Main Presentation
User Activity Detection in PDMA

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Contents

What is PDMA?

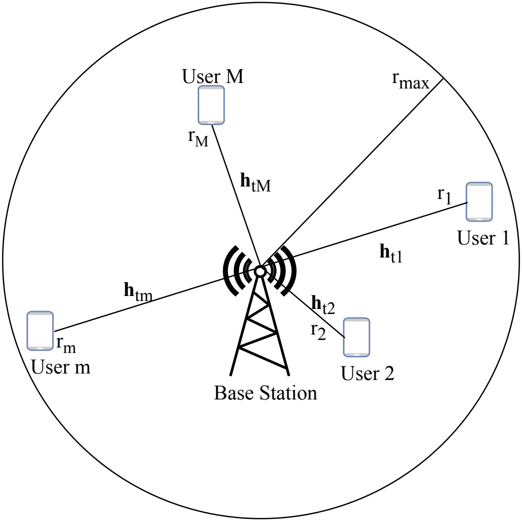
- Pattern Division Multiple Access
- Replication of packets across resource elements
- Enabled via successive interference cancellation
- With capture effect, throughput can be increased to greater than 1
- PDMA - binary patterns assigned to users

	User 1	User 2	User 3	User 4	User 5	User 6
Slot 1	✓	✓		✓		
Slot 2	✓		✓		✓	
Slot 3		✓				✓
Slot 4			✓			

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1M} \\ g_{21} & g_{22} & \cdots & g_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ g_{T1} & g_{T2} & \cdots & g_{TM} \end{bmatrix}$$

System Model

- Users transmit packet replicas according to the pattern matrix $\mathbf{G}$
- $\mathbf{y}_t = \sum_{m=1}^{M} g_{tm}\mathbf{h}_{tm}x_m + \mathbf{n}_t$
- Noise, $\mathbf{n}_t \sim \mathcal{CN}(\mathbf{0}_N, N_0\mathbf{I}_N)$
- Transmit packets x_m have $\mathbb{E}[x_m] = 0$ & $\mathbb{E}[|x_m|^2] = P$
- Channel gain, $\mathbf{h}_{tm} = \sqrt{\beta_m}\mathbf{v}_{tm}$
- Path loss, $\beta_m = (r_m/r_0)^{-\alpha}$
- Fading, $\mathbf{v}_{tm} \sim \mathcal{CN}(\mathbf{0}_N, \sigma_h^2\mathbf{I}_N)$



Channel Estimation

- Each packet header carries a pilot $x_m = \sqrt{P}$

$$\mathbf{y}_t^{pk} = \sum\nolimits_{i \in \mathcal{S}_k} g_{ti} \mathbf{h}_{ti} \sqrt{P} + \mathbf{n}_t^p \tag{1}$$

- MMSE channel estimates are recomputed every iteration as $\hat{\mathbf{h}}_{tm}^k \triangleq \mathbb{E}_{\mathbf{y}_t^{pk}} [\mathbf{h}_{tm}]$

$$\hat{\mathbf{h}}_{tm}^k = \frac{\sqrt{P}\sigma_h^2 g_{tm} \beta_m}{P\sigma_h^2 (\sum_{i \in \mathcal{S}_k} g_{ti}^2 \beta_i) + N_0} \mathbf{y}_t^{pk} =: \eta_{tm}^k \mathbf{y}_t^{pk} \tag{2}$$

- The estimation error $\tilde{\mathbf{h}}_{tm}^k \triangleq \hat{\mathbf{h}}_{tm}^k - \mathbf{h}_{tm}$ is uncorrelated with the received pilot signal and the estimate itself

- *Threshold based decoding*: SINR $\geq \gamma_{th} \geq 1$
- Under the common pilots scheme, MRC is performed

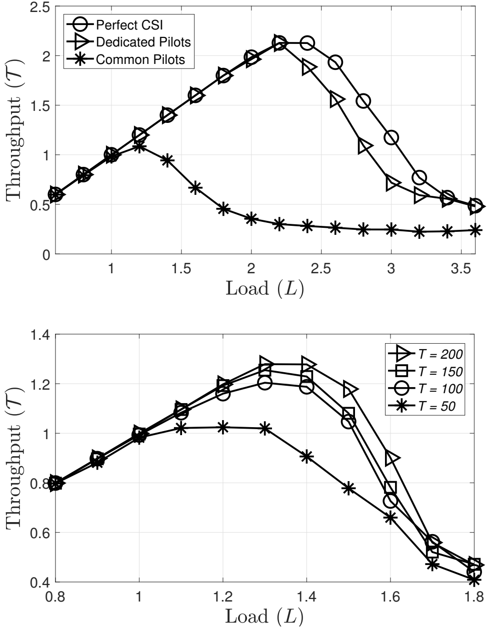
$$\tilde{y}_{tm}^{k} = \hat{\mathbf{h}}_{tm}^{kH}\mathbf{y}_{t}^{k} = \underbrace{\hat{\mathbf{h}}_{tm}^{kH}\hat{\mathbf{h}}_{tm}^{k}g_{tm}x_{m}}_{Signal} + \underbrace{\hat{\mathbf{h}}_{tm}^{kH}\mathbf{n}_{t}}_{Noise}$$
$$\underbrace{+\,\hat{\mathbf{h}}_{tm}^{kH}\sum_{i\in\mathcal{S}_{k}^{m}} g_{ti}\mathbf{h}_{ti}x_{i} - \hat{\mathbf{h}}_{tm}^{kH}\tilde{\mathbf{h}}_{tm}^{k}g_{tm}x_{m}}_{Interference} \quad (3)$$

- Effective SINR is calculated as

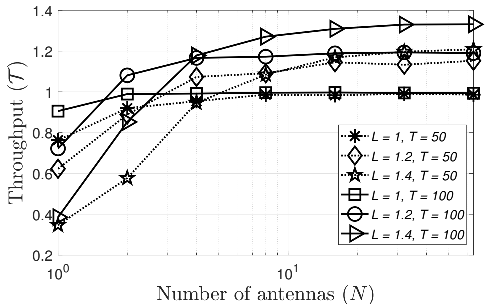
$$\mathrm{SINR}_{tm}^{k} = \frac{Pg_{tm}^{2}\|\hat{\mathbf{h}}_{tm}^{k}\|^{2}}{N_0 + P\left(\sum_{i\in\mathcal{S}_k} g_{ti}^{2}\delta_{ti}^{k} + \sum_{i\in\mathcal{S}_k^m} g_{ti}^{2}\|\hat{\mathbf{h}}_{ti}^{k}\|^{2}\right)} \quad (4)$$

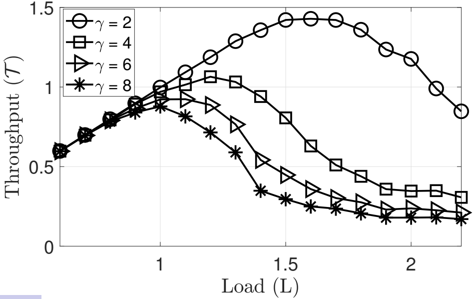
Results

- Parameters: $P = 1$, $\gamma = 4$, $\alpha = 3$, $N = 4$, $\sigma_h^2 = 1$, $T = 50$, $r_{max} = 1\text{km}$, $r_0 = 0.1\text{km}$, $N_0 = 10^{-4}$ & $N_{sim} = 100$
- Common pilots result in drop of 50% throughput w.r.t. of perfect CSI
- Dedicated pilots can recover loss at the cost of $M\bar{d} - T =$ 190 extra training symbols for $M = 60$



- For lower loads, increasing the number of antennas marginally increases the throughput and saturates thereafter
- For higher loads, increasing the number of antennas increases the throughput
- Decreasing the capture threshold yields higher throughputs as more users are decoded





User Activity Detection

Approach 1:

- Received Pilot: $\mathbf{y}_t^{pk} = \sqrt{P} \sum_{i \in \mathcal{S}_k} g_{ti} a_i \mathbf{h}_{ti} + \mathbf{n}_t^p$
- ML/MAP based detection

$$\mathbf{y}_t^{pk} \mid \mathbf{a} \sim \mathcal{CN}(\mathbf{0}_N, s_t \mathbf{I}_N)$$

$$s_t = P\sigma_h^2 \Big(\sum_{i \in \mathcal{S}_k} g_{ti}^2 a_i^2 \beta_i\Big) + N_0$$

$$\implies \|\mathbf{y}_t^{pk}\|^2 \mid \mathbf{a} \sim \frac{s_t}{2}\, \Gamma(N, 2)$$

- $\tilde{\mathbf{y}} = [\|\mathbf{y}_1^{pk}\|^2, \|\mathbf{y}_2^{pk}\|^2, \dots, \|\mathbf{y}_T^{pk}\|^2]^T$
- ML: $\max_{\mathbf{a}\in\{0,1\}^M} p(\tilde{\mathbf{y}}|\mathbf{a}) = \max_{\mathbf{a}\in\{0,1\}^M} \prod_{t=1}^{T} p(\tilde{y}_t|\mathbf{a})$

$$= \min_{\mathbf{a}\in\{0,1\}^M} \sum_{t=1}^{T} \left(\frac{\tilde{y}_t}{s_t} + \ln\left(\frac{s_t}{2}\right)\right)$$

$$\text{s.t.} \quad s_t = P\sigma_h^2 \mathbf{G}_{t:} \operatorname{diag}(\beta)\, \mathbf{a} + N_0, \ \forall t$$

- Non-convex integer programming problem
- Channel estimation as before

- MAP: Prior $p(\mathbf{a}) = \prod_{m=1}^{M} p(a_m), \quad p(a_m) = p_a^{a_m}(1-p_a)^{1-a_m}$

$$\max_{\mathbf{a}\in\{0,1\}^M} p(\tilde{\mathbf{y}}|\mathbf{a})p(\mathbf{a}) = \max_{\mathbf{a}\in\{0,1\}^M} \left(\prod_{t=1}^{T} p(\tilde{y}_t|\mathbf{a})\right)\left(\prod_{m=1}^{M} p(a_m)\right)$$

$$= \min_{\mathbf{a}\in\{0,1\}^M} \sum_{t=1}^{T}\left(\frac{\tilde{y}_t}{s_t} + \ln\left(\frac{s_t}{2}\right)\right) + \ln\left(\frac{1-p_a}{p_a}\right)\sum_{m=1}^{M} a_i$$

$$\text{s.t.} \quad s_t = P\sigma_h^2 \mathbf{G}_{t:} \operatorname{diag}(\beta)\, \mathbf{a} + N_0, \ \forall t$$

Approach 2:

- $\tau-$length pilots $\mathbf{p}_i$ used with $\|\mathbf{p}_i\|^2 = P$

$$\mathbf{Y}_t^{pk\,H} = \sum_{i=1}^{M} g_{ti} a_i \mathbf{p}_i \mathbf{h}_{ti}^H + \mathbf{N}_t^{p\,H}$$

$$= [g_{t1}\mathbf{p}_1, \ldots, g_{tM}\mathbf{p}_M] \begin{bmatrix} a_1 \mathbf{h}_{t1}^H \\ \vdots \\ a_M \mathbf{h}_{tM}^H \end{bmatrix} + \mathbf{N}_t^{p\,H}$$

$$\implies \underbrace{\mathbf{Y}_t^{pk\,H}}_{\tau \times N} = \underbrace{\mathbf{\Phi}_t}_{\tau \times M} \underbrace{\mathbf{X}_t}_{M \times N} + \underbrace{\mathbf{N}_t^{p\,H}}_{\tau \times N}$$

- Use CS methods to get activity coefficients only
- Discard channel estimates

- Channel estimation:

$$\mathbf{y}_{tm}^{pk} = \mathbf{Y}_t^{pk}\mathbf{p}_m = Pg_{tm}\hat{a}_m\mathbf{h}_{tm} + \mathbf{N}_t^p\mathbf{p}_m + \sum_{i\in\mathcal{S}_k^m} g_{ti}\hat{a}_i\mathbf{h}_{ti}\mathbf{p}_i^H\mathbf{p}_m$$

$$\implies \hat{\mathbf{h}}_{tm}^k = \mathbb{E}\left[\mathbf{h}_{tm}|\mathbf{y}_{tm}^{pk}\right] = \frac{Pg_{tm}\hat{a}_m\beta_m\sigma_h^2}{PN_0 + \sigma_h^2\sum_{i\in\mathcal{S}_k} g_{ti}^2\hat{a}_i^2\beta_i|\mathbf{p}_i^H\mathbf{p}_m|^2}\mathbf{Y}_t^{pk}\mathbf{p}_m$$

$$\implies \mathrm{SINR}_{tm}^k = \frac{Pg_{tm}^2\hat{a}_m^2\|\hat{\mathbf{h}}_{tm}^k\|^2}{N_0 + P\left(\sum_{i\in\mathcal{S}_k} g_{ti}^2\hat{a}_i^2\delta_{ti}^k + \sum_{i\in\mathcal{S}_k^m} g_{ti}^2\hat{a}_i^2\frac{\left|\hat{\mathbf{h}}_{tm}^{kH}\hat{\mathbf{h}}_{ti}^k\right|^2}{\|\hat{\mathbf{h}}_{tm}^k\|^2}\right)}$$

Approach 3:

- Channel hardening & asymptotic energy based detection

$$\frac{\|\mathbf{h}_{tm}\|^2}{N} \xrightarrow{N\to\infty} \beta_m \sigma_h^2$$

$$\frac{\mathbf{h}_{ti}^H \mathbf{h}_{tj}}{N} \xrightarrow{N\to\infty} 0$$

$$\implies \frac{\|\mathbf{y}_{tm}^{pk}\|^2}{N} - PN_0 \xrightarrow{N\to\infty} \sigma_h^2 \left(\sum_{i \in \mathcal{S}_k} g_{ti}^2 \beta_i a_i^2 |\mathbf{p}_i^H \mathbf{p}_m|^2 \right) =: E_{tm}$$

- $E_{tm} = \sigma_h^2 \mathbf{G}_{t:} \, \mathrm{diag}\,(\boldsymbol{\beta}) \, \mathrm{diag}\,(\mathbf{a}) \, \mathbf{P}'_{:m}$
- $\mathbf{P}'_{nm} = |\mathbf{p}_n^H \mathbf{p}_m|^2$

- Received energy matrix $\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} & \dots & E_{1M} \\ E_{21} & E_{22} & \dots & E_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ E_{T1} & E_{T2} & \dots & E_{TM} \end{bmatrix}$
- Collect asymptotic energies for all users across all REs

$$E_{tm} = \sigma_h^2 \, \mathbf{G}_{t:} \, \text{diag}\left(\beta\right) \, \text{diag}\left(\mathbf{a}\right) \, \mathbf{P}'_{:m}$$

$$\underbrace{\mathbf{E}}_{T\times M} = \underbrace{\sigma_h^2 \, \mathbf{G} \, \text{diag}\left(\beta\right)}_{T\times M} \underbrace{\text{diag}\left(\mathbf{a}\right)}_{M\times M} \underbrace{\mathbf{P}'}_{M\times M}$$

$\downarrow$ Khatri-Rao vectorization

$$\mathbf{e}_{TM\times 1} = \boldsymbol{\Phi}_{TM\times M} \, \mathbf{a}'_{M\times 1}$$

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