Coverage Analysis and Training Optimization for Uplink Cellular Networks with Practical Channel Estimation

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Outline

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- Area Spectral Efficiency
- Optimal Power Control and Training

PPP Preliminaries

- **Poisson Point Processes (PPP)**
  - First contact distribution
  - Thinning of PPP
- **Theorem**
  - Campbell's theorem
  - Probability generating functional (PGFL)

Motivation

- Uplink cellular network not being given adequate attention using stochastic geometric framework
- **Stochastic geometry**: A new tool
  - Takes into account the randomness present in cellular network
  - Provides simple mathematical tools for deriving network performance metrics
  - Gives useful design insights into the system
- **Channel estimation**: An important aspect
  - Channel estimation errors can't be ignored in practical systems
  - Need to optimize the training duration
- **Uplink power control**: To improve coverage
  - Optimal power control factor

System Model

- BS locations form PPP: ϕ_B with density λ_B
- MU locations form PPP: ϕ_M with density λ_M
- ϕ_M independent of ϕ_B
- Nearest neighbour connectivity
- Probability of Connection p_c:

$$p_c \approx 1 - \left(\frac{3.5}{3.5 + \frac{\lambda_M}{\lambda_B}} \right)^{3.5}$$

  - Observe the dependence of p_c on $\frac{\lambda_M}{\lambda_B}$
- BS serves a single MU in a given time frequency block
  - Only inter-cell interference, no intra-cell interference

- **Channel Model**
  - Coherence time L symbols:
    - L_τ symbols: Training duration
    - $L - L_\tau$ symbols: Data transmission
  - Distance dependent path loss, $\alpha \gg 2$
  - i.i.d. Rayleigh fading across users
- **Fractional Power Control**
  - Power control both during training and data transmission
  - Distance dependent fractional power control, $(R_u^\epsilon)^\alpha$, $\epsilon \in [0, 1]$
  - $\epsilon = 0$: No power control and $\epsilon = 1$: Perfect path loss compensation
  - Baseline power is assumed to be p^{-1}

Assumptions

1. MU locations connected to any BS in a given time frequency block form a PPP: ϕ_m
   - The density of PPP ϕ_m is $\lambda = p_c \lambda_B$
   - Consequence of independent thinning (approximation for tractability)
2. R_v for $v \in \phi_m(\lambda)$ the distance of interfering MUs form their tagged BSs are assumed to be independent
   - Dependence between R_v for $v \in \phi_m(\lambda)$ is very weak
3. No synchronization between training and data transmission phases among users is assumed
   - Generalized model
   - Captures the effect of pilot contamination

Goal

1. Derive an analytical expression for channel estimation error variance
2. Derive the uplink coverage probability expression for a typical BS-MU link
   - Study the coverage behaviour with Power control factor , ϵ and SINR threshold, θ
3. Area spectral efficiency (ASE)
4. Find ϵ_{opt}
5. Find $L_{\tau,opt}$

Problem Statement

- **Coverage Probability**: The probability that typical BS achieves a SINR threshold, θ

$$P_c(\epsilon, \theta, L_\tau) = \mathbb{P}(\text{SINR} > \theta)$$

- **ASE**: ASE is defined to be the total data transmitted in the uplink per unit area per channel use

$$\text{ASE}(\epsilon, L_\tau, R(\theta)) = \left(1 - \frac{L_\tau}{L}\right) R(\theta)\lambda P_c(\epsilon, \theta, L_\tau)$$

- **Optimal Power Control Factor,** ϵ_{opt}

$$\epsilon_{opt} = \arg\max_{\epsilon \in [0,1]} P_c(\epsilon, \theta, L_\tau)$$

- **Optimal Training Duration,** $L_{\tau,opt}$

$$L_{\tau,opt} = \arg\max_{L_\tau \in [0,L]} \left(1 - \frac{L_\tau}{L}\right) R(\theta)\lambda P_c(\epsilon_{opt}, \theta, L_\tau)$$

Two Phases

- Consider typical BS and MU pair and the BS located at origin

1. **Uplink Training**
   - Typical MU sends L_τ length training sequence
   - The BS obtains an estimate $\hat{h}_u$ of the channel h_u
2. **Uplink Data Transmission**
   - MU transmit data for rest $L - L_\tau$ symbol durations
   - BS makes use of $\hat{h}_u$ to estimate the transmitted symbol

Channel Estimation Error

- **Channel estimation error variance**, $\sigma_{e|r_u}$ conditioned on the first contact distance, $R_u = r_u$

$$\sigma^2_{e|r_u} = \frac{1}{1 + \frac{p^{-1} r_u^{\alpha(\epsilon-1)} L_\tau}{p^{-1}\mathcal{I}_v^\tau + \mathbb{E}[R_u^{\alpha\epsilon}]\sigma^2_{n_\tau}}}$$

where $\mathcal{I}_v^\tau = \mathbb{E}\left[\sum_{v\in\phi_m(\lambda)} (R_v^\epsilon)^\alpha D_v^{-\alpha} |h_v q_v|^2\right]$ is the interference term and $\sigma^2_{n_\tau}$ is the noise variance.

- Using Campbell's theorem, computing $\mathcal{I}_v^\tau$

$$\mathcal{I}_v^\tau = \int_0^\infty 2\pi\lambda (r_v^\epsilon)^\alpha \frac{r_u^{-\alpha+2}}{\alpha - 2} f_{R_v}(r_v) \mathrm{d}r_v$$

- The uplink **coverage probability** for a typical MU is given by

$$P_c(\epsilon,\theta,L_\tau) = \int_0^\infty \exp\left(-\frac{\theta\sigma^2_{e|r_u}}{1-\sigma^2_{e|r_u}}\right) \exp\left(-\frac{p\theta r_u^{\alpha(1-\epsilon)}\mathbb{E}[R_u^{\alpha\epsilon}]\sigma^2_{n_d}}{1-\sigma^2_{e|r_u}}\right)\mathcal{L}_{I_v^d}\left(\frac{\theta r_u^{\alpha(1-\epsilon)}}{1-\sigma^2_{e|r_u}}\right) f_{R_u}(r_u)\mathrm{d}r_u$$

  - $f_{R_u}(r_u)$ is the nearest neighbour distance distribution

- $\mathcal{L}_{I_v^d}(s)$ is the **Laplace transform** of the interference calculated at $s = \frac{\theta r_u^{\alpha(1-\epsilon)}}{1-\sigma^2_{e|r_u}}$

$$\mathcal{L}_{I_v^d}(s) = \exp\left(-2\pi\lambda\int_{r_u}^\infty\left(1-\int_0^\infty \frac{\pi\lambda_B}{1+st_v^{(\epsilon\alpha/2)}d_v^{-\alpha}} \exp(-\lambda_B\pi t_v)\mathrm{d}t_v\right)d_v\,\mathrm{d}d_v\right)$$

Table : System Parameters

BS density	0.24 $\mathrm{BS/km^2}$
MU density	0.80 $\mathrm{MU/km^2}$
Baseline transmit power	10 mW
Fractional power control factor	0.25, 0.75
Noise power (Training/ Data transmission)	-174 dBm
Path loss coefficient	2.5, 3.7
Training duration	10, 50 symbols
Coherence Duration	50, 200 symbols

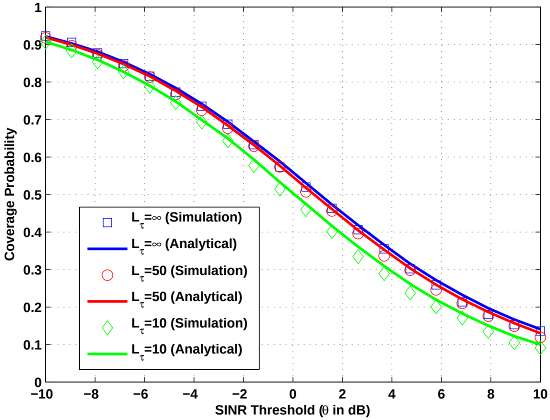

Figure : Coverage probability, P_c vs SINR threshold, θ for $\epsilon = 0.25$, $\lambda_B = 0.24$, $\lambda_M = 0.80$, $p^{-1} = 10$ mW, $\sigma_{n_d^2} = \sigma_{n_\tau^2} = -174$ dBm, $\alpha = 3.7$ and L_τ measured in symbols.

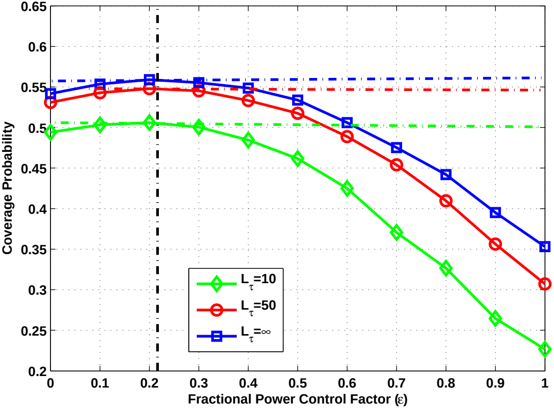

Figure : Coverage probability, P_c vs Fractional power control factor, ϵ, for perfect channel knowledge ($L_\tau = \infty$) and $L_\tau = 10, 50$, $\lambda_B = 0.24$, $\lambda_M = 0.80$, $\theta = 0$ dB, $p^{-1} = 10$ mW, $\sigma_{n_d^2} = \sigma_{n_\tau^2} = -174$ dBm, $\alpha = 3.7$ and L_τ measured in symbols.

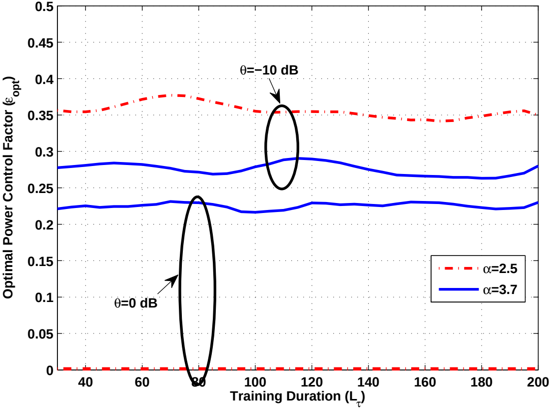

Figure : Optimal epsilon, ϵ_{opt} vs Training duration, L_τ for $\theta = 0$ dB, -10 dB, $\lambda_B = 0.24$, $\lambda_M = 0.80$, $p^{-1} = 10$ mW, $\sigma_{n_d^2} = \sigma_{n_\tau^2} = -174$ dBm, $\alpha = 2.5, 3.7$ and L_τ measured in symbols.

System Design Implications

$$\epsilon_{opt}, L_{\tau,opt} = \underset{\epsilon\in[0,1], L_\tau\in[0,L]}{\arg\max} \quad \text{ASE}$$

- **Optimal Fractional Power Control Parameter**, ϵ_{opt}

$$\epsilon_{opt} = \underset{\epsilon\in[0,1]}{\arg\max}\, P_c(\epsilon, \theta, L_\tau).$$

- **Optimal Training Duration**, $L_{\tau,opt}$ symbols

$$L_{\tau,opt} = \underset{L_\tau\in[0,L]}{\arg\max} \left(1 - \frac{L_\tau}{L}\right) R(\theta)\lambda P_c(\epsilon_{opt}, \theta, L_\tau).$$

- Use numerical computations to find ϵ_{opt} first
- Use ϵ_{opt} to numerically compute $L_{\tau,opt}$

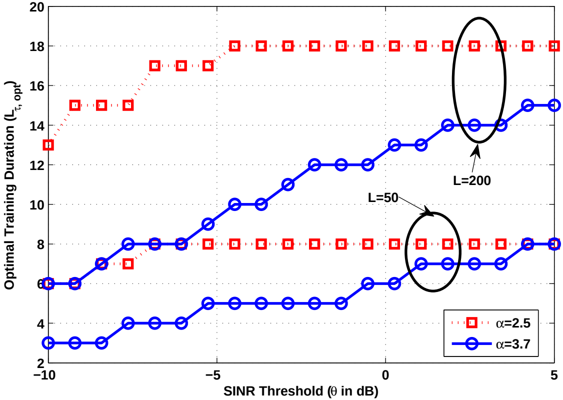

Figure : Optimal training duration $L_{\tau,opt}$ vs SINR threshold θ, for $\lambda_B = 0.24$, $\lambda_M = 0.80$, $p^{-1} = 10$ mW, $\sigma_{n_d^2} = \sigma_{n_\tau^2} = -174$ dBm, $\alpha = 2.5, 3.7$.

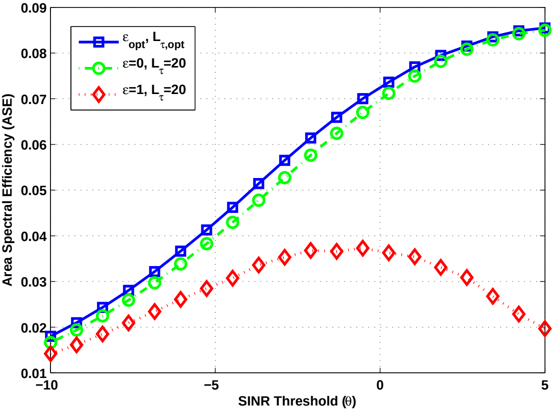

Figure : Area spectral efficiency, ASE vs SINR Threshold, θ for $\lambda_B = 0.24$, $\lambda_M = 0.80$, $p^{-1} = 10$ mW, $\sigma_{n_d^2} = \sigma_{n_\tau^2} = -174$ dBm, $\alpha = 3.7$ and $L = 200$

Thank You