ORDINARY KRIGING

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Contents

INTRODUCTION

- **Kriging**: method of interpolation that incorporates measures of error and uncertainty while determining estimates
- **Ordinary Kriging:** simplest form of kriging used when the mean and variance of the random process assumed is unknown

Best Linear Unbiased Estimate (B.L.U.E)

- Ordinary Kriging (OK) is:
  - **Best**: minimizes variance of errors in estimates
  - **Linear**: estimates are weighted linear combination of data
  - **Unbiased**: mean residual or error equal to zero

Few definitions

- Estimate: $\hat{u} = \sum_{j=1}^{n} w_j u_j$
- Error of i^{th} estimate: $r_i = \hat{u}_i - u_i$
- Average error of k estimates: $m_r = \frac{\sum_{i=1}^{k} r_i}{k}$
- Variance of error: $\sigma^2{}_R = \frac{\sum_{i=1}^{k}(r_i - mr)^2}{k}$

Random function model

- Values of stationary random function assumed at all sample points $\{U(x_1), U(x_2), \ldots, U(x_n)\}$

- Value at the point of estimation $U(x_0)$

- Estimate at location x_0: $\check{U}(x_0) = \sum_{j=1}^{n} w_j U(xi)$

- Error in estimate: $R(x_0) = \check{U}(x_0) - U(x_0)$

$$= [\sum_{j=1}^{n} w_j U(xi)] - U(x_0)$$

UNBIASEDNESS

- Expectation of error: $E\{R(x_0)\} = [\sum_{j=1}^{n} w_j \, E\{U(x_i)\}] - E\{U(x_0)\}$

- Stationary random process : $E\{U(x_i)\} = E\{U(x_0)\} = E\{U\}$

  $E\{R(x_0)\} = E\{U\}(\sum_{j=1}^{n} w_j \; - 1)$

- Unbiasedness: $E\{R(x_0)\} = 0$

  $(\sum_{j=1}^{n} w_j \; - 1) = 0$

$$\sum_{j=1}^{n} w_j = 1$$

Minimizing error variance

- Error in estimate: $\mathrm{R}(x_0) = \breve{\mathrm{U}}(\mathrm{x}_0) - U(x_0)$
- $Var\{R(x_0)\} = Cov\{\mathrm{U}(x_0)U(x_0)\} - 2Cov\{\breve{\mathrm{U}}(x_0)U(x_0)\} + Cov\{\breve{\mathrm{U}}(x_0)\breve{\mathrm{U}}(x_0)\}$

$$= \sigma^2 - 2\sum_{i=1}^{n} w_i\, C_{i0} + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j C_{ij}$$

$$C_{ij} = Cov\{U(x_i)U(x_j)\}$$

LAGRANGE MULTIPLIER

- Minimize $Var\{R(x_0)\}$ subject to $\sum_{j=1}^{n} w_j = 1$

- $Var\{R(x_0)\} \quad = \sigma^2 - 2\sum_{i=1}^{n} w_i\, C_{i0} + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j C_{ij} + 2\mu(\sum_{j=1}^{n} w - 1)$

$$\frac{\partial}{\partial w_i}(Var\{R(x_0)\}) = 2\sum_{j=1}^{n} w_j\, C_{ij} - 2Ci_0 + 2\mu = 0$$

$$\sum_{j=1}^{n} w_j\, C_{ij} + \mu = C_{i0}$$

for $i = 1,2,\ldots,n$

- This expression in matrix form:

$$\mathbf{Cw} = \mathbf{D}$$

$$\begin{bmatrix} C_{11} & \dots & C_{1n} & 1 \\ \vdots & \dots & \vdots & \vdots \\ C_{n1} & \dots & C_{nn} & 1 \\ 1 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix} = \begin{bmatrix} C_{10} \\ \vdots \\ C_{n0} \\ 1 \end{bmatrix}$$

- Solving for weights, we have $\mathbf{w} = \mathbf{C}^{-1}\mathbf{D}$

Covariance and Variogram Models

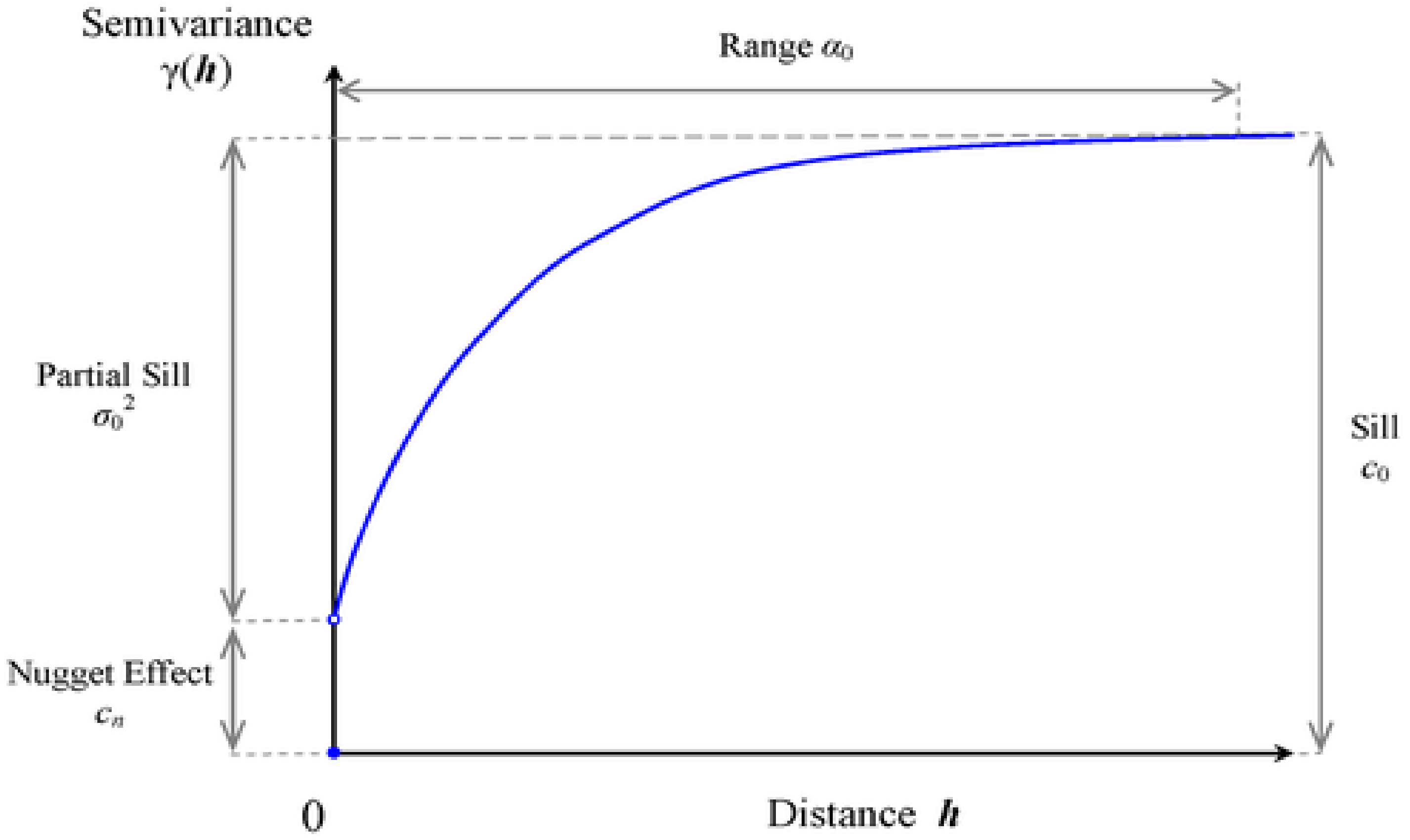
- Variogram $\gamma_{ij(h)} = \sigma^2 - \text{Cij}(h)$ where $h = ||x_i - xj||$
- In terms of Variogram, the OK system reduces to :

$$\sum_{j=1}^{n} w_j \Upsilon_{ij} - \mu = \Upsilon_{i0}$$

$$\sum_{j=1}^{n} w_j = 1$$

for i=1,2,…,n

A VARIOGRAM: SPHERICAL MODEL



KRIGING ALGORTIHM

for interpolation

Why ordinary kriging

- Shadowing :
  - Log normal distributed random variable
  - Spatially correlated
  - Mean unknown (values from dataset)

N=580, points to be interpolated=18032

↓

Construct variogram, fit the theoretical variogram

↓

Construct matrices, solve for weights

↓

Estimate values at 18032 points

↓

Write to text files

↓

Read them in the simulator

CONSTRUCTING VARIOGRAM

- An empirical variogram is constructed for the given data

$$\gamma(h) = \frac{1}{2\big(N(h)\big)} \sum_{N(h)} \big(U(xi) - U(xj)\big)^2$$

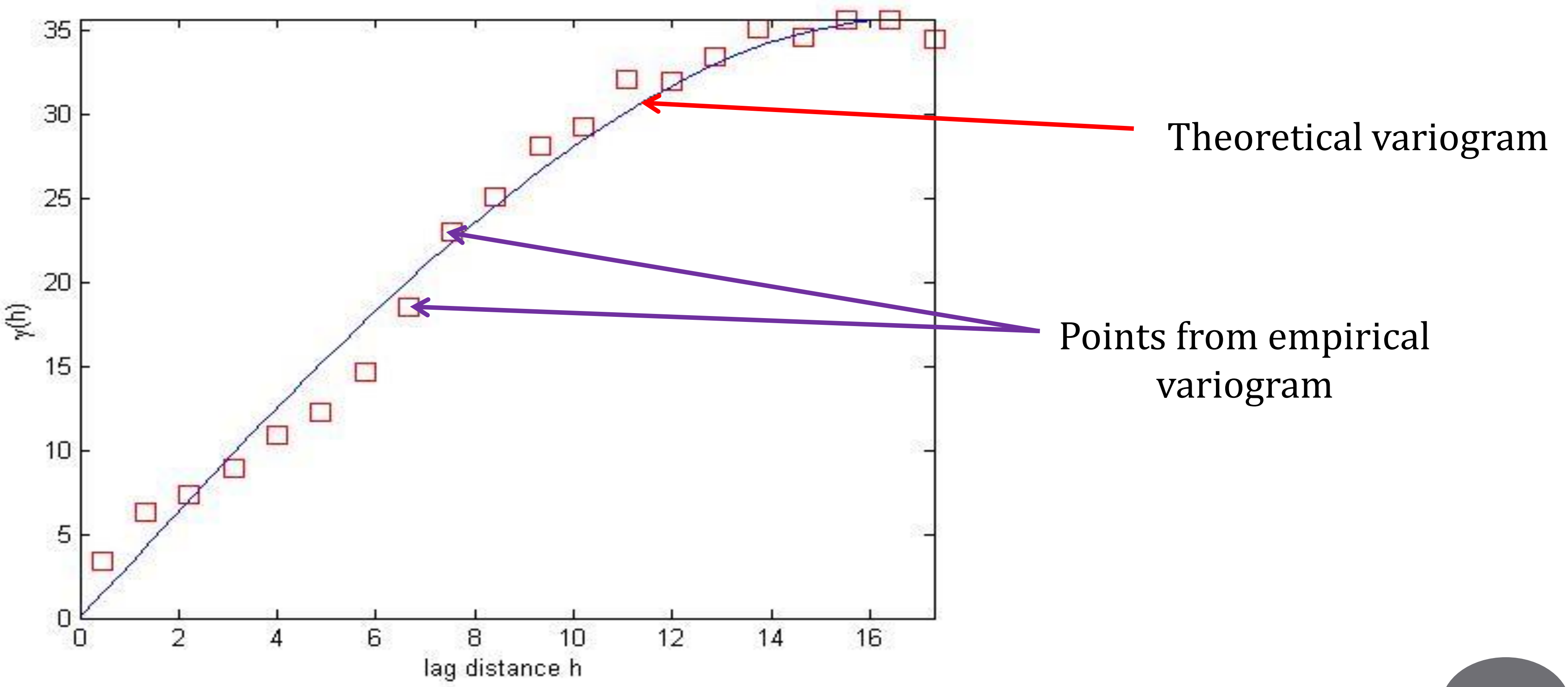
where $N(h)$ *:* set of pairs (i, j) such that $h - \Delta \leq \big||x_i - xj|\big| \leq h + \Delta$

- A model variogram (spherical): fit to the empirical one

$$\gamma(h)=\begin{cases} C\left(\frac{3}{2}\frac{h}{a}-\frac{1}{2}\frac{h^3}{a^3}\right) & h\le a \\ C & h>a \end{cases}$$

where *C*= *sill*, *a*=*range:* determined by least square fit

VARIOGRAM



35
30
25
20
15
10
5
0
0
2
4
6
8
10
12
14
16
lag distance h
γ(h)
Theoretical variogram
Points from empirical variogram

SOLVING FOR WEIGHTS:

$$\Gamma \mathbf{w} = \mathbf{D}$$

$$\begin{bmatrix} 0 & \gamma_{12} & \cdots & \gamma_{1n} & 1 \\ \gamma_{21} & 0 & \cdots & \gamma_{2n} & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ -\mu \end{bmatrix} = \begin{bmatrix} \gamma_{10} \\ \gamma_{20} \\ \vdots \\ \gamma_{n0} \\ 1 \end{bmatrix}$$

$$(n+1)\times(n+1) \qquad (n+1)\times 1 \qquad (n+1)\times 1$$

$$\mathbf{w} = \Gamma^{-1}\mathbf{D}$$

ESTIMATING VALUES

$$\breve{U}(x_0) = \sum_{j=1}^{n} w_j U(x_j)$$

$$n = 580$$

$$x_0 = 1,2, \ldots, 18032$$

END RESULT: SNAPSHOTS OF GUI (1/2)

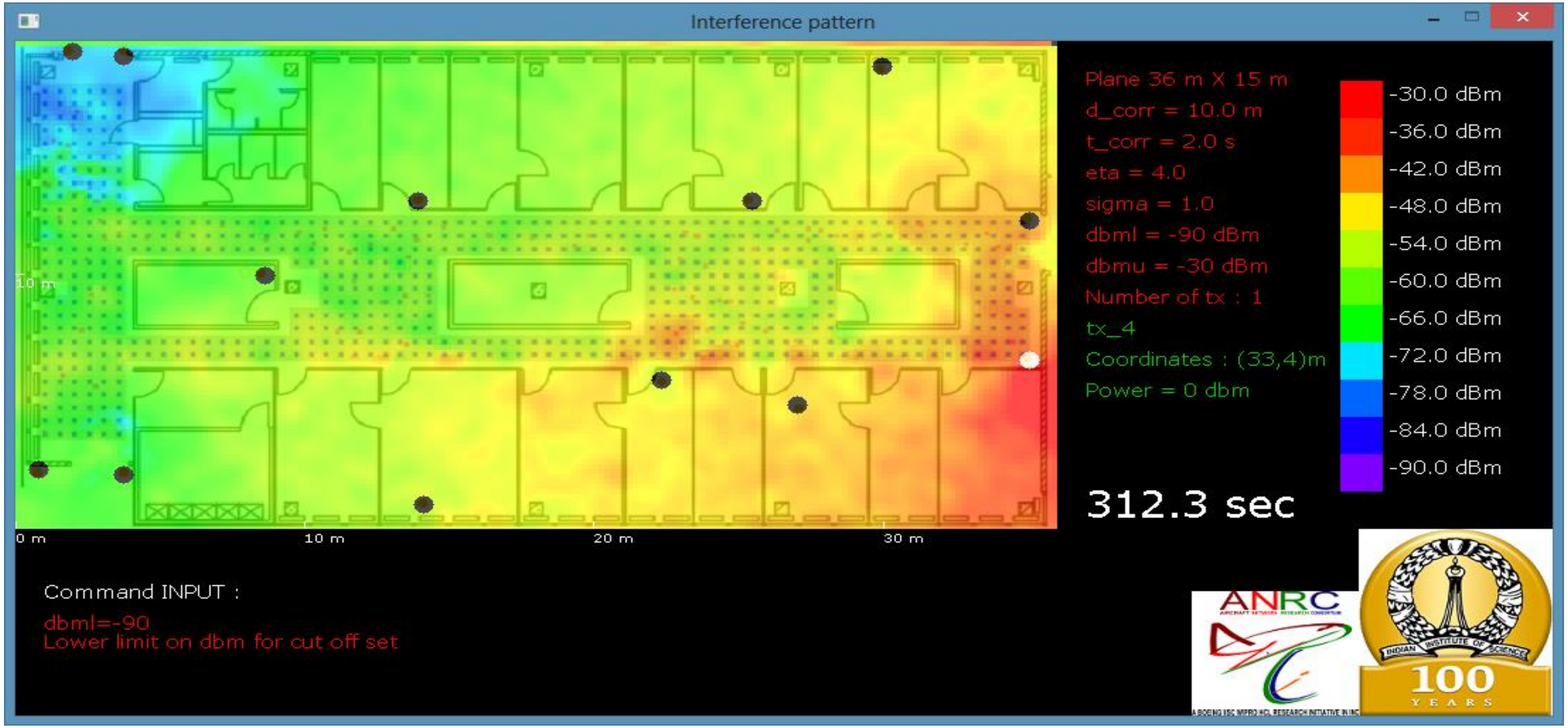


fig: AP 5 switched on

END RESULT: SNAPSHOTS OF GUI (2/2)

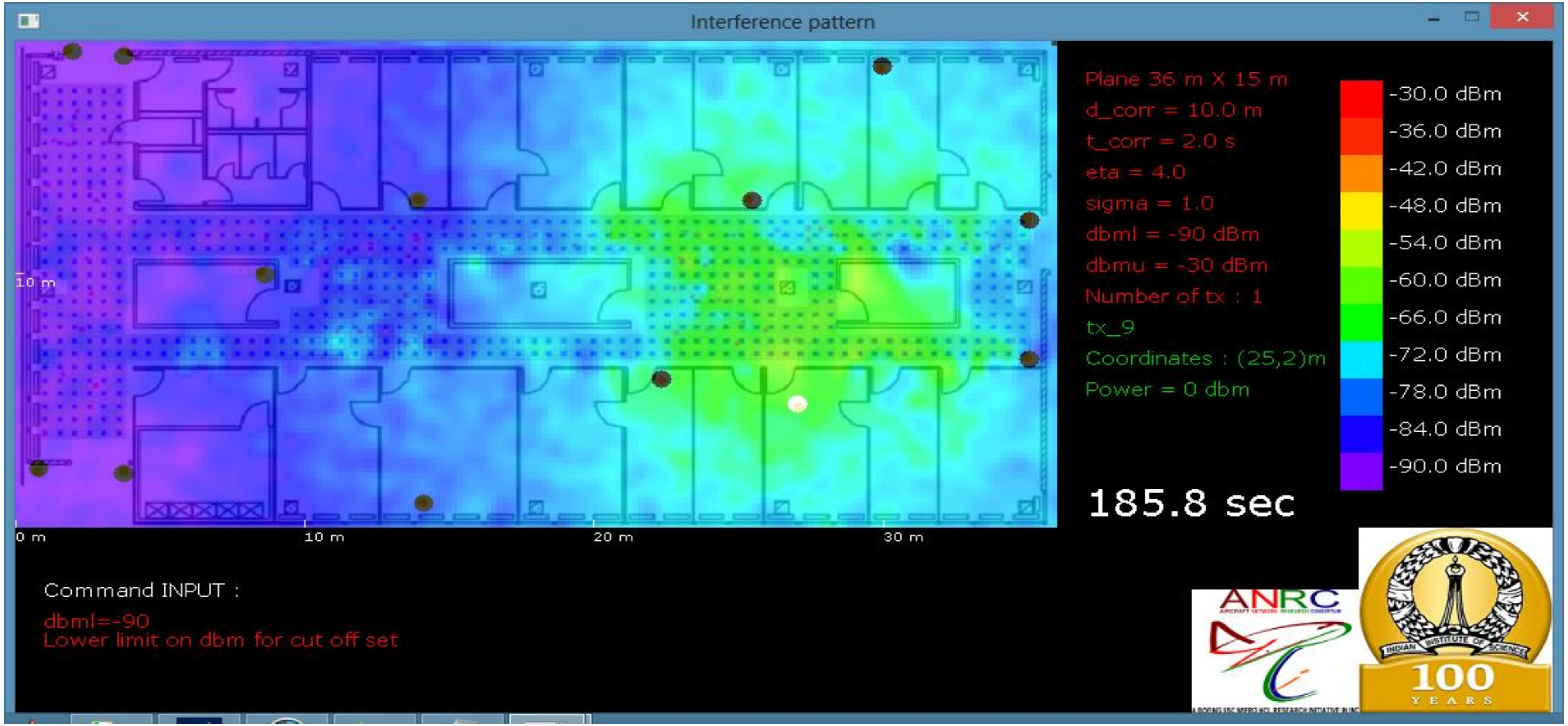


fig: AP 10 switched on

KRIGING V/S LINEAR INTERPOLATION

Visual effect is better with Kriging

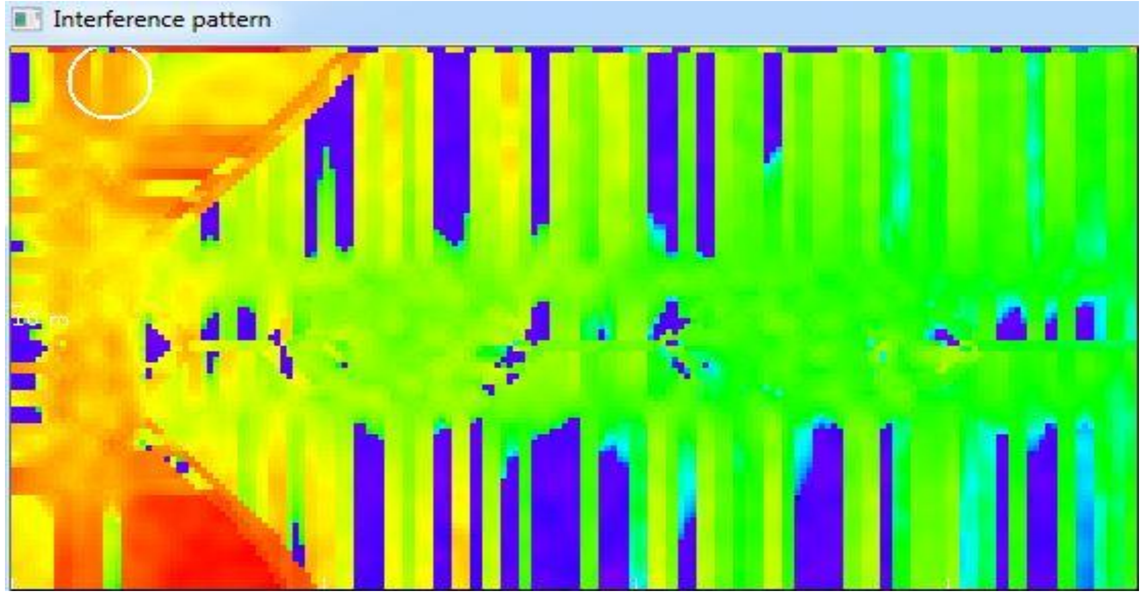


fig: Linear interpolation

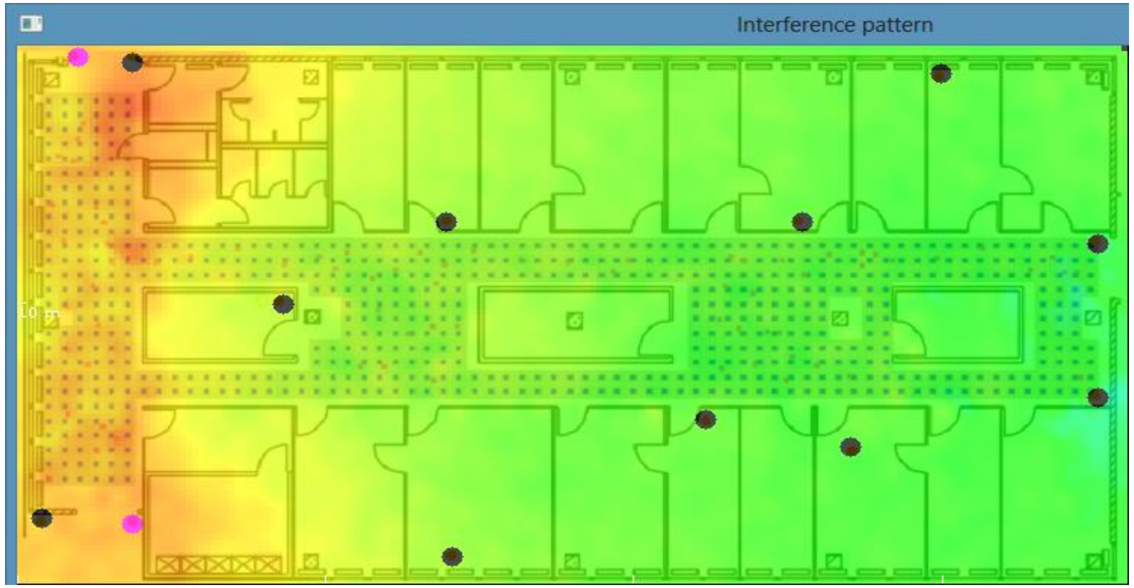


fig: Kriging

LESSER KNOWN POINTS USED:

LINEAR: MORE DEGRADATION

KRIGING: LESSER DEGRADATION

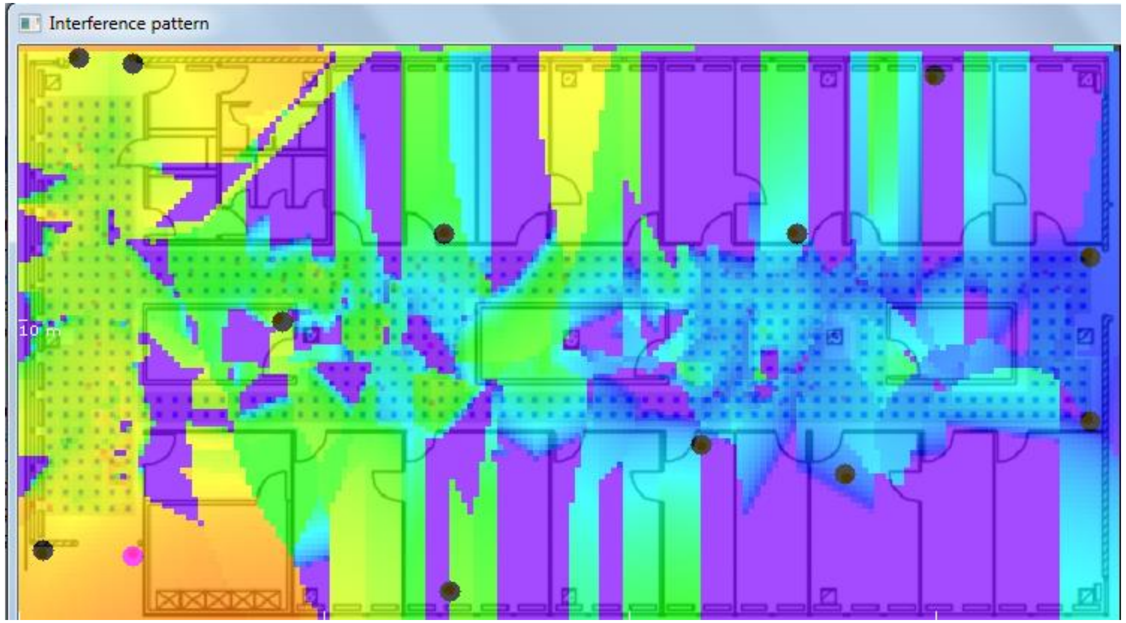


fig: Linear interpolation

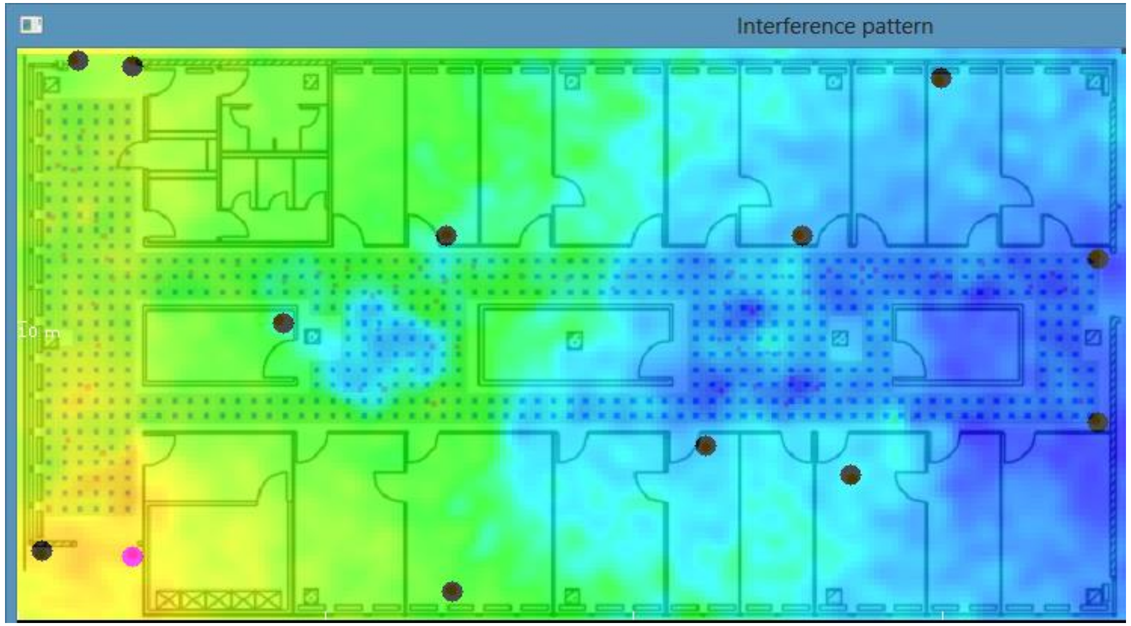


fig: Kriging

145 known points used for interpolation

10 KNOWN POINTS USED FOR INTERPOLATION

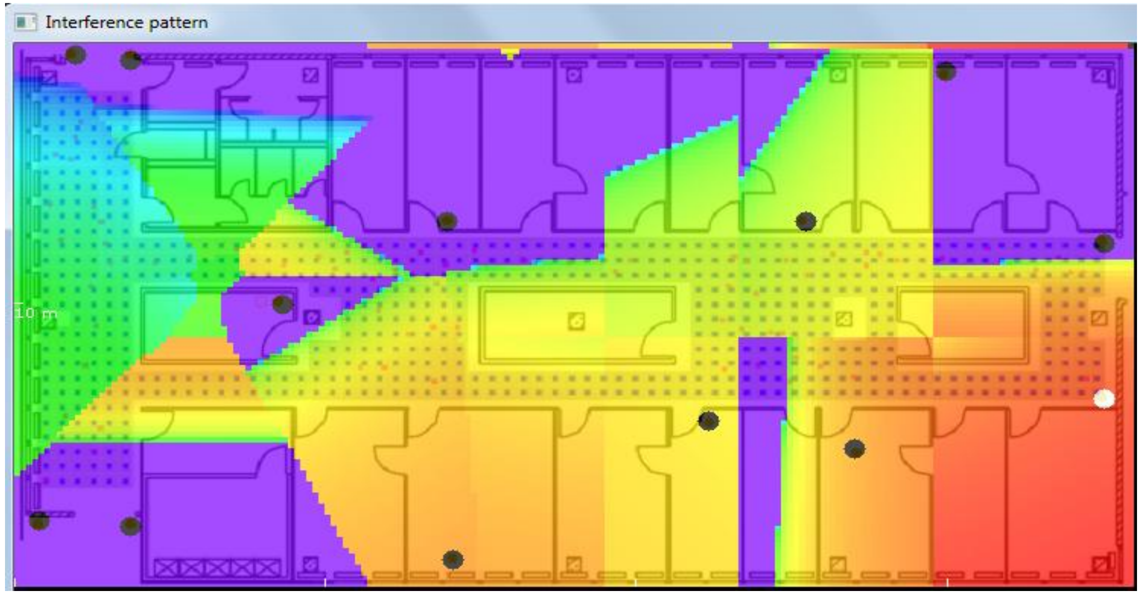


fig: Linear interpolation

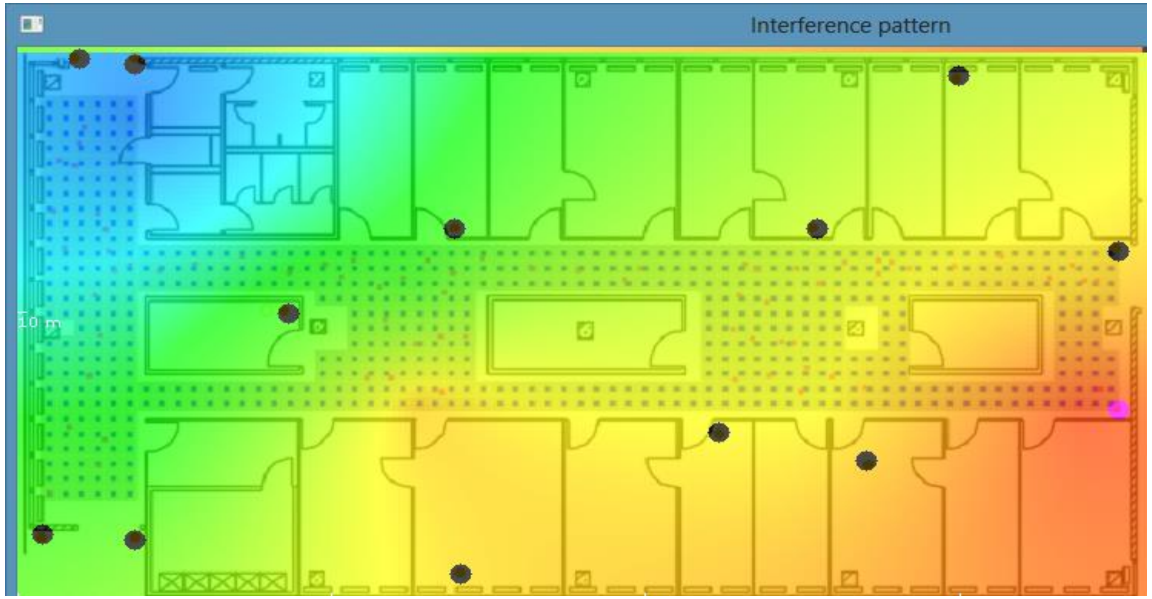


fig: Kriging

CONCLUSION

- Ordinary Kriging is the best linear unbiased estimator
- OK can be used even when mean and variance of the random process assumed are unknown
- It gives better visual result in comparison with linear interpolation

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