Main Presentation
Performance Analysis of Hybrid Massive MIMO Downlink under Imperfect Channel Reciprocity

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Outline

Introduction: Hybrid Beamforming

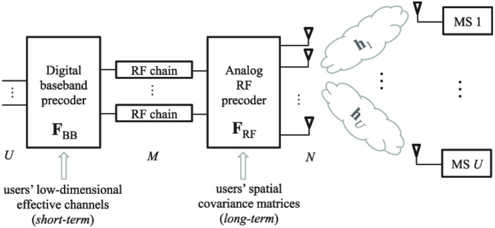

Figure: Typical System Architecture

Ⓐ Why **hybrid beamforming**?

- In full-dimensional DSP architecture, the number of RF chains equals the number of BS antennas. Therefore, it is not at all cost efficient in massive MIMO systems [1].
- In hybrid beamfoermer architecture, number of RF chains are much smaller than number of BS antennas.
- It is cost efficient as well as the complexity of DSP circuitry reduces significantly.
- Power consumption is also less.

Introduction: Hybrid Beamforming

B What are the drawbacks of hybrid beamforming?

- Reduction in achievable rate due to RF chain constraint.
- Phase quantization error of the low dimension analog beamformer [2].

C Benefits of using massive MIMO

- It has bee shown that using a large number of BS antennas can mitigate the effects of finite precision quantization in the analog stage of an hybrid beam-former.

Key Note Even in the presence of an asymptotically large number of BS antennas, the effects of finite level quantization can never be completely nullified [3].

Introduction: Reciprocity Imperfection

1. Why do we make perfect reciprocity assumption?
   - In general, massive MIMO systems operate in TDD mode.
   - Here the BS learns the channel via uplink channel training.
   - It saves the large amount of training overhead required for downlink training.
2. Why is this assumption not valid?
   - Perfect reciprocity assumption needs the uplink and downlink channels (over-the-air channel) to be identical and the perfect calibration of the RF chains both at the BS and at the users side.
   - The former requirement can be safely assumed.
   - However, the transceiver RF chains at both the BS and users' end are never identical.
   - Due to
     - (i.) imperfect clock synchronization
     - (ii.) variable cable lengths of antennas
     - (iii.) manufacturing defects of low noise amplifiers along with other hardware asymmetries
     - (iv.) imperfect calibrations.

Problem Statement and System Model

Goal

Investigate the effects of reciprocity calibrations imperfections on the rates achievable by hybrid massive MIMO systems, and discuss the relative merits of different downlink training approaches for mitigating these effects.

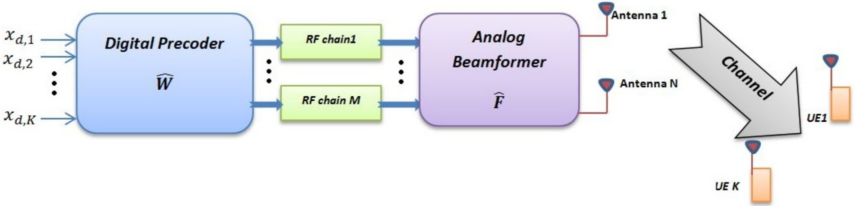

Figure: Hybrid Beamforming Architecture: We consider a single cell massive MIMO system, where the base station (BS) is equipped with M transmit RF-chains and N antennas, that are used to serve K ($K < M << N$) single antenna user nodes.

Channel Models: Perfect Reciprocity

1. Channel training & estimation:
   - We follow the round-robin channel estimation technique. In each round of uplink channel training, M out of N BS antennas will be selected.
   - Total $K \times \lceil \frac{N}{M} \rceil$ rounds of pilot training will be needed.
   - Let us consider the mth cycle ($m \in \{1, 2, \ldots, \lceil \frac{N}{M} \rceil\}$).
   - Now, the MMSE estimate of $h_{i,k}^{m}$, denoted as $\hat{h}_{i,k}^{m}$, can be expressed as

$$h_{i,k}^{m} = \sigma_k \hat{h}_{i,k}^{m} + \bar{\sigma}_k \tilde{h}_{i,k}^{m},$$

   with $\sigma_k = \sqrt{\dfrac{\beta_k P_k}{\beta_k P_k + N_o}}$ and $\bar{\sigma}_k = \sqrt{1 - \sigma_k^2}$.

2. Design of the analog beamformer ($\hat{\mathbf{F}} \in \mathbb{C}^{N \times M}$)

$$[\hat{\mathbf{F}}]_{p,q} = \frac{1}{\sqrt{N}} \exp(j\hat{\theta}_{p,q}),$$

   - $\hat{\theta}_{p,q}$ is the phase of the (p, q)th entry of $\hat{\mathbf{H}}_u$.

Channel Models:Imperfect Reciprocity

1. Channel Models:
    - The uplink channel from the kth user to the ith BS antenna: $h_{u,ik} = u_{t,k} b_{r,i} h_{i,k}$.
    - The downlink between the ith BS antenna and the kth user: $h_{d,ik} = u_{r,k} b_{t,i} h_{ik}$.
    - $u_{t,k}$, $u_{r,k}$, $b_{r,i}$, and $b_{t,i}$ are i.i.d across all antennas and all users with finite nth order moment denoted as $\eta_{u,t,n}$, $\eta_{u,r,n}$, $\eta_{b,r,n}$ and $\eta_{b,t,n}$, respectively.
2. Calibration error
    - The BS and all users perform over-the-air calibration of the transmit and receive RF chains to obtain the estimates its associated complex values gain coefficients.
    - The downlink channel can be estimated from the uplink channel estimate as,

$$\hat{h}_{d,ik} = \left(\frac{\hat{b}_{t,i}}{\hat{b}_{r,i}}\right)\left(\frac{\hat{u}_{r,k}}{\hat{u}_{t,k}}\right)\hat{h}_{u,ik}.$$

    - These calibrations are also going to be imperfect, with magnitude calibration errors ($\delta_{b,i}$ & $\delta_{u,k}$) and phase calibration errors ($\phi_{b,i}$ & $\phi_{u,k}$)
    - $\delta_{b,i} e^{j\phi_{b,i}} \triangleq \frac{b_{t,i}}{b_{r,i}} \frac{\hat{b}_{r,i}}{\hat{b}_{t,i}}$ & $\delta_{u,k} e^{j\phi_{u,k}} \triangleq \frac{u_{r,k}}{u_{t,k}} \frac{\hat{u}_{t,k}}{\hat{u}_{r,k}}$

Composite Channel Matrix

Overall downlink channel in terms of uplink channel

$$\mathbf{H}_d = \mathbf{D}_b \hat{\mathbf{H}}_d \mathbf{D}_u \mathbf{A} + \tilde{\mathbf{H}}_d \bar{\mathbf{A}},$$

where, $\mathbf{D}_b \triangleq \text{diag}[\delta_{b,1} e^{j\phi_{b,1}}, \ldots, \delta_{b,N} e^{j\phi_{b,N}}]^T$, $\mathbf{D}_u \triangleq \text{diag}[\delta_{u,1} e^{j\phi_{u,1}}, \ldots, \delta_{u,K} e^{j\phi_{u,K}}]^T$ $\mathbf{A} = \text{diag}[\sigma_1, \ldots, \sigma_K]$ and $\tilde{h}_{d,ik} \sim \mathcal{CN}(0, \eta_{b,t,2}\eta_{u,r,2})$ being the i.i.d entries of $\tilde{\mathbf{H}}_d$.

1. Digital Precoder Design.

$$\hat{\mathbf{W}}_{MRT} \triangleq \hat{\mathbf{G}}^H = \hat{\mathbf{F}}^H \hat{\mathbf{H}}_d^H,$$

- $\hat{\mathbf{G}} = \hat{\mathbf{H}}_d \hat{\mathbf{F}} \in \mathbb{C}^{K \times M}$.
- Depending on Case I and Case II, the channel matrix equations will change.
- For imperfect reciprocity case, the analog beamformer matrix will be designed based on the composite downlink channel matrix.

Achievable Downlink Rate with MRT under Perfect Channel Reciprocity

- Processed signal after MRT ($\mathbf{w}_k = \hat{\mathbf{F}}^H \hat{\mathbf{h}}_{d,k}^H$)

$$\begin{aligned} z_{d,k} = & \sqrt{P_{t,k}\beta_k}\sigma_k \hat{\mathbf{h}}_{d,k}\hat{\mathbf{F}}\hat{\mathbf{F}}^H\hat{\mathbf{h}}_{d,k}^H x_{d,k} + \sqrt{P_{t,k}\beta_k}\bar{\sigma}_k \tilde{\mathbf{h}}_{d,k}\hat{\mathbf{F}}\hat{\mathbf{F}}^H\hat{\mathbf{h}}_{d,k}^H x_{d,k} \\ & + \sum_{i=1,i\neq k}^{K} \sqrt{P_{t,i}\beta_k}\mathbf{h}_{d,k}\hat{\mathbf{F}}\hat{\mathbf{F}}^H\hat{\mathbf{h}}_{d,i}^H x_{d,i} + n_k. \end{aligned}$$

- DE of the downlink SINR

$$\gamma_{d,k_{\text{MRT}}} - \frac{P_{t,k}\beta_k\sigma_k^2 M^2}{P_{t,k}\beta_k\bar{\sigma}_k^2 M + \sum_{i=1,i\neq k}^{K} P_{t,i}\beta_k M + \sigma_n^2} \xrightarrow{\text{a.s.}} 0,$$

- Achievable downlink rate

$$R_{\text{MRT}} = \frac{T_c - \tau_p}{T_c}\sum_{k=1}^{K}\log_2(1+\gamma_{d,k_{\text{MRT}}}),$$

where T_c and τ_p are total coherence interval of the channel and training period length, respectively.

Achievable Downlink Rate with MRT under Imperfect Channel Reciprocity

- Substitute $\mathbf{w}_k = \hat{\mathbf{F}}_{\text{im}}^H \hat{\mathbf{h}}_{d,k}^H$
- The received signal by the kth user

$$\begin{aligned} z_{d,k}[n] =& \sigma_k d_{u,k} \sqrt{\beta_k P_{d,s,k}} \hat{\mathbf{h}}_{d,k} \mathbf{D}_b \hat{\mathbf{F}}_{\text{im}} \hat{\mathbf{F}}_{\text{im}}^H \hat{\mathbf{h}}_{d,k}^H x_{d,k}[n] \\ &+ \bar{\sigma}_k \sqrt{\beta_k P_{d,s,k}} \tilde{\mathbf{h}}_{d,k} \hat{\mathbf{F}}_{\text{im}} \hat{\mathbf{F}}_{\text{im}}^H \hat{\mathbf{h}}_{d,k}^H x_{d,k}[n] \\ &+ \sum_{i=1, i \neq k}^{K} \sqrt{\beta_k P_{d,s,i}} \hat{\mathbf{h}}_{d,i} \hat{\mathbf{F}}_{\text{im}} \hat{\mathbf{F}}_{\text{im}}^H \hat{\mathbf{h}}_{d,k}^H x_{d,k}[n] + \sqrt{N_o} n_k. \end{aligned}$$

- Following DE convergences can be shown
  - $$\left| \hat{\mathbf{h}}_{d,k} \mathbf{D}_b \hat{\mathbf{F}}_{\text{im}} \hat{\mathbf{F}}_{\text{im}}^H \hat{\mathbf{h}}_{d,k}^H \right|^2 - M^2 (E[\delta \cos \phi_b])^2 \eta_{b,t,2}^2 \eta_{u,r,2}^2 \xrightarrow{a.s.} 0.$$
  - $$\left| \tilde{\mathbf{h}}_{d,k} \hat{\mathbf{F}}_{\text{im}} \hat{\mathbf{F}}_{\text{im}}^H \hat{\mathbf{h}}_{d,k}^H \right|^2 - M \eta_{b,t,2}^2 \eta_{u,r,2}^2 \xrightarrow{a.s.} 0.$$

Achievable Downlink Rate with MRT under Imperfect Channel Reciprocity

- Therefore, the SINR of the k user can be written as

$$\gamma_{d,k_{\mathrm{MRT}}} - \frac{P_{d,s,k}\beta_k\sigma_k^2 d_{u,k}^2 M^2 (E[\delta\cos\phi_b])^2 \eta_{b,t,2}^2\eta_{u,r,2}^2}{P_{d,s,k}\beta_k\bar{\sigma}_k^2 M\eta_{b,t,2}^2\eta_{u,r,2}^2 + \sum_{i=1,i\neq k}^{K} P_{d,s,i}\beta_k M\eta_{b,t,2}^2\eta_{u,r,2}^2 + \sigma_n^2} \xrightarrow{\text{a.s.}} 0.$$

- Under the assumption of perfect calibration, i.e., $\delta_{b,i} = 1 = \delta_{u,i}$ and $\phi_b = 0 = \phi_u$

$$\gamma_{d,k_{\mathrm{MRT}}} - \frac{P_{d,s,k}\beta_k\sigma_k^2 M^2 \eta_{b,t,2}^2\eta_{u,r,2}^2}{P_{d,s,k}\beta_k\bar{\sigma}_k^2 M\eta_{b,t,2}^2\eta_{u,r,2}^2 + \sum_{i=1,i\neq k}^{K} P_{d,s,i}\beta_k M\eta_{b,t,2}^2\eta_{u,r,2}^2 + \sigma_n^2} \xrightarrow{\text{a.s.}} 0$$

Limited Downlink Training Mode

- Since the SINR saturation occurs due to a mismatch in the gain and phase of the effective downlink channel and its expected value, the BS can transmit downlink pilots over the effective channels to let the UEs obtain their estimates.
- the SINR achievable at the kth user

$$\gamma_k^D - \frac{M P_{d,s,k}\beta_k a_k^2 c_k^2 d_{u,k}^2 E^2[\delta \cos\phi_b]}{P_{d,s,k}\beta_k(\bar{a}_k^2 + M\bar{c}_k^2 a_k^2 d_{u,k}^2 E^2[\delta\cos\phi_b]) + \sum_{i=1,i\neq k}^{K} P_{d,s,i}\beta_k + N_0} \xrightarrow{\text{a.s.}} 0.$$

- The achievable rate in this case can be given as,

$$R_k^D = \frac{T - K\left(\lceil \frac{N}{M} \rceil + 1\right)}{T}\log_2(1+\gamma_k^D).$$

- We note that the SINR still saturates as a function of the number of RF chains. However, the denominator term now depends on the coefficient $\bar{c}_k$, that can be made arbitrarily small by scaling up downlink pilot power.

$[\bar{c}_k = \sqrt{\frac{N_0}{M a_k^2 d_{u,k}^2 \beta_k P_{d,p,k}(1-\delta_b\cos\phi_b)^2 + N_0}}.]$

Full Downlink Training

- Alternatively, the BS can use full downlink training, involving transmission of downlink pilots from all the N BS antennas over N training slots.
- These pilots are used by the UEs to obtain estimates of the downlink channel coefficients, that are communicated to the BS over a Q bit rate limited channel such that $h_{d,ik}$, can be expressed in terms of the channel estimate $\hat{h}_{u,ik}$ as

$$h_{d,ik} = e_k \hat{h}_{d,ik} + \bar{e}_k \tilde{h}_{d,ik} \tag{1}$$

  $\tilde{h}_{d,ik}$ representing the ZMCSCG distributed error due to estimation inaccuracies and quantization noise, such that $E[\tilde{h}_{d,ik}\hat{h}^*_{d,ik}] = 0$.
- Also, it is easy to show that $\bar{e}_k = \sqrt{(1-2^{-Q})}\sqrt{\frac{N_0}{\beta_k \mathcal{E}_{p,k}+N_0}} + 2^{-\frac{Q}{2}}$.
- Again the BS uses the available estimates to obtain the analog and digital beamforming matrices, and transmit the data symbols to the UEs.

Full Downlink Training

- Again using DE analysis, it is easy to show that

$$e_k \sqrt{\beta_k P_{d,s,k}} \hat{\mathbf{h}}_{d,k}^T \mathbf{D}_b \hat{\mathbf{F}} \hat{\mathbf{F}}^H \hat{\mathbf{h}}_{d,k}^* - M e_k \sqrt{\beta_k P_{d,s,k}} \xrightarrow{\text{a.s.}} 0, \tag{2}$$

  that is the value available at the UEs.

- Consequently, the achievable SINR for the kth UE takes the form

$$\gamma_k^F - \frac{M P_{d,s,k} \beta_k e_k^2}{e_k P_{d,s,k} \beta_k + \sum_{i=1, i \neq k}^{K} P_{d,s,i} \beta_k + N_0} \xrightarrow{\text{a.s.}} 0. \tag{3}$$

- It can be observed that in this case, while the numerator term grows linearly with the number of RF chains at the BS, no such behavior is shown by the noise and interference terms in the denominator.

- High massive MIMO array gain. achieved at the cost of full downlink training resulting in longer training durations and shorter data transmission durations.

Simulation Results

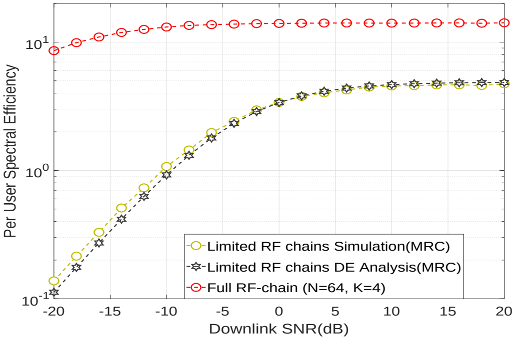

Figure: Monte-Carlo simulation for per user downlink rate with MRT processing under perfect channel reciprocity

Observations

- Monte-Carlo simulation closely follows the derived expression.
- We get the upper bound of the downlink rate considering full RF chain.

Simulation Results

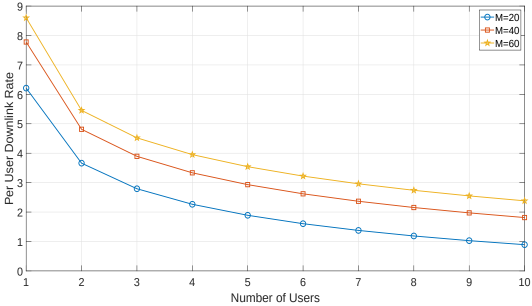

Figure: Average Downlink Rate versus Number of Users

Observations

- As the number of user increases, the downlink rate also decreases.
- System performance can be improved by increasing number of RF-chains, given a fixed number of users.

Simulation Results

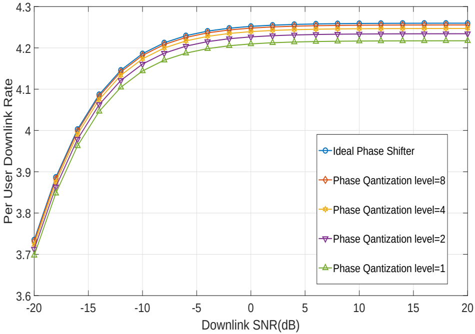

Figure: Average Downlink Rate versus Downlink SNR under different level of quantization at the analog beamformer.

Observations

- Ideal phase shifter achieves optimal performance.
- Increasing the number of quantization steps, the performance can be significantly improved.

Simulation Results

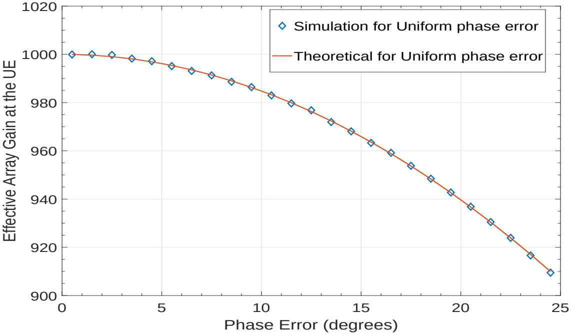

Figure: Effective array gain at the UE versus phase error(Number of BS antenna is assumed to be 512).

Observations

- It can be observed that the simulation closely follow theoretical plot.
- As the range of phase error distributions is getting dynamic, the effective array gain at the UE also deteriorates.

Simulation Results

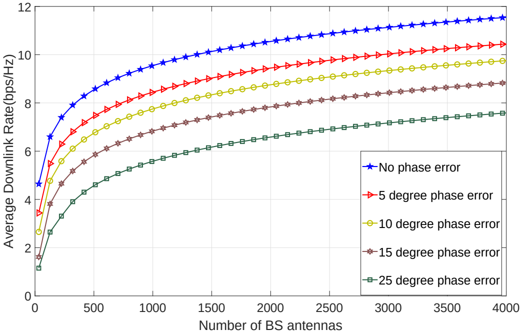

Figure: Achievable Downlink Rates under different degrees of calibration errors under MRT(Number of BS antenna is assumed to be 512, the number of user and the number of RF chains are both assumed to be 10. The pilot SNR and downlink SNR are assumed to be 10 dB).

Observations

- It can be observed that the increase in phase calibration error results in significant decrease in the achievable rate.

Summary of Work & Conclusions

Hybrid beamforming under perfect channel reciprocity

- RZF precoding delivers better performance than MRT precoding.
- Investigated the effect of phase quantization noise of the analog beamformer
- The achievable downlink rate increases as we increase the quantization level till the optimal value.
- Downlink rate increases with the increase in number of RF-chains.

Downlink rates under imperfect channel reciprocity

- Performance evaluation under MRT precoding
- Downlink rate decreases with the increase in phase calibration error
- Increase in BS antenna increases the downlink rate per user

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Thank You