Multiple Transmitter Localization and Communication Footprint Identification using Sparse Reconstruction Techniques

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27th April 2011

Outline

- Introduction to Transmitter Localization
- Problem Definition
- Proposed Schemes for Transmitter Localization
- Simulation Results

Transmitter Localization and Communication Footprint Construction

- Transmitter Localization: Locating transmitters in a given area
- Communication Footprint: Area around the transmitter where its signal can be received with good fidelity
- Applications
    - Spectrum Enforcement: Identifying pirate radios
    - Cognitive Radio Networks: White space detection

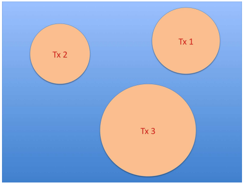
Past Works

- Approach: Using power measurements from sensors deployed around the transmitter
- [Nelson '06, '09] - Minimize the difference between true received power and estimated received power at sensors
- [Nasif '09] - Minimize net MSE in power estimate and location estimate at sensors
- Assumes number of transmitters and their transmit powers to be known

Past Works - Using Sparsity

- [Cevher '08, Feng '09] - Considers spatial sparsity of targets, but need RSS based dictionary
- [Bazerque '10] - Cooperative approach, considers sparsity in narrow-band nature of transmissions and spatially sparse active transmitters
- All these methods depend on RSS measurements being sent to a central node
- Disadvantage: Dense deployment of sensors will lead to delay in localization

Our Approach

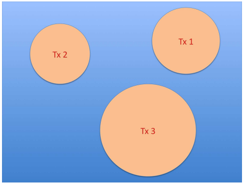


- Deploy a number of low-cost sensors over the geographical area
- Sensors detect presence/absence of primary at their locations and convey 1-bit information to a Fusion Center (FC) over a control channel
- Construct the spectrum usage map at FC by clustering the alarming sensors

How is Our Approach Different From Past Work?

- Number of transmitters and transmit powers are unknown
  - We limit the maximum number of transmitters and transmit power range
- 1-bit transmissions to reduce delay
  - The 1-bit transmissions could be repeated some number of times
  - Supports a dense deployment of sensors
- Communication footprint construction in addition to transmitter localization

Round Robin Scheme



- Query each sensor in round-robin manner
- Disadvantage: Delay in map construction is proportional to number of sensors
- Footprint map is a sparse image, hence Compressive Sensing can be used to reduce delay

Brief Introduction to Compressive Sensing (CS)

- Consider a set of equations

$$\underset{M}{y} = \underset{(M \times L)}{\Phi} \; \underset{L}{s} \qquad M < L$$

- In general, there are infinitely many solutions
- CS theory: If s is sparse and Φ satisfies Restricted Isometry Property (RIP), then s can be uniquely recovered [Donoho '06] [Candes '05, '06]
- RIP: Every set of s columns of Φ are nearly orthonormal
- Gaussian and Bernoulli ensemble satisfy RIP

Recovery algorithms for CS

- ℓ_1 minimization
  - $\min_{\hat{s}} \|\hat{s}\|_1$ s.t. $y = \Phi \hat{s}$
  - $M > \mathcal{O}(K \log(L/K))$ [Donoho '06]
- OMP
  - Iterative algorithm
  - Finds Best K columns of Φ which have maximum correlation with y
  - $M > \mathcal{O}(K \log(L))$ [Tropp '07]

Problem Definition

- T transmitters are located at $l_i = (x_i, y_i)$, with radius of circular radio footprint of r_i, for $i = 1, .., T$
- L sensors deployed uniformly at random locations in the geographical area convey their 1-bit information to the FC
- Objective: Estimate T, l_i and r_i and construct the circular footprints at the FC with minimum delay
- Performance metrics:
  - Relative error in area of reconstructed footprint to original footprint (Hamming distance)
  - MSE in transmitter localization

Contributions

- Proposing a Sensors to FC Communication protocol which fits into CS framework
- Proposing two schemes for estimating l_i and r_i
- Proposing a method for identifying the number of transmitters T
- Design of the number of sensors to be deployed and their power thresholds

Sensor to FC Communication Protocol

- Alarming sensors synchronously transmit a ‘1’ to the FC M times
- Each time sensors transmit, they pre-rotate the bit by a pseudo-random binary phase shift $\{0,\ \pi\}$
- Fusion center knows these binary phase shifts
- Channel from sensors to FC is assumed to be constant for M transmissions

Mathematical Model of Sensors to FC Communication

Measurement vector y at FC

$$\underset{M}{y} = \underset{(M \times L)}{X} \underset{L}{h} + w$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \frac{1}{\sqrt{M}} \begin{bmatrix} x_1 e^{j\theta_{11}} & x_2 e^{j\theta_{12}} & \dots & x_L e^{j\theta_{1L}} \\ x_1 e^{j\theta_{21}} & x_2 e^{j\theta_{22}} & & x_L e^{j\theta_{2L}} \\ \vdots & & \ddots & \vdots \\ x_1 e^{j\theta_{M1}} & x_2 e^{j\theta_{M2}} & \dots & x_L e^{j\theta_{ML}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}$$

where
$w_i \sim \mathcal{CN}(0, \sigma^2)$ (receiver noise),
$x_j \in \{0,\ 1\}$ is decision at j^{th} sensor,
h_j is channel from j^{th} sensor to FC, and

$$\theta_{ij} = \begin{cases} \pi & \text{w. p. } 0.5 \\ 0 & \text{w. p. } 0.5 \end{cases}$$

Equivalence to CS Measurement Equation

$$y = \frac{1}{\sqrt{M}} \begin{bmatrix} +1 & -1 & \dots & +1 \\ -1 & +1 & \dots & +1 \\ & & \ddots & \\ +1 & +1 & \dots & -1 \end{bmatrix} \begin{bmatrix} x_1 h_1 \\ x_2 h_2 \\ \vdots \\ x_L h_L \end{bmatrix} + w$$

CS measurement equation

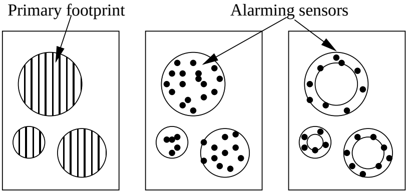
$$\underset{M}{y} = \underset{(M \times L)}{\Phi} \underset{L}{s} + w$$

- Φ is a Bernoulli ensemble
- s is sparse because $[x_1 x_2 \dots x_L]$ is sparse

Contributions

- Sensors to FC Communication protocol to fit into CS framework
- Two schemes for estimating locations l_i and radius r_i at FC
- Method for identifying number of transmitters T
- Design of the number of sensors to be deployed and their power thresholds

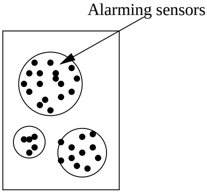
Schemes for radio map reconstruction



(a) Footprint (b) Scheme 1 (c) Scheme 2

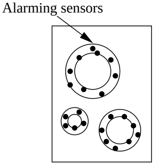
- Proposed schemes based on alarming sensors
    - Scheme 1 - Sensors that are within circular boundaries around transmitters
    - Scheme 2 - Sensors that are within annuli around transmitters

Scheme 1



- K-means algorithm to cluster the alarming sensors
- Location Estimate - K-means centroid
- Radius Estimate - Distance of the farthest sensor to cluster center

Scheme 2



- Sensors in annulus are alarming sensors
- K-means algorithm to cluster the alarming sensors
- Trilateration
    - Associate a representative power to all sensors and draw power contours
    - Location Estimate - Average of intersections obtained by trilateration
- Circular Regression - Steepest descent method
- Radius Estimate - Distance of the farthest sensor to cluster center

Contributions

- Sensors to FC Communication Protocol
- Two schemes for estimating locations l_i and radius r_i at FC
- Method for identifying number of transmitters T
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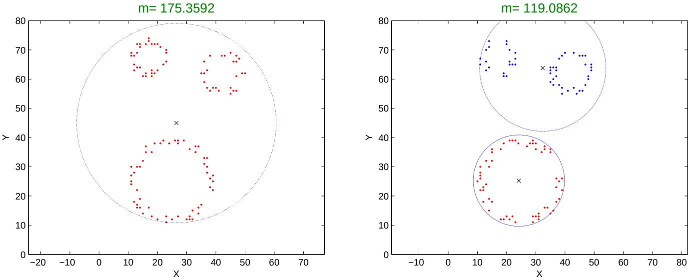
Identifying Number of Transmitters T

- K-means clustering needs number of clusters, K as input
- Calinski and Harbasz (CH) Index [CH '74]
  - Depends on inter cluster and intra cluster distances
- Hartigan Method [Hartigan '75]
  - Depends on intra cluster distances
- Sensitive to size of clusters and distance between clusters

Proposed Metric

- Clusters are annulus shaped or circular shaped
- Fit K circles to clusters
- Average of the deviation of points from the circular fit
- Metric, $m = \frac{1}{N_i} \sum_i \left\{ \sum_{j=1}^{N_i} (\sqrt{(x_{ji} - a_i)^2 + (y_{ji} - b_i)^2} - r_i)^2 \right\}$ (a_i, b_i), and r_i, are center and radius of i^{th} circular fit, N_i - Number of points in i^{th} cluster.
- First minimum of the metric identifies true number of clusters

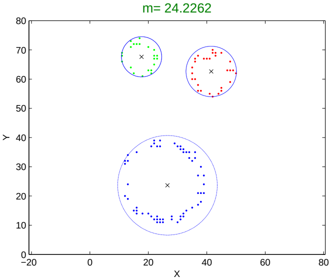
Example: Estimating Number of Transmitters



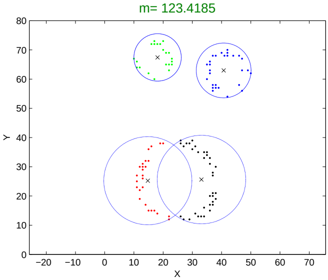
(a) K = 1

(b) K = 2

Example: Estimating Number of Transmitters



(c) K = 3



(d) K = 4

Algorithm for Estimating the Number of Transmitters

Step 1 Initialize $K = 1$ transmitter.

Step 2 Perform K-means clustering. Fit K circles.

Step 3 Compute the metric
$m = \frac{1}{N_i} \sum_i \{ \sum_{j=1}^{N_i} (\sqrt{(x_{ji} - a_i)^2 + (y_{ji} - b_i)^2} - r_i)^2 \}$
(a_i, b_i), and r_i, are center and radius of i^{th} circular fit.

Step 4 Increment K. Repeat *Step* 2 and *Step* 3 until the first minimum of the metric (m) is obtained.

Step 5 Output the K that corresponds to the first minimum.

Contributions

- Sensors to FC Communication Protocol
- Two schemes for estimating locations l_i and radius r_i at FC
- Method for identifying number of transmitters T
- Design of the number of sensors to be deployed and their power thresholds

Number of Sensors to be Deployed - Scheme 1

- Assumptions:
  - T_{max} - Maximum number of transmitters in the area
  - P_{max} and P_{min} - Maximum and minimum powers with which a transmitter can operate
- K_{max} - Maximum number of alarming sensors when T_{max} transmitters operate at P_{max}
- $z \triangleq \frac{K_{max}}{L}$, sparsity constraint - $0 < z \leq \kappa$
- $Pr\{\text{missing a transmitter of } P_{min} \text{ power }\} < p_m$
- To minimize number of transmissions $K_{max} \log(L/K_{max})$

Number of Sensors to be Deployed - Scheme 1

- Optimization problem:

$$\min_{L,z} Lz \log\left(1/z\right)$$
$$\text{subject to} \;\; 0 < z \leq \kappa, \text{and } L \geq -\frac{a}{\log(1-bz)}$$

where $a = \log(1/p_m)$, $b = \frac{(P_{min}/P_{max})^{2/\eta}}{T_{max}}$ and $z \triangleq \frac{K_{max}}{L}$

- $L_{opt} = -\frac{a}{\log(1-b\kappa)}$ and $z_{opt} = \kappa$

Number of Sensors to be Deployed - Scheme 2

- Objective: To find number of sensors L, threshold of sensors τ_i and τ_o
- Relative width of the annulus is fixed to δ
- Optimization problem:

$$\min_{L,z} Lz \log\left(1/z\right)$$

$$\text{subject to } \left(\frac{P_{max}}{P_{min}}\right)^{2/\eta}\left(1-p_m^{1/L}\right)T_{max} \leq z \leq \kappa,$$

- $L_{opt} = \frac{\log p_m}{\log\left(1-\frac{\kappa}{T_{max}}\left(\frac{P_{min}}{P_{max}}\right)^{2/\eta}\right)}$, $z_{opt} = \kappa$, $\rho \triangleq \frac{\tau_i}{\tau_o} = (1+\delta)^{\eta}$

MSE in Localization - Scheme 1

(X_s, Y_s) - location of transmitter, (X_i, Y_i) - location of i^{th} sensor
d is radius of footprint in an area of A
m - number of sensors in annulus when L sensors are deployed
Estimate of centroid - $(\sum_{i=1}^{m} X_i/m, \sum_{i=1}^{m} Y_i/m)$

$$MSE = \mathbb{E}_m \left\{ \frac{1}{m^2} \mathbb{E}_{X_i, Y_i} \left\{ \sum_{i=1}^{m} (X_s - X_i)^2 + \sum_{i=1}^{m} (Y_s - Y_i)^2 \right\} \right\}$$

$X_s - X_i = r\cos\theta,\ Y_s - Y_i = r\sin\theta$, where $\theta \sim \mathcal{U}(0, 2\pi)$, $r = \sqrt{z}d$ where $z \sim \mathcal{U}[0, 1]$

$$\begin{aligned} MSE &= \mathbb{E}_m \left\{ \frac{1}{m^2} \mathbb{E}_r \left\{ mr^2 \right\} \right\}, \\ &= \mathbb{E}_m \left\{ \frac{1}{m} \right\} \frac{d^2}{2}. \\ &\approx \frac{A}{2\pi L} \ (\text{using } \mathbb{E}_m \{1/m\} \approx 1/\mathbb{E}_m \{m\}). \end{aligned}$$

Simulation Results

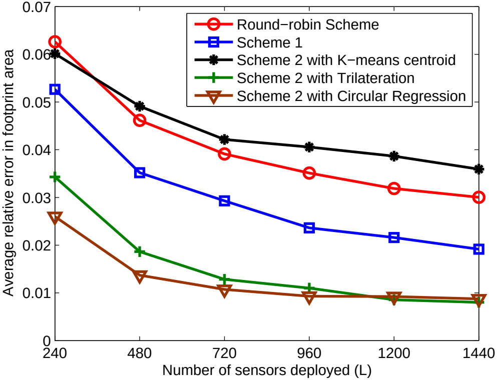
- Deployment of L sensors in a rectangular geographical area with $N = 4800$ grid locations and $T = 3$ transmitters
- Footprints cover 23% of the total area
- Performance measure: Relative error in footprint area (Hamming distance)

Table: Footprint Identification Performance of Different Schemes

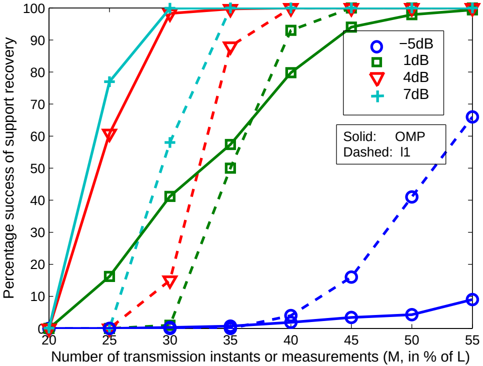
Schemes	L	$\bar{S}$	M	**Relative error in area**
Scheme 1	960	214	558	0.0236
Scheme 1	480	120	336	0.0352
Scheme 2	**960**	**122**	**336**	**0.0110**
$Round - robin$	336	-	336	0.0383
$Round - robin$	558	-	558	0.0302
$Round - robin$	960	-	960	0.0220

Relative Error in Area Vs Number of Sensors Deployed, L

- $Average\ Receive\ SNR = 4dB\ per\ sensor$



Success in Localization Vs Number of Transmissions, M



Comparison of Power Budget: Numerical Example

- Consider $L = 960$ sensors, Non-coherent On-Off keying communication protocol
- Round-robin Scheme: A Receive SNR of $14dB$ is required to ensure prob. of bit error of 10^{-3}
- This requires $14dB \times 120$, i.e. $35dB$ of receive SNR
- Scheme 2 requires $4dB \times 120$, i.e. $25dB$ of receive SNR

Identification of Number of Clusters

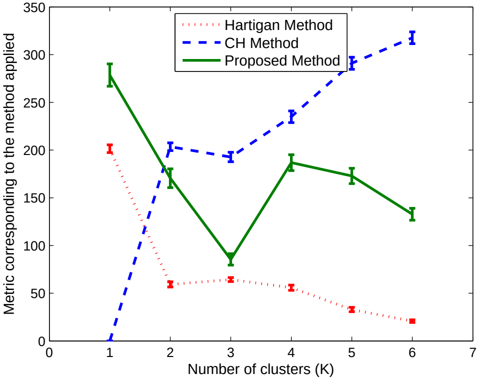
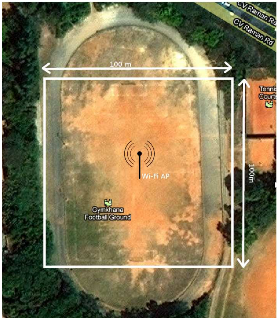


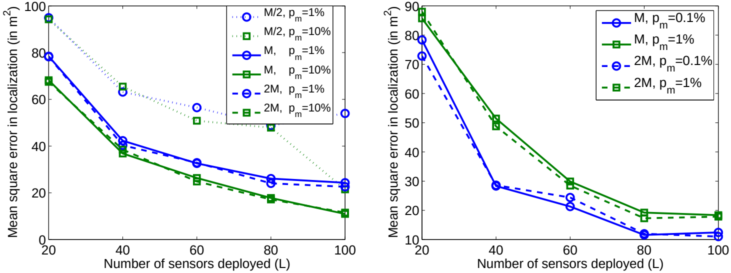
Figure: Comparison of the proposed method with CH and Hartigan methods

Evaluation of Proposed Schemes with Experimental Data

- Wi-Fi AP as transmitter with transmit power: $24dBm$, Frequency channel: 11^{th} channel of $2.4GHz$ band
- Laptop with Wi-Fi card was used as receiver
- Power measurements at randomly chosen 250 locations in $(100m \times 100m)$ area



MSE Vs Number of Sensors Deployed



(a) Scheme 1 (b) Scheme 2

Figure: MSE Vs L

Summary

- Alarming sensors employ M consecutive 1 bit transmissions with pseudo random binary phase shifts
- At the FC, these phase-shifts act as elements of a CS measurement matrix which enables the use of sparse recovery methods
- Proposed an iterative method to estimate number of transmitters
- K-means algorithm is used to cluster alarming sensors and thereby locate transmitters
- Scheme 2 performs the best in terms of footprint area error performance among the methods considered

Future Work

- Shadowing and Rayleigh fading between the transmitter and sensor is not considered in current setup
- Standard deviation of shadowing can range from 4 to 12, that makes circular boundaries to be highly distorted
- Need to modify the schemes to handle these

Publications

- Venugopalakrishna Y. R., Chandra R. Murthy, D. Narayana Dutt, and Sneha Latha Kottpalli, "Multiple Transmitter Localization and Communication Footprint Identification Using Sparse Reconstruction Techniques", accepted for presentation at ICC 2011, Kyoto, Japan.

Thank You

K-means clustering of alarmed sensors

- K-means algorithm - unsupervised technique for clustering data
- Algorithm for finding K clusters
    - Step 1 - Initialisation - Randomly picks K centroids, and forms K clusters using the data points that are close to each of these centroids
    - Step 2 - Find new centroids corresponding to each of these clusters and clusters the data again
    - Repeat Step 2 till centroids converge