Target Self-Localization to an Area

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  - Algorithms that are computationally efficient.

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- **Motivation:**
  - Localization in indoor environments is challenging.
  - Advances in WSN has enabled low-cost infrastructure deployment.
  - Algorithms that are computationally efficient.
- **Applications:**
  - Tracking position of a target on a factory floor or in a hospital (intrusion detection, fire alarm).
  - Enabling Coginitve Radio spectrum through geo-location of WSDs.

Random node deployment strategies

Figure: Source:ResearchGate

Comparison of deployment strategies

	Coverage	*Connectivity*	*Connected coverage*
Constant diffusion	++	++	++
Continuous diffusion	+	+	+
R-random diffusion	±	±	±
Simple diffusion	−	−	−
Exponential diffusion	−−	−−	−−

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Notation:

- K - Number of beacons
- M - Number of power thresholds
- δ - Required Degree of Accuracy/Size of Grid Cells
- L - Number of grid points in each dimension ($L \triangleq \lceil \frac{1}{\delta} \rceil$)
- λ - Beacon Density (K/L^2)

Illustration

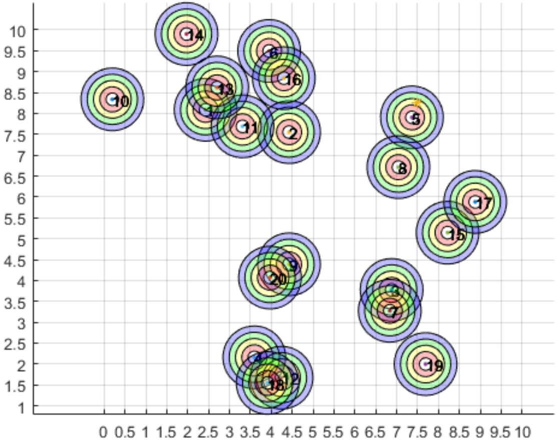

Figure: Measurement process for Target Self-Localization

System Model

- Beacon node b_i transmits with a power P_0. RSS is observed at the target node $P_{rx,i} \triangleq \min(P_0, P_0(d_0/d_i)^{\eta})$.

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- The reading corresponding to b_i and $\mathcal{I}^{(j)}$ is set as follows:

$$y_i^{(j)} \triangleq \begin{cases} 1, & P_{th}^{(j-1)} > P_{rx,i} \geq P_{th}^{(j)} \\ 0, & \text{else.} \end{cases} \tag{1}$$

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Objective: (i) Minimize the area uncertainty, or (ii) Minimize beacon density required to meet the desired localization accuracy (with high probability).

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- Let $P_{\boldsymbol{\nu}}$ be the probability that the target present at (x_t, y_t) has a reading $\boldsymbol{\nu}$. Averaging over both target and beacon deployment, the the average area uncertainty at (x_t, y_t) is:

$$\Omega = \sum_{\boldsymbol{\nu} \in \mathcal{V}} \mathbb{E}\left[P_{\boldsymbol{\nu}}^2\right]. \quad (2)$$

Performance Analysis ($M = 1$)

Theorem

When K beacon nodes, each with a power contour of radius r, are distributed uniformly at random in $\mathcal{A}$, the average area uncertainty in localizing the target is given by

$$\Omega_a(q) \approx \left[q^2 + (1-q)^2\right]^K \tag{3}$$

where $q \triangleq \mathbb{E}[X]$ and X is the r.v. representing coverage area of a single beacon. Further, $q^* = 1/2$ minimizes (3), and the corresponding beacon radius is $r^* = 0.512$ and the average area uncertainty is $\Omega_a(q^*) = (1/2)^K$.

Performance Analysis

Proof.

Suppose the first l entries of the reading $\boldsymbol{\nu}$ are '1' and the remaining $(K - l)$ entries are '0'. Since beacons are i.i.d. uniformly over $\mathcal{A}$, the probability of observing the reading $\boldsymbol{\nu}$ is $P_{\boldsymbol{\nu}} = X^l(1 - X)^{K-l}$.

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There are $\binom{K}{l}$ combinations of readings with l ones and $K - l$ zeros. Therefore, the expectation of $\sum_{\boldsymbol{\nu} \in \mathcal{V}} P_{\boldsymbol{\nu}}^2$ over the target location, i.e., the average area uncertainty in localization is given by

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$$\begin{aligned}\Omega &= \mathbb{E}\left[\sum_{l=0}^{K}\binom{K}{l}(X^2)^l((1-X)^2)^{K-l}\right],\\ &= \mathbb{E}\left[\left(X^2+(1-X)^2\right)^K\right]. \qquad (4)\end{aligned}$$

□

Performance Analysis

Proof.

Further, by Jensen's inequality, the lower bound on (4) is given by

$$\begin{aligned}\Omega &\geq \left(\mathbb{E}\left[X^2\right] + \mathbb{E}\left[(1-X)^2\right]\right)^K, \\ &= \left(q^2 + (1-q)^2 + 2\ \mathrm{Var}\left[X\right]\right)^K \triangleq \Omega_{lb},\end{aligned}$$

where $q \triangleq \mathbb{E}[X]$. In comparison to $q^2 + (1-q)^2$, the variance term is nearly flat across different values of r:

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$q^* = 1/2$ minimizes (5) over $q \in [0, 1]$, and the corresponding beacon radius is $r^* = 0.512$, computed using

$$q = (1/2)r^4 - (8/3)r^3 + \pi r^2. \tag{6}$$

Performance Analysis

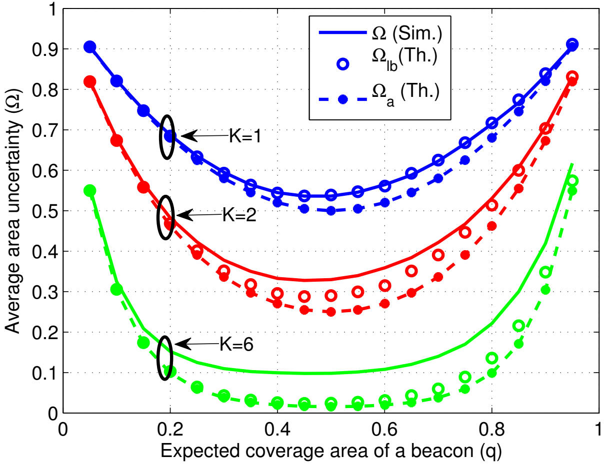
Theorem

When K beacon nodes, each with M power contours of radii $r_1 < r_2 < \ldots < r_m < \ldots < r_M$, are distributed uniformly at random in $\mathcal{A}$, the average area uncertainty in localizing the target is given by

$$\Omega_a \approx \left[q_1^2 + \sum_{m=2}^{M} (q_m - q_{m-1})^2 + (1 - q_M)^2 \right]^K \tag{7}$$

where $q_m \triangleq \mathbb{E}[X_m]$, $m = 1, 2, \ldots, M$, and X_m is an r.v. representing the area coverage of a single beacon with radius r_m. The quantities $q_m^* = \frac{m}{M+1}$, $m = 1, 2, \ldots, M$, minimize (5), and the corresponding average area uncertainty is $\Omega_a^* = \left(\frac{1}{M+1}\right)^K$. Note that, the beacon radii r_m^*, $m = 1, 2, \ldots, M$, is obtained by inverse-mapping the q_m^* using (6).

Result



Ω (Sim.)
Ω_lb (Th.)
Ω_a (Th.)
K=1
K=2
K=6
Average area uncertainty (Ω)
Expected coverage area of a beacon (q)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

Result

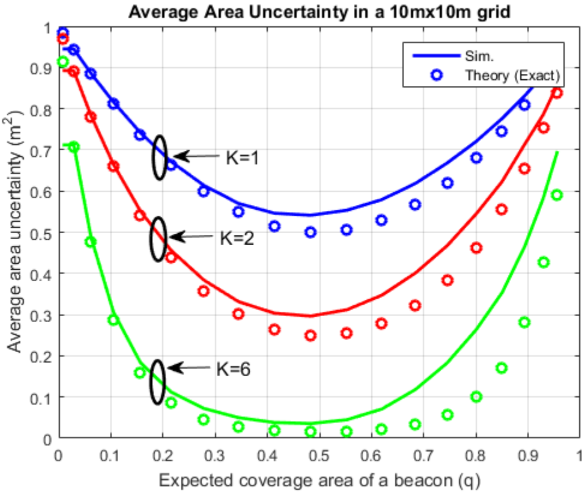


Figure: Outer loop Target, Inner loop Beacons

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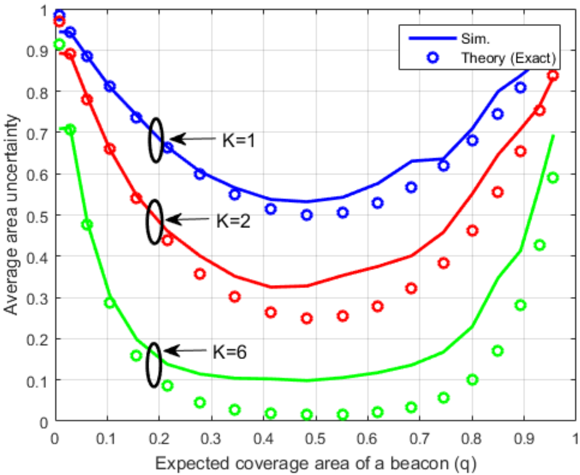

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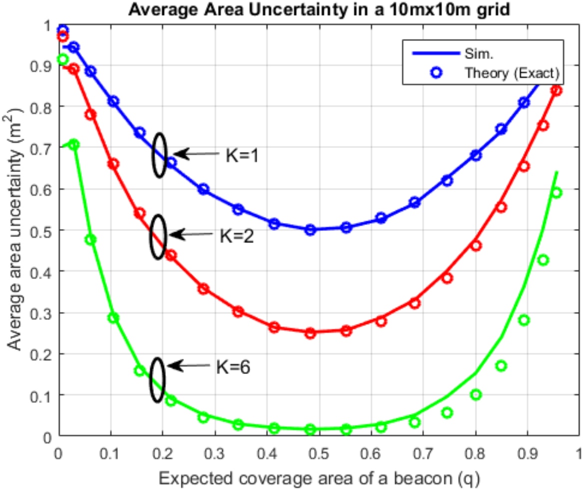

Figure: Joint deployment

2 - Column Matching Algorithm

- For the test corresponding to the j^{th} threshold interval of the i^{th} beacon's signal, the grid points in the annulus $\mathcal{A}_i^{(j)}$ are tested. Let it be represented by $\mathbf{a}_i^{(j)} \in \{0,1\}^{1 \times C}$, where $C \triangleq L_1 L_2$

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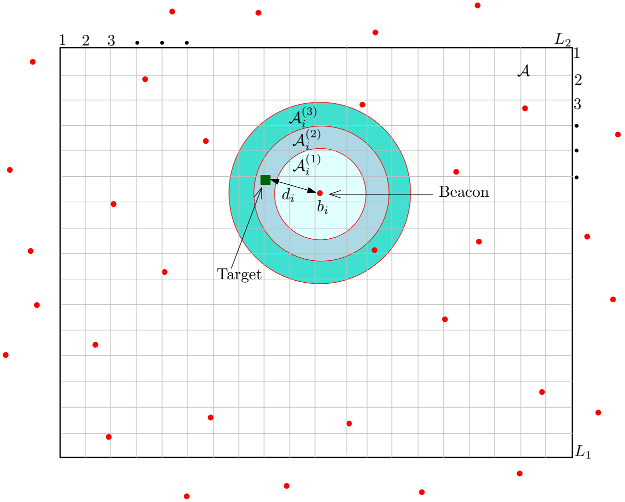
- The entries corresponding to the points being tested are set to 1 and the remaining entries are set to 0.

Illustration



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- The measurement process:

$$\mathbf{y} = \mathbf{A}\mathbf{x}, \tag{8}$$

$\mathbf{x} \in \{0, 1\}^{C \times 1}$ - true position of the target.

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- The measurement process:

$$\mathbf{y} = \mathbf{A}\mathbf{x}, \tag{8}$$

  $\mathbf{x} \in \{0, 1\}^{C \times 1}$ - true position of the target.
- The Column Matching Algorithm attempts to match the columns of $\mathbf{A}$ with test result vector $\mathbf{y}$:

$$\mathcal{K} = \text{supp}\ \{\max\{\mathbf{y}^t\mathbf{A} - \mathbb{1}_{algo}(\mathbf{y}^c)^t\mathbf{A}\}\}, \tag{9}$$

Column Matching Algorithm (xnor)

```
tar =

    7.4871    8.2558

Target is able to detect:
     5


estimate_xnor_centroid =

    7.5000
    8.0000

Elapsed time is 0.479449 seconds.
>>
```

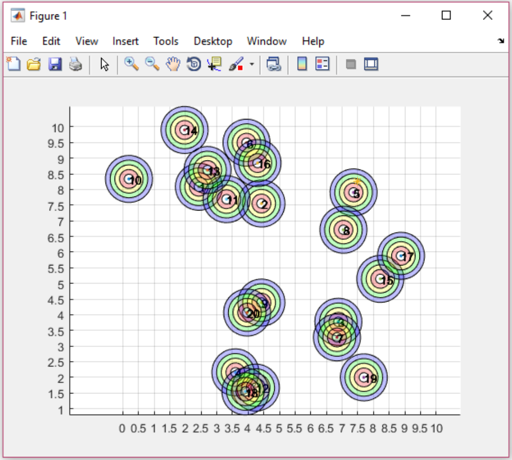

Figure: Target Localization in a 10x10 grid. Target shown by a yellow star.

Performance Analysis

Lemma

When the beacon nodes are distributed as PPP with intensity λ*, the number of beacon nodes with power contours of radius* r *intersecting any vertical/horizontal line segment* $\mathcal{S}$ *is Poisson distributed with mean* $\mu_1 = \lambda(2r)$*. The total number of such intersections* N *on the line segment* $\mathcal{S}$ *is approximately Poisson distributed with mean* $\lambda(4r - \pi r^2)$*.*

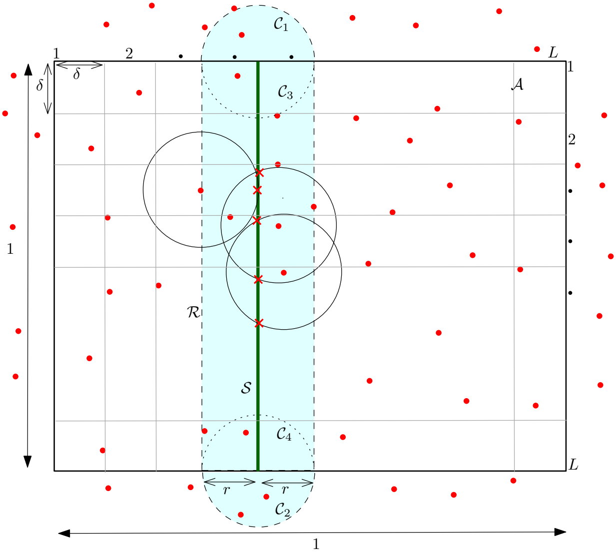

Figure: Illustration of the beacon power contours intersecting a line

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Proof.

Consider a region $\mathcal{R}$ formed by a rectangular strip of size $1 \times 2r$. The average number of beacon nodes that intersect $\mathcal{S}$ is

$$\mu_1 = \lambda(\text{Area of } \mathcal{R}) = \lambda(2r). \tag{10}$$

The mean of the number of intersections on $\mathcal{S}$ is given by

$$\mu = 2\lambda(2r - \pi r^2) + \lambda(\pi r^2) = \lambda(4r - \pi r^2). \tag{11}$$

Performance Analysis

Lemma

The cumulative distribution function (cdf) of the largest among the spacings between successive ordered uniform r.v.s in the range $[0,1]$ *is given by*

$$Pr(V_{(n+1)} \leq \delta) = 1 - \sum_{k=1}^{\min(n+1, L-1)} (-1)^{k-1} \binom{n+1}{k} (1 - k\delta)^n, \qquad (12)$$

where $n \geq 0$, $\delta \in (0,1)$ *and* $L \triangleq \lceil \frac{1}{\delta} \rceil$.

Performance Analysis

Proof.

The probability of the occurrence of at least one of the events $V_i > \delta$ can be expressed as (Boole's formula)

$$Pr\left\{\bigcup_{i=1}^{n+1}(V_i > \delta)\right\} = \sum_{i} Pr(V_i > \delta) - \sum_{i<j} Pr(V_i > \delta, V_j > \delta) + \ldots + (-1)^n Pr(V_1 > \delta, V_2 > \delta, \ldots, V_{n+1} > \delta). \quad (13)$$

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The joint distribution of k events $V_1 > \delta, V_k > \delta$ is symmetrical in V_i. The union event $\cup_{i=1}^{n+1}(V_i > \delta)$ is the same as $(V_{(n+1)} > \delta)$.

$$Pr(V_{(n+1)} > \delta) = \sum_{k=1}^{\min(n+1,L-1)} (-1)^{k-1}\binom{n+1}{k}(1-k\delta)^n, \quad (14)$$

Performance Analysis

Theorem

The average probability of the largest spacing between successive intersections being less than or equal to the size of the grid cell, when the number of intersections N is Poisson distributed with mean μ, is given by

$$\mathbb{E}\left[Pr(V_{(N+1)} \leq \delta)\right] = 1 - \sum_{k=1}^{L-1} \frac{e^{-k\delta\mu}\left[\mu(1-k\delta)+k\right]\left[-\mu(1-k\delta)\right]^{k-1}}{k!}, \tag{15}$$

where $\delta \triangleq \frac{1}{L}$ is the size of the grid cell.

Performance Analysis

Proof.

$$\mathbb{E}\left[Pr(V_{(N+1)} > \delta)\right] = \sum_{n=0}^{\infty} Pr(V_{(n+1)} > \delta) Pr(N = n)$$

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&= \sum_{k=1}^{L-1} \sum_{n=k-1}^{\infty} (-1)^{k-1} \binom{n+1}{k} (1 - k\delta)^n \frac{e^{-\mu}\mu^n}{n!} \\
&= e^{-\mu} \sum_{k=1}^{L-1} \frac{(-1)^{k-1}}{k!} \sum_{n=k-1}^{\infty} \frac{(n+1)}{(n+1-k)!} [\mu(1 - k\delta)]^n
\end{aligned}
$$

□

Proof.

$$= e^{-\mu} \sum_{k=1}^{L-1} \frac{(-1)^{k-1}}{k!} \left[\sum_{n=k-1}^{\infty} \frac{(n+1-k)}{(n+1-k)!} [\mu(1-k\delta)]^n + \sum_{n=k-1}^{\infty} \frac{k}{(n+1-k)!} [\mu(1-k\delta)]^n \right] \tag{16}$$

□

Performance Analysis

Proof.

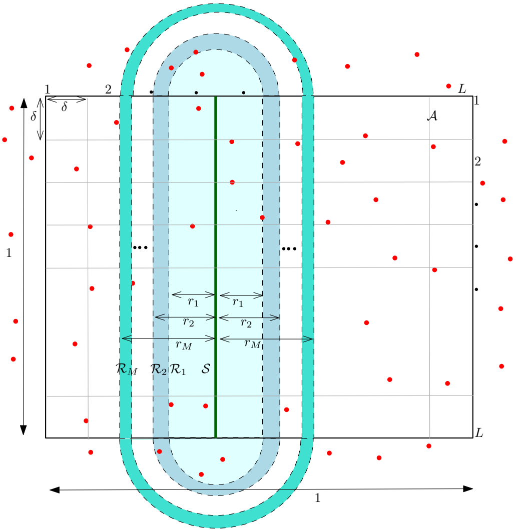
$$= e^{-\mu} \sum_{k=1}^{L-1} \frac{(-1)^{k-1}}{k!} \left[\sum_{n=k-1}^{\infty} \frac{(n+1-k)}{(n+1-k)!} [\mu(1-k\delta)]^n + \sum_{n=k-1}^{\infty} \frac{k}{(n+1-k)!} [\mu(1-k\delta)]^n \right] \quad (16)$$

The inner summation terms of (16) are Taylor series expansions of the scaled exponential function in $\mu(1 - k\delta)$, so

$$\mathbb{E}\left[Pr(V_{(N+1)} > \delta)\right] = e^{-\mu} \sum_{k=1}^{L-1} \frac{(-1)^{k-1}}{k!} [[\mu(1-k\delta)]^k + k[\mu(1-k\delta)]^{k-1}] e^{\mu(1-k\delta)}. \quad (17)$$

□

Evaluating μ



Probability of Localization

- For a given δ, $\mathbb{E}\left[Pr(V_{(N+1)} > \delta)\right]$ can be upper bounded by the first term of the summation in (17), leading to the lower bound:

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- For small δ (< 0.2) and relatively large μ (> 33):

$$\mathbb{E}\left[Pr(V_{(N+1)} \leq \delta)\right] \approx 1 - \mu e^{-\delta\mu} = 1 - (4\lambda\bar{r}M)e^{-\delta(4\lambda\bar{r}M)} \qquad (18)$$

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$$\mathbb{E}\left[Pr(V_{(N+1)} \leq \delta)\right] \approx 1 - \mu e^{-\delta\mu} = 1 - (4\lambda\bar{r}M)e^{-\delta(4\lambda\bar{r}M)} \quad (18)$$

- Parameters λ, $\bar{r}$ and M alone affect $\mathbb{E}\left[Pr(V_{(N+1)} \leq \delta)\right]$ through their product.

Probability of Localization

- For a given δ, $\mathbb{E}\left[Pr(V_{(N+1)} > \delta)\right]$ can be upper bounded by the first term of the summation in (17), leading to the lower bound:

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- **Practical Interest:** Choosing the optimal beacon density to meet a given localization accuracy with high probability.

3 - Simulation Setup

- We consider a square area $\mathcal{A}$ of size (a, a), with $a = 10$.
- Area $\mathcal{A}$ divided into grid cell fine-ness varying from 5×5 to 100×100
- Location of the target, beacon nodes are chosen uniformly at random over $\mathcal{A}$.
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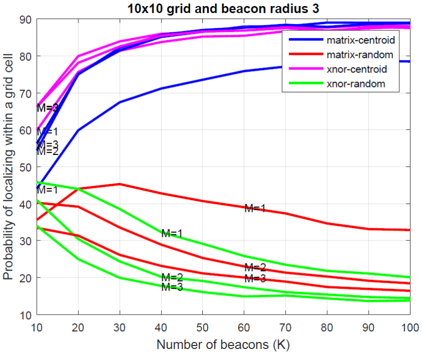
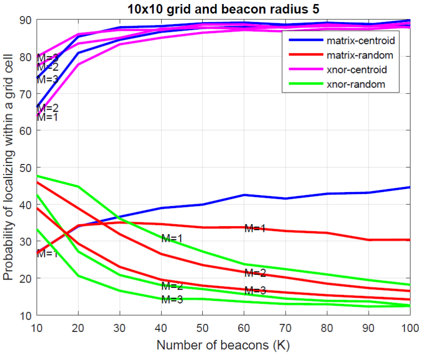
- **Goal 1**: Verifying the minimum average area uncertainty.
- **Goal 2**: Selecting the 'best' localization algorithm.
- **Goal 3**: To compute beacon density required for achieving target localization to a desired accuracy for a specified number of the instantiations (say, 90%) while varying parameters.

Performance comparison: Matrix vs Xnor, Centroid vs Random (Coarse Grid)





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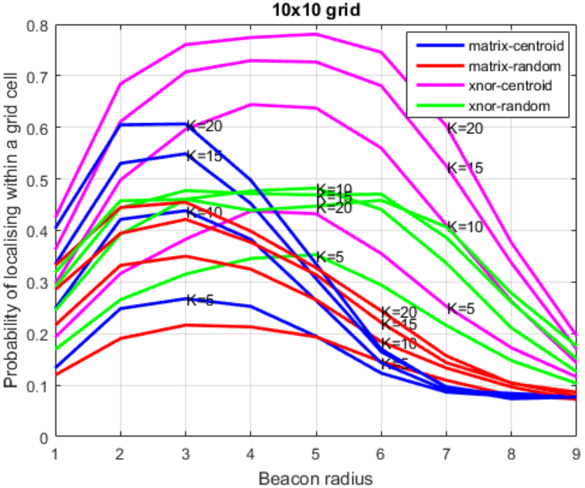

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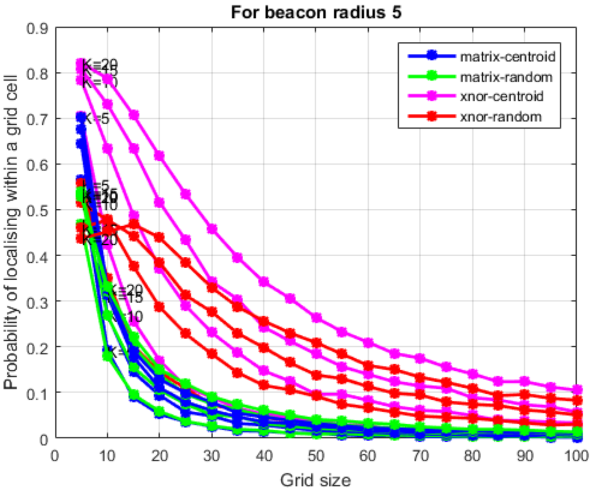
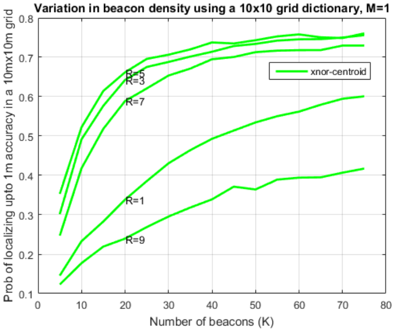
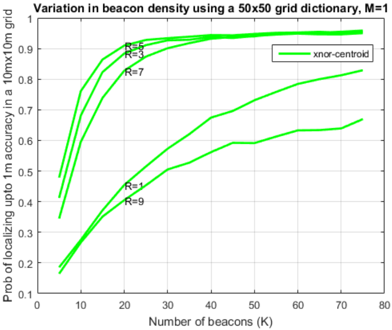
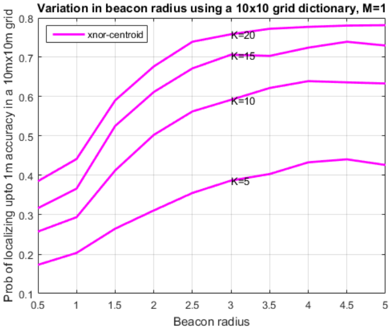


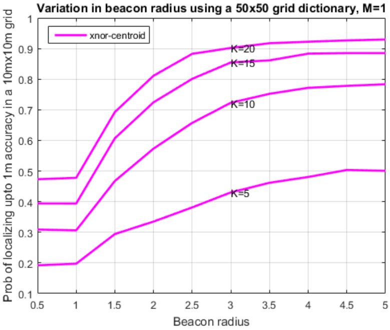
Coarse vs Fine grid





Coarse vs Fine grid





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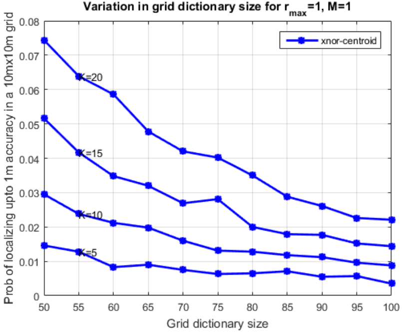
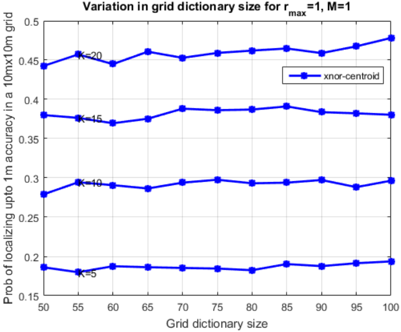
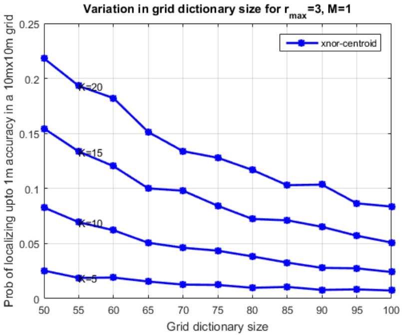
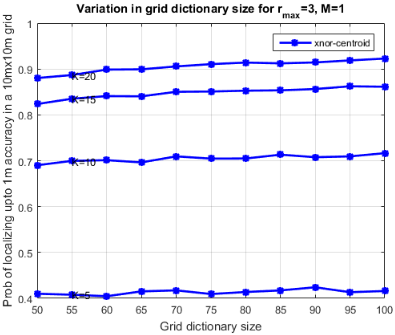




Coarse vs Fine grid approach





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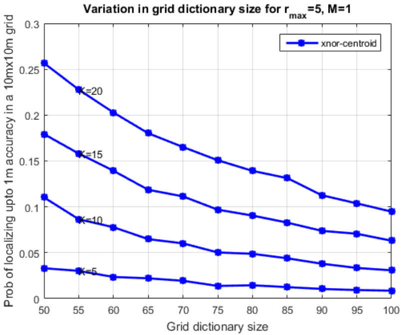
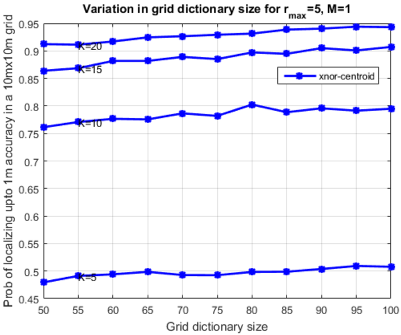
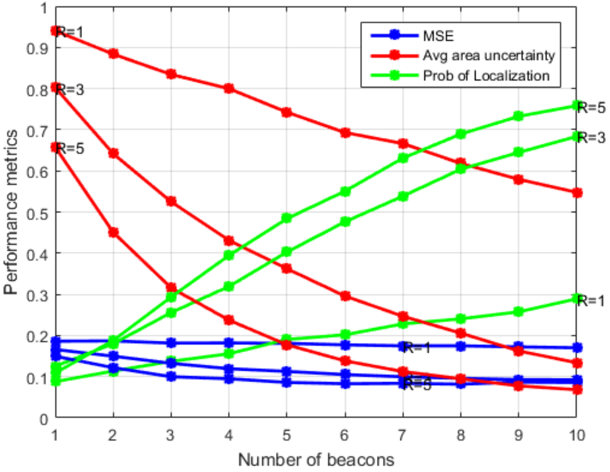




Performance Metric comparison



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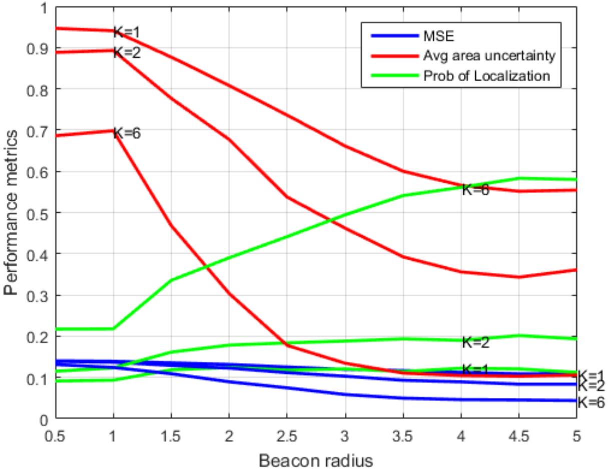
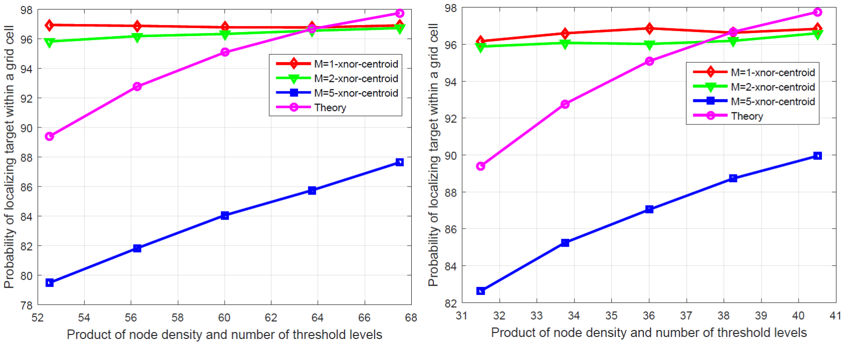


Additional Plots



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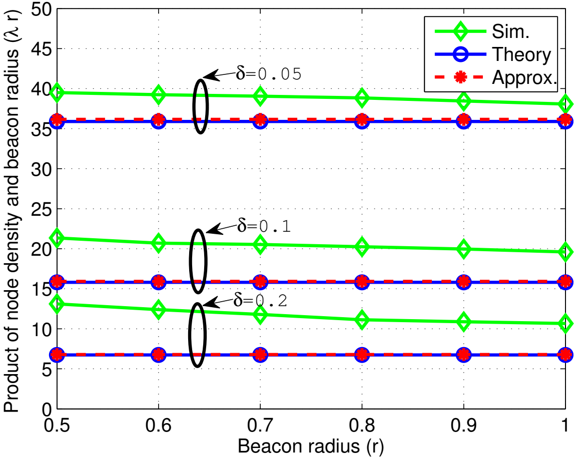
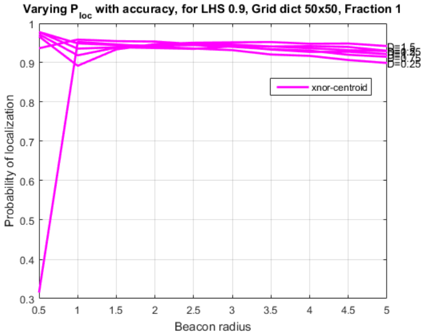
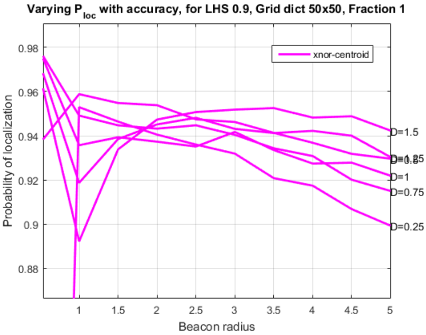
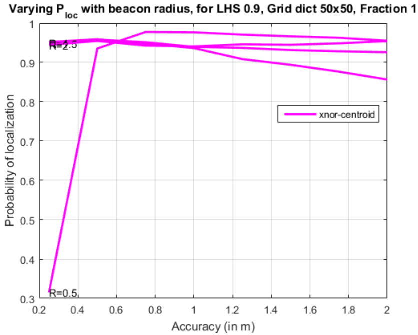
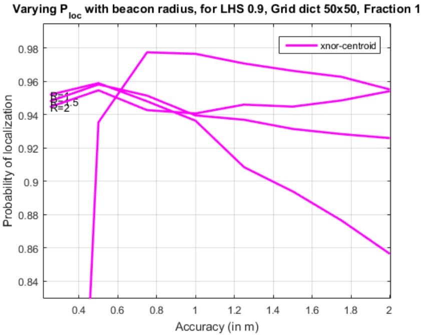



Varying P_{loc} with accuracy, for LHS 0.9, Grid dict 50x50, Fraction 1
Probability of localization
Beacon radius
xnor-centroid
D=1.5
D=0.25
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Varying P_{loc} with accuracy, for LHS 0.9, Grid dict 50x50, Fraction 1
Probability of localization
Beacon radius
xnor-centroid
D=1.5
D=1
D=0.75
D=0.25
0.98
0.96
0.94
0.92
0.9
0.88
1
1.5
2
2.5
3
3.5
4
4.5
5

Results


Varying P_{loc} with beacon radius, for LHS 0.9, Grid dict 50x50, Fraction 1
Probability of localization
Accuracy (in m)
xnor-centroid
R=1.5
R=2
R=0.5
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Varying P_{loc} with beacon radius, for LHS 0.9, Grid dict 50x50, Fraction 1
Probability of localization
Accuracy (in m)
xnor-centroid
R=2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98

Comparison of deployment strategies

	Constant diffusion	*Simple diffusion*	*R-random*
Delivery rate	+	+	+
Consumed energy per packet	+	±	±
End-to-end delay	±	+	±
Fault-tolerance related to detection errors	−	±	+
Fault-tolerance related to transient errors	−	+	±
Fault-tolerance related to global errors	−	+	+
Network lifespan based on coverage	±	+	+
Network lifespan based on connectivity	−	+	+
Network lifespan based on the quality of surveillance	−	±	+

Figure: Source:ResearchGate

4 - Future Work

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- In a stochastic Energy Harvesting setting, a reading of "0" could arise for two reasons. Given this dilemma, what is a good algorithm for estimating the target's location?
- What is the optimal trade-off between number of power thresholds, beacon energy consumption (transmission range) and required localization accuracy in the above setting?