A Simple Tx Diversity Scheme with CSIT

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May 19th, 2012

Outline

1. Introduction
   - Tx Diversity
2. Tx Precoding
   - Channel Inversion
   - Equivalent Channel
3. New Precoding
   - New Precoder
   - Avg. Tx Power
   - Simulation Results

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Diversity and Multiplexing Gain

- Diversity and Multiplexing gain for a block fading Rayleigh MIMO system are defined as

$$d \triangleq \lim_{SNR \to \infty} \frac{-\partial \log P_e}{\partial \log SNR}$$

$$r \triangleq \lim_{SNR \to \infty} \frac{\partial R}{\partial \log SNR}$$

- Diversity can be obtained by either Rx diversity or Tx diversity or both.
  - E.g., MRC Rx diversity = N_r, MRT Tx diversity = N_t
- Rx diversity based schemes result in a maximum diversity of $N_r N_t$.
- Can we get better than $d = N_r N_t$?

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- Rx diversity based schemes result in a maximum diversity of $N_r N_t$.
- Can we get better than $d = N_r N_t$? Yes.

Alamouti Scheme- I

- 2 Tx antenna is needed. Any number of Rx antennas is supported.
- For i^{th} Rx antenna, the received vector is represented as

$$[y_{1i}\ y_{2i}] = [h_{1i}\ h_{2i}] \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + [n_{1i}\ n_{2i}]$$

where $[h_{1i}\ h_{2i}]$ represent the channel gains from 2 Tx ant. to i^{th} Rx ant.

Alamouti Scheme- II

- The decoder estimates the symbols as

$$\hat{x}_1 = h_{1i}^* y_{1i} + h_{2i} y_{2i}^*$$

$$\hat{x}_2 = h_{2i}^* y_{1i} - h_{1i} y_{2i}^*$$

Alamouti Scheme- III

- The signal model can be equivalently written as

$$\begin{bmatrix} y_{1i} \\ y_{2i}^* \end{bmatrix} = \sqrt{\frac{k\rho}{2}} \begin{bmatrix} h_{1i} & h_{2i} \\ h_{2i}^* & -h_{1i}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{1i} \\ n_{2i}^* \end{bmatrix}, \qquad (1)$$

$$\mathbf{y}' = \sqrt{\frac{k\rho}{2}}\tilde{\mathbf{H}}\mathbf{x} + \mathbf{n}'$$

- Equivalent channel matrix $\tilde{\mathbf{H}}$ is an orthogonal matrix [1].
- This scheme achieves full diversity $N_r N_t$.

[1] See Exercise 9.4 in [8]

Tx Diversity Schemes

Scenario	CSI Condition	Maximum Diversity Order d	References
SIMO	$CSIR$	N_r	[4]
SIMO	$CSIR, C\hat{S}IT$	$2N_t, \infty^*$	[6, 3]*
MISO	$CSIR$	N_t	[2, 7]
MISO	$C\hat{S}IR, C\hat{S}IT$	$N_t^2(N_t^2 + N_t + 1) + N_t$	[9]
MIMO	$CSIR$	$N_t N_r$	[11]
MIMO	$C\hat{S}IR$	$N_t N_r \left\lceil \frac{rT_c}{T_c - L_{tr}} \right\rceil$	[10]
MIMO	$CSIR, CSIT$	∞	[1]
MIMO	$CSIR, C\hat{S}IT$	$N_r N_t (N_r N_t + 2)$	[1]
MIMO	$C\hat{S}IR, C\hat{S}IT$	$2N_r N_t$	[1]

Table : Summary of maximum diversity order in Rayleigh fading MIMO Channels.

Diversity-Multiplexing Gain Trade off

- Tse and Viswanath [8] had shown that, there exists a trade-off between Diversity and Multiplexing gain that can be achieved.
- A certain combinations of (r, d) is only possible for the given (N_r, N_t) configuration.

$$(d, r) = (k, (N_t - k)(N_r - k)), \; k = 0, 1, \ldots, \min(N_r, N_t)$$

- Alamouti scheme operates at one of the operating points $(N_r N_t, 0)$.
- To realize other operating points, new codes can be designed or Tx precoding can be done.

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Channel inversion in SIMO

- Consider the SIMO channel

$$\mathbf{y} = \sqrt{\frac{k\rho}{N_t}}\mathbf{h}x + \mathbf{n}$$

  where ρ refers to total Tx power, k is a constant to ensure average power constraint.

- Since $\|\mathbf{h}\|^2$ is a χ^2_{2d} random variable, $\mathbb{E}\left[\frac{1}{\|\mathbf{h}\|^2}\right] = \frac{1}{2(d-1)}$ is finite.
- One can choose $k = \frac{1}{\|\mathbf{h}\|^2}$ to obtain equivalent AWGN channel at the Rx, $\Rightarrow d = \infty$.
  - This is not possible for SISO case since average Tx power is not finite.

Channel inversion in MIMO

For the MIMO channel also one can invert the channel by choosing precoding P matrix suitably. That is,

$$\mathbf{y} = \sqrt{\frac{\rho}{N_t}}\mathbf{HPx} + \mathbf{n}$$

Let $\mathbf{P} = \sqrt{\frac{N_t}{\rho}}\mathbf{H}^\dagger$ where $\mathbf{H}^\dagger$ refers to the pseudo-inverse of $\mathbf{H}$.

$$\mathbb{E}[\mathbf{x}^H\mathbf{P}^H\mathbf{Px}] = \|\mathbf{x}\|^2 \frac{tr[\mathbf{P}^H\mathbf{P}]}{\min(N_r, N_t)} = \frac{\|\mathbf{x}\|^2}{\min(N-r, N_t)}\mathbb{E}\left[\sum_i \frac{1}{\sigma_i^2}\right]$$

which is not finite for Rayleigh block fading channel since the smallest eigenvalue of $\mathbf{H}^H\mathbf{H}$ is χ_2^2 distributed.

NOTE: Mean value of inverse of χ_2^2 is not finite.

Tx Diversity using Precoding

- Recall that the effective channel matrix $\tilde{\mathbf{H}}$ is orthogonal.
- Use $\mathbf{P} = \tilde{\mathbf{H}}^H$. That is,

$$\hat{\mathbf{x}} = \sqrt{\frac{k\rho}{N_t}}\tilde{\mathbf{H}}\mathbf{P}\mathbf{x} + \mathbf{n} = \sqrt{\frac{k\rho\alpha^2}{N_t}}\mathbf{x} + \mathbf{n} \qquad (2)$$

where $\alpha = |h_1|^2 + |h_2|^2 + \ldots + |h_d|^2$ is a χ^2_{2d} random variable.

Orthogonality of $\tilde{\mathbf{H}}$- I

Lemma 1

The equivalent channel matrix constructed for real square O-STBC is orthogonal.

Proof: The two equivalent representations of the received vector $\mathbf{y}$ in terms of $\mathbf{h}$ and $\mathbf{x}$ can be written as

$$\mathbf{y} = \sqrt{\frac{k\rho}{N_t}}\mathbf{X}\mathbf{h} = \sqrt{\frac{k\rho}{N_t}}\tilde{\mathbf{H}}\mathbf{x} \tag{3}$$

Orthogonality of $\tilde{\mathbf{H}}$- II

Multiplying by $\mathbf{X}^T$ on both sides, we get

$$\alpha\mathbf{h} = \mathbf{X}^T\tilde{\mathbf{H}}\mathbf{x}$$

where $\mathbf{X}^T\mathbf{X} = \alpha\mathbf{I}$.

- There exists a linear transformation between $\mathbf{h}$ and $\mathbf{x}$ which indicates that the columns of $\tilde{\mathbf{H}}$ are linearly independent.

Orthogonality of $\tilde{\mathbf{H}}$- III

- Due to the structure of O-STBC codes, $\mathbf{X}_1$ and $\mathbf{X}_2$ are orthogonal matrices by construction. Moreover, it can be shown that $\mathbf{x}_{2,j}^T\mathbf{x}_{1,i} = -\mathbf{x}_{2,i}^T\mathbf{x}_{1,j}$ and $\mathbf{x}_{1,i}^T\mathbf{x}_{2,i} = \mathbf{x}_{1,j}^T\mathbf{x}_{2,j}$ [5]. That is,

$$\mathbf{y}_1^T\mathbf{y}_2 = \mathbf{h}^T\mathbf{X}_1^T\mathbf{X}_2\mathbf{h} = \sum_i\sum_j h_ih_j\mathbf{x}_{1,i}^T\mathbf{x}_{2,j} = \sum_i h_i^2\mathbf{x}_{1,i}^T\mathbf{x}_{2,i} = \mathbf{x}_{1,i}^T\mathbf{x}_{2,i}\sum_i h_i^2, \tag{4}$$

$$\mathbf{y}_1^T\mathbf{y}_2 = \mathbf{x}_1^T\left(\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}\right)\mathbf{x}_2. \tag{5}$$

Orthogonality of $\tilde{\mathbf{H}}$- IV

- Eqns. (4) and (5) are equal, if and only if $\tilde{\mathbf{H}}$ is orthogonal and $\mathbf{h}_i^T \mathbf{h}_i = \mathbf{h}_j^T \mathbf{h}_j$.
- Note that, the converse part is true since one of the columns of $\mathbf{X}_1$ is same as $\mathbf{x}_1$ and one of the columns of $\mathbf{X}_2$ is same as $\mathbf{x}_2$.

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Proposed Precoder: Assumptions

- Only block fading Rayleigh channels are considered.
- CSI is available only at the Tx.
- Signalling is assumed to use orthogonal STBC with energy normalized constellations.
- $N_r > 1$ for complex O-STBC and $N_r > 2$ for real O-STBC.

Proposed Precoder - I

Theorem 1

The optimum precoding matrix for Tx diversity with CSIT in the case of generic $N_t \times N_r$ MIMO channel is given by

$$\mathbf{P} = \left(\sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i + \Delta \right)^H ,$$

where Δ is a correction matrix defined by

$$\Delta^H = \left(\sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i \right)^{-1} \left(c\mathbf{I} - \left[\sum_{i=1}^{N_r} \sum_{j=1}^{N_r} \tilde{\mathbf{H}}_i \tilde{\mathbf{H}}_j^H \right] \right)$$

Proposed Precoder - II

Proof:

- c is a design parameter which controls average Tx power, and $\tilde{\mathbf{H}}_i$ refers to the equivalent channel matrix for i^{th} Rx antenna.
- First, we will prove that

$$\sum_{i=1}^{N_r} \sum_{j \neq i} \tilde{\mathbf{H}}_i \tilde{\mathbf{H}}_j^H \tag{6}$$

is a diagonal matrix.

Proposed Precoder - III

- To prove (6), consider the product between the column vectors of $\tilde{\mathbf{H}}_i$ and $\tilde{\mathbf{H}}_j$.

$$< \mathbf{h}_j^l, \mathbf{h}_i^m > = - < \mathbf{h}_i^m, \mathbf{h}_j^l >$$

$$< \mathbf{h}_i^l, \mathbf{h}_j^l > = < \mathbf{h}_i^k, \mathbf{h}_j^k >$$

$$\sum_{i=1}^{N_r} \mathbf{H}_i^H \sum_{j=1}^{N_r} \mathbf{H}_j = \gamma \mathbf{I}$$

for some real value γ.

Proposed Precoder - IV

- To obtain infinite diversity order, we need

$$\left(\tilde{\mathbf{H}}_1 + \tilde{\mathbf{H}}_2\right)\left(\tilde{\mathbf{H}}_1 + \tilde{\mathbf{H}}_2 + \Delta\right)^H = c\mathbf{I}$$

$$\Rightarrow \left(\tilde{\mathbf{H}}_1 + \tilde{\mathbf{H}}_2\right)\Delta^H = c\mathbf{I} - \left(\tilde{\mathbf{H}}_1 + \tilde{\mathbf{H}}_2\right)\left(\tilde{\mathbf{H}}_1 + \tilde{\mathbf{H}}_2\right)^H$$

Proposed Precoder - V

$$\Delta^H = \left(\sum_{i=1}^{2} \tilde{\mathbf{H}}_1\right)^{-1} \left(c\mathbf{I} - \left[\left(\sum_{i=1}^{2} \tilde{\mathbf{H}}_i\right)\left(\sum_{i=1}^{2} \tilde{\mathbf{H}}_i\right)^H\right]\right)$$

- In general

$$\Delta^H = \left(\sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i\right)^{-1} \left(c\mathbf{I} - \left[\left(\sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i\right)\left(\sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i\right)^H\right]\right), \quad (7)$$

which concludes the proof.

Average Tx Power Constraint - I

- The average Tx power can be written as

$$P_{avg} = \frac{k\rho}{N_t} \mathbb{E}_{\mathbf{h},\mathbf{x}} \left[\mathbf{x}^H \left(\Delta + \sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i \right) \left(\Delta + \sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i \right)^H \mathbf{x} \right].$$

- Using the definition of Δ, it can written that

$$\left[\left(\Delta + \sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i \right) \left(\Delta + \sum_{i=1}^{N_r} \tilde{\mathbf{H}}_i \right)^H \right] = \left[\Delta\Delta^H + c\mathbf{I} + \Delta \left(\sum_{i} \tilde{\mathbf{H}}_i \right)^H \right]. \quad (8)$$

Average Tx Power Constraint - II

- Since $\sum_i \tilde{\mathbf{H}}_i$ is orthogonal we can write, $\eta \mathbf{I} \triangleq \left(\sum_i \tilde{\mathbf{H}}_i\right) \left(\sum_i \tilde{\mathbf{H}}_i\right)^H$.
- The average Tx power can be simplified as follows. Using (7), it can be written that

$$\begin{aligned} \Delta^H &= \frac{\left(\sum_i \tilde{\mathbf{H}}_i\right)^H}{\eta} \left[c\mathbf{I} - \left(\sum_i \tilde{\mathbf{H}}_i\right) \left(\sum_i \tilde{\mathbf{H}}_i\right)^H \right] \\ &= \underbrace{\frac{c-\eta}{\eta}}_{\Gamma} \left(\sum_i \tilde{\mathbf{H}}_i\right)^H \qquad (9) \end{aligned}$$

Average Tx Power Constraint - III

- Hence, $\Delta\Delta^H$ can be written as

$$\Delta\Delta^H = \Gamma^2 \left(\sum_i \tilde{\mathbf{H}}_i\right)\left(\sum_i \tilde{\mathbf{H}}_i\right)^H \tag{10}$$

- Substituting (9) and (10) in (8), we get

$$\mathbf{P}\mathbf{P}^H = \left[c\mathbf{I} + (\Gamma^2 + \Gamma)\left(\sum_i \tilde{\mathbf{H}}_i\right)\left(\sum_i \tilde{\mathbf{H}}_i\right)^H\right] \tag{11}$$

Average Tx Power Constraint - IV

- By choosing $k = \frac{d-2}{c^2}$, to meet the average power constraint, we get

$$P_{avg} \quad = \quad \frac{\rho\,(d-2)}{c^2 N_t}\mathbb{E}_{\mathbf{x}}[\mathbf{x}^H\mathbf{x}]\frac{c^2}{d-2} = \rho \qquad (12)$$

  Hence, the receiver can use the fixed channel gain value of $\sqrt{\frac{k\rho c^2}{N_t}}$ for decoding the symbols.

Simulation Results-I

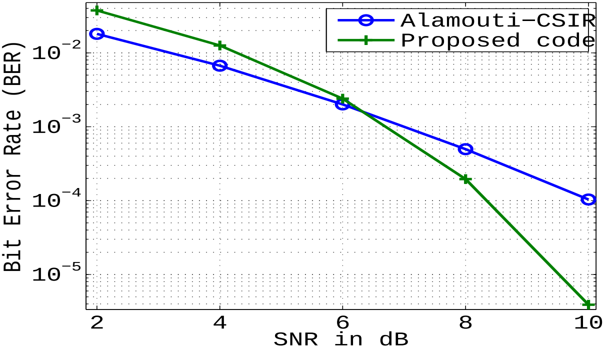

Figure : Comparison of 2×2 Alamouti scheme with perfect CSIR and proposed

Simulation Results-II

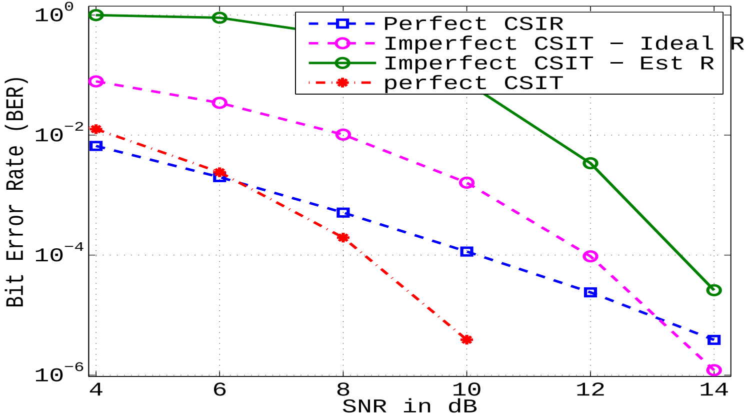

Figure : Comparison of 2×2 Alamouti scheme with perfect CSIR and proposed

References- I

[1] V. Aggarwal and A. Sabharwal, *Power controlled feedback and training for two-way mimo channels*, IEEE Transactions on Information Theory, (2010), pp. 3310–3331.

[2] S. M. Alamouti, *A simple diversity technique for wireless communications*, IEEE Jl. Selected Areas of Communication, 16 (1998), pp. 1451–1458.

[3] B. N. Bharath and C. R. Murthy, *On the diversity-multiplexing gain tradeoff for tdd-simo system*, Arxiv:submit/0246301[cs.IT], (2011).

[4] S. N. Diggavi, N. Al-Dhahir, A. Stamoulis, and A.R.Calderbank, *Great expectations : The value of spatial diversity in wireless networks*, Proceedings of the IEEE, 92 (2004), pp. 219–270.

References- II

[5] E. G. LARSSON AND P. STOICA, *Space-Time Block Coding for Wireless Communications*, Cambridge University Press, first ed., 2003.

[6] C. STEGER, A. KHOSHNEVIS, A. SABHARWAL, AND B. AAZHANG, *The case for transmitter training*, in Proceedings of ISIT, 2006, pp. 35–39.

[7] V. TAROKH, H. JAFARKHANI, AND A. CALDERBANK, *Space-time block codes from orthogonal designs*, IEEE Transactions on Information Theory, 45 (1999), pp. 1456–1467.

[8] D. TSE AND P. VISWANATH, *Fundamentals of Wireless Communication*, Cambridge University Press, first ed., 2005.

References- III

[9] X. J. ZHANG AND K. B. LETAIEF, *Power control and channel training for mimo channels: A dmt perspective*, IEEE Transactions on Wireless Communications, (2011), pp. 2080–2088.

[10] L. ZHENG, *Diversity-Multiplexing Tradeoff: A Comprehensive View of Multiple Antenna Systems*, PhD thesis, Univ. of California, Berkeley, 2002.

[11] L. ZHENG AND D. TSE, *Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels*, IEEE Transactions on Information Theory, 49 (2003), pp. 1073–1096.