Transmitter Localization using Received Signal Strength Measurements

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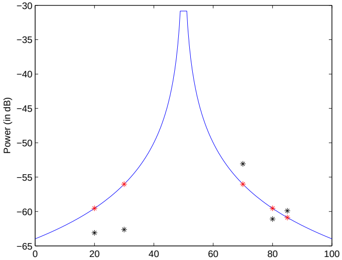
Outline

- Introduction to the Problem
- System Model
- Maximum Likelihood (ML) Estimator
- Approximation of ML Estimator (a past work)
- Proposed Estimator

Introduction to Transmitter Localization

- Single transmitter, transmit power known and position unknown
- N sensors are placed at uniformly random locations
- Sensors report Received Signal Strengths (RSS) to a central node
- Need to locate the transmitter

System Model

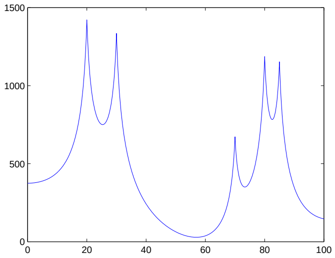


- $P_i = P_0 - 10\gamma \log \frac{\|\theta - \phi_i\|}{d_0} + m_i,\ i = 1 \ldots N$
- $m_i \sim \mathcal{N}(0, \sigma^2)$ in dB
- P_i's are assumed independent

ML Estimator

- $P_i \sim \mathcal{N}(P_0 - 10\gamma \log \frac{\|\theta - \phi_i\|}{d_0}, \sigma^2)$
- $\Pr(P_1, .., P_N|\theta) = \frac{1}{\sqrt{2\pi\sigma^2}^N} \exp(-\sum \frac{(P_i - (P_0 - 10\gamma \log \frac{\|\theta-\phi_i\|}{d_0}))^2}{(2\sigma^2)})$
- $\theta^* = \arg\max_\theta \Pr(P_1, .., P_N|\theta)$
- $\theta^* = \arg\min_\theta \sum (P_i - (P_0 - 10\gamma \log \frac{\|\theta-\phi_i\|}{d_0}))^2$

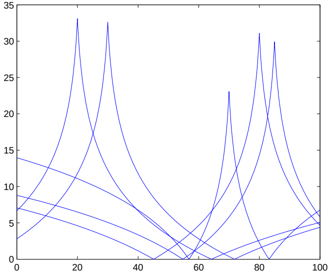
ML Estimator - Cost function



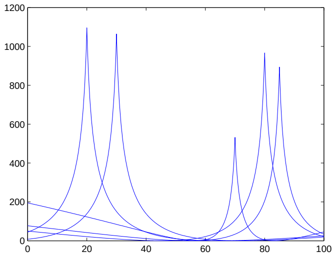
Approximation to ML [Ouyang et al., IEEEVT Mar 2010]

- $\theta^* = \arg\min_\theta \max_i |(P_i - (P_0 - 10\gamma \log \frac{\|\theta - \phi_i\|}{d_0}))|$
- $|(P_i - (P_0 - 10\gamma \log \frac{\|\theta - \phi_i\|}{d_0}))| \equiv |\log \frac{\|\theta - \phi_i\|^2}{A_i}|$ where $A_i = d_0^2 10^{P_0 - P_i/5\gamma}$
- $|\log \frac{\|\theta - \phi_i\|^2}{A_i}| = \max(\log \frac{\|\theta - \phi_i\|^2}{A_i}, \log \frac{A_i}{\|\theta - \phi_i\|^2})$
- $|\log \frac{\|\theta - \phi_i\|^2}{A_i}| = \log(\max(\frac{\|\theta - \phi_i\|^2}{A_i}, \frac{A_i}{\|\theta - \phi_i\|^2}))$
- Convex estimator development by Semidefinite relaxation

Approximate ML - Cost function



Least Squares - Individual Costs



Convex Costs

- Individual cost is convex on either sides of ϕ_i
- Consider the part that is in the direction of transmitter
- Consider the part that is in the direction of sensor receiving highest power (s_1)
- $C_i = (P_i - (P_0 - 10\gamma \log \frac{\|\theta-\phi_i\|}{d_0}))^2|_{\theta:\phi_i \rightarrow s_1} + I_i|_{\theta:\phi_i \leftarrow s_1} \triangleq B_i + I_i$ where I_i is an indicator function
- cost function$= \sum C_i$

Still there is non-convexity

- $\frac{d^2 B_i}{d\theta^2} = 2b[a + b - b \ln \|\theta - \phi_i\|]/(\theta - \phi_i)^2$
  where $a = P_0 - P_i + (10\gamma \ln d_0 / \ln 10)$, $b = 10\gamma / \ln 10$
- Inflection point, $\theta = ed_0 e^{(P_0 - P_i)/(b)}$
- A continuous function of finite energy can be expressed as sum of a convex and a concave function
- Say c is the minimum value of $\frac{d^2 B_i}{d\theta^2}$, then
  $B_i = B_i + \frac{d(\theta - \phi_i)^2}{2} + (-\frac{d(\theta - \phi_i)^2}{2})$
- Second derivate is a monotonically decreasing function, therefore minimum is in the extremal point

New cost function

- Cost function can be expressed as $\sum B_i^{vex} + \sum B_i^{cave}$
- Convex-Concave procedure (CCCP) can be used to solve such problems

Thank you