Error Exponent Analysis of Energy-Based Bayesian Spectrum Sensing Under Fading Channels

Sanjeev G., Chandra R. Murthy and V. Sharma
{sanjeev.g,cmurthy,vinod}@ece.iisc.ernet.in
http://ece.iisc.ernet.in/~cmurthy/SPCLab

Dept. of ECE, Indian Institute of Science, Bangalore, India

Dec. 2011

Outline

Spectrum Sensing (SS) Basics

- The problem of detecting whether a primary is on or off
  - Can be modelled as a hypothesis test
- Main challenges: Fading and the hidden node problem
  - Decentralized detection
- Various types of detection schemes (FSS, SeqD) and detectors (MFD, ED, and FBD)
- ED is the simplest to implement, computationally inexpensive, albeit the performance is least
- Our goal : The error exponents at the individual sensors and at the fusion center (FC) with ED, under fading.

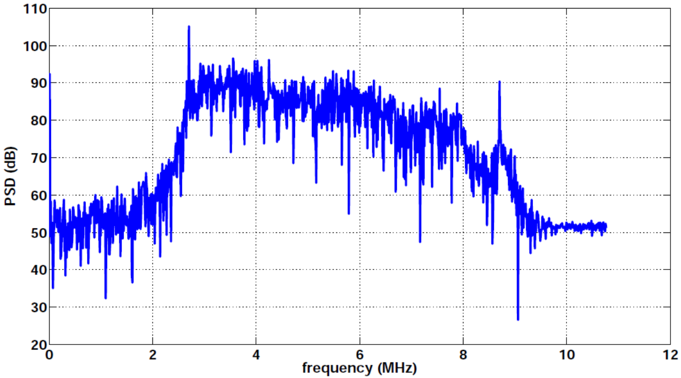
System Model

- N sensors with M observations each. At sensor level, under low SNR regime:

$$\begin{aligned} \mathcal{H}_0 : V_y &\sim \mathcal{N}\left(0, \frac{1}{M}\right) \\ \mathcal{H}_1 : V_y &\sim \mathcal{N}\left(|h|^2 P, \frac{1}{M}\right), \end{aligned}$$

- $V_y \triangleq \frac{1}{M}\sum_{i=1}^{M} |Y_i|^2$, where Y_i is the i-th observation at each sensor. P is the average primary signal power.
- Interest : Wideband (WB) primary signals with a strong pilot.
- NB signals undergo Rayleigh fading, WB signals undergo Lognormal fading [Shellhammer et al. 2006]

Primary Signals of Interest - An Example



Assumptions

- The primary transmit power P (after the effects of shadowing and path loss) and noise variance σ_n^2 are known at the sensors through a calibration phase
- The observations at each sensor are recorded within a coherence time, and are conditionally independent
- The channel between the individual sensors and FC is error free

Relation to Past Work

- [Tsitsiklis. 1988], [Chamberland et al. 1988] Error exponents in decentralized setup with large N.
- [Bai et al. 2010], [Wang et al. 2010] Error exponents with N for centralized setup under fading.
- [Bianchi et al. 2010] Error exponents with N for decentralized setup under fading and NP framework.
- Our work considers the Error exponent in a decentralized setup under Bayesian framework, for a finite N.

The Bayesian ED Problem

- Bayesian problem $\Rightarrow$ minimize p_e. LRT is optimal.
- $LR(V_y) = \dfrac{\frac{1}{\sqrt{2\pi/M}} \int_{|h|^2} \exp\left(-\frac{M(V_y - |h|^2 P)^2}{2}\right) f_{|h|^2}(|h|^2)\, d|h|^2}{\frac{1}{\sqrt{2\pi/M}} \exp\left(-\frac{MV_y^2}{2}\right)}.$
- Closed form analysis of p_e becomes difficult.

Error Exponents

- Error exponent gives the exponential rate of decay on p_e. Mathematically,

$$\epsilon_e \triangleq -\lim_{M\to\infty} \frac{\log p_e}{M} \quad \text{and} \quad \epsilon_E^{(N)} \triangleq -\lim_{M\to\infty} \frac{\log P_E}{M}.$$

- Turns out that the error exponents under both WB and NB sensing are zero. Therefore, in terms of the error exponents, WB and NB sensing problems are equivalent.
- Discount the low channel gains?

Error Exponent With A Confidence Level

Definition

Error exponent with confidence q: Let S_q denote a set of channel instantiations such that $\mathcal{P}(|h|^2 \in S_q) = q$. The highest error exponent achievable over all possible choices of S_q is defined to be the error exponent with a confidence q.

- This novel concept is used to answer the question of WB vs. NB SS.

EECL at the Sensor Level

Theorem

For the above SS problem, a positive error exponent of $(|h_0|^2 P)^2/8$ *is achievable with a confidence level* q*, where* $|h_0|^2$ *satisfies* $\mathcal{P}\left(|h|^2 > |h_0|^2\right) = q$*.*

- Note that the above is valid for a general fading model. Also, note that $EECL = 0$, with $q = 1$.
- It follows that,
  - For Rayleigh fading case, an EECL of $\frac{(\alpha_0 P)^2}{8}$ is achievable with confidence $\exp(-\alpha_0)$.
  - For lognormal shadowing case, an EECL of $\frac{(\ell_0 P)^2}{8}$ is achievable with confidence $1 - Q\left(\frac{\log(\ell_0/P)}{\sigma_s}\right)$.

Outline of the Proof

- Let $\alpha = |h|^2$. It is straightforward to show that

$$p_e = \pi_0 Q(x\sqrt{M}) + \pi_1 \int_{-\infty}^{x} \int_{\alpha_0}^{\infty} f_{\mathcal{N}}\left(v - \alpha P, \frac{1}{\sqrt{M}}\right) \frac{f_\alpha(\alpha)}{q} d\alpha dv.$$

- Further simplification yields

$$\frac{q\pi_0}{\pi_1} = \int_{\alpha_0}^{\infty} \exp\left(M\left(x\alpha P - \frac{\alpha^2 P^2}{2}\right)\right) f_\alpha(\alpha) d\alpha.$$

- The rest of the proof involves showing that $x \rightarrow \frac{\alpha_0 P}{2}$ as $M \rightarrow \infty$ and examining the exponent of p_f (obtained through direct analysis) which is $\frac{x^2}{2}$.

EECL at the FC

- Question : Can EECL be improved with a decentralized detection scheme?
- OR rule is considered for its simplicity and analytical tractability. The FC exploits even if one of the N sensors experiences a “good” channel state.
- Lower bounds on the EECL at the FC are derived. Bounds are tight when $q \to 1$.

Theorem

When the channel between the primary and sensors is Rayleigh distributed, given that the FC combines decisions from N sensors using the OR rule, the error exponent with confidence level q at the FC for the SS problem is lower bounded by $(\alpha_{min}P)^2/8$ *with confidence q, where* α_{min} *satisfies*

$$\alpha_{min} = 2\left(\frac{1-q}{C_N}\right)^{\frac{1}{N}}, \quad C_N \triangleq \sum_{k=0}^{N}\binom{N}{k}\frac{\mathcal{V}_k}{2^k},$$

where $\mathcal{V}_k = \pi^{k/2}/\Gamma\left(1+\frac{k}{2}\right)$ *is the volume of a k dimensional unit sphere.*

Theorem

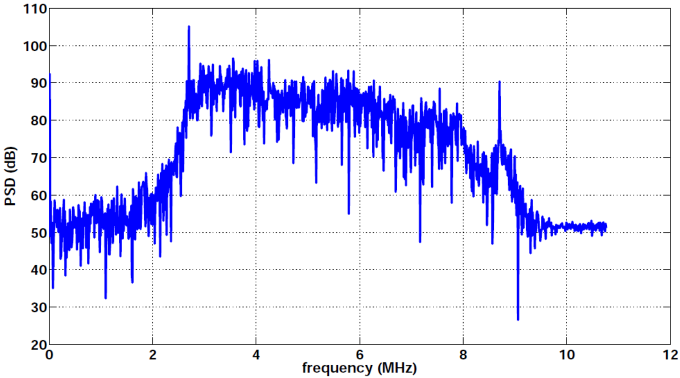
When the channel between the primary and sensors is lognormal distributed, given that the FC combines the decisions from N sensors using the OR rule, the error exponent with confidence level q at the FC for the SS problem is lower bounded by $\frac{(\ell_{min}P)^2}{8}$ *with confidence q, where* ℓ_{min} *satisfies*

$$\sum_{k=0}^{N} \binom{N}{k} D_A^k D_B^{N-k} \frac{\mathcal{V}_k}{2^k} = 1 - q,$$

$$\text{with } D_A \triangleq \frac{1}{2\sigma_s\sqrt{2\pi}} \exp\left(- \frac{\left(\log\left(\frac{\ell_{min}}{P}\right)\right)^2}{2\sigma_s^2} \right),$$

and $D_B \triangleq Q\left(\frac{1}{\sigma_s}\log\left(\frac{2P}{\ell_{min}}\right)\right)$.

Wideband vs. Narrowband Sensing (1/2)



Wideband vs. Narrowband Sensing (2/2)

Theorem

Let P_{NB} and P_{WB} denote the ratios of average powers to their bandwidth in the NB and the WB detector, respectively. Then, with the same confidence level, if

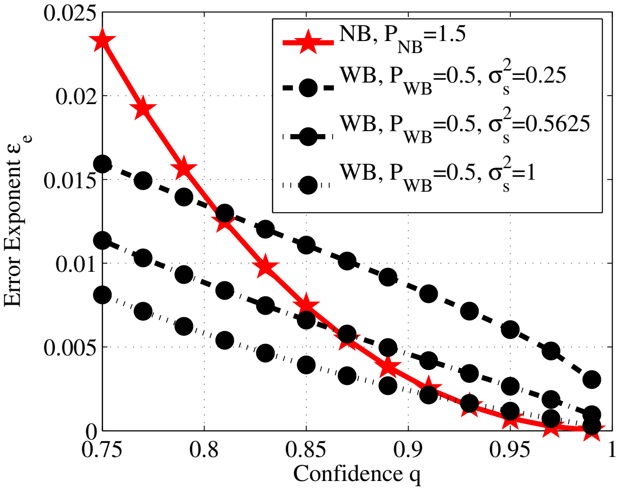
$$\left(\frac{P_{NB}}{P_{WB}}\right)^2 > \left(\frac{\ell_0}{\alpha_0}\right)^2,$$

then the pilot based NB sensing performs better than WB sensing, in the EECL sense.

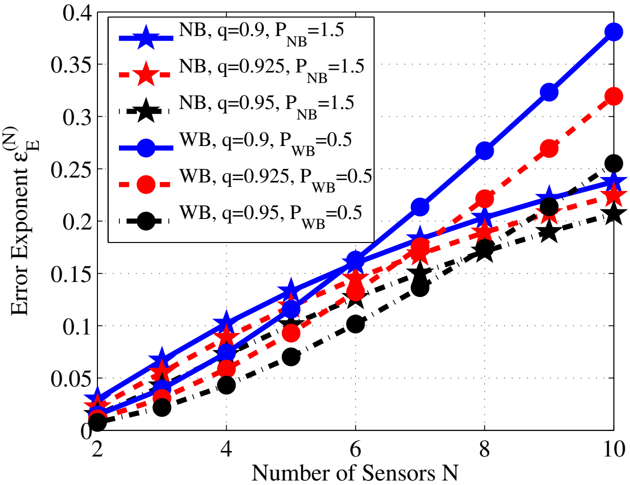
Similarly at the FC, NB SS is better that WB sensing when

$$\left(\frac{P_{NB}}{P_{WB}}\right)^2 > \left(\frac{\ell_{min}}{\alpha_{min}}\right)^2.$$

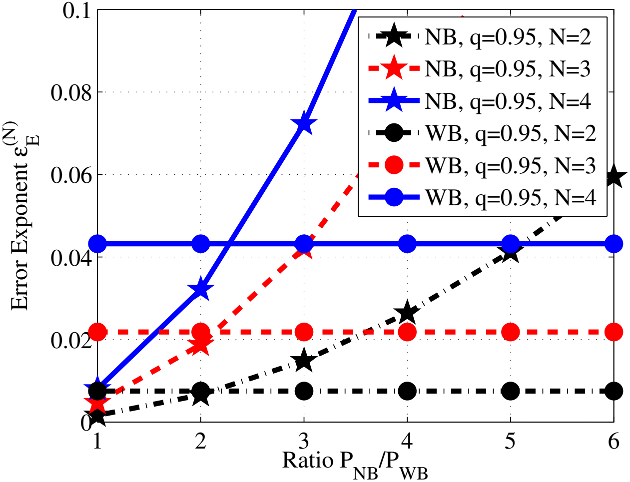
Variation of ϵ_e at sensors with confidence q.



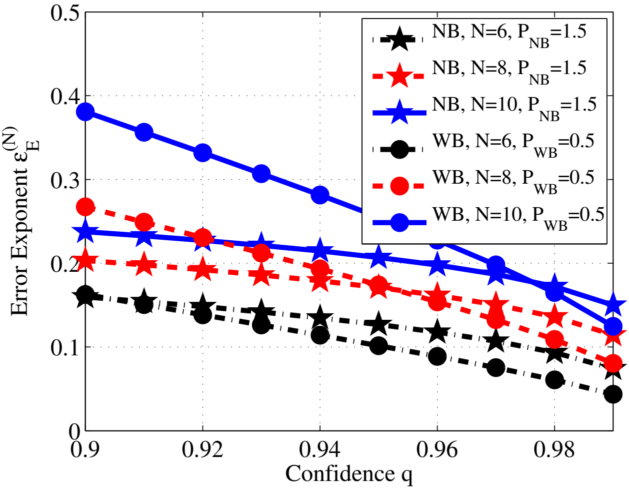
Variation of $\epsilon_E^{(N)}$ with N.



Variation of $\epsilon_E^{(N)}$ with P_{NB}/P_{WB}.



Variation of $\epsilon_E^{(N)}$ with q.



Summary

- Proposed a novel concept called *error exponent with a confidence level*. Using this, it was shown that the ED achieves a zero error exponent under a general fading model.
- This generalized concept was extended to the decentralized setup and performance under the OR rule was studied in detail.
- The question of WB vs. NB sensing was successfully answered using this novel metric.

The End

Thank you very much!
Questions?