phi}_k} \geq \bar{\gamma}_k, \forall \frac{\|\bar{\mathbf{h}}_k^H \bar{\mathbf{g}}_k\|_2^2}{\bar{\phi}_k} \geq \bar{a}_k, \forall k \in \mathcal{K},$$

$$\sum_{l=1, l \neq K}^{K} \|\bar{\mathbf{h}}_k^H \bar{\mathbf{g}}_l\|_2^2 + \sigma^2 \leq \bar{\phi}_k, \forall k \in \mathcal{K},$$

Final Sum Rate Maximization Problem

As the problem is nonconvex, we can approximate the constraint by

$$\frac{\|\bar{h}_k^H \bar{g}_k\|_2^2}{\bar{\phi}_k} \geq \bar{\Phi}_k^{(l)}\left(\bar{g}_k \bar{\phi}_k\right) \triangleq \frac{2\Re\left(\left(\bar{g}_k^{(l)}\right)^H \bar{h}_k \bar{h}_k^H \bar{g}_k\right)}{\bar{\phi}_k^{(l)}} - \left(\frac{\|\bar{h}_k^H \bar{g}_k^{(l)}\|_2}{\bar{\phi}_k^{(l)}}\right)^2 \bar{\phi}_k,$$
$$\forall k \in \mathcal{K},$$

Here l denotes the lth iteration. We can write

$$\bar{\Phi}_k^{(l)}\left(\bar{g}_k, \bar{\phi}_k\right) \geq \bar{\gamma}_k,\ \bar{\Phi}_k^{(l)}\left(\bar{g}_k, \bar{\phi}_k\right) \geq \bar{a}_k,\ \forall k \in \mathcal{K}.$$

Contd.

We solve the following convex problem to obtain the optimal solution

$$\max_{\{\bar{\mathbf{g}}_k, \bar{a}_k, \bar{b}_k, \bar{\phi}_k\}} \sum_{k=1}^{K} \bar{b}_k,$$

$$\text{s.t. } 1 + \bar{a}_k \geq e^{\bar{b}_k}, \forall k \in \mathcal{K}, \sum_{k=1}^{K} \|\hat{F}\bar{g}_k\|_2^2 \leq P$$

$$\sum_{l=1, l\neq k}^{K} \|\bar{\mathbf{h}}_k^H \bar{\mathbf{g}}_l\|_2^2 + \sigma^2 \leq \bar{\phi}_k, \forall k \in \mathcal{K},$$

$$\bar{\Phi}_k^{(I)}\left(\bar{g}_k, \bar{\phi}_k\right) \geq \bar{\gamma}_k, \ \bar{\Phi}_k^{(I)}\left(\bar{g}_k, \bar{\phi}_k\right) \geq \bar{a}_k, \ \forall k \in \mathcal{K}.$$

Solution of the Sum Rate Maximization Problem

Algorithm for optimal solution

Let the value of our objective function is $\bar{\tau}$ and $\bar{\tau}^{(l)}$is the value of $\bar{\tau}$ at the lth iteration.

1: Let $l = 0$, take some initial points as $\bar{\Gamma}^{(l)}$ and get $\bar{\tau}^{(l)}$.
2: Solve the convex problem with $\bar{\Gamma}^{(l)}$, and obtain new values of the $\bar{\Gamma}$ and $\bar{\tau}$.
3: If $|\bar{\tau} - \bar{\tau}^{(l)}| \leq \zeta$, then $\bar{\tau}, \bar{\Gamma}$ will be our optimal solution, otherwise $\bar{\Gamma}^{(l)} \leftarrow \bar{\Gamma}, \bar{\tau}^{(l)} \leftarrow \bar{\tau}$ and go to step 2.

Random Baseband Precoder

- The baseband precoder is of size $S \times K$ genereted using random numbers and then the columns are unit normalized.
- The complex numbers are distributed uniformly in $[-0.5, 0.5]$.

Simulation Result

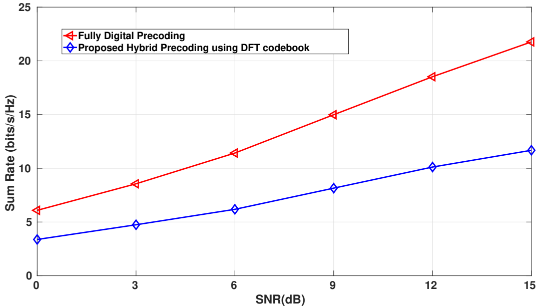


Figure: Sum Rate vs SNR, $M = N = 16, K = 4$

- ULA of transmit antennas $d = \lambda/2$
- $N_{cl} = 6, N_{ray} = 8$ AoD$\sim$Laplacian, mean of $\phi_{m_p} \sim \mathcal{U}[-\pi, \pi)$, $\sigma_\phi = 7.5^o$
- $\zeta = 10^{-3}$
- $\lambda_L = 0$ & $\lambda_U = 10$.

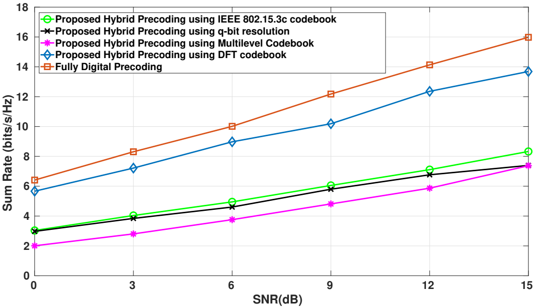


Figure: Sum Rate vs SNR, $M = N = 16, S = 4, K = 2$

- ULA of transmit antennas $d = \lambda/2$
- $N_{cl} = 6, N_{ray} = 8$ AoD∼Laplacian, mean of $\phi_{m_p} \sim \mathcal{U}[-\pi, \pi)$, $\sigma_\phi = 7.5^o$
- $\zeta = 10^{-3}$
- $\lambda_L = 0$ & $\lambda_U = 10$.

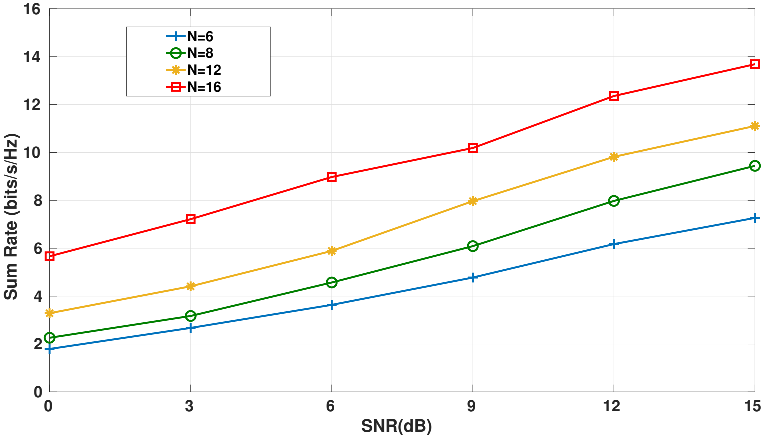


Figure: Sum Rate vs SNR, $M = 16, S = 4, K = 2$

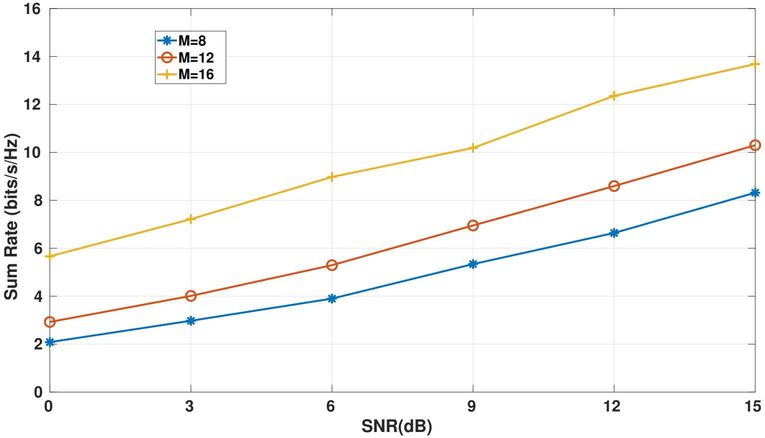


Figure: Sum Rate vs SNR, $N = 16, S = 4, K = 2$

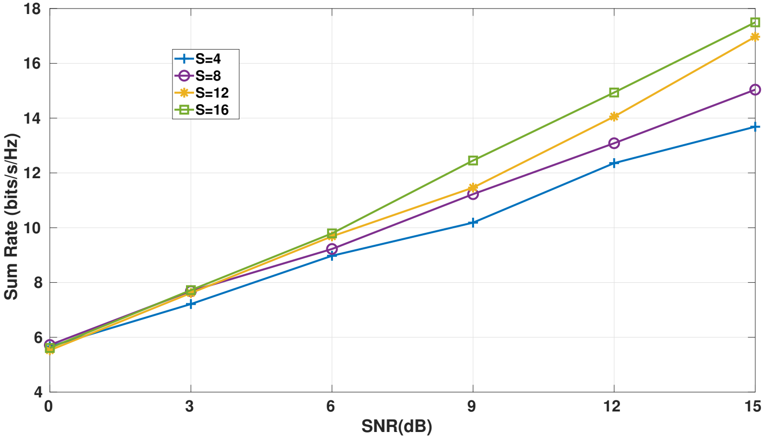


Figure: Sum Rate vs SNR, $M = N = 16, K = 2$

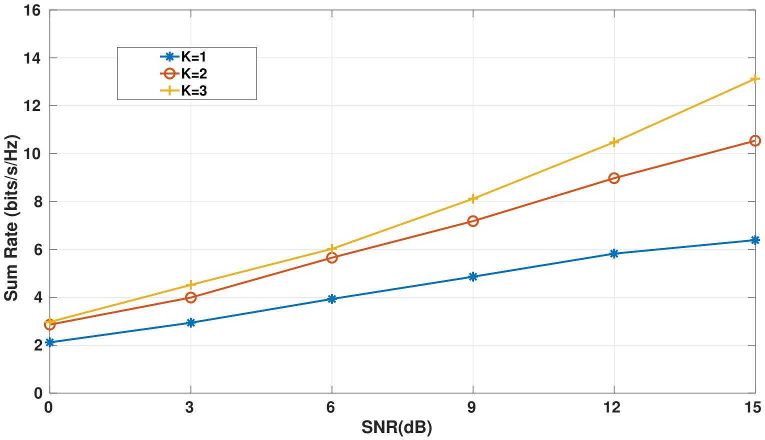


Figure: Sum Rate vs SNR using Random RF unitary codebooks, $M = N = 16, S = 4$

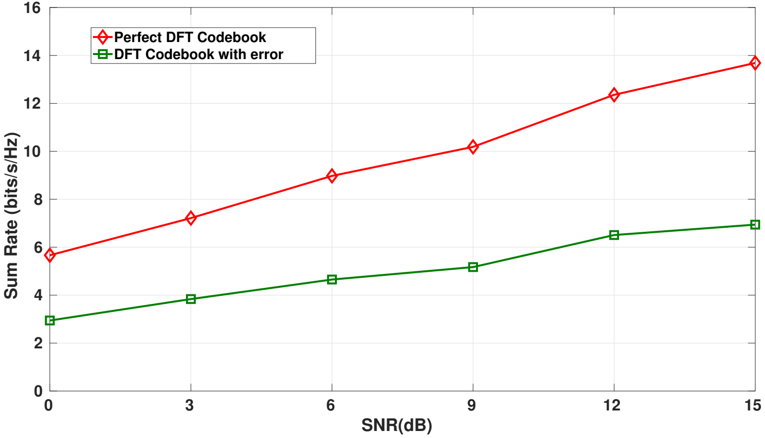


Figure: Sum Rate vs SNR using DFT codebooks checking Robustness, $M = N = 16, S = 4, K = 2, c = 2$

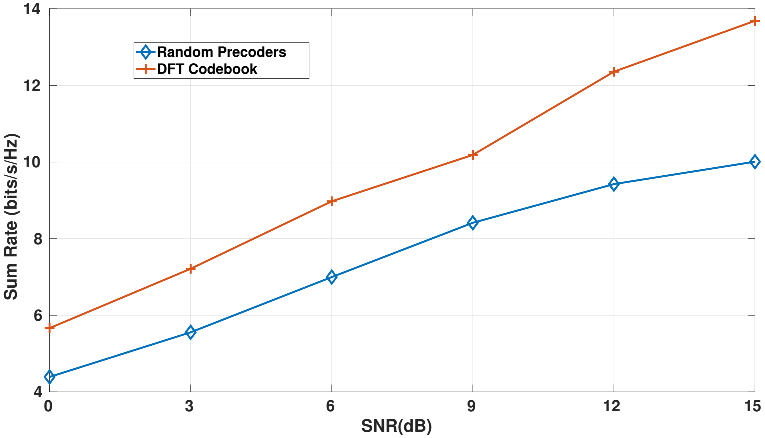


Figure: Sum Rate vs SNR, $M = N = 16, S = 4, K = 2$

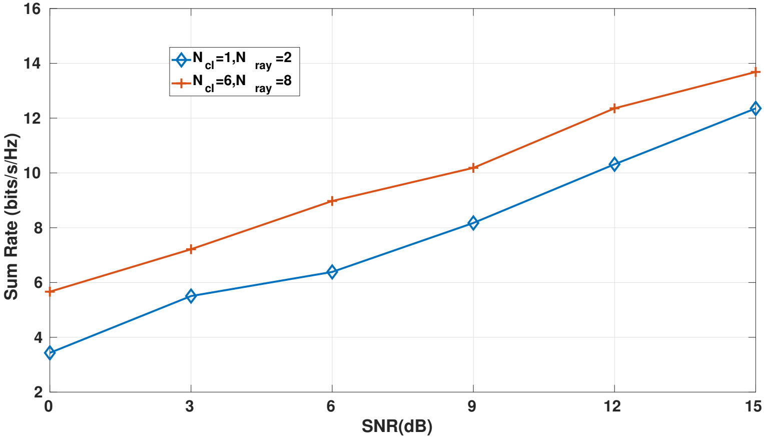


Figure: Sum Rate vs SNR, $M = N = 16, S = 4, K = 2$

Conclusions and future work

- We have seen that the generating the baseband precoder based on the channel and RF precoder chosen from the DFT codebook gives us the better performance among different codebooks.
- We have also seen the performance of the system when their is analog error in the codebook due to hardware mismatch.
- We can extend this work to multiple antenna users or allocating equal power to all users to design the hybrid precoders.

Bibliography

S. He, J. Wang, Y. Huang, and W. Hong, "Codebook based hybrid precoding for millimeter wave multiuser systems," *IEEE Trans. Signal Process.*, vol. 65, no. 20, pp. 5289–5304, Oct. 15 2017.

S. Noh, M. D. Zoltowski, and D. J. Love, "Multi-resolution codebook and adaptive beamforming sequence design for millimeter wave beam alignment," *IEEE Trans. Wireless Commun.*, vol. 16, no. 9, pp. 5689–1683, Sep. 2017.

Thank You