Spectrum Cartography under Spatially Correlated Shadowing

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Overview

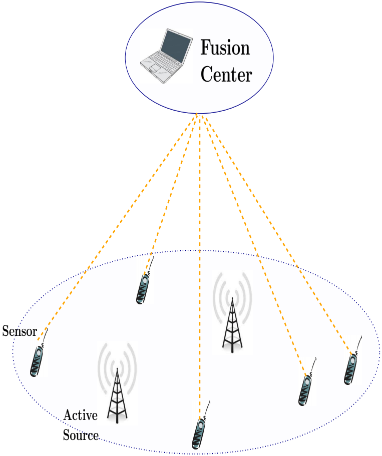
Spectrum Cartography

- Constructing maps across space and time using spectrum measurements
- Two types:
  - Power distribution across frequency
  - Channel gain across frequency between each node and any given point in space
- **Focus**: Expected value that reflects long term effects on the power of a signal, distributed in space

Problem

- Primary network
  - Number of transmitters
  - Transmitter Locations
  - Transmit Powers

  } Unknowns
- Sensors the power received over a given frequency bin at their location
- measurements are sent to a fusion center
- **Aim**: Reconstruct the spatial power map at the fusion center using measurement - sensor location pairs



Motivation: Spectrum Management

- Monitoring radio spectrum utilization
- Applications:
    - Dynamic spectrum allocation
    - Radio planning
    - Monitoring spectrum usage

Cognitive Radio Network

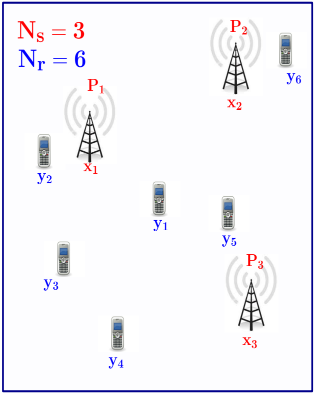
- Cognitive Radio: context-aware intelligent radio
- Adaptive spectrum sharing without disturbing primary users
- Secondary users scan for instantaneous availability of spectrum bands
- Requires knowledge of the spatial and temporal traffic statistics of different services

Other Applications

- Radio planning
  - Interference analysis
  - Resource allocation
  - Identification of coverage holes in the service area for placement of new Access Points or Base Stations.
- Monitoring spectrum
  - Detect any unauthorized transmissions in licensed band

Problem Setup

- Primary network
  - N_s stationary, mutually independent transmitting sources
  - Transmitter locations $\mathcal{X} = \{\mathbf{x}_s\}_{s=1}^{N_s}$
  - Transmit power $\mathcal{P} = \{P_s\}_{s=1}^{N_s}$
- Secondary network
  - N_r coordinated sensors
  - Sensor locations $\mathcal{Y} = \{\mathbf{y}_r\}_{r=1}^{N_r}$
- **Objective**: Reconstruct the power map over the entire area



Received Signal Strength

- Signal received at sensor r is a superposition of the signals from all the transmitters
- The average received power at $\boldsymbol{y}_r$ is

$$\phi(r) = \sum_{s=1}^{N_s} P_s H_{sr} \qquad r = 1, 2, ..., N_r$$

H_{sr} is the random attenuation of power

Power Attenuation Model

- H_{sr} can be characterized using two multiplicative components: $H_{sr} = \rho(\|\boldsymbol{x}_s - \boldsymbol{y}_r\|)\xi(\boldsymbol{y}_r)$
  - Distance dependent path loss $\rho(d) = \min\left\{1, \left(\frac{d_0}{d}\right)^\eta\right\}$
    - η - path loss exponent, d_0 - reference distance
    - d - distance between transmitter and receiver
  - Shadowing fading or large scale fading ξ
- Measurements are

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- Measurements are

$$\phi(r) = \sum_{s=1}^{N_s} P_s H_{sr} \quad \Longrightarrow \quad \phi(r) = \sum_{s=1}^{N_s} P_s \rho\left(\|\boldsymbol{y}_r - \boldsymbol{x}_s\|\right)\xi(r)$$

Power Attenuation Model

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    - η - path loss exponent, d_0 - reference distance
    - d - distance between transmitter and receiver
  - Shadowing fading or large scale fading ξ
- Measurements in decibel (dB) scale are

$$\phi_{\mathrm{dB}}(r) = 10\log_{10}\left[\sum_{s=1}^{N_s} P_s\rho\left(\|\mathbf{y}_r - \mathbf{x}_s\|\right)\right] + \xi_{\mathrm{dB}}(r)$$

$$r = 1, 2, ..., N_r$$

Shadowing Model

- Shadowing ξ follows log-normal distribution
- ξ_{dB} is spatially correlated Gaussian random process
- Zero mean, variance σ^2
- Widely accepted Gudmundson's model for correlation
- Correlation between any two points $\boldsymbol{u}$ and $\boldsymbol{v}$

$$R(\boldsymbol{u}, \boldsymbol{v}) = e^{-\|\boldsymbol{u}-\boldsymbol{v}\| \ln 2 / d_{\mathrm{cor}}}$$

d_{cor} - decorrelation distance

Goal

- Reconstruction algorithm for the power map of entire area
- Inputs: Sensor observations and sensor locations
- Assumption: Knowledge of environment dependent parameters - decorrelation distance and path loss exponent

Spatial Interpolation

- Deterministic methods
  - Extent of similarity (Inverse Distance Weighted)
  - The degree of smoothing (Splines based methods)
- Stochastic methods - kriging

Why Stochastic Methods?

- Incorporates the concept of randomness in function to be interpolated
- Exploits knowledge of statistical model of function
- Indication of estimation error

Two-part reconstruction framework

- Power received at any point in space can be treated as a random field with two components

$$\phi_{\mathrm{dB}}(r) = \underbrace{10\log_{10}\left[\sum_{s=1}^{N_s} P_s \rho\left(\|\boldsymbol{y}_r - \boldsymbol{x}_s\|\right)\right]}_{\text{Pathloss}} + \underbrace{\xi_{\mathrm{dB}}(r)}_{\text{Shadowing}}$$

  - Trend component - path loss part
  - Residual component - shadowing part
- First and second order statistics
  - Mean - unknown
  - Covariance - known
- **Strategy**: Estimate the deterministic component first and random part of using this estimate

Stage 1: Path loss Component Estimation

- The path loss model gives a parametric form to measurements

$$\phi_{\mathrm{dB}}(r) = \underbrace{10\log_{10}\left[\sum_{s=1}^{N_s} P_s \rho\left(\|\boldsymbol{y}_r - \boldsymbol{x}_s\|\right)\right]}_{\text{Parametric form}} + \underbrace{\xi_{\mathrm{dB}}(r)}_{\text{Correlated Gaussian term}}$$

- Parameters
  - Transmitter locations $\{\boldsymbol{x}_s\}_{s=1}^{N_s}$
  - Transmit power $\{P_s\}_{s=1}^{N_s}$

Why not conventional approaches?

- Classical approaches:
  - Least Squares, Maximum Likelihood, Maximum A Posteriori
- Difficult!
  - Logarithm of sum of unknown parametric components
- Consider measurements in linear scale to avoid the logarithm?
  - Shadowing becomes multiplicative

Dictionary Based Estimation

- An alternate approach: Basis Expansion Model (BEM)*
- Sample the parameter to be estimated over the range of possible values to form an over-complete basis
- Candidate set of N source locations $\mathcal{Z} = \{z_i\}_{i=1}^{N}$, based on a virtual network grid

Virtual Grid Model

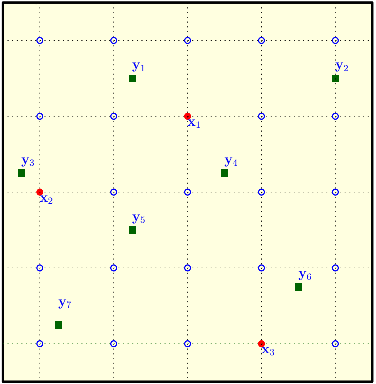

Figure: Wireless network with $N_s = 3$ transmitters, $N_r = 7$ sensors and N =25 candidate locations

Path loss Component Representation

- Path loss component in linear scale,

$$\sum_{s=1}^{N_s} P_s \rho(\|\mathbf{y}_r - \mathbf{x}_s\|) = \begin{bmatrix} \rho(\|\mathbf{z}_1 - \mathbf{y}_r\|) & \rho(\|\mathbf{z}_2 - \mathbf{y}_r\|) & \dots & \rho(\|\mathbf{z}_N - \mathbf{y}_r\|) \end{bmatrix} \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_N \end{bmatrix}}_{\boldsymbol{\theta}}$$

  - $\boldsymbol{\theta}$ is N_s sparse power vector

Path loss Component Representation

- Path loss component in linear scale,

$$\begin{bmatrix} \sum_{s=1}^{N_s} P_s \rho\left(\|\mathbf{y}_1 - \mathbf{x}_s\|\right) \\ \sum_{s=1}^{N_s} P_s \rho\left(\|\mathbf{y}_2 - \mathbf{x}_s\|\right) \\ \vdots \\ \sum_{s=1}^{N_s} P_s \rho\left(\|\mathbf{y}_N - \mathbf{x}_s\|\right) \end{bmatrix} = \underbrace{\begin{bmatrix} \rho\left(\|\mathbf{z}_1 - \mathbf{y}_1\|\right) & \rho\left(\|\mathbf{z}_2 - \mathbf{y}_1\|\right) & \dots & \rho\left(\|\mathbf{z}_N - \mathbf{y}_1\|_2\right) \\ \rho\left(\|\mathbf{z}_1 - \mathbf{y}_2\|\right) & \rho\left(\|\mathbf{z}_2 - \mathbf{y}_2\|\right) & \dots & \rho\left(\|\mathbf{z}_N - \mathbf{y}_2\|\right) \\ \vdots & & & \\ \rho\left(\left\|\mathbf{z}_1 - \mathbf{y}_{N_r}\right\|\right) & \rho\left(\left\|\mathbf{z}_2 - \mathbf{y}_{N_r}\right\|\right) & \dots & \rho\left(\left\|\mathbf{z}_N - \mathbf{y}_{N_r}\right\|\right) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_N \end{bmatrix}}_{\boldsymbol{\theta}}$$

  - $\boldsymbol{\theta}$ is N_s sparse power vector

Path loss Component Representation

- Measurements in dB scale,

$$\underbrace{\begin{bmatrix} \phi_{\text{dB}}(1) \\ \phi_{\text{dB}}(2) \\ \vdots \\ \phi_{\text{dB}}(N_r) \end{bmatrix}}_{\boldsymbol{\phi}_{\text{dB}}} = 10\log_{10}\{\boldsymbol{A\theta}\} + \underbrace{\begin{bmatrix} \xi_{\text{dB}}(1) \\ \xi_{\text{dB}}(2) \\ \vdots \\ \xi_{\text{dB}}(N_r) \end{bmatrix}}_{\boldsymbol{\xi}_{\text{dB}}}$$

  - $\boldsymbol{\theta}$ is N_s sparse power vector

Measurement Model

- The measurements can be written as

$$\phi_{\mathrm{dB}} = 10\log_{10}(\boldsymbol{A\theta}) + \xi_{\mathrm{dB}}$$

- Sparse solution reveals the location of sources and their transmit power
- Model is not linear
- Traditional compressive sensing techniques do not apply directly

Exploiting Spatial Correction

- Measurement vector $\phi_{\text{dB}} \sim \mathcal{N}(\boldsymbol{\mu}, \sigma^2 \boldsymbol{\Gamma})$
  - Mean vector $\boldsymbol{\mu} \in \mathbb{R}^{N_r}$ is path loss component in dB scale

$$\boldsymbol{\mu}_i = 10 \log_{10} \left(\sum_{s=1}^{N_s} P_s \rho \left(\| \boldsymbol{y}_i - \boldsymbol{x}_s \| \right) \right)$$

  - σ^2 is shadowing variance
  - Shadowing correlation matrix $\boldsymbol{\Gamma} \in \mathbb{R}^{N_r \times N_r}$ is defined using modified Gudmundson's model

Greedy Approach

- Locate a source in each iteration
  - Search over the candidate locations set to find the location that maximizes a likelihood function
  - Evaluate corresponding transmitter power to update transmit power estimate
- Update the likelihood function to add the contribution of the identified source
- Repeat until no candidate can further improve likelihood

Likelihood Function

- The ML estimate of transmitter locations over $\mathcal{Z}$, given estimates till $k-1$th iteration $\{\boldsymbol{\mathcal{I}}^{k-1}, \tilde{\boldsymbol{\theta}}^{k-1}\}$

$$\begin{bmatrix} l_{\mathrm{ML}} & \hat{\theta}_{\mathrm{ML}} \end{bmatrix} = \underset{1 \le l \le N, \hat{\theta} > 0}{\operatorname{argmax}} \; f(\phi_{\mathrm{dB}} | l, \hat{\theta}, \boldsymbol{\mathcal{I}}^{k-1}, \hat{\boldsymbol{\theta}}^{k-1}) \qquad (1)$$

- Probability density function f
  - Correlated Gaussian distribution
  - Covariance $\sigma^2 \boldsymbol{\Gamma}$
  - Mean $\boldsymbol{\mu}(l, \tilde{\theta} | \boldsymbol{\mathcal{I}}^k, \tilde{\boldsymbol{\theta}}^k) = 10 \log_{10} \left(\boldsymbol{A} \tilde{\boldsymbol{\theta}}^k + \tilde{\theta} \boldsymbol{A}_l \right)$

Optimization Problem

$$l_{\mathrm{ML}} = \underset{1 \le l \le N}{\operatorname{argmin}} \left\{ \left[\phi_{\mathrm{dB}} - \boldsymbol{\mu}(l, \hat{\theta}_{\mathrm{ML}}(l) | \mathcal{I}^{k-1}, \hat{\boldsymbol{\theta}}^{k-1}) \right]^{\mathsf{T}} \boldsymbol{\Gamma}^{-1} \left[\phi_{\mathrm{dB}} - \boldsymbol{\mu}(l, \hat{\theta}_{\mathrm{ML}}(l) | \mathcal{I}^{k-1}, \hat{\boldsymbol{\theta}}^{k-1}) \right] \right\} \quad (2)$$

$$\hat{\theta}_{\mathrm{ML}}(l) = \underset{\hat{\theta} > 0}{\operatorname{argmin}} \left\{ \left[\phi_{\mathrm{dB}} - \boldsymbol{\mu}(l, \hat{\theta} | \mathcal{I}^{k-1}, \hat{\boldsymbol{\theta}}^{k-1}) \right]^{\mathsf{T}} \boldsymbol{\Gamma}^{-1} \left[\phi_{\mathrm{dB}} - \boldsymbol{\mu}(l, \hat{\theta} | \mathcal{I}^{k-1}, \hat{\boldsymbol{\theta}}^{k-1}) \right] \right\} \quad (3)$$

Wrong Approach!

- Does not work: In kth iteration, it tries explain all measurements with k sources when there are actually N_s sources
- Once a wrong location is chosen, the error propagates through the subsequent iterations

Modify Likelihood Function

- Modification: Insert a damping term in the cost function
- The path loss function is a decaying function
- Forgetting term gives exponentially less weight to measurements which are away from the candidate location
- Damping term for measurement at $\boldsymbol{y}_r$ and candidate location $\boldsymbol{z}$: $e^{-\lambda||\boldsymbol{y}_r - \boldsymbol{z}||}$, $\lambda > 0$

Modified Likelihood Function

- The negative log likelihood function for candidate location index l and transmit power $\tilde{\theta}$ in the kth iteration is given by

$$L_k(l,\hat{\theta}) = \left[\phi_{\mathrm{dB}} - \boldsymbol{\mu}(l,\hat{\theta}|\mathcal{I}^{k-1}\hat{\boldsymbol{\theta}}^{k-1})\right]^{\mathsf{T}} \boldsymbol{F}_l \boldsymbol{\Gamma}^{-1} \boldsymbol{F}_l \left[\phi_{\mathrm{dB}} - \boldsymbol{\mu}(l,\hat{\theta}|\mathcal{I}^{k-1},\hat{\boldsymbol{\theta}}^{k-1})\right] \quad (4)$$

- The forgetting term matrix $\boldsymbol{F}_l \in \mathbb{R}^{N_r \times N_r}$ is a diagonal matrix with (i,i)th entry as $e^{-\lambda||\boldsymbol{y}_i - \boldsymbol{z}_l||}$

Power Estimation

- Power estimate can be now approximated as

$$\Theta(l) \approx \left(\frac{\mathbf{1}^{\mathrm{T}} \boldsymbol{F}_l \boldsymbol{\Gamma}^{-1} \boldsymbol{F}_l \left[\phi_{\mathrm{dB}} - 10 \log_{10}(\boldsymbol{A}_l)\right]}{\mathbf{1}^{\mathrm{T}} \boldsymbol{F}_l \boldsymbol{\Gamma}^{-1} \boldsymbol{F}_l \mathbf{1}} \right)^{+}$$

- Estimate of power if there is a transmitting source at candidate location l

Algorithm Input and Output

- Inputs from sensor network
  - Measurements in dB scale: $\phi_{\text{dB}} \in \mathbb{R}^{N_r \times 1}$
  - Sensor locations $\{\boldsymbol{y}_r\}_{r=1}^{N_r}$

Algorithm Input and Output

- Inputs from sensor network
  - $\phi \in \mathbb{R}^{N_r \times 1}$
  - $\{\boldsymbol{y}_r\}_{r=1}^{N_r}$

- Outputs
  - Source location indices $\mathcal{I}$
  - Transmit power estimates $\hat{\theta}$

Algorithm Input and Output

- Inputs from sensor network
  - $\phi \in \mathbb{R}^{N_r \times 1}$
  - $\{\boldsymbol{y}_r\}_{r=1}^{N_r}$
- Knowledge of environment parameters
  - Shadowing decorrelation distance d_{cor}
  - Path loss exponent η
  - Reference Distance d_0

- Outputs
  - $\mathcal{I}$
  - $\hat{\theta}$

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- Outputs
  - $\mathcal{I}$
  - $\hat{\boldsymbol{\theta}}$
- Parameters
  - Candidate locations $\mathcal{Z} = \{\mathbf{z}_i\}_{i=1}^{N}$
  - Forgetting factor parameter λ

Algorithm Input and Output

- Inputs from sensor network
  - $\phi \in \mathbb{R}^{N_r \times 1}$
  - $\{\boldsymbol{y}_r\}_{r=1}^{N_r}$
- Knowledge of environment parameters
  - d_{cor}
  - η
  - d_0

- Outputs
  - $\boldsymbol{\mathcal{I}}$
  - $\hat{\boldsymbol{\theta}}$
- Parameters
  - $\mathcal{Z} = \{\boldsymbol{z}_i\}_{i=1}^{N}$
  - λ

Greedy Algorithm

Input: ϕ_{dB},$\mathcal{Y}$,$\mathcal{Z}$ measurements, sensor locations, candidate set
Output: $\mathcal{I}$,$\tilde{\boldsymbol{\theta}}$ index set, power estimates

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Input: ϕ_{dB},$\mathcal{Y}$,$\mathcal{Z}$ measurements, sensor locations, candidate set
Output: $\mathcal{I}$,$\tilde{\boldsymbol{\theta}}$ index set, power estimates
Initialization: $k \leftarrow 1$, $\mathcal{I} = \{\}$, $\tilde{\boldsymbol{\theta}} = \mathbf{0}$,
$\Omega_0 \leftarrow 1, 2, \ldots, N$

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Repeat

$\mathcal{I}(k) \leftarrow \min_{l \in \Omega_{k-1}} L_k(l, \tilde{\theta}_l)$ best among eligible candidates

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Repeat

$\mathcal{I}(k) \leftarrow \min_{l \in \Omega_{k-1}} L_k(l, \tilde{\theta}_l)$ best among eligible candidates

$\boldsymbol{\theta}(\mathcal{I}(k)) \leftarrow \tilde{\Theta}_{\mathcal{I}(k)}$ power estimate

Greedy Algorithm

Input: ϕ_{dB},$\mathcal{Y}$,$\mathcal{Z}$ measurements, sensor locations, candidate set
Output: $\mathcal{I}$,$\tilde{\boldsymbol{\theta}}$ index set, power estimates
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$k \leftarrow k + 1$

Greedy Algorithm

Input: ϕ_{dB},$\mathcal{Y}$,$\mathcal{Z}$ — measurements, sensor locations, candidate set

Output: $\mathcal{I}$,$\tilde{\boldsymbol{\theta}}$ — index set, power estimates

Initialization: $k \leftarrow 1$, $\mathcal{I} = \{\}$, $\tilde{\boldsymbol{\theta}} = \mathbf{0}$,

$\Omega_0 \leftarrow 1, 2, \ldots, N$

Repeat

$\mathcal{I}(k) \leftarrow \min_{l \in \Omega_{k-1}} L_k(l, \tilde{\theta}_l)$ — best among eligible candidates

$\boldsymbol{\theta}(\mathcal{I}(k)) \leftarrow \tilde{\Theta}_{\mathcal{I}(k)}$ — power estimate

$k \leftarrow k + 1$

$\Omega_k \leftarrow \{l : L_k(l, \tilde{\theta}_l) < L_k(l, 0)\}$ — set of eligible candidates

Until $\Omega_k = \{\}$ — stop: no candidate improves likelihood

Stage 2: Shadowing Component Estimation

- Remove path loss component from the measurements to obtain shadowing observations

$$\hat{\hat{\xi}}_{\mathrm{dB}} = \phi_{\mathrm{dB}} - 10\log_{10}(\boldsymbol{A}_{\mathcal{I}}\tilde{\boldsymbol{\theta}})$$

- Mean square error optimal estimate of shadowing is obtained by *Simple Kriging*
- Linear interpolation

$$\tilde{\xi}_{\mathrm{dB}}(\boldsymbol{u}) = \sum_{r=1}^{N_r} G_r \hat{\hat{\xi}}_{\mathrm{dB}}(r) \tag{5}$$

Shadowing Component Estimation

- Interpolation weights are given by Wiener-Hopf equation
- Estimate at a point $\boldsymbol{u}$ is

$$\tilde{\xi}_{\mathrm{dB}}(\boldsymbol{u}) = \gamma_{\boldsymbol{u}\mathcal{Y}}^{\mathsf{T}}(\sigma^2\boldsymbol{\Gamma})^{-1}\hat{\hat{\boldsymbol{\xi}}}_{\mathrm{dB}} \tag{6}$$

$\gamma_{\boldsymbol{u}\mathcal{Y}} \in \mathbb{R}^{N_r \times 1}$, $\gamma_{\boldsymbol{u}\mathcal{Y}}(i) = \sigma^2 e^{-\Delta \|\boldsymbol{y}_i - \boldsymbol{u}\|/d_{\mathrm{cor}}}$

- Lower bound on MSE error

$$\epsilon(\boldsymbol{u}) = \sigma^2 - \gamma_{\boldsymbol{u}\mathcal{Y}}^{\mathsf{T}}(\sigma^2\boldsymbol{\Gamma})^{-1}\gamma_{\boldsymbol{u}\mathcal{Y}} \tag{7}$$

Power Map Estimate

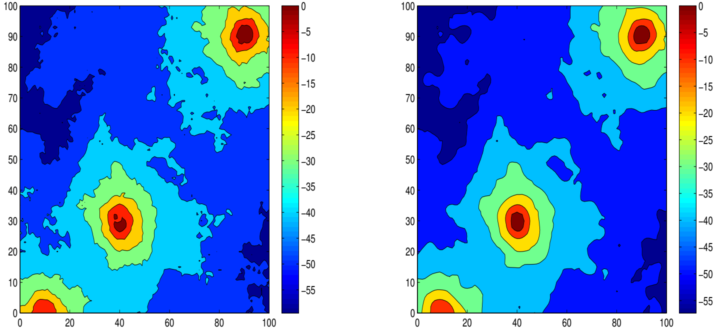
- Combine shadowing estimate with path loss estimate at all points to obtain the estimated power map

$$\tilde{\phi}(\boldsymbol{u}) = 10\log_{10}\left(\sum_{s\in\mathcal{I}} \tilde{\boldsymbol{\theta}}(s)\rho\left(\|\boldsymbol{u}-\boldsymbol{z}_s\|\right)\right) + \frac{1}{\sigma^2}\gamma_{\boldsymbol{u}\boldsymbol{\mathcal{Y}}}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\hat{\hat{\boldsymbol{\xi}}}_{\mathrm{dB}} \tag{8}$$

Simulation Setup

- 100 m $\times$ 100 m
- 3 sources transmitting at unit power
- Path loss model: $\min\{1, \Delta/d^{\eta}\}$
  - Path loss exponent $\eta = 4$
  - Reference parameter $\Delta = 60\text{m}^4$
- Shadowing decorrelation distance $d_{\text{cor}} = 25$ m
- Grid spacing = 10 m

Reconstructed Map



(a) Power map generated by 3 sources

(b) Power distribution recovered by the algorithm with $N_r = 1000$, $N = 121$

Figure: Power distribution over an area of $10^4 \mathrm{m}^2$, $\sigma = 4\mathrm{dB}$

Algorithm Performance

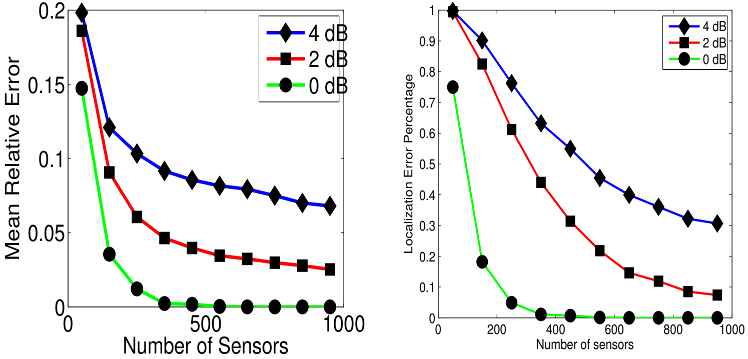


Figure: Performance of algorithm with varying number of sensors with $\lambda = 1$ and $N = 121$

Forgetting Factor λ

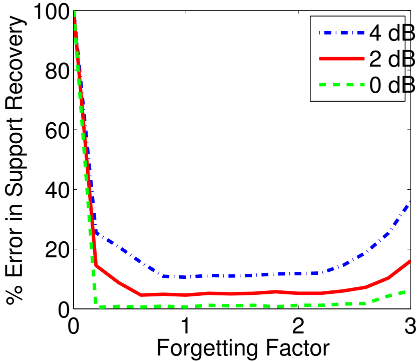


Figure: Performance of algorithm with varying forgetting factor λ with $N_r = 1000$ with no shadowing component

Error Analysis

- Interpolation problems - Mean and mean square error at any arbitary point in the operational area
- Difficulty: Complicted sequential nature of greedy algorithm
- Analysis under simplified assumptions
- Lower bound on statistics of error

Simplifying Assumptions

- Assumptions:
  1. Algorithm picks the candidate location closer to each of the actual transmitter location
  2. Transmit power estimates are exactly equal to actual transmit power when shadowing is absent
- Error can be due to
  1. Quantization of source location
  2. Shadowing term

Defintions

- Grid dimensions: ϑ_1, $\vartheta 2$
- Parameter vector: $\boldsymbol{\Upsilon} \in \mathbb{R}^{3N_s \times 1}$
  $\boldsymbol{\Upsilon} \triangleq \begin{bmatrix} P_1 & \boldsymbol{x}_1^{\mathsf{T}} & P_2 & \boldsymbol{x}_2^{\mathsf{T}} & \dots & P_{N_r} & \boldsymbol{x}_{N_r}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}}$

Notations

- $\kappa = \frac{10}{\ln 10}$
- $\boldsymbol{B}_s \triangleq \frac{\boldsymbol{F}_{\iota_s}\boldsymbol{\Gamma}^{-1}\boldsymbol{F}_{\iota_s}}{\boldsymbol{1}^\mathsf{T}\boldsymbol{F}_{\iota_s}\boldsymbol{\Gamma}^{-1}\boldsymbol{F}_{\iota_s}\boldsymbol{1}}$, where ι_s is the index of candidate location corresponding to s$^{\text{th}}$ transmitter.
- $\zeta_s \triangleq \acute{\kappa}\frac{\eta}{2}\begin{bmatrix}\frac{\boldsymbol{y}_1-\boldsymbol{x}_S}{\|\boldsymbol{y}_1-\boldsymbol{x}_S\|^2} & \frac{\boldsymbol{y}_2-\boldsymbol{x}_S}{\|\boldsymbol{y}_2-\boldsymbol{x}_S\|^2} & \cdots & \frac{\boldsymbol{y}_{N_R}-\boldsymbol{x}_S}{\|\boldsymbol{y}_{N_R}-\boldsymbol{x}_S\|^2}\end{bmatrix}^\mathsf{T} \in \mathbb{R}^{N_R\times 2}$, for $s = 1, 2, \ldots, N_s$
- $\boldsymbol{R} \triangleq \text{bdaig}\left\{\boldsymbol{R}_s^\mathsf{T}\boldsymbol{R}_s, 1 \leq s \leq N_S\right\} \in \mathbb{R}^{3N_S\times 3N_S}$ with $\boldsymbol{R}_S = \begin{bmatrix}2\zeta_s^\mathsf{T}\boldsymbol{B}_s\boldsymbol{1} & \boldsymbol{I}\end{bmatrix} \in \mathbb{R}^{2\times 3}$.
- $\boldsymbol{M} = \begin{bmatrix}\boldsymbol{B}_1\boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{B}_2\boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} & \ldots & \boldsymbol{B}_{N_S}\boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}^\mathsf{T} \in \mathbb{R}^{3N_S\times N_R}$

Greedy Algorithm Performance

- Under the above stated assumptions and when $\vartheta_1 = \vartheta_2 = \vartheta$
  1. Estimates are unbiased
  2. Covariance matrix $\boldsymbol{C} = \frac{\vartheta^2}{12}\boldsymbol{R} + \sigma^2 \boldsymbol{M}\boldsymbol{\Gamma}\boldsymbol{M}^\mathsf{T}$
- Two terms representing to quantization error and error due to shadowing.
- Smaller grid size and shadowing variance result in better performance.

- When $\vartheta_1 \neq \vartheta_2$ transmit power etimates are unbiased
- Bias of s$^{\text{th}}$ transmitter is

$$\mathbb{E}\left\{\Delta P_s\right\} = \acute{\kappa}\frac{\eta}{24}(\vartheta_1^2 - \vartheta_2^2)\mathbf{1}^{\mathsf{T}}\boldsymbol{B}_s\boldsymbol{\tau}, \qquad (9)$$

where $\boldsymbol{\tau} \in \mathbb{R}^{N_R \times 1}$, $\boldsymbol{\tau}_r = \frac{\left(\boldsymbol{y}_{r,1} - \boldsymbol{x}_{s,1}\right)^2 - \left(\boldsymbol{y}_{r,2} - \boldsymbol{x}_{s,2}\right)^2}{\|\boldsymbol{y}_r - \boldsymbol{x}_S\|^4}$

Cramér Rao Bound

CRB $= \sigma^2 \left(\boldsymbol{D}^{\mathsf{T}} \boldsymbol{\Gamma}^{-1} \boldsymbol{D} \right)^{-1}$, where $\boldsymbol{D} \in \mathbb{R}^{N_R \times 3N_S}$ with

$$\boldsymbol{D}_{ij} = d_0 \frac{10^{\frac{1}{10}[P_s - \mu_i(\boldsymbol{\Upsilon})]}}{\|\boldsymbol{y}_i - \boldsymbol{x}_s\|^{\eta}} \begin{cases} 1 & \text{for } j \bmod 3 = 1 \\ \acute{\kappa}\eta \frac{(\boldsymbol{y}_{i,1} - \boldsymbol{x}_{s,1})}{\|\boldsymbol{y}_i - \boldsymbol{x}_s\|^2} & \text{for } j \bmod 3 = 2, \; s = \left\lceil \frac{j}{3} \right\rceil \\ \acute{\kappa}\eta \frac{(\boldsymbol{y}_{i,2} - \boldsymbol{x}_{s,2})}{\|\boldsymbol{y}_i - \boldsymbol{x}_s\|^2} & \text{for } j \bmod 3 = 0, \end{cases}$$

Cramér Rao Bound

- For any unbiased estimator,
  - $\mathbb{E}\left\{\|\boldsymbol{x}_s - \hat{\boldsymbol{x}}_s\|^2\right\} \geq \left[\boldsymbol{J}^{-1}\right]_{3s-1,3s-1} + \left[\boldsymbol{J}^{-1}\right]_{3s,3s}$
  - $\mathbb{E}\left\{\left(P_s - \hat{P}_s\right)^2\right\} \geq \left[\boldsymbol{J}^{-1}\right]_{3s-2,3s-2}$
- No efficient estimator exists
  - Signal model is not affine in the parameters

Spectral map estimation error - Sensor Location

- There is an exact match between measured value and reconstructed map at sensor locations

$$\hat{\phi}(\boldsymbol{y}_r) = \mu(\boldsymbol{y}_r; \hat{\boldsymbol{\Upsilon}}) + \boldsymbol{g}_{\boldsymbol{y}_r}\hat{\boldsymbol{\xi}} \quad (10)$$

$$= \mu(\boldsymbol{y}_r; \hat{\boldsymbol{\Upsilon}}) + \hat{\xi}[r] \quad (11)$$

$$= \phi[r] \quad (12)$$

More Defintions

- Gradient of the path loss component,
  $$\boldsymbol{d}_{\boldsymbol{u}} = \left(\frac{d\mu(\boldsymbol{u};\hat{\boldsymbol{\Upsilon}})}{d\hat{\boldsymbol{\Upsilon}}}\right)_{\hat{\boldsymbol{\Upsilon}}=\boldsymbol{\Upsilon}} = \begin{bmatrix}\boldsymbol{d}_{\boldsymbol{u}1} & \boldsymbol{d}_{\boldsymbol{u}2} & \dots & \boldsymbol{d}_{\boldsymbol{u}N_S}\end{bmatrix} \in \mathbb{R}^{3N_S\times 1}$$
  with $\boldsymbol{d}_{\boldsymbol{us}} = \mathring{m}_{\boldsymbol{us}} \begin{bmatrix}1 & \frac{\eta\acute{\kappa}(\boldsymbol{u}-\boldsymbol{x}_s)^{\mathsf{T}}}{\|\boldsymbol{u}-\boldsymbol{x}_s\|^2}\end{bmatrix}^{\mathsf{T}} \in \mathbb{R}^{3\times 1}$ and
  $$\mathring{m}_{\boldsymbol{us}} = d_0 \frac{10^{\frac{1}{10}[P_s-\mu(\boldsymbol{u};\boldsymbol{\Upsilon})]}}{\|\boldsymbol{u}-\boldsymbol{x}_s\|^{\eta}}$$
- $\boldsymbol{K}_{\boldsymbol{u}} = \mathrm{bdiag}\left\{\frac{1}{\mathring{m}_{\boldsymbol{us}}}\boldsymbol{d}_{\boldsymbol{us}}\boldsymbol{d}_{\boldsymbol{us}}^{\mathsf{T}}, 1 \le s \le N_S\right\}$

Map Error at an arbitrary point $\boldsymbol{u}$

- Mean error is
$\frac{1}{2}\text{tr}\left\{\boldsymbol{C}\left(\boldsymbol{K_u} - \sum_{r=1}^{N_R}\boldsymbol{g_u}[r]\boldsymbol{K_{y_r}}\right)\right\}$
$-\frac{1}{2\acute{\kappa}}\{\boldsymbol{d}_{\boldsymbol{u}}^{\mathsf{T}}\boldsymbol{C}\boldsymbol{d_u} + \frac{1}{2\acute{\kappa}}\boldsymbol{g}_{\boldsymbol{u}}^{\mathsf{T}}\text{diag}\left\{\boldsymbol{D}\boldsymbol{C}\boldsymbol{D}^{\mathsf{T}}\right\}$
- Mean square error
$\left(\boldsymbol{d_u} - \boldsymbol{D}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\gamma_{\boldsymbol{u}}\right)^{\mathsf{T}}\boldsymbol{C}\left(\boldsymbol{d_u} - \boldsymbol{D}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\gamma_{\boldsymbol{u}}\right) + \sigma^2\left(1 - \gamma_{\boldsymbol{u}}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\gamma_{\boldsymbol{u}}\right).$

Special Cases

- Single Transmitter
  - Unbiased estimators
- Shadowing is uncorrelated
  - Mean error $= \frac{1}{2}\text{tr}\left\{\boldsymbol{C}\boldsymbol{K}_{\boldsymbol{u}}\right\} - \frac{1}{2\acute{\kappa}}\boldsymbol{d}_{\boldsymbol{u}}^{\mathsf{T}}\boldsymbol{C}\boldsymbol{d}_{\boldsymbol{u}}$,
  - Mean square error $= \boldsymbol{d}_{\boldsymbol{u}}^{\mathsf{T}}\boldsymbol{C}\boldsymbol{d}_{\boldsymbol{u}} + \sigma^2$ for $\forall \boldsymbol{u} \notin \{\boldsymbol{y}_r\}_{r=1}^{N_R}$

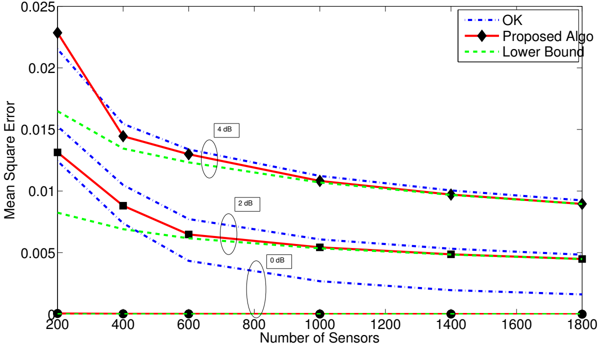


Figure: Performance of algorithm with varying number of sensors N_r for shadowing variances $\sigma = 0$ dB, 2 dB and 4 dB

Conclusion

- Goal: Reconstruction of spatial power map using power measurements at N_r sensors at known locations
- Unknown number of transmitters, transmit powers, Lognormal shadowing
- Proposed a 2-step solution:
  1. Greedy algorithm for ML estimation of transmitter locations and powers
  2. Exploit spatial correlation to reconstruct power map using Kriging
- Bounds on error in estimation and Cramér Rao Bound for estimation problem