# Infinite Diversity Order Techniques Using CSIT

T. Ganesan

gana@ti.com

SPC Lab

Jan 5th, 2013

# Outline

1. Introduction
   - Tx Diversity
2. Tx Precoding with CSIT
   - Channel Inversion
   - Equivalent Channel
3. New Precoding
   - New Precoder 1
   - New Precoder 2
   - Multi-user Channels
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# Diversity and Multiplexing Gain

- Diversity and Multiplexing gain for a block fading Rayleigh fading MIMO system are defined as

$$d \triangleq \lim_{SNR \to \infty} \frac{-\partial \log P_e}{\partial \log SNR}$$

$$r \triangleq \lim_{SNR \to \infty} \frac{\partial R}{\partial \log SNR}$$

- Diversity can be obtained by either Rx diversity or Tx diversity or both.
  - E.g., MRC Rx diversity = $N_r$, MRT Tx diversity = $N_t$
- Rx diversity based schemes result in a maximum diversity of $N_r N_t$.
- Can we get better than $d = N_r N_t$ ?

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- Rx diversity based schemes result in a maximum diversity of $N_r N_t$.
- Can we get better than $d = N_r N_t$ ? Yes.

# Alamouti Scheme : I

- 2 Tx antenna is needed. Any number of Rx antennas is supported.
- For $i^{\text{th}}$ Rx antenna, the received vector is represented as

$$[y_{1i}\ y_{2i}] = [h_{1i}\ h_{2i}] \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + [n_{1i}\ n_{2i}]$$

where $[h_{1i}\ h_{2i}]$ represent the channel gains from 2 Tx ant. to $i^{\text{th}}$ Rx ant.

# Alamouti Scheme : II

- The decoder estimates the symbols as

$$\hat{x}_1 = h_{1i}^* y_{1i} + h_{2i} y_{2i}^*$$

$$\hat{x}_2 = h_{2i}^* y_{1i} - h_{1i} y_{2i}^*$$

# Alamouti Scheme : III

- The signal model can be equivalently written as

$$\begin{bmatrix} y_{1i} \\ y_{2i}^* \end{bmatrix} = \sqrt{\frac{k\rho}{2}} \begin{bmatrix} h_{1i} & h_{2i} \\ h_{2i}^* & -h_{1i}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{1i} \\ n_{2i}^* \end{bmatrix}, \tag{1}$$

$$\mathbf{y}' = \sqrt{\frac{k\rho}{2}} \tilde{\mathbf{H}} \mathbf{x} + \mathbf{n}'$$

- Equivalent channel matrix $\tilde{\mathbf{H}}$ is an orthogonal matrix [1].
- This scheme achieves full diversity $N_r N_t$.

[1] See Exercise 9.4 in [1]

# Tx Diversity Schemes

| Scenario | CSI Condition | Maximum Diversity Order d | References |
|---|---|---|---|
| SIMO | $CSIR$ | $N_r$ | [2] |
| SIMO | $CSIR, C\hat{S}IT$ | $2N_t, \infty^*$ | [3, 4]* |
| MISO | $CSIR$ | $N_t$ | [5, 6] |
| MISO | $C\hat{S}IR, C\hat{S}IT$ | $N_t^2(N_t^2 + N_t + 1) + N_t$ | [7] |
| MIMO | $CSIR$ | $N_t N_r$ | [8] |
| MIMO | $C\hat{S}IR$ | $N_t N_r \left[\frac{rT_c}{T_c - L_{tr}}\right]$ | [9] |
| MIMO | $CSIR, CSIT$ | $\infty$ | [10] |
| MIMO | $CSIR, C\hat{S}IT$ | $N_r N_t (N_r N_t + 2)$ | [10] |
| MIMO | $C\hat{S}IR, C\hat{S}IT$ | $2N_r N_t$ | [10] |

Table : Summary of maximum diversity order in Rayleigh fading MIMO Channels.

# Diversity-Multiplexing Gain Trade-off with CSIR

- Zheng and Tse [1] had shown that, there exists a trade-off between Diversity and Multiplexing gain that can be achieved.
- A certain combinations of $(r, d)$ is only possible for the given $(N_r, N_t)$ configuration.

$$(d, r) = (k, (N_t - k)(N_r - k)),\ k = 0, 1, \ldots, \min(N_r, N_t)$$

- Alamouti scheme operates at one of the operating points $(0, N_r N_t)$.
- To realize other operating points, new codes can be designed or Tx precoding can be done.

# Channel inversion in SIMO

- Consider the SIMO channel

$$\mathbf{y} = \sqrt{\frac{k\rho}{N_t}}\mathbf{h}x + \mathbf{n}$$

where $\rho$ refers to total Tx power, $k$ is a constant to ensure average power constraint.

- Since $\|\mathbf{h}\|^2$ is a $\chi^2_{2d}$ random variable, $\mathbb{E}\left[\frac{1}{\|\mathbf{h}\|^2}\right] = \frac{1}{2(d-1)}$ is finite.
- One can choose $k = \frac{1}{\|\mathbf{h}\|^2}$ to obtain equivalent AWGN channel at the Rx, $\Rightarrow d = \infty$.
  - This is not possible for SISO case since average Tx power is not finite.

# Channel inversion in MIMO

For the MIMO channel also one can invert the channel by choosing precoding $P$ matrix suitably. That is,

$$\mathbf{y} = \sqrt{\frac{\rho}{N_t}}\mathbf{H}\mathbf{P}\mathbf{x} + \mathbf{n}$$

Let $\mathbf{P} = \sqrt{\frac{N_t}{\rho}}\mathbf{H}^{\dagger}$ where $\mathbf{H}^{\dagger}$ refers to the pseudo-inverse of $\mathbf{H}$.

$$\mathbb{E}[\mathbf{x}^H\mathbf{P}^H\mathbf{P}\mathbf{x}] = \|\mathbf{x}\|^2\frac{tr[\mathbf{P}^H\mathbf{P}]}{\min(N_r, N_t)} = \frac{\|\mathbf{x}\|^2}{\min(N-r, N_t)}\mathbb{E}\left[\sum_i \frac{1}{\sigma_i^2}\right]$$

which is not finite for Rayleigh block fading channel since the smallest eigenvalue of $\mathbf{H}^H\mathbf{H}$ is $\chi_2^2$ distributed.

NOTE: Mean value of inverse of $\chi_2^2$ is not finite.

# Orthogonality of $\tilde{\mathbf{H}}$ for $\mathbf{h} \in \mathbb{R}^{N_t}$ : I

**Lemma 1**

*The equivalent channel matrix constructed for real square O-STBC is orthogonal.*

**Proof:** The two equivalent representations of the received vector $\mathbf{y}$ in terms of $\mathbf{h}$ and $\mathbf{x}$ can be written as

$$\mathbf{y} = \sqrt{\frac{k\rho}{N_t}}\mathbf{X}\mathbf{h} = \sqrt{\frac{k\rho}{N_t}}\tilde{\mathbf{H}}\mathbf{x} \tag{2}$$

# Orthogonality of $\tilde{\mathbf{H}}$ for $\mathbf{h} \in \mathbb{R}^{N_t}$ : II

Multiplying by $\mathbf{X}^T$ on both sides, we get

$$\alpha \mathbf{h} = \mathbf{X}^T \tilde{\mathbf{H}} \mathbf{x}$$

where $\mathbf{X}^T \mathbf{X} = \alpha \mathbf{I}$.

- There exists a linear transformation between $\mathbf{h}$ and $\mathbf{x}$ which indicates that the columns of $\tilde{\mathbf{H}}$ are linearly independent.

# Orthogonality of $\tilde{\mathbf{H}}$ for $\mathbf{h} \in \mathbb{R}^{N_t}$ : III

- Due to the structure of O-STBC codes, $\mathbf{X}_1$ and $\mathbf{X}_2$ are orthogonal matrices by construction. Moreover, it can be shown that $\mathbf{x}_{2,j}^T \mathbf{x}_{1,i} = -\mathbf{x}_{2,i}^T \mathbf{x}_{1,j}$ and $\mathbf{x}_{1,i}^T \mathbf{x}_{2,i} = \mathbf{x}_{1,j}^T \mathbf{x}_{2,j}$ [11]. That is,

$$\mathbf{y}_1^T \mathbf{y}_2 = \mathbf{h}^T \mathbf{X}_1^T \mathbf{X}_2 \mathbf{h} = \sum_i \sum_j h_i h_j \mathbf{x}_{1,i}^T \mathbf{x}_{2,j} = \sum_i h_i^2 \mathbf{x}_{1,i}^T \mathbf{x}_{2,i} = \mathbf{x}_{1,i}^T \mathbf{x}_{2,i} \sum_i h_i^2, \tag{3}$$

$$\mathbf{y}_1^T \mathbf{y}_2 = \mathbf{x}_1^T \left( \tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right) \mathbf{x}_2. \tag{4}$$

# Orthogonality of $\tilde{\mathbf{H}}$ for $\mathbf{h} \in \mathbb{R}^{N_t}$ : IV

- Eqns. (4) and (5) are equal, if and only if $\tilde{\mathbf{H}}$ is orthogonal and $\mathbf{h}_i^T \mathbf{h}_i = \mathbf{h}_j^T \mathbf{h}_j$.
- Note that, the converse part is true since one of the columns of $\mathbf{X}_1$ is same as $\mathbf{x}_1$ and one of the columns of $\mathbf{X}_2$ is same as $\mathbf{x}_2$.

■

# Proposed Precoder: Assumptions

- Only block fading Rayleigh channels are considered.
- CSI is available only at the Tx.
- Signaling is assumed to use orthogonal STBC using symbols from unit energy constellations.
- $N_t > 3$ for real O-STBC cases.

# Proposed Precoder 1 : I

## Tx Precoder-1 for $N_r = 1$

The precoding matrix that converts the $N_t \times 1$ MISO channel into fixed gain SISO AWGN channel is given by

$$\mathbf{P} = \frac{1}{\alpha}\tilde{\mathbf{H}}^H,$$

where $\alpha$ is related to $\tilde{\mathbf{H}}$ as $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H = \alpha\mathbf{I}$.

- Recall that the effective channel matrix $\tilde{\mathbf{H}}$ is orthogonal.
- Use $\mathbf{P} = \tilde{\mathbf{H}}^H$. That is,

$$\hat{\mathbf{x}} = \sqrt{\frac{k\rho}{N_t}}\tilde{\mathbf{H}}\mathbf{P}\mathbf{x} + \mathbf{n} = \sqrt{\frac{k\rho}{N_t}}\mathbf{x} + \mathbf{n} \quad (5)$$

where $\alpha = |h_1|^2 + |h_2|^2 + \ldots + |h_d|^2$ is a $\chi^2_{2d}$ random variable.

# Average Tx Power Constraint : I

- The average Tx power can be written as

$$P_{\text{avg}} = \frac{k\rho}{N_t}\,\mathbb{E}_{\mathbf{h},\mathbf{x}}\left[\mathbf{x}^H\mathbf{P}\mathbf{P}^H\mathbf{x}\right].$$

$$P_{\text{avg}} = \frac{k\rho}{N_t}\,\mathbb{E}_{\mathbf{x}}\left[\mathbf{x}^H\mathbf{x}\right]\;\mathbb{E}_{\mathbf{x}}\left[\frac{1}{\alpha}\right].$$

- It can be shown that

$$\mathbb{E}_{\mathbf{h}}\left[\frac{1}{\alpha}\right] = \frac{1}{N_t - 2},\ \text{for } N_t > 2.$$

- Hence, one can pick $k = (N_t - 2)$ and get $P_{\text{avg}} = \rho$.

# Proposed Precoder 1 for Multiple Rx Antennas : I

**Tx Precoder-1 for $N_r > 1$**

Adopt Antenna selection at the Rx and use the selected (say $i^{\text{th}}$) MISO channel $\mathbf{h}_i$ to compute the precoding matrix $\mathbf{P}$.

- The average Tx power can be written as

$$P_{\text{avg}} = \frac{k\rho}{N_t} \, \mathbb{E}_{\mathbf{x}} \left[\mathbf{x}^H \mathbf{x}\right] \; \mathbb{E}_{\mathbf{x}} \left[\frac{1}{\alpha}\right],$$

where $\alpha = \max[\alpha_1, \alpha_2, \ldots, \alpha_{N_r}]$

# Proposed Precoder 1 for Multiple Rx Antennas : II

- It can be shown that

$$\mathbb{E}\left[\frac{1}{\alpha}\right] = \begin{cases} \frac{2^{1-N_t}}{\Gamma\left(\frac{N_t}{2}\right)} \sum_{m=0}^{\infty} \frac{\Gamma(N_t-1+m)}{2^m \Gamma\left(\frac{N_t}{2}+m\right)}, & \text{for real } \mathbf{h}_i \\ \frac{2^{2-2N_t}}{\Gamma(N_t)} \sum_{m=0}^{\infty} \frac{\Gamma(2N_t-1+m)}{2^m \Gamma(N_t+m)}, & \text{for complex } \mathbf{h}_i \end{cases}. \quad (6)$$

- Hence, one can pick $k = 1/\mathbb{E}\left[\frac{1}{\alpha}\right]$ and get $P_{\text{avg}} = \rho$.

# Proposed Precoder 2 : I

The model for the signal at the receiver is given by

$$y = \sqrt{\frac{k\rho}{N_t}}\mathbf{h}^H\mathbf{P}\tilde{\mathbf{x}} + n, \tag{7}$$

# Proposed Precoder 2 : II

## Tx Precoder-2 for $N_r = 1$

Assuming $N_t \geq 2$, the precoding matrix that converts the $N_t \times 1$ MISO channel $\mathbf{h}$ into fixed gain SISO AWGN channel is given by

$$\mathbf{P} = \mathbf{Q}\mathbf{U}$$

where $\mathbf{Q}$ is related to $\mathbf{H}$ as $\mathbf{h} = \mathbf{Q}\mathbf{R}$, $\mathbf{U}$ is an arbitrary non-diagonal unitary matrix and $\tilde{\mathbf{x}}$ is chosen such that $R^H\mathbf{U}\tilde{\mathbf{x}} = \mathbf{x}$, where $\mathbf{x} \in \mathbb{C}$ is the data vector to be sent.

# Average Tx Power Constraint : I

- The average Tx power can be written as

$$
\begin{aligned}
P_{\text{avg}} &= \frac{k\rho}{N_t}\ \mathbb{E}_{\mathbf{x},\mathbf{h}}\left[\mathbf{x}^H\mathbf{x} + \mathbf{x}^H(\mathbf{I}-\mathbf{R}_{u1})^H\mathbf{R}_{u2}^{-H}\mathbf{R}_{u2}^{-1}(\mathbf{I}-\mathbf{R}_{u1})\mathbf{x}\right] \\
&= \frac{k\rho}{N_t}\ \left\{tr\left(\mathbb{E}_{\mathbf{x}}[\mathbf{x}\mathbf{x}^H]\right) + tr\left(\mathbb{E}_{\mathbf{h}}\left[\mathbf{R}_{u2}^{-1}(\mathbf{I}-\mathbf{R}_{u1})\mathbb{E}_{\mathbf{x}}[\mathbf{x}\mathbf{x}^H](\mathbf{I}-\mathbf{R}_{u1})^H\mathbf{R}\right.\right.\right. \\
&= \frac{k\rho}{N_t}\ \left\{N_r + tr\left(\mathbb{E}_{\mathbf{h}}\left[\mathbf{R}_{u2}^{-1}(\mathbf{I}-\mathbf{R}_{u1})(\mathbf{I}-\mathbf{R}_{u1})^H\mathbf{R}_{u2}^{-H}\right]\right)\right\}.
\end{aligned}
$$

# Average Tx Power Constraint : II

- For simplicity, we can choose $\mathbf{U}_{11} = \mathbf{I}_{N_r}/\sqrt{N_t}$ and $\mathbf{U}_{12} = -\mathbf{I}_{N_r}/\sqrt{N_t}$.

  Now, we get

$$
\begin{aligned}
& tr\left(\mathbb{E}_{\mathbf{h}}\left[\mathbf{R}_{u2}^{-1}(\mathbf{I}-\mathbf{R}_{u1})(\mathbf{I}-\mathbf{R}_{u1})^H\mathbf{R}_{u2}^{-H}\right]\right) \\
&= \mathbb{E}_{\mathbf{h}}\left[\frac{1}{N_r} tr\left(\mathbf{R}_1^{-H}\mathbf{R}_1^{-1}\right) - \frac{2}{\sqrt{N_r}} tr\left(\mathbf{R}_1^{-1}\right) + N_r\right] \\
&= tr\left(\mathbb{E}_{\mathbf{h}}\left[\mathbf{R}_1^{-H}\mathbf{R}_1^{-1}\right]\right) - 2tr\left(\mathbb{E}_{\mathbf{h}}\left[\mathbf{R}_1^{-1}\right]\right) + N_r. \qquad (9)
\end{aligned}
$$

# Average Tx Power Constraint : III

- Using Lemma 6 in [12] and approximating the diagonal elements of $\mathbf{R}_1$ as $\chi_{N_t}-$distributed random variables, we get

$$P_{\text{avg}} \leq \frac{k\rho}{N_t} \left[ 2N_r + 1 - 2N_r \frac{\Gamma(N_t - 0.5)}{\Gamma(N_t)} \right]. \tag{10}$$

- And, $k$ can be chosen as

$$k \geq \frac{N_t}{\left[ 2N_r + 1 - 2N_r \frac{\Gamma(N_t - 0.5)}{\Gamma(N_t)} \right]}, \tag{11}$$

to satisfy the average transmit power constraint.

# Proposed Precoder 2 for $Nr > 1$ : I

- The model for the signal at the receiver is given by

$$\mathbf{y} = \sqrt{\frac{k\rho}{N_t}}\mathbf{H}^H\mathbf{P}\tilde{\mathbf{x}} + \mathbf{n}, \tag{12}$$

# Proposed Precoder 2 for $Nr > 1$ : II

**Tx Precoder-2 for $N_r > 1$**

Assuming $N_t \geq 2$, the precoding matrix that converts the $N_t \times 1$ MISO channel $\mathbf{h}$ into fixed gain $N_r$ parallel AWGN channels is given by

$$\mathbf{P} = \mathbf{QU}$$

where $\mathbf{Q}$ is related to $\mathbf{H}$ as $\mathbf{h} = \mathbf{QR}$, $\mathbf{U}$ is an arbitrary non-diagonal unitary matrix and $\tilde{\mathbf{x}}$ is chosen such that $R^H \mathbf{U} \tilde{\mathbf{x}} = \mathbf{x}$, where $\mathbf{x} \in \mathbb{C}^{N_r}$ is the data vector to be sent.

# Proposed Precoder 2 for $Nr > 1$ : III

- The average power constraint can be obtained by choosing $k$ such that

$$k = \frac{N_t}{N_r \left\{ N_t + tr\left( \mathbb{E}_{\mathbf{h}}\left[ \mathbf{R}_{u2}^{-1}(\mathbf{I} - \mathbf{R}_{u1})(\mathbf{I} - \mathbf{R}_{u1})^H \mathbf{R}_{u2}^{-H} \right] \right) \right\}}, \quad (13)$$

# MAC Channel with $N_r = 1$ and Scheme-1 : I

- For $M$ user MAC channel, the received signal $\mathbf{y} \in \mathbb{R}^L$ can be written as

$$\mathbf{y} = \sum_{i=1}^{M} \sqrt{\frac{k\rho_i}{N_t}} \tilde{\mathbf{H}}^{(i)} \mathbf{P}^{(i)} \mathbf{x}_i + \mathbf{n}, \tag{14}$$

where $\mathbf{n} \in \mathbb{C}^L$ , $\mathbf{x}_i \in \mathbb{R}_i^L$ is the O-STBC data vector, and $\rho_i$ denotes the average transmit power from the $i^{\text{th}}$ user, $\mathbf{P}^{(i)}$ denotes the precoding matrix employed by the $i^{\text{th}}$ transmitter corresponding to its channel to the receiver, $\tilde{\mathbf{H}}^{(i)}$ is the equivalent channel matrix.

# MAC Channel with $N_r = 1$ and Scheme-2 : I

- For $M$ user MAC channel, the received signal $y \in \mathbb{C}$ can be written as

$$y = \sum_{i=1}^{M} \sqrt{\frac{k\rho_i}{N_t}} \mathbf{h}_i^H \mathbf{P}_i \tilde{\mathbf{x}}_i + n, \tag{15}$$

where $\mathbf{h}_i \in \mathbb{C}^{N_t}$ denotes the channel of the $i^{\text{th}}$ user , $\tilde{\mathbf{x}}_i \in \mathbb{C}^{N_t}$ denotes an extended data vector, and $\mathbf{P}_i \in \mathbb{C}^{N_t \times N_t}$ denote the precoding matrix from $i^{\text{th}}$ user.

# Broadcast Channel with $N_r = 1$ and Scheme-2 : I

- For $M$ user broadcast channel, let $\mathbf{x} = [\sqrt{\rho_1}s_1 \sqrt{\rho_2}s_2 \ldots \sqrt{\rho_M}s_M]^T$ denote the vector containing the messages intended to the $M$ users
- $\sum_i \rho_i = \rho$,
- $s_i$ comes from a constellation with unit energy.
- $\tilde{\mathbf{x}} \in \mathbb{C}^{N_t}$ denote an extended message vector, derived from $\mathbf{x} \in \mathbb{C}^M$

# Broadcast Channel with $N_r = 1$ and Scheme-2 : II

- Hence, one can write the signal model as

$$\mathbf{y} = \sqrt{\frac{k}{N_t}}\mathbf{H}^H\mathbf{P}\tilde{\mathbf{x}} + \mathbf{n} \tag{16}$$

where $\mathbf{P} \in \mathbb{C}^{N_t \times N_t}$ is a common precoding matrix for all users, $k$ is a normalization constant and $\mathbf{n} \in \mathbb{C}^M$ denotes the noise vector at the $M$ receivers.

# Interference Channel with $N_r = 1$ and Scheme-2 : I

- For 2 user Interference channel, one can write the signal model as

$$\mathbf{y} = \sqrt{\frac{k}{N_t}}\mathbf{H}^H\mathbf{P}\tilde{\mathbf{x}} + \mathbf{n} \tag{17}$$

where $\mathbf{P} \in \mathbb{C}^{N_t \times N_t}$ is a common precoding matrix for all users, $k$ is a normalization constant and $\mathbf{n} \in \mathbb{C}^M$ denotes the noise vector at the $M$ receivers.

# Simulation Results : I

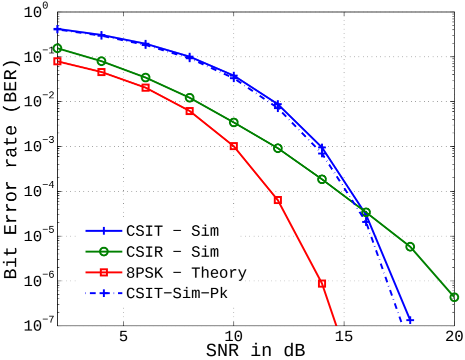


Figure : Comparison of O-STBC receivers with perfect CSIR and proposed scheme with perfect CSIT for $2 \times 2$ system with 8-PSK constellation. Dotted line shows the performance with peak power limited to 13.22 *dB* more than average transmit power.

# Simulation Results : II

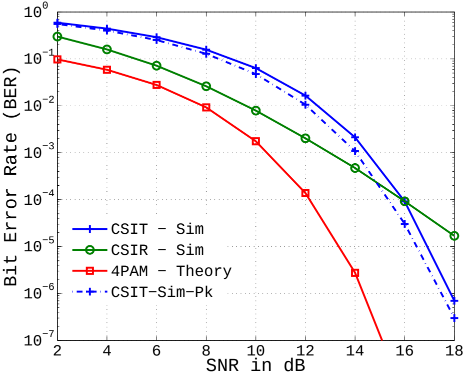


Figure : Comparison of O-STBC receivers with perfect CSIR and proposed scheme with perfect CSIT for $4 \times 2$ system with 4-PAM constellation. Dotted line shows the performance with peak power limited to 12.55 $dB$ more than average transmit power.

# Simulation Results : III

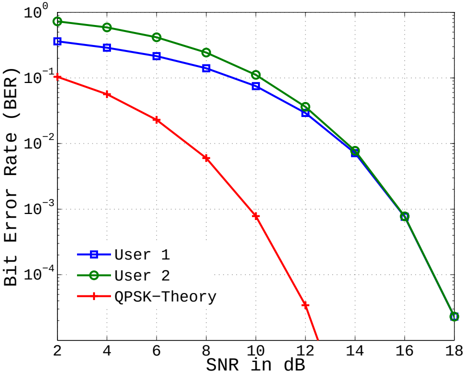


Figure : BER performance of user 1 and user 2 in a $2 \times 2$ MAC channel with $\rho_1 = \frac{3\text{SNR}}{4}$ and $\rho_2 = \frac{\text{SNR}}{4}$.

# Simulation Results : IV

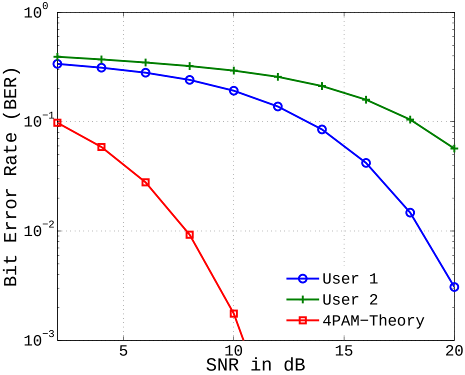


Figure : BER performance of user 1 and user 2 in a $N_t = 4$ and $N_r = 1$ per user BC channel with $\rho_1 = \frac{3\text{SNR}}{4}$ and $\rho_2 = \frac{\text{SNR}}{4}$.

# Simulation Results : V

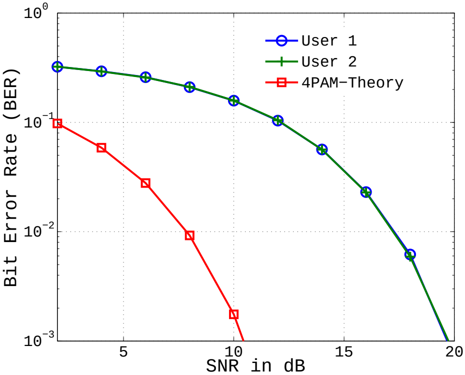


Figure : BER performance of user 1 and user 2 in a $N_t = 4$ and $N_r = 1$ per user interference channel equal total power allocated to both transmitters.

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