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Geethu Joseph

SPC Lab, IISc

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Blind Deconvolution Using Convex Programming

Ali Ahmed, Benjamin Recht and Justin Romberg

- Problem: Recovering two unknown vectors, $\boldsymbol{w}$ and $\boldsymbol{x}$ from their circular convolution.
- Structural assumptions: $\boldsymbol{w}, \boldsymbol{x} \in \mathbb{R}^L$ are members of known subspaces whose dimensions are K and N

$$\boldsymbol{w} = \boldsymbol{B}\boldsymbol{h}, \quad \boldsymbol{x} = \boldsymbol{C}\boldsymbol{m}.$$

- Observations: $\boldsymbol{y} = \boldsymbol{w} * \boldsymbol{x}, \quad \boldsymbol{y}[l] = \sum_{l'=1}^{L} \boldsymbol{w}[l']\boldsymbol{x}[L-l']$.
- DFT of $\boldsymbol{y}$ is related to $\boldsymbol{h}\boldsymbol{m}^*$ through a linear transform $\mathcal{A}$ which depends on $\boldsymbol{B}$ and $\boldsymbol{C}$,

$$\hat{\boldsymbol{y}} = \boldsymbol{F}\boldsymbol{y} = \mathcal{A}(\boldsymbol{h}\boldsymbol{m}^*)$$

- A linear inverse problem over the (non-convex) set of rank-1 matrices.

Blind Deconvolution Using Convex Programming

Approach and Results

- Convexify the problem using the nuclear norm, as a proxy for rank, $\min \|\boldsymbol{X}\|_* \quad \text{subject to } \hat{\boldsymbol{y}} = \mathcal{A}(\boldsymbol{X}).$
- Condition for the unique solution is $\boldsymbol{hm}^*$ when
  1. $\boldsymbol{B}$ is an arbitrary deterministic $L \times K$ matrix with orthonormal columns,
  2. entries of $\boldsymbol{C}$ are iid, $\boldsymbol{C}[l, n] \sim \mathcal{N}(0, L^{-1})$,

  - maximum of N and K is almost on the order of L
  - basis vectors of $\boldsymbol{w}$ are spread out in the frequency domain
- Guarantees for recovery in the presence of noise, for the above model.
- Application: Multipath Channel Protection Using Random Codes, $L \log^3 L \gtrsim N + K$.

On the Theorem of Uniform Recovery of Random Sampling Matrices

Joel Andersson and Jan-Olov Strömberg

- Suppose the RIC of order $2s$, δ_{2s} of a sensing matrix $\boldsymbol{A} \in \mathcal{C}^{m\times N}$satisfies

$$\delta_{2s} < \frac{4}{\sqrt{41}} \approx 0.62$$

  every s-sparse vector $\boldsymbol{x} \in \mathcal{C}^N$ is recovered by l_1-minimization.
- This is an improvement of the best known result, which had $\delta_{2s} < 0.4931$.
- Improved version of result which is not linear in s,

$$\delta_{2s} = \begin{cases} \max\left\{\sqrt{\frac{4-5/s}{9-5/s}}, \frac{4}{\sqrt{41}}\right\}, & 2 \leq s, 5 \bmod s \neq 0 \\ \frac{2}{3}, & 2 \leq s, 5 \bmod s \neq 0 \end{cases}$$

On the Theorem of Uniform Recovery of Random Sampling Matrices

- Let $\mathcal{D} \subset \mathbb{R}^d$, ν a probability measure on $\mathcal{D}$, $\{\psi_j\}_{j=1}^N$ is a bounded orthogonal system of complex-valued functions on $\mathcal{D}$ if $\forall j, k \in [N]$

$$\int_{\mathcal{D}} \psi_j(t)\overline{\psi_k(t)}d\nu(t) = \delta_{jk} \quad \text{and} \quad \|\psi_j\|_\infty \leq K.$$

- $\boldsymbol{A} \in \mathcal{C}^{m\times N}$ be a random sampling matrix associated to a bounded orthonormal system,
  - i.e., $a_{lk} = \psi_k(t_l)$ and $\{t_1, t_2, \ldots, t_m\} \subset \mathcal{D}$ are selected independently at random with respect to ν
  - Eg., random partial Fourier matrix, $a_{lk} = \frac{1}{\sqrt{N}}\exp\{-2\pi ilk/N\}$
- Conditions on m, K, n for guarantees on the effective RIC of $\boldsymbol{A}$, $\tilde{\delta}_s \leq \delta$ when $K \geq 1$.

Other Papers

- **A Diversity Analysis for Distributed Interference Alignment Using the Max-SINR Algorithm**
  - T. Xu and X.-G. Xia
- **Interference Channels With Half-Duplex Source Cooperation**
  - R. Wu, V. M. Prabhakaran, P. Viswanath, and Y. Wang
- **Degrees of Freedom for a MIMO Gaussian K-Way Relay Channel: Successive Network Code Encoding and Decoding**
  - N. Lee and J. Chun
- **Efficient Capacity Computation and Power Optimization for Relay Networks**
  - F. Parvaresh and R. Etkin
- **Delay-Limited Cooperative Communication With Reliability Constraints in Wireless Network**
  - R. Urgaonkar and M. J. Neely