Efficient Energy Detection in Decentralized Sensor Networks using Ordered Transmissions - A Life Testing Approach

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System Model and Assumptions (1/2)

- N sensors with M observations each. Hypothesis testing at each sensor

$$\begin{aligned} \mathcal{H}_0 : Y_i &= s_i + n_i \sim \mathcal{CN}(0, \sigma_s^2 + \sigma_n^2) \\ \mathcal{H}_1 : Y_i &= n_i \sim \mathcal{CN}(0, \sigma_n^2), \\ & i \in \{1, 2 \cdots, M\} \end{aligned} \quad (1)$$

- Energy Detection (ED) is optimal. Let

$$E_j \triangleq \frac{1}{M} \sum_{i=1}^{M} |Y_i|^2, \quad j = 1, 2, \cdots, N \quad (2)$$

- No fading
- σ_s^2 is known

System Model and Assumptions (2/2)

$$\begin{aligned} \mathcal{H}_0 : E_j &\sim \Gamma_D\left(M, \frac{2(\sigma_s^2+\sigma_n^2)}{M}\right) \\ \mathcal{H}_1 : E_j &\sim \Gamma_D\left(M, \frac{2\sigma_n^2}{M}\right) \end{aligned} \quad (3)$$

- Each sensor calculates its E_j and transmits it after time $T_j = KE_j$ units. Therefore, E_js arrive at the FC in order
- Let $E_{(j)}$ represent the j^th ordered statistic i.e., $E_{(1)} \leq E_{(2)} \leq \cdots \leq E_{(N)}$
- Goal : Efficient ED using observations $E_{(j)}$ from only r-out-of-N sensors.
- Therefore, efficiency $\Rightarrow$ saving in number of transmissions

Existing "efficient" techniques

- Censoring sensors scheme
- Sadler and Blum's scheme

In this work...

- We present a new scheme based on an approach used in life testing
- For no fading, known σ_n^2, and σ_s^2 cases, an expression for r_{opt} is given, which satisfies $P_D = 1 - \beta$, and $P_F \leq \alpha$, simultaneously
- We generalize some of the existing results (for exponential case) in life testing, for gamma distributions

Life Testing

Joint PDF lemma

Lemma

The joint PDF of first r ordered statistics of T_j is given by

$$f_{T,r}(t_{(1)},\cdots,t_{(r)}) = \frac{N!}{(N-r)!}\left\{\frac{\left(\prod_{j=1}^{r} t_{(j)}\right)^{M-1}}{\left(\frac{\sigma^2}{M}\right)^{M}\Gamma(M)} \times \exp\left(-\frac{M}{\sigma^2}\sum_{j=1}^{r}\frac{1}{t_{(j)}}\right)\right\}^{r}\left(1-\frac{\Gamma\left(M,\frac{M}{t_{(r)}\sigma^2}\right)}{\Gamma M}\right)^{N-r} \quad (4)$$

- $\sigma^2 = \sigma_n^2$ under $\mathcal{H}_1$ and $\sigma^2 = \sigma_n^2 + \sigma_s^2$, under $\mathcal{H}_0$.

Maximum Likelihood Estimate

Lemma

The MLE of σ^2 from $T_{(1)}, \cdots, T_{(r)}$ is given by the value of σ^2 which satisfies

$$\frac{(N-r)t_{(r)}^M}{\Gamma(M)\left(\frac{\sigma^2}{M}\right)^{M-1}\sum_{m=0}^{M-1}\frac{1}{m!}\left(\frac{Mt_{(r)}}{\sigma^2}\right)^m} + \sum_{j=1}^{r} t_{(j)} - r.\sigma^2 = 0 \quad (5)$$

- Denote the solution to above equation as $\widehat{\sigma}^2_{r,N}$
- The detection strategy : $\widehat{\sigma}^2_{r,N} \gtrless \tau$
- For $M = 1$, the result rolls back to exponential case, for which $\widehat{\sigma}^2_{r,N}$ is an efficient estimate of $\widehat{\sigma}^2_{r,N}$, and is a sufficient statistic for ED

A Conjecture

Conjecture

The statistic $\widehat{\sigma}^2_{r,N}$ has the same distribution as $\widehat{\sigma}^2_{r,r}$ i.e.,

$$\widehat{\sigma}^2_{r,N} \sim \Gamma_D\left(rM, \frac{\sigma^2}{rM}\right) \tag{6}$$

- $\sigma^2 = \sigma_n^2$ under $\mathcal{H}_1$ and $\sigma^2 = \sigma_n^2 + \sigma_s^2$, under $\mathcal{H}_0$.

Choosing the r_{opt}

Lemma

When both σ_s^2 and σ_n^2 are known, the detector $\widehat{\sigma}_{r,N}^2 \gtrless \tau$ meets the criteria

(a) $P_D \triangleq \mathcal{P}\{\mathcal{H}_0|\mathcal{H}_0\} = 1 - \beta$, *and*

(b) $P_F \triangleq \mathcal{P}\{\mathcal{H}_0|\mathcal{H}_1\} \leq \alpha$,

when τ and r are chosen such that

(i) $\tau = (\sigma_s^2 + \sigma_n^2)\gamma_{inc}^{-1}\left(\beta, rM, \frac{1}{rM}\right)$, *and*

(ii) $\frac{\gamma_{inc}^{-1}\left(\beta, rM, \frac{1}{rM}\right)}{\gamma_{inc}^{-1}\left(1-\alpha, rM, \frac{1}{rM}\right)} \geq \frac{\sigma_n^2}{\sigma_s^2+\sigma_n^2}$

Proof.

- Following the conjecture, $\widehat{\sigma}^2_{r,N} \sim \Gamma_D\left(rM, \frac{1}{rM}\right)$,
  $\Rightarrow \frac{\widehat{\sigma}^2_{r,N}}{\sigma^2} \triangleq W \sim \Gamma_D\left(rM, \frac{1}{rM}\right)$.
- Need $P_D = \mathcal{P}\{\widehat{\sigma}^2_{r,N} > \tau | \sigma^2 = \sigma_s^2 + \sigma_n^2\} = \mathcal{P}\left\{W > \frac{\tau}{\sigma_s^2+\sigma_n^2}\right\} = 1 - \beta$.
  Taking the inverse, we get the expression for τ
- Need $P_F = \mathcal{P}\{\widehat{\sigma}^2_{r,N} > \tau | \sigma^2 = \sigma_n^2\} = \mathcal{P}\left\{W > \frac{\tau}{\sigma_n^2}\right\} \leq \alpha$
  $\Rightarrow \mathcal{P}\left\{W \leq \frac{\tau}{\sigma_n^2}\right\} \geq 1 - \alpha$
  Taking the inverse, $\frac{\tau}{\sigma_n^2} \geq \gamma^{-1}_{inc}\left(1-\alpha, rM, \frac{1}{rM}\right)$
  Substituting for τ gives the condition to choose r_{opt}

□

Future Work

- Comparison with Censoring sensors, and Sadler-Blum schemes
- A suboptimal test : $t_{(r)} \gtrless \tau_1$
- Extension of the test to the general ED problem [Urkowitz67]

$$\begin{aligned} \mathcal{H}_0 : E_j &\sim \chi^2_{2M}(2\rho) \\ \mathcal{H}_1 : E_j &\sim \chi^2_{2M}(0) \end{aligned} \quad (7)$$

- A sequential version of the test