Performance Analysis and Training Optimization for Uplink Cellular Networks with Power Control and Channel Estimation Errors

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Outline

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PPP Preliminaries

- **Poisson Point Processes (PPP)**
  - First contact distribution
  - Thinning of PPP
  - Slivnyak's theorem: Reduced palm distribution
- **Theorem**
  - Campbell's theorem
  - Probability generating functional (PGFL)

Motivation

- Uplink cellular network not being given adequate attention using stochastic geometric framework
- **Stochastic geometry**: A new tool
  - Takes into account the randomness present in cellular network
  - Provides simple mathematical tools for deriving network performance metrics
  - Gives useful design insights into the system
- **Channel estimation**: An important aspect
  - Channel estimation errors can't be ignored in practical systems
  - Need to optimize the training duration
- **Uplink power control**: To improve coverage
  - Optimal power control factor

System Model

- BS locations form PPP: ϕ_B with density λ_B
- MU locations form PPP: ϕ_M with density λ_M
- ϕ_M independent of ϕ_B
- Nearest neighbour connectivity
- Probability of Connection p_c:

$$p_c \approx 1 - \left(\frac{3.5}{3.5 + \frac{\lambda_M}{\lambda_B}} \right)^{3.5}$$

  - Observe the dependence of p_c on $\frac{\lambda_M}{\lambda_B}$
- BS serves a single MU in a given time frequency block
  - Only inter-cell interference, no intra-cell interference

System Model

- **Channel Model**
  - Coherence time L symbols:
    - L_τ symbols: Training duration
    - $L - L_\tau$ symbols: Data transmission
  - Distance dependent path loss, $\alpha \gg 2$
  - i.i.d. Rayleigh fading across users
- **Fractional Power Control**
  - Power control both during training and data transmission
  - Distance dependent fractional power control, $(R_u^\epsilon)^\alpha$, $\epsilon \in [0, 1]$
  - $\epsilon = 0$: No power control and $\epsilon = 1$: Perfect path loss compensation
  - Baseline power is assumed to be μ^{-1}

Assumptions

1. MU locations connected to any BS in a given time frequency block form a PPP: ϕ_m
   - The density of PPP ϕ_m is $\lambda = p_c \lambda_B$
   - Consequence of independent thinning (approximation for tractability)
2. R_v for $v \in \phi_m(\lambda)$ the distance of interfering MUs form their tagged BSs are assumed to be independent
   - Dependence between R_v for $v \in \phi_m(\lambda)$ is very weak
3. No synchronization between training and data transmission phases among users is assumed
   - Generalized model
   - Captures the effect of pilot contamination

Goal

1. Derive an analytical expression for channel estimation error variance
2. Derive the uplink coverage probability expression for a typical MU
    - Simplify the coverage expression for various practically useful scenarios
    - Study the coverage behavior for against L_τ, λ_B and SINR threshold, θ
3. Derive analytical expression for the ergodic capacity
4. Optimize ϵ to find ϵ_{opt} using the capacity expression for different L_τ
5. Use ϵ_{opt} to find the optimal training duration $L_{\tau,opt}$

Problem Statement

- **Coverage Probability**: The probability that typical BS achieves a SINR threshold, θ

$$P_c(\epsilon, \theta, L_\tau) = \mathbb{P}(\mathsf{SINR} > \theta)$$

- **Ergodic Capacity**: Average rate achieved by typical BS

$$C(\epsilon, L_\tau) = \frac{(L - L_\tau)}{L} \mathbb{E}[\ln(1 + \mathsf{SINR})]$$

- **Optimal Power Control Factor,** ϵ_{opt} and **Optimal Training Duration,** $L_{\tau,opt}$

$$\epsilon_{opt}, L_{\tau,opt} = \arg\max_{\epsilon, L_\tau} \left(1 - \frac{L_\tau}{L}\right) C(\epsilon, L_\tau)$$

Two Phases

- Consider typical BS and MU pair and the BS located at origin
- Invoke Slivnyak's theorem

1. **Uplink Training**
   - Typical MU sends L_τ length training sequence
   - The BS obtains an estimate $\hat{h}_u$ of the channel h_u
2. **Uplink Data Transmission**
   - MU transmit data for rest $L - L_\tau$ symbol durations
   - BS makes use of $\hat{h}_u$ to estimate the transmitted symbol

- **Channel estimation error variance**, $\sigma_{e|r_u}$ conditioned on the first contact distance, $R_u = r_u$

$$\sigma^2_{e|r_u} = \frac{1}{1 + \frac{\mu^{-1} r_u^{\alpha(\epsilon-1)} L_\tau}{\mu^{-1}\mathcal{I}_v^\tau + \sigma^2_{n_\tau}}}$$

where $\mathcal{I}_v^\tau = \mathbb{E}\left[\sum_{v\in\phi_m(\lambda)} (R_v^\epsilon)^\alpha D_v^{-\alpha} |h_v q_v|^2\right]$ is the interference term and $\sigma^2_{n_\tau}$ is the noise variance.

- Using Campbell's theorem, computing $\mathcal{I}_v^\tau$

$$\mathcal{I}_v^\tau = \int_0^\infty 2\pi\lambda (r_v^\epsilon)^\alpha \frac{r_u^{-\alpha+2}}{\alpha-2} f_{R_v}(r_v) \mathrm{d}r_v$$

Coverage Probability

- The uplink **coverage probability** for a typical MU is given by

$$P_c(\epsilon,\theta,L_\tau) = \int_0^\infty \exp\left(-\frac{\theta\sigma^2_{e|r_u}}{1-\sigma^2_{e|r_u}}\right)\exp\left(-\frac{\mu\theta r_u^{\alpha(1-\epsilon)}\sigma^2_{n_d}}{1-\sigma^2_{e|r_u}}\right) \mathcal{L}_{I_v^d}\left(\frac{\theta r_u^{\alpha(1-\epsilon)}}{1-\sigma^2_{e|r_u}}\right) f_{R_u}(r_u)\mathrm{d}r_u$$

  - $f_{R_u}(r_u)$ is the nearest neighbour distance distribution
- $\mathcal{L}_{I_v^d}(s)$ is the **Laplace transform** of the interference calculated at $s=\frac{\theta r_u^{\alpha(1-\epsilon)}}{1-\sigma^2_{e|r_u}}$

$$\mathcal{L}_{I_v^d}(s) = \exp\left(-2\pi\lambda\int_{r_u}^\infty\left(1-\mathbb{E}_{R_v}\left[\frac{1}{1+s(R_v^\epsilon)^\alpha d_v^\alpha}\right]\right)d_v\mathrm{d}d_v\right)$$

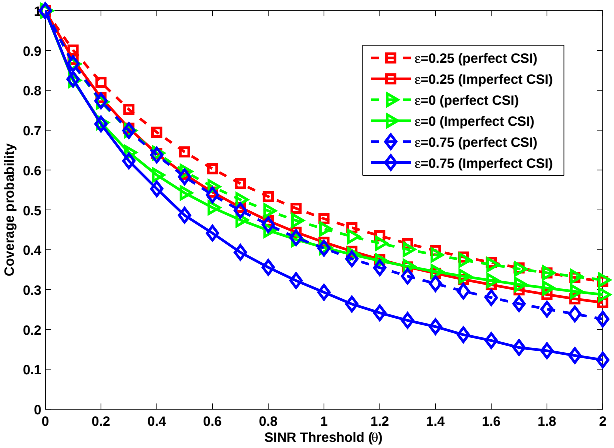

Figure: SINR threshold, θ vs Coverage probability, P_C, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_T^2}} = 20\text{dB}, \lambda_B = 0.05/m^2$, $\lambda_M = 0.3/m^2$, $\lambda_B = 0.05/m^2$, $L_T = 10$ symbols and $\alpha = 3.5$

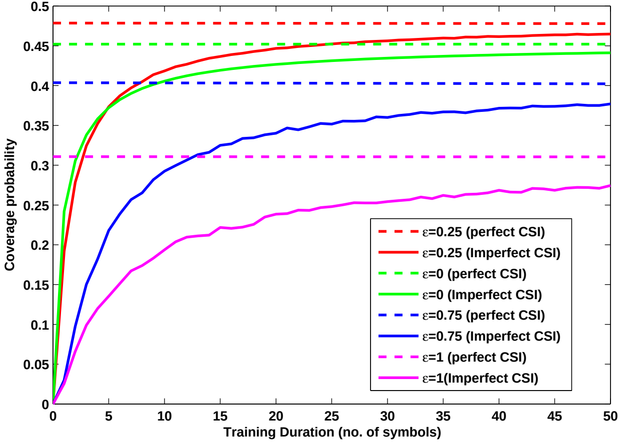

Figure: Training duration, L_τ vs Coverage probability, P_C, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_\tau^2}} = 20\text{dB}$, $\lambda_B = 0.05/m^2$, $\lambda_M = 0.3/m^2$, $\alpha = 3.5$ and $\theta = 1$

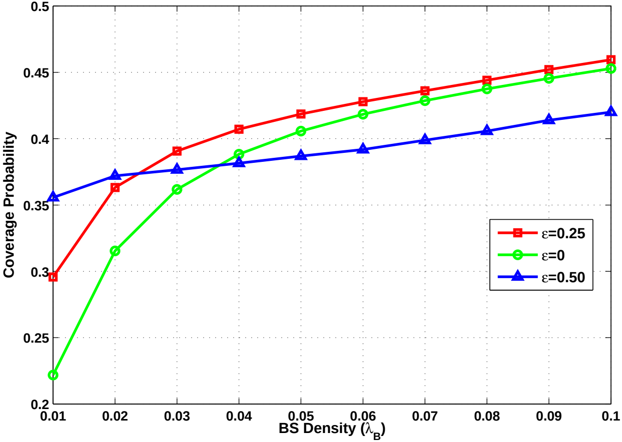

Figure: BS Density, λ_B vs Coverage probability, P_C, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_\tau^2}} = 20\text{dB}$, $\lambda_M = 0.3/m^2$, $\theta = 1$, $L_T = 10$ symbols and $\alpha = 3.5$

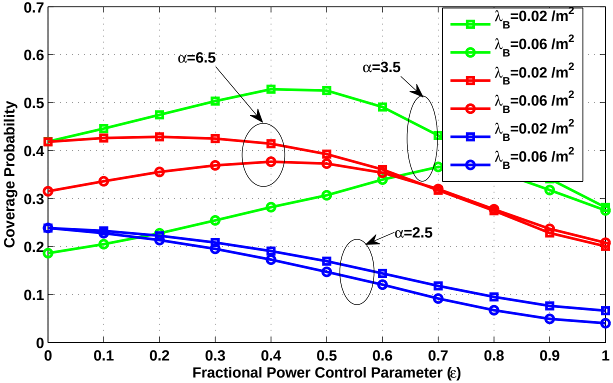

Figure: Fractional power control parameter, ϵ vs Coverage probability, P_C, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_\tau^2}} = 20\text{dB}$, $\lambda_M = 0.3/m^2$, $\theta = 1$, $L_T = 10$ symbols

Ergodic Capacity

- The **average achievable rate** in the uplink for the typical MU-BS pair

$$C_{\text{eff}}(\epsilon, L_\tau) \triangleq \left(1 - \frac{L_\tau}{L}\right) C(\epsilon, L_\tau),$$

where $C(\epsilon, L_\tau) \triangleq \mathbb{E}[\ln(1 + \text{SINR})]$ is

$$C(\epsilon, L_\tau) = \int_{r_u > 0} f_{R_u}(r_u) \int_{t>0} \exp\left(-\frac{(e^t - 1)\sigma^2_{e|r_u}}{1 - \sigma^2_{e|r_u}}\right)$$
$$\exp\left(-\frac{\mu(e^t - 1) r_u^{\alpha(1-\epsilon)} \sigma^2_{n_d}}{1 - \sigma^2_{e|r_u}}\right) \mathcal{L}_{I_v^d}\left(\frac{(e^t - 1) r_u^{\alpha(1-\epsilon)}}{1 - \sigma^2_{e|r_u}}\right) \mathrm{d}t \mathrm{d}r_u,$$

where $\mathcal{L}_{\mathcal{I}_v^d}(s)$ is **Laplace transform** of the interference term, evaluated at $s = \frac{(e^t - 1) r_u^{\alpha(1-\epsilon)}}{1 - \sigma^2_{e|r_u}}$.

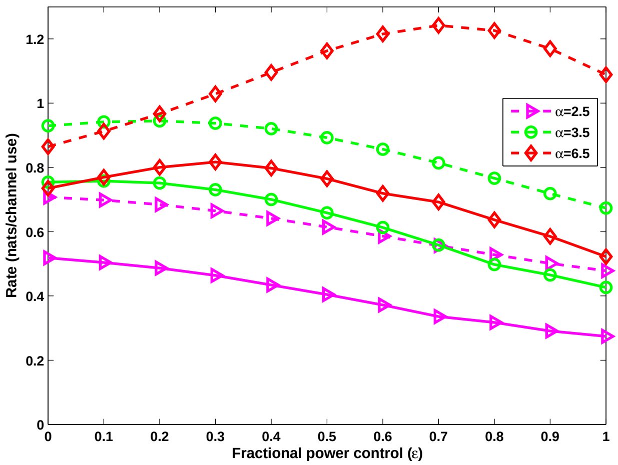

Figure: Fractional power control parameter, (ϵ), vs Rate, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_\tau^2}} = 20\text{dB}$, $L_\tau = 10$, $L = 100$, $\lambda_B = 0.05/m^2$, $\lambda_M = 0.3/m^2$, $\alpha = 3.5$ and $\theta = 1$

System Design Implications

- **Optimal Fractional Power Control Parameter**, ϵ_{opt}

$$\epsilon_{opt} = \arg\max_{\epsilon} C(\epsilon, L_\tau)$$

- **Optimal Training Duration**, $L_{\tau,opt}$ symbols

$$L_{\tau,opt} = \arg\max_{L_\tau} \left(1 - \frac{L_\tau}{L}\right) C(\epsilon_{opt}, L_\tau)$$

- Use numerical computations to find ϵ_{opt} first
- Use ϵ_{opt} to numerically compute $L_{\tau,opt}$

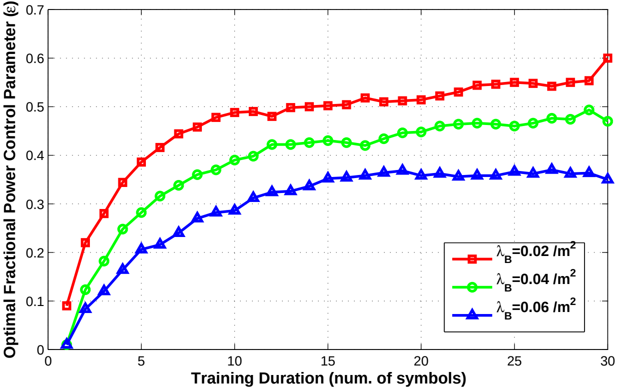

Figure: Training duration, L_τ vs Optimal fractional power control factor, ϵ_{opt}, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_\tau^2}} = 20\text{dB}$, $\lambda_M = 0.3/m^2$, $\alpha = 3.5$ and $\theta = 1$

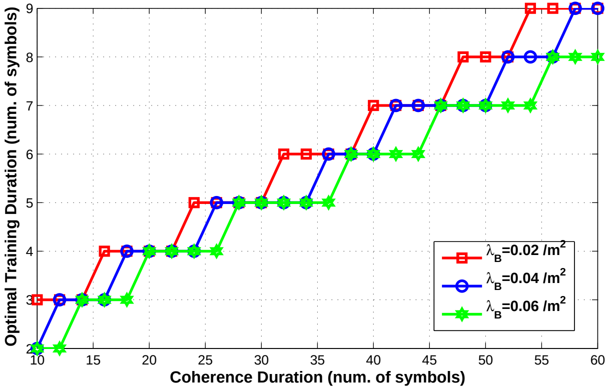

Figure: Optimal training duration, $L_{\tau,opt}$ vs Coherence duration, L, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_\tau^2}} = 20\text{dB}$, $\lambda_M = 0.3/m^2$, $\alpha = 3.5$ and $\theta = 1$

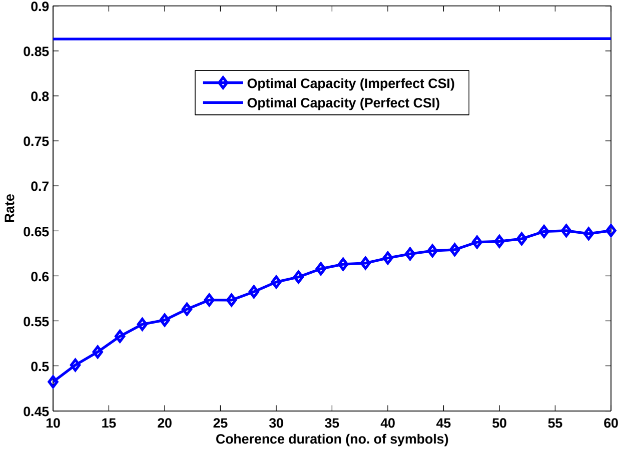

Figure: Optimal Rate vs Coherence duration, L, for $\frac{\mu^{-1}}{\sigma_{n_d^2}} = \frac{\mu^{-1}}{\sigma_{n_\tau^2}} = 20\text{dB}$, $\lambda_B = 0.06/m^2$, $\lambda_M = 0.3/m^2$, $\alpha = 3.5$ and $\theta = 1$

Thank You