Main Presentation
A new look at the throughput of IRSA/PDMA systems

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Contents

What is PDMA?

- Pattern Division Multiple Access
- Orthogonal vs non-orthogonal access
- Enabled via successive interference cancellation
- With capture effect, throughput can be increased to greater than 1
- PDMA - binary patterns assigned to users

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1M} \\ g_{21} & g_{22} & \cdots & g_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ g_{T1} & g_{T2} & \cdots & g_{TM} \end{bmatrix} \in \{0,1\}^{T \times M} \quad (1)$$

What is IRSA?

- Irregular Repetition Slotted Aloha is a multiple access protocol
- Similar setup as PDMA with packets replicated across slots
- Enabled via successive interference cancellation
- Repetition factors (d_m) chosen via distributions
- Uniformly randomly choose d_m slots to transmit
- Truncated soliton distribution proven to push throughput close to 1 in the case of no capture effect[1]:

$$\mathrm{P}(d_m = d) = \begin{cases} \frac{1-a}{2z}, & d = 2 \\ \frac{1}{d(d-1)z}, & 3 \leq d \leq k \end{cases} \tag{2}$$

[1] K. R. Narayanan and H. D. Pfister, "Iterative collision resolution for slotted ALOHA: An optimal uncoordinated transmission policy"

Received Signal

- M users (single antenna) access T resource elements (REs) to communicate with a base station (BS) having N antennas
- Users are static and the BS knows their locations
- Users transmit a replica of packet in a subset of the REs
- Subset determined by a pattern matrix $\mathbf{G}$ (known by BS)

$$\mathbf{y}_t = \sum_{m=1}^{M} \mathbf{h}_{tm} g_{tm} x_m + \mathbf{n}_t \quad \in \mathbb{C}^{N\times 1} \tag{3}$$

- Noise $\mathbf{n}_t \sim \mathcal{CN}(\mathbf{0}_N, N_0\mathbf{I}_N)$
- Transmit packets x_m have $\mathbb{E}[x_m] = 0$ & $\mathbb{E}[|x_m|^2] = P$
- Transmit diversity offered by the setup $\Rightarrow$ complete failure avoided

Example frame I

	User 1	User 2	User 3	User 4	User 5	User 6
Slot 1	✓	✓		✓		
Slot 2	✓		✓		✓	
Slot 3		✓	✓			✓

- $M = 6$ users, $T = 3$ slots $\Rightarrow$ Load, $L = M/T = 2$
- Repetition factor/ transmit diversity order $\mathbf{d} = [2, 2, 2, 1, 1, 1]^T$
- Collision factor $\mathbf{c} = [3, 3, 3]^T$

Example frame II

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$\Rightarrow \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} & \mathbf{0}_N & \mathbf{h}_{14} & \mathbf{0}_N & \mathbf{0}_N \\ \mathbf{h}_{21} & \mathbf{0}_N & \mathbf{h}_{23} & \mathbf{0}_N & \mathbf{h}_{25} & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{h}_{32} & \mathbf{h}_{33} & \mathbf{0}_N & \mathbf{0}_N & \mathbf{h}_{36} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{bmatrix} \tag{5}$$

Channel Model

- Channel gain $h_{tmn} = \sqrt{\beta_m} v_{tmn}$
- Path loss coefficient $\beta_m = r_m^{-\alpha}$
- Distance of the mth user from the base station r_m
- Path loss exponent α
- For a sub-6 GHz system with a rich scattering environment, the fading coefficients are assumed to be IID
- Fading coefficient $\mathbf{v}_{tm} = [v_{tm1}, v_{tm2}, \ldots, v_{tmN}]^T \sim \mathcal{CN}(\mathbf{0}_N, \sigma^2 \mathbf{I}_N)$

Channel Estimates I

- Each packet header carries a pilot $x_m = \sqrt{P}$

$$\Rightarrow \mathbf{y}_t^p = \left(\sum_{m=1}^{M} \mathbf{h}_{tm} g_{tm} \right) \sqrt{P} + \mathbf{n}_t^p \tag{6}$$

- Without capture effect, $T > M \Rightarrow \mathbf{G}$ is tall $\Rightarrow$ Overdetermined system
- With capture effect, $T < M \Rightarrow \mathbf{G}$ is fat $\Rightarrow$ Underdetermined system
- MMSE estimate is calculated

$$\mathbb{E}\left[\|\mathbf{h}_{s,t}\|^2\right] = \mathrm{Tr}\left(\sum_{i=1}^{M} g_{ti}^2 \beta_i \left(\sigma^2 \mathbf{I}_N \right) \right) =: N\sigma_{s,t}^2 \tag{7}$$

Channel Estimates II

$$\mathbb{E}\left[\mathbf{h}_{tm}^H \mathbf{y}_t^p\right] = N\sqrt{P} g_{tm} \beta_m \sigma^2 \quad (8)$$

$$\mathbb{E}\left[\|\mathbf{n}_t^p\|^2\right] = \mathrm{Tr}\left(N_0 \mathbf{I}_N\right) = NN_0 \quad (9)$$

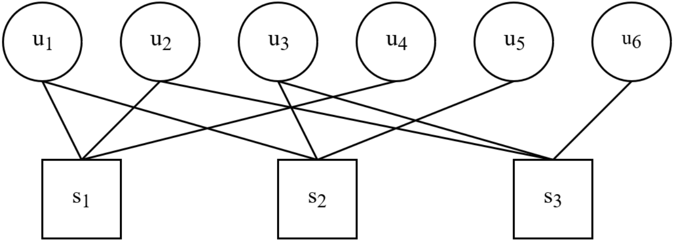
$$\Rightarrow \hat{\mathbf{h}}_{tm} = \frac{\mathbb{E}\left[\mathbf{h}_{tm}^H \left(\mathbf{y}_t^p/\sqrt{P}\right)\right]}{\mathbb{E}\left[\left(\mathbf{y}_t^p/\sqrt{P}\right)^H \left(\mathbf{y}_t^p/\sqrt{P}\right)\right]} \frac{\mathbf{y}_t^p}{\sqrt{P}} \quad (10)$$

$$= \left(\frac{\sqrt{P} g_{tm} \beta_m \sigma^2}{P\sigma_{s,t}^2 + N_0}\right) \mathbf{y}_t^p =: \sigma_{tm}^2 \mathbf{y}_t^p \quad (11)$$

- MMSE estimation error $\tilde{\mathbf{h}}_{tm} = \hat{\mathbf{h}}_{tm} - \mathbf{h}_{tm}$ has the property

$$\mathbb{E}\left[\hat{\mathbf{h}}_{tm} \tilde{\mathbf{h}}_{tm}^H\right] = \mathbf{0}, \quad \mathbb{E}\left[\mathbf{y}_t^p \tilde{\mathbf{h}}_{tm}^H\right] = \mathbf{0} \quad (12)$$

Graph decoding



- SIC based decoding $\equiv$ message passing on a Tanner graph
- Circles are users nodes and squares are RE nodes
- Decoding involves removing edges from graph based some criteria
- Decoding failure if any iteration fails to remove atleast 1 edge or after some maximum iterations

Threshold model

- Capture threshold based decoding model
- Probability of decoding the mth users packet in the tth RE in any decoding iteration is

$$p_{tm} = \begin{cases} 1, & \mathrm{SINR}_{tm} \geq b \\ 0, & \mathrm{SINR}_{tm} < b \end{cases} \tag{13}$$

- Threshold chosen to be $b \geq 1$ for a narrowband system
- Higher threshold of $b = 2$ or 4 ensures high power of the user for decoding

SINR calculation

- Characterizing throughput/rate as well as decoding requires calculation of SINR
- Receiver combines signal via an estimate

$$\tilde{y}_{tm} = \hat{\mathbf{h}}_{tm}^H \mathbf{y}_t = \hat{\mathbf{h}}_{tm}^H \left(\sum_{i=1}^{M} g_{ti} \mathbf{h}_{ti} x_i + \mathbf{n}_t \right)$$
$$= \hat{\mathbf{h}}_{tm}^H \hat{\mathbf{h}}_{tm} g_{tm} x_m - \hat{\mathbf{h}}_{tm}^H \tilde{\mathbf{h}}_{tm} g_{tm} x_m$$
$$+ \hat{\mathbf{h}}_{tm}^H \sum_{i \neq m} g_{ti} \mathbf{h}_{ti} x_i + \hat{\mathbf{h}}_{tm}^H \mathbf{n}_t \quad (14)$$

- Receiver has side information $\mathbf{z} = \hat{\mathbf{h}}_{tm}$
- Calculate the SINR conditioned on estimate

$$\mathbb{E}_{\mathbf{z}}\left[|\tilde{y}_{tm}|^2\right] = \mathbb{E}_{\mathbf{z}}\left[\left| \begin{matrix} \|\hat{\mathbf{h}}_{tm}\|^2 g_{tm} x_m - \hat{\mathbf{h}}_{tm}^H \tilde{\mathbf{h}}_{tm} g_{tm} x_m \\ + \hat{\mathbf{h}}_{tm}^H \sum_{i \neq m} g_{ti} \mathbf{h}_{ti} x_i + \hat{\mathbf{h}}_{tm}^H \mathbf{n}_t \end{matrix} \right|^2\right] \quad (15)$$

- Two cross-power components $\mathbb{E}_{\mathbf{z}}\left[\hat{\mathbf{h}}_{tm}\mathbf{h}_{ti}^H x_m x_i^*\right]$ & $\mathbb{E}_{\mathbf{z}}\left[\tilde{\mathbf{h}}_{tm}\mathbf{h}_{ti}^H x_m x_i^*\right]$ are both zero as $\mathbb{E}_{\mathbf{z}}\left[x_m x_i^*\right] = \mathbb{E}_{\mathbf{z}}\left[x_m\right]\mathbb{E}_{\mathbf{z}}\left[x_i^*\right] = 0$
- Third cross-power component also becomes zero

$$\mathbb{E}_{\mathbf{z}}\left[\tilde{\mathbf{h}}_{tm}\right] = \mathbb{E}_{\mathbf{z}}\left[\hat{\mathbf{h}}_{tm} - \mathbf{h}_{tm}\right] = \hat{\mathbf{h}}_{tm} - \mathbb{E}_{\mathbf{z}}\left[\mathbf{h}_{tm}\right] = \mathbf{0}_N \quad (16)$$

- Power of received signal is the sum of powers of terms

$$\mathbb{E}_{\mathbf{z}}\left[|\tilde{y}_{tm}|^2\right] = \mathbb{E}_{\mathbf{z}}\left[\left|\|\hat{\mathbf{h}}_{tm}\|^2 g_{tm} x_m\right|^2\right] + \mathbb{E}_{\mathbf{z}}\left[\left|\hat{\mathbf{h}}_{tm}^H \tilde{\mathbf{h}}_{tm} g_{tm} x_m\right|^2\right] + \mathbb{E}_{\mathbf{z}}\left[\left|\hat{\mathbf{h}}_{tm}^H \sum_{i \neq m} g_{ti} \mathbf{h}_{ti} x_i\right|^2\right] + \mathbb{E}_{\mathbf{z}}\left[\left|\hat{\mathbf{h}}_{tm}^H \mathbf{n}_t\right|^2\right] \tag{17}$$

- Signal power:

$$\mathbb{E}_{\mathbf{z}}\left[\left|\|\hat{\mathbf{h}}_{tm}\|^2 g_{tm} x_m\right|^2\right] = P\|\hat{\mathbf{h}}_{tm}\|^4 g_{tm}^2 \tag{18}$$

- For some $i \in \{1, 2, \cdots, M\}$

$$\mathbb{E}_{\mathbf{z}}\left[\mathbf{h}_{ti}\mathbf{h}_{ti}^{H}\right] = \mathbb{E}\left[\mathbf{h}_{ti}\mathbf{h}_{ti}^{H}\right] - \mathbb{E}\left[\mathbf{h}_{ti}\hat{\mathbf{h}}_{tm}^{H}\right]\left(\mathbb{E}\left[\hat{\mathbf{h}}_{tm}\hat{\mathbf{h}}_{tm}^{H}\right]\right)^{-1}\mathbb{E}\left[\hat{\mathbf{h}}_{tm}\mathbf{h}_{ti}^{H}\right]$$

$$= \beta_i\sigma^2\mathbf{I}_N - \frac{g_{ti}\beta_i}{g_{tm}\beta_m}\sigma_{tm}^2\sqrt{P}g_{ti}\beta_i\sigma^2\mathbf{I}_N$$

$$= \beta_i\sigma^2\frac{P\sigma^2\left(\sum_{j\neq i} g_{tj}^2\beta_j\right) + N_0}{P\sigma^2\left(\sum_j g_{tj}^2\beta_j\right) + N_0}\mathbf{I}_N =: \delta_{ti}\mathbf{I}_N \tag{19}$$

$$\Rightarrow \mathbb{E}_{\mathbf{z}}\left[\mathbf{h}_{tm}\mathbf{h}_{tm}^{H}\right] = \delta_{tm}\mathbf{I}_N \tag{20}$$

- First interference power (caused by estimation error):

$$\mathbb{E}_{\mathbf{z}}\left[\left|\hat{\mathbf{h}}_{tm}^H\tilde{\mathbf{h}}_{tm}g_{tm}x_m\right|^2\right] = Pg_{tm}^2\hat{\mathbf{h}}_{tm}^H\,\mathbb{E}_{\mathbf{z}}\left[\tilde{\mathbf{h}}_{tm}\tilde{\mathbf{h}}_{tm}^H\right]\hat{\mathbf{h}}_{tm} \tag{21}$$

$$\begin{aligned}\mathbb{E}_{\mathbf{z}}\left[\tilde{\mathbf{h}}_{tm}\tilde{\mathbf{h}}_{tm}^H\right] &= \hat{\mathbf{h}}_{tm}\hat{\mathbf{h}}_{tm}^H - \mathbb{E}_{\mathbf{z}}\left[\mathbf{h}_{tm}\right]\hat{\mathbf{h}}_{tm}^H - \hat{\mathbf{h}}_{tm}\,\mathbb{E}_{\mathbf{z}}\left[\mathbf{h}_{tm}^H\right] + \mathbb{E}_{\mathbf{z}}\left[\mathbf{h}_{tm}\mathbf{h}_{tm}^H\right] \\ &= \mathbb{E}_{\mathbf{z}}\left[\mathbf{h}_{tm}\mathbf{h}_{tm}^H\right] - \hat{\mathbf{h}}_{tm}\hat{\mathbf{h}}_{tm}^H = \delta_{tm}\mathbf{I}_N - \hat{\mathbf{h}}_{tm}\hat{\mathbf{h}}_{tm}^H \end{aligned} \tag{22}$$

$$\begin{aligned}\Rightarrow \mathbb{E}_{\mathbf{z}}\left[\left|\hat{\mathbf{h}}_{tm}^H\tilde{\mathbf{h}}_{tm}g_{tm}x_m\right|^2\right] &= Pg_{tm}^2\hat{\mathbf{h}}_{tm}^H\left[\delta_{tm}\mathbf{I}_N - \hat{\mathbf{h}}_{tm}\hat{\mathbf{h}}_{tm}^H\right]\hat{\mathbf{h}}_{tm} \\ &= Pg_{tm}^2\|\hat{\mathbf{h}}_{tm}\|^2\left[\delta_{tm} - \|\hat{\mathbf{h}}_{tm}\|^2\right] \end{aligned} \tag{23}$$

- Second interference power (caused by other users):

$$\mathbb{E}_{\mathbf{z}}\left[\left|\hat{\mathbf{h}}_{tm}^{H}\sum_{i\neq m} g_{ti}\mathbf{h}_{ti}x_i\right|^2\right] = \hat{\mathbf{h}}_{tm}^{H}\,\mathbb{E}_{\mathbf{z}}\left[\sum_{i\neq m} g_{ti}^2\mathbf{h}_{ti}\mathbf{h}_{ti}^{H}\left|x_i\right|^2\right]\hat{\mathbf{h}}_{tm}$$

$$= P\|\hat{\mathbf{h}}_{tm}\|^2\left(\sum_{i\neq m} g_{ti}^2\delta_{ti}\right) \quad (24)$$

- Noise power:

$$\mathbb{E}_{\mathbf{z}}\left[\left|\hat{\mathbf{h}}_{tm}^{H}\mathbf{n}_t\right|^2\right] = \hat{\mathbf{h}}_{tm}^{H}\,\mathbb{E}_{\mathbf{z}}\left[\mathbf{n}_t\mathbf{n}_t^{H}\right]\hat{\mathbf{h}}_{tm} = \|\hat{\mathbf{h}}_{tm}\|^2 N_0 \quad (25)$$

- Effective SINR is

$$\begin{aligned}\mathrm{SINR}_{tm} &= \frac{Pg_{tm}^2\|\hat{\mathbf{h}}_{tm}\|^4}{P\|\hat{\mathbf{h}}_{tm}\|^2\left(\sum_i g_{ti}^2\delta_{ti}\right) - Pg_{tm}^2\|\hat{\mathbf{h}}_{tm}\|^4 + \|\hat{\mathbf{h}}_{tm}\|^2 N_0} \\ &= \frac{Pg_{tm}^2\|\hat{\mathbf{h}}_{tm}\|^2}{P\left(\sum_i g_{ti}^2\delta_{ti}\right) - Pg_{tm}^2\|\hat{\mathbf{h}}_{tm}\|^2 + N_0}\end{aligned} \tag{26}$$

- Signal power of the *m*-th user acts as interference also
- Further decoding and rate/throughput analysis can be performed using this SINR
- Shannon capacity/density evolution can be applied to characterize rate/throughput

References

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Thank You!