Sparse Signal Recovery(SSR) Based Algorithms for Data Decoding in Media Based Modulation

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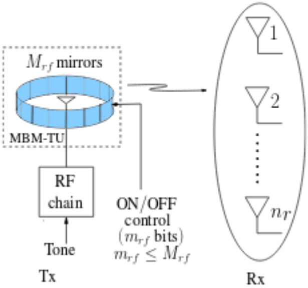
Agenda

- What is MBM
- Sparsity in MBM
- Structure of sparsity in GSM-MBM
- SSR algorithm for MBM
- Simulation results
- Future work

MBM: Media Based Modulation

- Varying the end to end channel based on the input is called Media Based Modulation.
- Carrier is modulated after leaving the transmitter by changing RF properties of the medium.
- All others traditional modulations are referred as Source Based Modulations(SBM).
- Small perturbation near the tx in a rich scattering environment results an independent end-to-end channel. RF mirrors are used for creating perturbations.



- If r_s bits are used for SBM and r_m bit for MBM, total $r_s + r_m$ can be transmitted by combining SBM and MBM, and receiver will receive one of the points from constellation of $2^{(r_s+r_m)}$ points.

Advantages of MBM

- Increasing the spectral efficiency without increasing energy unlike SBM, where increasing r_s results exponential increase in energy.
- Deep fades do not have persisting effect because of Constellation diversity. As constellation size increases, this converts static multi-path fading channel into non-fading AWGN.
- In a $1xD$ SIMO-MBM system received vector spans in D receive dimension unlike SIMO-SBM which spans in single complex dimension, which is equivalent to SIMO-SBM with D times bandwidth.
- Possibility of choosing subset of channel similar to multi user diversity gain in scheduling.

Disadvantages of MBM

- Random arrangements of constellation points and all points are used with equal probability. While in SBM constellation can be used with non uniform probability to realize shaping gain.
- MBM is Linear Time variant, can trouble the traditional channel equalization techniques
- Signal in single dimension at the input is spread across the multiple dimension at output.

GSM-MBM:

- MBM is combined with generalized spatial modulation(GSM) is called GSM-MBM
- A subset of N_a tx antennas are active out of N_t rf chains at any time, a constellation symbol from signal set of size M is sent on each active chain, and $N_r f$ mirrors per antenna are used for perturbing thje environment near the active antennas.
- A total of $\log \binom{N_t}{N_{rf}} + N_a N_{rf} + n_a \log_2^M$ bits are sent per channel use(bpcu)

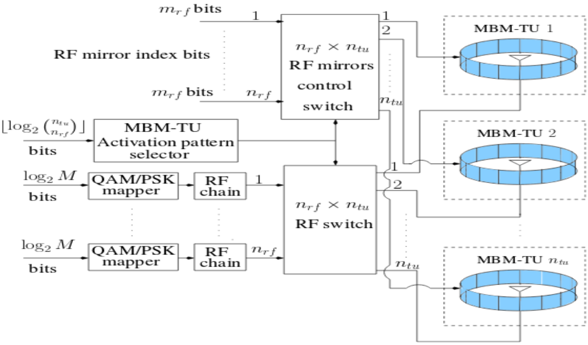


Fig. 2. GSM-MBM transmitter.

system model:

$$\mathbf{y} = \Phi\mathbf{x} + \mathbf{w}$$

observation vector, $\mathbf{y} \in \mathbb{C}^M$,
vector to be estimated, $\mathbf{x} \in \mathbb{C}^N$
$\mathbf{w}$ is AWGN noise vector i.e. $\sim \mathcal{N}(0, \sigma^2 \mathbf{I}_N)$.

- In GSM-MBM system model, $M = N_{rx}$ is number of receive antennas, $N = N_t * 2^{N_{rf}}$ where N_t is no.of transmit antennas, N_{rf} is no.of rf mirrors per antenna.

Goal: Estimate the unknown vector $\mathbf{x}$

- In GSM-MBM model only N_a number of rf chains are active, and corresponding to each active antenna, one mirror pattern is activated out of $2^{N_{rf}}$ mirror patterns.
- The vector $\mathbf{x}$ has inherent structured sparsity.

- The vector **x** is block sparse with only N_a number of active blocks and within each active block of size $2^{N_{rf}}$ only one mirror pattern is active at anytime. Following figure depicts structure of **x**

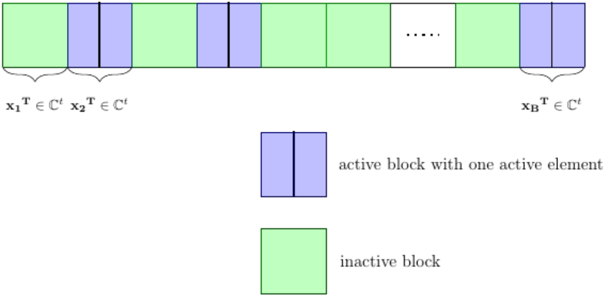


Figure 1: Block sparse vector with one active alement in each active block

- The above system can be modeled as
$$\mathbf{y} = \Phi\mathbf{DEz} + \mathbf{w}$$
where $\mathbf{D} = diag(\mathbf{d}) \otimes \mathbf{I}$, $\mathbf{d} = [d_1, d_2, \ldots, d_B]^T$,
$\mathbf{E} = diag(\mathbf{e}), \mathbf{e} = [e_{11}, e_{12}, \ldots, e_{BM}]^T$,
$\mathbf{z} = [z_{11}, z_{12}, \ldots, z_{BM}]^T, z_{ij} \sim \mathcal{N}(0, \gamma_i)$
- Structured sparsity is controlled by variables d_i,e_{ij} $i \in [1, B]$, $j \in [1, M]$ having prior distributions as follows:

$$\mathbf{p}(d_1, \ldots, d_B) = \frac{\exp\left\{-\alpha\left(N_a - \sum_{i=1}^{B} d_i\right)^2\right\}}{d_{const}},$$

$$\mathbf{p}(e_{11}, \ldots, e_{BM}/\mathbf{d}) = \prod_{i=1}^{B} \mathbf{p}(e_{i1}, \ldots, e_{iM}/d_i) = \prod_{i=1}^{B} \frac{\exp\left\{-\alpha\left(d_i - \sum_{j=1}^{M} e_{ij}\right)^2\right\}}{e_{const}},$$

Problem formulation:

$$\begin{aligned}
\widehat{\mathbf{d}}_{MAP} &= arg\ \underset{\mathbf{d}}{max}\ \ln \mathbf{p}(\mathbf{d}|\mathbf{y}, \Phi, \mathbf{e}, \mathbf{z}; \sigma^2) \\
&= arg\ \underset{\mathbf{d}}{max}\ \ln \mathbf{p}(\mathbf{y}|\mathbf{d}, \mathbf{e}, \mathbf{z}; \sigma^2)\mathbf{p}(\mathbf{d}) \\
&= arg\ \underset{\mathbf{d}}{max}\ \ln \mathbf{p}(\mathbf{y}|\mathbf{d}, \mathbf{e}, \mathbf{z}; \sigma^2) + \ln \mathbf{p}(\mathbf{d})
\end{aligned}$$

where,

$$\ln \mathbf{p}(\mathbf{y}|\mathbf{d}, \mathbf{z}; \sigma^2) = -\frac{\|\mathbf{y} - \Phi\mathbf{DEz}\|^2}{2\sigma^2} - \frac{1}{2} \ln \left(2\pi\sigma^2\right)$$

$$\ln \mathbf{p}(\mathbf{d}) = -\alpha \left(N_a - \sum_{i=1}^{B} d_i\right)^2 - \ln(d_{const})$$

Let us define $\mathbf{f}(\mathbf{d}) \triangleq \frac{\|\mathbf{y}-\Phi\mathbf{DEz}\|^2}{2\sigma^2} + \alpha \left(N_a - \sum_{i=1}^{B} d_i\right)^2$, then

$$\widehat{\mathbf{d}}_{MAP} = arg\ \underset{\mathbf{d}}{max}\ -\mathbf{f}(\mathbf{d}) \quad (1)$$

$$= arg\ \underset{\mathbf{d}}{min}\ \mathbf{f}(\mathbf{d}) \quad (2)$$

$$
\begin{aligned}
\widehat{\mathbf{e}}_{MAP} &= arg \max_{\mathbf{e}} \; \ln \mathbf{p}(\mathbf{e}|\mathbf{y},\mathbf{d},\mathbf{z};\sigma^2) \\
&= arg \max_{\mathbf{e}} \; \ln \mathbf{p}(\mathbf{y}|\mathbf{d},\mathbf{e},\mathbf{z};\sigma^2)\mathbf{p}(\mathbf{e}/\mathbf{d})\mathbf{p}(\mathbf{d}) \\
&= arg \max_{\mathbf{d}} \; \ln \mathbf{p}(\mathbf{y}|\mathbf{d},\mathbf{e},\mathbf{z};\sigma^2) + \ln \mathbf{p}(\mathbf{e}/\mathbf{d}) + \ln \mathbf{p}(\mathbf{d})
\end{aligned}
$$

where,

$$
\ln \mathbf{p}(\mathbf{e}/\mathbf{d}) = \sum_{i=1}^{B} -\alpha \left(d_i - \sum_{i=1}^{M} e_{ij} \right)^2 - \ln(e_{const})
$$

Let us define $\mathbf{f}(\mathbf{e}) \triangleq \frac{\|\mathbf{y}-\Phi\mathbf{DEz}\|^2}{2\sigma^2} + \sum_{i=1}^{B} \alpha \left(d_i - \sum_{i=1}^{M} e_{ij} \right)^2$

$$
\widehat{\mathbf{e}}_{MAP} = arg \max_{\mathbf{e}} \; -\mathbf{f}(\mathbf{e}) \tag{3}
$$

$$
= arg \min_{\mathbf{e}} \mathbf{f}(\mathbf{e}) \tag{4}
$$

$$
\begin{aligned}
\widehat{\mathbf{z}}_{MAP} &= arg \max_{\mathbf{z}} \; \ln \mathbf{p}(\mathbf{z}|\mathbf{y}, \mathbf{d}, \mathbf{e}; \sigma^2) \\
&= arg \max_{\mathbf{z}} \; \ln \mathbf{p}(\mathbf{y}|\mathbf{d}, \mathbf{e}, \mathbf{z}; \sigma^2)\mathbf{p}(\mathbf{z}) \\
&= arg \max_{\mathbf{d}} \; \ln \mathbf{p}(\mathbf{y}|\mathbf{d}, \mathbf{e}, \mathbf{z}; \sigma^2) + \ln \mathbf{p}(\mathbf{z})
\end{aligned}
$$

Solving the optimization problems:
Steepest Descent method is used to solve optimization problems of (2) and (4).
Updating **d** using Steepest descent method as follows:

$$\mathbf{d}_{t+1} = \mathbf{d}_t - \mu_d \frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t}$$

$$\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t} = \frac{\Phi^H(\Phi \mathbf{DEz} - y)(\mathbf{Ez})^H}{\sigma^2} + 2\alpha \left(\sum_{i=1}^{B} d_i - Na \right) (\mathbf{1})_{N\times 1}$$

Step size μ can calculated by setting $\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mu_d}|_{\mathbf{d}=\mathbf{d}_{t+1}} = 0$

$$\mu_d = \frac{\frac{\text{real}\{(\mathbf{y}^H \Phi \left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t} \right) \mathbf{Ez} - (\mathbf{\Phi DEz})^H \Phi \left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t} \right) \mathbf{Ez}\}}{\sigma^2} + 2\alpha \left(Na - \sum_{i=1}^{B} d_i \right) \left(\sum_{i=1}^{B} \frac{\partial \mathbf{f}(d_i)}{\partial d_i}|_{d_i = d_{i\,t}} \right)}{\frac{\left(\Phi \left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t} \right) \mathbf{Ez} \right)^H \Phi \left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t} \right) \mathbf{Ez}}{\sigma^2} + 2\alpha \left(\sum_{i=1}^{B} \frac{\partial \mathbf{f}(d_i)}{\partial d_i}|_{d_i = d_{i\,t}} \right)^2}$$

Updating $\mathbf{e}$ using Steepest descent method as follows:

$$\mathbf{e}_{t+1} = \mathbf{e}_t - \mu_e \frac{\partial \mathbf{f}(\mathbf{e})}{\partial \mathbf{e}}|_{\mathbf{e}=\mathbf{e}_t}$$

$$\frac{\partial \mathbf{f}(\mathbf{e})}{\partial \mathbf{e}}|_{\mathbf{e}=\mathbf{e}_t} = \frac{(\Phi\mathbf{D})^H(\Phi\mathbf{DEz} - y)(\mathbf{z})^H}{\sigma^2} + 2\alpha\sum_{i=1}^{B}\left(\sum_{j=1}^{M} e_{ij} - d_i\right)(\mathbf{1})_{N\times 1}$$

Step size μ can calculated by setting $\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mu_e}|_{\mathbf{d}=\mathbf{d}_{t+1}} = 0$

$$\mu_e = \frac{\frac{\text{real}\{(\mathbf{y}^H\Phi\left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t}\right)\mathbf{Ez} - (\Phi\mathbf{DEz})^H\Phi\left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t}\right)\mathbf{Ez}\}}{\sigma^2} + 2\alpha\left(Na - \sum_{i=1}^{B} d_i\right)\left(\sum_{i=1}^{B}\frac{\partial \mathbf{f}(d_i)}{\partial d_i}|_{d_i=d_{i\,t}}\right)}{\frac{\left(\Phi\left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t}\right)\mathbf{Ez}\right)^H\Phi\left(\frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}}|_{\mathbf{d}=\mathbf{d}_t}\right)\mathbf{Ez}}{\sigma^2} + 2\alpha\left(\sum_{i=1}^{B}\frac{\partial \mathbf{f}(d_i)}{\partial d_i}|_{d_i=d_{i\,t}}\right)^2}$$

Estimation of z using Type-II ML:

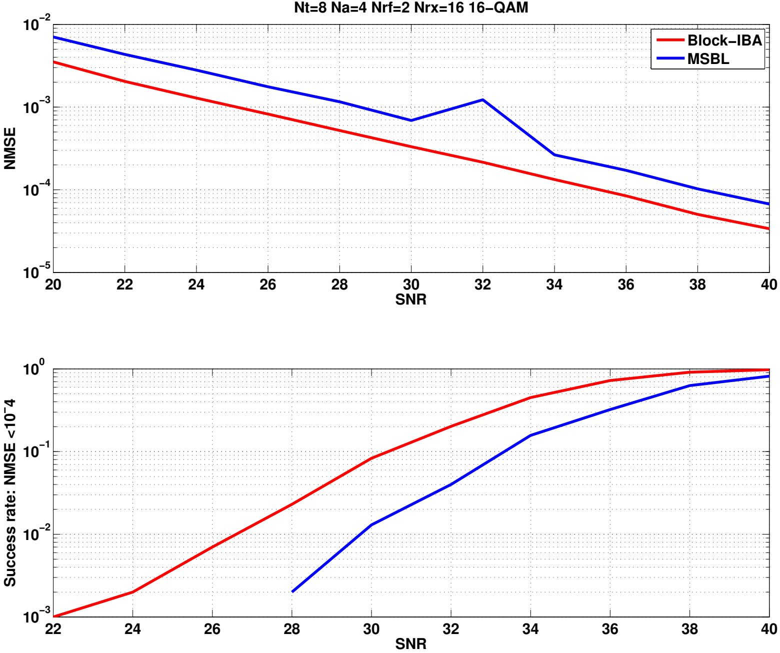
- We assume a $\mathbf{z}$ has Gaussian distribution with mean zero and variance Γ, where $\Gamma = diag(\gamma_1, \ldots, \gamma_N)$
- Given $\mathbf{d}$, $\mathbf{e}$ the likelihood function of $\mathbf{z}$ is also a Gaussian with distribution $\sim \mathcal{N}(\Phi\mathbf{DEz}, \sigma^2\mathbf{I_N})$

- The posterior distribution $\mathbf{p}\left(\mathbf{z}|\mathbf{y}, \mathbf{d}, \mathbf{e}; \sigma^2\right)$ is multivariate Gaussian with mean μ_z and covariance Σ_0
  where
  $\mu_z = \sigma^{-2}\sigma_0 \left(\Phi\mathbf{DE}\right)^H \mathbf{y}$
  $\Sigma_0 = \Gamma^{-1} - \Gamma^{-1} \left(\Phi\mathbf{DE}\right)^H \left(\sigma^2\mathbf{I}_N + \Phi\mathbf{DE}\Gamma^{-1} \left(\Phi\mathbf{DE}\right)^H\right)^{-1} \Phi\mathbf{DE}\Gamma^{-1}$
- Using Type-II ML estimator, the update for Γ can be expressed as $\Gamma = |\mu_z|^2 + diag(\Sigma_0)$

Iterative Bayesian Algorithm:

- Initialize $\mathbf{z}$ with LS square solution
- Initialize $\mathbf{d}$ and $\mathbf{e}$ from $mathbf\, z_{init}$
- while $|\mathbf{z}_t - \mathbf{z}_{t-1}| < \epsilon$
  Update $\mathbf{d}$ using steepest descent
  Update $\mathbf{e}$ using steepest descent
  Update $\mathbf{z}$ using EM-SBL
  threshold $\mathbf{d}$, $\mathbf{e}$

Simulation results:



Future work

- penalty for forcing entries of **d**, **e** to either 0 or 1
- Handling of concave part in objective function which results from penalty added for forcing $d_i, e_{ij} \in \{0, 1\}$