

# Universal Hashing for Information Theoretic Security

Himanshu Tyagi and Alexander Vardy

## Abstract

The information theoretic approach to security entails harnessing the correlated randomness available in nature to establish security. It uses tools from information theory and coding and yields provable security, even against an adversary with unbounded computational power. However, the feasibility of this approach in practice depends on the development of efficiently implementable schemes. In this article, we review a special class of practical schemes for information theoretic security that are based on *2-universal hash families*. Specific cases of secret key agreement and wiretap coding are considered, and general themes are identified. The scheme presented for wiretap coding is *modular* and can be implemented easily by including an extra pre-processing layer over the existing transmission codes.

## Index Terms

2-Universal hash family, information theoretic security, modular coding schemes, secret key agreement, wiretap codes.

## I. INTRODUCTION

Random variations in physical observations constitute a valuable resource for facilitating security in engineering systems. Authentication keys can be extracted from noisy recordings of biometric signatures [80], [55]; unique signatures for hardware devices can be generated by implanting a *physically uncloneable function* (PUF), implemented using random manufacturing variations in the period of a ring-oscillator [75], [30]; secret keys extracted from the random fade of a wireless communication channel can be used for cryptographic applications [106]; various physical layer security techniques can be used to mitigate the security threats in cyberphysical systems and ad-hoc networks [98], [56]; and wiretap codes can be used for protection against side-channel attacks [16]. The information theoretic approach for security

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entails developing a systematic theory for designing and analyzing security primitives based on harnessing physical randomness. In this approach, we treat physical observations as a source of randomness hidden from the attacker and study the design of optimal codes for accomplishing specific security objectives. One limitation of this approach is the assumption that the attacker does not have a complete access to or cannot manipulate the correlated randomness used for implementing security. In lieu, we can provide information theoretic security guarantees which hold even when the attacker has unlimited computational power.

The origin of information theoretic security, as well as of theoretical cryptography, lies in the seminal paper of Shannon [87]. This paper shows that in order to securely transmit an  $m$ -bit random message over an insecure public channel the transmitter and the receiver must share an  $m$ -bit *perfect secret key*<sup>1</sup>, *i.e.*,  $m$  uniformly distributed bits that are concealed from an eavesdropper with access to the public channel. This requirement of large perfect secret keys is impractical, and thus, the result of [87] is largely considered a negative result. Following the pioneering work of Diffie and Hellman [27], modern cryptography circumvents this restriction by relaxing the security requirement from information theoretic security to security against a computationally bounded adversary. However, a different remedy is possible in situations where the transmitter and the receiver have access to correlated randomness, which is available to the eavesdropper only in part. Specifically, it was shown by Wyner in [103] that if the eavesdropper can access only a noisy version of the observations of the legitimate receiver, secure transmission<sup>2</sup> is feasible without requiring any additional resources. Furthermore, it was shown in [12], [70], [2] that information theoretically secure secret keys can be extracted from correlated random observations by communicating over an insecure, public communication channel. These works constitute the basic foundations of information theoretic security, suggesting that the requirement of large secret keys for the feasibility of information theoretic security can be circumvented if correlated randomness is available.

Inspired by these results, practical schemes for information theoretically secure message transmission and secret key agreement have been proposed, utilizing the correlated randomness available in the physical communication channel (*cf.* [6], [69], [21], [104]) or the correlated randomness extracted from physical observations (*cf.* [80], [55], [75], [30]). However, most of the practical schemes proposed have either no theoretical guarantees of performance or are suboptimal. In fact, even for the basic problems of *secret*

<sup>1</sup>Shannon [87] established the necessity of an  $m$ -bit perfect secret key only for the case when a one-time-pad is used for encryption. The general necessary condition for any scheme was shown in [68] (see, also, [57, Problems 2.12 and 2.13], [50], [96, Section VI]).

<sup>2</sup>Strictly speaking, we are concerned with the transmission of confidential messages in the presence of passive eavesdroppers.

*key agreement* and coding for a *wiretap channel*, optimal practical schemes are few and have emerged only over the last decade (see [14], [28], [105], [81], [19] for optimal schemes for secret key agreement and the review article [35] for references on optimal codes for a wiretap channel).

In this article, we review a class of practical coding schemes for attaining information theoretically secure secret key agreement as well as for information theoretically secure message transmission in a wiretap channel model. Specifically, we focus on schemes that use *2-universal hash families* (UHF) [17] (see Section III for the definition of a UHF) as a building block. This restriction in scope is for two reasons: First, UHFs are easy to implement and are ideally suited for lightweight cryptography (*cf.* [60], [107]), and second, while review articles are available that cover the role of error-correcting codes in physical layer security (*cf.* [72], [35]), the UHF based schemes for wiretap channels are recent and are not well-known.

The remainder of this article is organized as follows. We begin by describing the secret key agreement and the wiretap coding problem in the next section. In the subsequent section, we define a UHF and discuss its basic properties and some practical implementations. In the final two sections, we review UHF based coding schemes for secret key agreement and wiretap channels.

## II. PRIMITIVES FOR INFORMATION THEORETIC SECURITY

In this section, we describe two basic primitives for information theoretic security. Both rely on the correlation in the random observations of legitimate parties; however, the form of correlation is different in each. The first of these, namely secret key agreement, is concerned with extracting shared secret bits from noisy correlated random data. The second, coding for wiretap channels, focuses on sending data over a noisy channel when a passive eavesdropper observes noisy versions of the transmissions. The two problems seem to be different in their scope and objective. Yet similar schemes based on error-correcting codes and UHF will be seen to be optimal for both in many cases.

Note that the basic cryptographic primitives of oblivious transfer [77] and bit commitment [15], too, have information theoretically secure counterparts; see, for instance, [20], [73], [100], [4], [79], [96] and [101], [52], [78], [96], respectively, for treatments of information theoretically secure oblivious transfer and bit commitment. However, there are only a few practical schemes available (*cf.* [52]), and they will not be reviewed here.

### A. Secret key agreement

Discrete, correlated random variables  $X$  and  $Y$ , with arbitrary but known distribution  $P_{XY}$ , are observed by the first and the second party, respectively. The parties seek to agree on random, unbiased bits. These

correlated random variables correspond to random physical observations and can be derived, for instance, from different noisy recordings of the same biometric fingerprint, or from the random fade observed in a wireless communication channel. The parties also have access to a public communication channel such as a shared public server, or a broadcast channel, or any other insecure communication network. They can use this communication channel to exchange bits with each other; however, the bits exchanged will be available to a (passive) eavesdropper. The mode of communication allowed depends on the application at hand. For instance, in the biometric and PUF applications, only one sided communication from  $X$  to  $Y$  is available since  $Y$  corresponds to a later (in time) recording of  $X$  itself. In general, the parties can execute an interactive communication protocol  $\Pi$  with multiple rounds of interaction and possibly randomized communication in each round<sup>3</sup>. The goal is to derive a secret key  $K$  consisting of bits  $(K_1, \dots, K_l)$  such that (i) with large probability, both parties can recover  $K$  accurately; (ii) bits  $(K_1, \dots, K_l)$  are almost independent and unbiased; and (iii) an eavesdropper with access to the communication  $\Pi$  and a side information  $Z$  cannot ascertain any information about  $K$ .

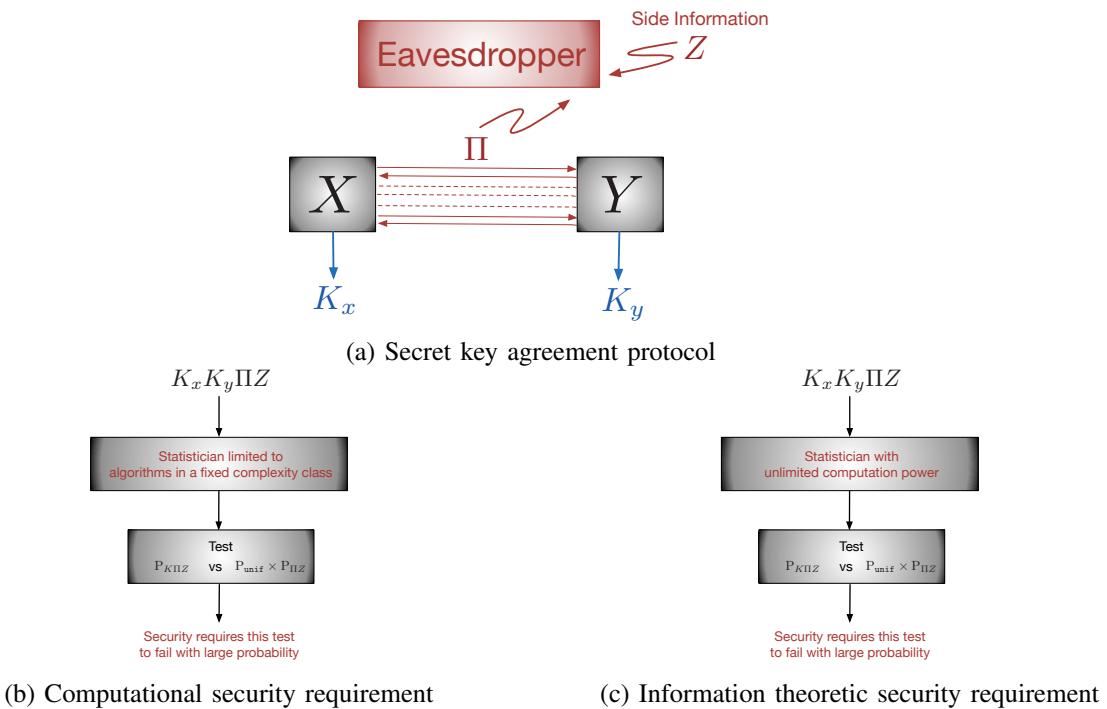


Fig. 1: Illustration of secret key agreement

Condition (i) above constitutes the *recoverability* requirement. Parties must form estimates  $K_x$  and  $K_y$

<sup>3</sup>The communicated data  $\Pi$  is sometimes referred to as *helper data*.

of  $K$  such that

$$\Pr(K_x = K_y = K) \geq 1 - \epsilon,$$

for a suitably small parameter  $\epsilon$ .

Conditions (ii) and (iii) above constitute the *security* requirement. Traditional notion of cryptographic security is computational and requires (*cf.* [31]) that a computationally bounded adversary with access to efficient algorithms for solving problems in a particular complexity class, but not beyond it, cannot reliably distinguish if the observed outputs of the secret key agreement protocol  $(K, \Pi, Z)$  are coming from the real protocol or an ideal one with all values of  $K$  equally likely for each realization of  $(\Pi, Z)$ . In contrast, [12], [70], [2] initiated the study of the secret key agreement problem under *information theoretic security* where the computationally bounded adversary above is replaced by an unrestricted one with access to any statistical test<sup>4</sup>. Formally, it is required that the *statistical distance* between the joint distribution  $P_{K\Pi Z}$  and  $P_{\text{unif}} \times P_{\Pi Z}$  is small. Two popular measures of statistical distance that have been used in secret key agreement literature are the K-L divergence [70], [2], [11], [22], [25], [26], [40]

$$D(P||Q) = \sum_i P_i \log \frac{P_i}{Q_i},$$

and the total variation distance [84], [82], [41]

$$\|P - Q\|_1 = \frac{1}{2} \sum_i |P_i - Q_i|.$$

For concreteness, we shall consider security of secret keys under the total variation distance and require

$$\|P_{K\Pi Z} - P_{\text{unif}} \times P_{\Pi Z}\|_1 \leq \delta,$$

where  $P_{\text{unif}}$  is a uniform distribution on  $l$ -bits. Figure 1 illustrates the setup and a comparison of the computational and information theoretic security criteria. For given values of recoverability and security parameters  $\epsilon$  and  $\delta$ , we seek to design secret key agreement protocols that yield as many bits of secret key  $K$  as possible, *i.e.*, the largest possible value of  $l$  above.

The theoretical limits of the length of secret keys possible have been studied extensively: [70] and [2] considered the case when the underlying observations are *independent and identically distributed* (IID) and, under a weaker notion of security than that above, characterized the *secret key capacity*, *i.e.*, the maximum rate of secret key length per observation; [11], [22], [3], [71] provide basic tools for attaining the stronger notion of security above without any loss of performance; [25] establishes the secret key

<sup>4</sup>For another connection between binary hypothesis testing and secret key agreement, see [94], [96].

capacity for a multiparty version of the problem; [84], [82] derive bounds on the secret key length for the single-shot case above, when only one-sided communication is allowed; [94], [43], [45] give the best-known bounds for the general problem above, stressing on the role of interactive communication. However, none of these works give an efficient secret key agreement scheme. The literature on constructive coding schemes, on the other hand, is narrow and has focused mostly on the case with one-sided communication. In this article, we will discuss a class of constructive schemes for secret key agreement that rely on UHFs.

### B. Coding for wiretap channel

The problem of wiretap coding is that of transmitting a message with confidentiality from an eavesdropper with side-information. Specifically, a sender seeks to communicate a message  $M$  to a receiver by using transmissions over a noisy communication channel  $T$  with inputs from a set  $\mathcal{X}$  and outputs from a set  $\mathcal{Y}$ . For each input  $x$  to  $T$ , the receiver observes an output  $y$  with a given probability density  $T(y|x)$ . Furthermore, for each transmission  $x$  an eavesdropper observes the output  $z$  of another communication channel  $W$ . It is required that while the legitimate receiver decodes  $M$  with a low probability of error, while the message remains concealed from the eavesdropper (or the *wire-tapper*). See Figure 2 for an illustration.

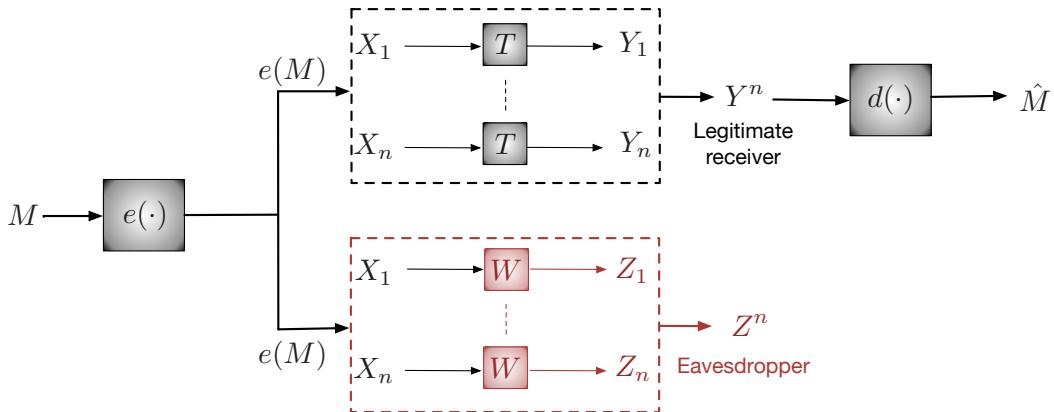


Fig. 2: Illustration of wiretap channel

An  $(n, k)$  code for this wiretap channel consists of a (stochastic) encoder  $e : \{0, 1\}^k \rightarrow \mathcal{X}^n$  and a decoder  $d : \mathcal{Y}^n \rightarrow \{0, 1\}^k$ . A random message  $M$  is sent as  $e(M)$  and decoded as  $\hat{M} = d(Y^n)$ , where  $Y^n = (Y_1, \dots, Y_n)$  denotes the outputs for  $n$  independent uses of the channel  $T$  for inputs  $(X_1, \dots, X_n) = e(M)$ . At the same time, an eavesdropper gets to observe the outputs  $Z^n$  corresponding to transmitting the inputs  $X^n$  over the channel  $W$ . It is required that the code  $(e, d)$  ensures high reliability, i.e.  $P(M \neq \hat{M}) \approx 0$  (it is required that  $P(M \neq \hat{M})$  goes to 0 sufficiently rapidly in  $n$ ), and ensures

security under an appropriate notion. The rate of this code is  $(k/n)$ ; the maximum possible asymptotic rate of a wiretap code is called the *wiretap capacity* of  $(T, W)$ .

This basic model was introduced by Wyner in [103] where he considered a *degraded wiretap channel* where  $W = V \circ T$  for some stochastic mapping  $V$ , *i.e.*, the eavesdropper's observation is a further noisy version of the legitimate receiver's observation, and for an input  $x$  the eavesdropper's channel produces an output  $z$  with probability  $W(z|x) = \sum_y V(z|y)T(y|x)$ . For this important special case, Wyner characterized the wiretap capacity under the *weak security* requirement given by

$$\lim_{n \rightarrow \infty} \frac{1}{n} I(M \wedge Z^n) = 0,$$

where the message  $M$  is a uniform random variable and  $I(U \wedge V)$  denotes the mutual information between random variables  $U$  and  $V$  [24]. Later, Csiszár and Körner characterized  $C$  for all discrete, memoryless wiretap channels [23].

Interestingly, the wiretap capacity remains unchanged even if we drop the normalization by  $n$  in the weak security condition above and require *strong security* [22]

$$\lim_{n \rightarrow \infty} I(M \wedge Z^n) = 0,$$

for a uniform message  $M$ . A still more demanding notion of security introduced<sup>5</sup> in [10] requires security not only for a uniform message  $M$  but any random message  $M$  and is given by

$$\lim_{n \rightarrow \infty} \max_{P_M} I(M \wedge Z^n) = 0.$$

In fact, [10] extended the cryptographic notion of *semantic security* (*cf.* [31]) to the wiretap channel and showed that it is implied by the security requirement above. In this article, we shall use the term *semantic security* synonymously with the security requirement above, keeping in mind that, in fact, we are demanding something even stronger than semantic security.

It remains an open question if the wiretap capacity can be achieved under semantic security, in general. However, for specific wiretap channels, codes that achieve wiretap capacity while ensuring semantic security have been proposed recently (*cf.* [42], [65], [8], [10], [63]). In particular, the schemes in [42], [40], [8], [47], [41], [95] rely on UHFs and are discussed below.

<sup>5</sup>This notion of security is termed *mutual information security* in [10] and *source universality* in [47].

### III. 2-UNIVERSAL HASH FAMILIES

The key primitive that underlies all the schemes that will be discussed in this article is a UHF. Universal hashing was introduced by Carter and Wegman in their seminal work [17] as a multipurpose tool for theoretical computer science and was applied for privacy amplification first in [12]. A UHF is, roughly speaking, a family of functions such that the random mapping obtained by uniformly choosing a function from this family is *almost* invertible. In information theory, as in theoretical computer science, many of the proofs are completed using a random mapping or binning or coloring of elements of a set. It turns out that most of the tasks that can be done using a completely random mapping can also be done by a randomly selected member of a UHF. Moreover, while implementing a random mapping is not practical, structured implementations of certain UHFs are available (*cf.* [60], [107], [49] and [41, Appendix II]). Thus, UHFs constitute an efficiently implementable substitute for random mappings.

Formally, a family  $\mathcal{F}$  consisting of mappings  $f : \mathcal{X} \rightarrow \{1, \dots, 2^k\}$  is a ( $k$ -bit) UHF if for every  $x \neq x'$

$$\frac{1}{|\mathcal{F}|} |\{f \in \mathcal{F} : f(x) = f(x')\}| \leq 2^{-k}, \quad (1)$$

*i.e.*, the random mapping  $F$  chosen uniformly over  $\mathcal{F}$  maps two distinct values to the same output with probability less than  $2^{-k}$ .

The diverse applications of UHFs in information theoretic security include: secret key agreement (*cf.* [11], [84], [40]), quantum key distribution (*cf.* [82]), biometric and hardware security (*cf.* [28]), and coding for wiretap channels (*cf.* [37], [42], [8], [47]); see [90] for other applications in cryptography. In these applications, the importance of a UHF lies in the role it plays in randomness extraction in source and channel models. In a source model, we consider a randomness which is observed by a legitimate party and is generated by a fixed distribution. On the other hand, in a channel model, the randomness is observed by an adversary and its distribution is controlled by a legitimate party. Basic results were first derived for source models and, later, variants of these basic results were derived for channel models; we shall review the results for both these cases below.

#### A. Source models

In a source model, the available random observation and eavesdropper's observation are modeled by correlated random variables  $(X, Z)$ . In applications such as secret key agreement, we seek to design a primitive that extracts from  $X$  uniformly distributed random bits that are almost independent of  $Z$ . UHFs described above provide a constructive tool for realizing such a primitive. First, we consider the special case of a constant  $Z$ . The main result here is the *leftover hash lemma* which shows roughly that

the output of a randomly chosen member of a  $k$ -bit UHF applied to a random variable  $X$  constitutes uniformly random bits, provided that  $k$  is smaller than a threshold. Different versions of *leftover hash lemma* are available in literature, each with a slightly different choice of this threshold (*cf.* [54], [53], [11], [36], [90], [82], [84]). We review a version due to [82], [84] where the aforementioned threshold for randomness extraction is given by the *smooth min-entropy*  $H_{\min}^{\epsilon}(P_X)$  of the underlying random variable  $X$ , defined as follows [82], [84]: The *min-entropy* of  $X$  is given by [85]

$$H_{\min}(P_X) = \min_x -\log P_X(x),$$

and the  $\epsilon$ -smooth min-entropy of  $X$  is defined as [83], [84], [82]

$$H_{\min}^{\epsilon}(P_X) = \sup_{Q: \|P-Q\|_1 \leq \epsilon} H_{\min}(Q).$$

The leftover hash lemma uses a randomly selected member of a given UHF. In order to facilitate this random selection, we assume that a random seed  $S$  distributed uniformly over a discrete set  $\mathcal{S}$  is available to both the legitimate party as well as the eavesdropper. While bounding the leaked information of the extracted bits, eavesdropper's knowledge of the random seed is taken into account as well.

**Lemma 1 (Leftover hash: No side information).** *Consider random variables  $X$  taking values in a finite set  $\mathcal{X}$ . Then, for a  $k$ -bit UHF consisting of mappings  $\{f_s, s \in \mathcal{S}\}$  and a random seed  $S$  distributed uniformly over the set  $\mathcal{S}$ , it holds for every  $\epsilon \in [0, 1)$  that*

$$\|P_{f_S(X)S} - P_{\text{unif}} \times P_S\|_1 \leq \epsilon + \frac{1}{2} \sqrt{2^{k-H_{\min}^{\epsilon}(X)}}.$$

The first instance of a variant of this result, for the special case  $\epsilon = 0$ , appeared in [53] (see, also, [36] for further strengthening of this result). The term “leftover hash lemma” appeared in [54] where a strengthening of the result of [53] was given with Rényi entropy of order 2 in place of min-entropy. The form given above is a special case of a general result in [82], [84] for the case where, in addition to the random seed  $S$ , the eavesdropper observes a (possibly continuous-valued) random variable  $Z$ . In this general version, the threshold  $H_{\min}^{\epsilon}(X)$  is replaced by the  $\epsilon$ -smooth conditional min-entropy given by [83], [82]

$$H_{\min}^{\epsilon}(P_{XZ}|Z) = \sup_{Q_{XZ}: \|P_{XZ}-Q_{XZ}\|_1 \leq \epsilon} H_{\min}(Q_{XZ}|Z),$$

where  $H_{\min}(Q_{XZ}|Z)$  denotes the conditional min-entropy

$$H_{\min}(P_{XZ}|Z) = \sup_{Q_Z: \text{supp}(P_Z) \subset \text{supp}(Q_Z)} H_{\min}(P_{XZ}|Q_Z)$$

and, for  $P_Z$  and  $Q_Z$  with densities  $f_P$  and  $f_Q$  (with respect to a measure  $\mu$  on  $\mathcal{Z}$ ), respectively,

$$H_{\min}(P_{XZ}|Q_Z) = \inf_{x \in \mathcal{X}, z \in \text{supp}(Q_Z)} -\log \frac{P_{X|Z}(x|z) f_P(z)}{f_Q(z)}.$$

Note that smooth min-entropies replaces Shannon entropies as a measure of randomness in the context of randomness extraction (see [11, Section VI] for further discussion). However, for IID observations  $X^n$ , Shannon entropy constitutes the leading asymptotic term in smooth min-entropy of  $P_{X^n}$  (*c.f.* [82]). We depict the result of [82], [84] in Figure 3. Below, we recall a further generalization where the side information  $Z$  available to the eavesdropper consists of a finite-valued random variable  $Z_1$  and a continuous-valued random variable  $Z_2$ ; see, for instance, [45, Appendix B]) for a proof.

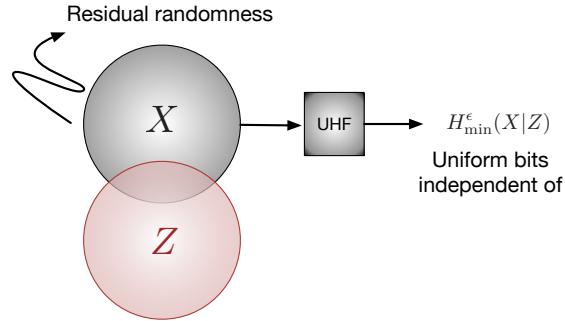


Fig. 3: Leftover hash property of UHFs

**Lemma 2 (Leftover hash).** Consider random variables  $X, Z_1, Z_2$  taking values, respectively, in a finite set  $X$ , a (possibly uncountable) set  $\mathcal{Z}_1$ , and a finite set  $\mathcal{Z}_2$ . Then, for a  $k$ -bit UHF consisting of mappings  $\{f_s, s \in \mathcal{S}\}$  and a random seed  $S$  distributed uniformly over the set  $\mathcal{S}$ , it holds for every  $\epsilon \in [0, 1)$  that

$$\|P_{f_S(X)Z_1Z_2S} - P_{\text{unif}} \times P_{Z_1Z_2S}\|_1 \leq \epsilon + \frac{1}{2} \sqrt{|\mathcal{Z}_2| 2^{k - H_{\min}^{\epsilon}(P_{XZ_1}|Z_1)}}.$$

In essence, the result above says that  $H_{\min}^{\epsilon}(P_{XZ_1}|Z_1) - \log |\mathcal{Z}_2|$  almost uniform bits which are almost independent of  $(Z_1, Z_2)$  can be extracted from  $X$ . To measure “almost” uniformity and independence, the results above use the total variation distance. An alternative form of the leftover hash lemma, with the K-L divergence replacing the variation distance, was derived in [11] and is reviewed below.

**Lemma 3 (Leftover hash: Divergence form).** Consider random variables  $X, Z$  taking values, respectively, in a finite set  $X$  and a (possibly uncountable) set  $\mathcal{Z}$ . Then, for a  $k$ -bit UHF consisting of mappings  $\{f_s, s \in \mathcal{S}\}$  and a random seed  $S$  distributed uniformly over the set  $\mathcal{S}$ , it holds that

$$D(P_{f_S(X)ZS} \| P_{\text{unif}} \times P_{ZS}) \leq \frac{2^{k - H_{\min}(P_{XZ}|P_Z)}}{\ln 2}.$$

Note that by Pinsker's inequality (*cf.* [24]), the K-L divergence form yields the total variation distance form (up to a constant factor). On the other hand, using the continuity of entropy in total variation distance, a K-L divergence form was derived using the total variation distance form above in [40] (see, also, [25, Lemma 1]). Both the total variation distance form and the K-L divergence form of the leftover hash lemma given above combine the requirement of almost uniformity of  $f_S(X)$  and security of  $f_S(X)$  from an observer of  $(Z, S)$  into a single criterion. In fact, the result in [11] shows that  $k - H(f_S(X)|ZS) = k - H(f_S(X)) + I(f_S(X) \wedge ZS)$  is bounded above by  $2^{k-H_{\min}(P_{XZ}|P_Z)} / \ln 2$ , which in turn implies that the mutual information  $I(f_S(X) \wedge ZS)$  is bounded above by the same quantity<sup>6</sup>. In the information theory literature, traditionally, mutual information has been used as a measure of information leakage<sup>7</sup> (*cf.* [87], [103], [23]), and the result above says that the information about  $f_S(X)$  leaked to the eavesdropper is small as long as  $k$  is sufficiently smaller than  $H_{\min}(P_{XZ}|P_Z)$ . It was noted in [37, Appendix III] that, under an almost uniformity assumption for  $f_S(X)$ , a bound on  $I(f_S(X) \wedge ZS)$  yields a bound on the total variation distance  $\|P_{f_S(X)ZS} - P_{\text{unif}} \times P_{ZS}\|_1$ . On the other hand, a counterexample was given to show that a small  $\|P_{f_S(X)ZS} - P_{\text{unif}} \times P_{ZS}\|_1$  need not guarantee a small  $I(f_S(X) \wedge ZS)$ .

In practice, one is interested in characterizing the optimal tradeoff between information leakage and the range-size  $k$  of the UHF used. For the case of IID observations  $X^n$ , [38] considered the optimal  $k = k_n(\epsilon)$  required to attain a given leakage  $\epsilon$  as a function of  $n$  and studied the second-order asymptotic term, for both the total variation distance and the K-L divergence criteria (see the textbook [33] and the references therein for a treatment of general sources beyond IID). In a different regime, [40] studied the exponential decrease in the leakage for increasing  $n$ , for a fixed rate  $k = nR$  for the total variation distance based leakage and the mutual information leakage, with a focus on the latter; optimal exponents for decay rate of the total variation distance based leakage as a function of  $n$  were obtained in [41].

### B. Channel models

Another class of models relevant for the wiretap channel entails a channel  $V : \mathcal{X} \rightarrow \mathcal{Z}$  between the legitimate party and the eavesdropper. For each input  $x \in \mathcal{X}$  selected by the legitimate party, the eavesdropper observes a random variable  $Z \in \mathcal{Z}$  with distribution  $V_x$ . The goal is to determine a stochastic map (a channel)  $\Gamma : \mathcal{M} \rightarrow \mathcal{X}$  such that for the composite channel  $V' = V \circ \Gamma$ , with inputs from  $\mathcal{M}$  and outputs in  $\mathcal{Z}$ , it holds that

<sup>6</sup>The quantity  $k - H(f_S(X)|ZS)$  was defined as a security index in [25] and was noted to equal  $D(P_{f_S(X)ZS} \| P_{\text{unif}} \times P_{ZS})$ .

<sup>7</sup>Bounds on leakage measured by Rényi information quantities were derived recently in [48].

- (i) For a uniformly distributed input  $M$  of  $\Gamma$ , the random variable  $M$  is almost independent of the output  $Z$  of  $V'$  observed by the eavesdropper; and
- (ii)  $M$  can be determined from the input  $X$  of  $V$ .

Note that in the source model discussed in the previous section, the distribution of  $X$  is fixed and a uniformly distributed  $M$  is obtained as  $F(X)$ , the output of a randomly chosen member  $F$  of a UHF. In contrast, in the channel model we fix the distribution of  $M$  and seek to design  $\Gamma$  such that the two properties above hold. Here, too, a constructive scheme can be obtained using a UHF satisfying certain “balanced” conditions. Specifically, we consider a UHF  $\{f_s : \mathcal{X} \rightarrow \{0, 1\}^k, s \in \mathcal{S}\}$  satisfying the following balanced condition: For every seed  $s \in \mathcal{S}$  and  $m \in \{0, 1\}^k$ ,

$$|\{x \in \mathcal{X} \mid f_s(x) = m\}| = 2^b.$$

The condition above says that for each member of the UHF, the cardinality of each inverse-image set is the same. We call a UHF satisfying the condition above a  $b$ -balanced UHF.

A  $b$ -balanced UHF can be used to design the aforementioned stochastic map  $\Gamma : \mathcal{M} \rightarrow \mathcal{X}$  as follows: For each  $m \in \mathcal{M}$ , choose  $X$  uniformly over  $f_S^{-1}(m)$ , where the random seed  $S$  is chosen uniformly over  $\mathcal{S}$ . The next result is a counterpart of the leftover hash lemma for the channel model and shows that the requirement (i) above holds if  $b$  is less than a threshold. The first instance of such a result appears in [42, Section V]. The weaker version below uses a different threshold which is often easier to evaluate. Specifically, the threshold in the lemma below is given by the *smooth max-information* of the channel, which is defined as follows: Consider a subnormalized channel  $V : \mathcal{X} \rightarrow \mathcal{Z}$  with a finite input alphabet  $\mathcal{X}$  and such that for each  $x \in \mathcal{X}$  the measure  $V(\cdot \mid x)$  on  $\mathcal{Z}$  has a density  $\omega(z|x)$  with respect to a measure  $\mu$  on  $\mathcal{Z}$ . The *max-information* of  $V$  is given by

$$I_{\max}(V) = \log \int \max_{x \in \mathcal{X}} \omega(z|x) d\mu.$$

For a subset  $\mathcal{T}$  of  $\mathcal{X} \times \mathcal{Z}$ , denote by  $V_{\mathcal{T}}$  the subnormalized channel corresponding to the density

$$\omega_{\mathcal{T}}(z \mid x) = \begin{cases} \omega(z|x), & (x, z) \in \mathcal{T}, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

The  $\epsilon$ -smooth max-information of  $V$ ,  $I_{\max}^{\epsilon}(V)$ , is given by the infimum of  $I_{\max}(V_{\tau})$  over all sets  $\mathcal{T} \subset \mathcal{X} \times \mathcal{Z}$  such that

$$V(\{z : (x, z) \in \mathcal{T}\} \mid x) \geq 1 - \epsilon, \quad \text{for all } x \in \mathcal{X}. \quad (3)$$

Note that the smoothing operation in the definition of smooth max-information is different from the one used in defining smooth max-entropy above, but is similar to the definition of smoothing in [84].

**Lemma 4 (Leftover hash: Channel model).** *Given a channel  $V : \mathcal{X} \rightarrow \mathcal{Z}$ , with a finite input set  $\mathcal{X}$  and arbitrary output set  $\mathcal{Z}$ , and an  $b$ -balanced  $k$ -bit UHF  $\{f_s : s \in \mathcal{S}\}$ , suppose that for each  $m \in \mathcal{M} = \{0, 1\}^k$  and  $s \in \mathcal{S}$  the input  $X$  of  $V$  is chosen uniformly over  $f_s^{-1}(m)$ . Then, for a random variable  $M$  distributed uniformly on  $\mathcal{M}$ ,*

$$I(M \wedge Z, S) \leq \frac{1}{\ln 2} \cdot 2^{-(b - I_{\max}^{\epsilon}(V))} + \epsilon k, \quad (4)$$

where the seed  $S$  is distributed uniformly over  $\mathcal{S}$ .

*Proof:* Consider a  $b$ -balanced UHF  $\{f_s, s \in \mathcal{S}\}$ . We first prove the bound in (4) for the special case of  $\epsilon = 0$ . To that end, note first that the conditional density of  $Z$  (w.r.t.  $\mu$ ) given  $M = m$  and  $S = s$  is given by

$$\begin{aligned} \frac{dP_{Z|M,S}}{d\mu}(z|m, s) &= \sum_{x \in f_s^{-1}(m)} \frac{1}{|f_s^{-1}(m)|} \cdot \omega(Z | x) \\ &= 2^{-b} \sum_{x \in \mathcal{X}} \mathbb{1}(f_s(x) = m) \omega(z | x), \end{aligned} \quad (5)$$

where the equality is by definition of a  $b$ -balanced UHF. Similarly, since  $M$  and  $S$  are independent and  $M$  is distributed uniformly over  $\{0, 1\}^k$ , the conditional density of  $Z$  (w.r.t.  $\mu$ ) given  $S = s$  is given by

$$\begin{aligned} \frac{dP_{Z|S}}{d\mu}(z|s) &= 2^{-b-k} \sum_{m \in \mathcal{M}} \sum_{x \in \mathcal{X}} \mathbb{1}(f_s(x) = m) \omega(z | x) \\ &= 2^{-b-k} \sum_{x \in \mathcal{X}} \omega(z | x), \end{aligned} \quad (6)$$

where we have used  $\sum_{m \in \mathcal{M}} \mathbb{1}(f_s(x) = m) = 1$ . By (5) and (6), we get

$$\begin{aligned} I(M \wedge Z, S) &= I(M \wedge Z | S) \\ &= \mathbb{E} \log \frac{dP_{Z|M,S}}{dP_{Z|S}} \\ &= \mathbb{E} \log \frac{2^{-b} \sum_{x' \in \mathcal{X}} \mathbb{1}(f_S(x') = M) \omega(Z | x')}{2^{-b-k} \sum_{x'' \in \mathcal{X}} \omega(Z | x'')} \\ &= \mathbb{E} \log \frac{2^k \sum_{x' \in \mathcal{X}} \mathbb{1}(f_S(x') = M) \omega(Z | x')}{\sum_{x'' \in \mathcal{X}} \omega(Z | x'')} \end{aligned} \quad (7)$$

$$= \frac{2^{-b-k}}{|\mathcal{S}|} \int_{\mathcal{Z}} \sum_{s \in \mathcal{S}, m \in \mathcal{M}, x \in \mathcal{X}} \left[ \mathbb{1}(f_s(x) = m) \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}} \mathbb{1}(f_s(x') = m) \omega(z | x')}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \right] \mu(dz). \quad (8)$$

Since the summand inside  $[\cdot]$  is nonzero only for  $m = f_s(x)$ , we can replace the term  $\mathbb{1}(f_s(x') = m)$  with  $\mathbb{1}(f_s(x') = f_s(x))$  to obtain

$$\begin{aligned} I(M \wedge Z, S) &= \frac{2^{-b-k}}{|\mathcal{S}|} \int_{\mathcal{Z}} \sum_{s \in \mathcal{S}, m \in \mathcal{M}, x \in \mathcal{X}} \mathbb{1}(f_s(x) = m) \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}} \mathbb{1}(f_s(x') = f_s(x)) \omega(z | x')}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \mu(dz) \\ &= \frac{2^{-b-k}}{|\mathcal{S}|} \int_{\mathcal{Z}} \sum_{s \in \mathcal{S}, x \in \mathcal{X}} \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}} \mathbb{1}(f_s(x') = f_s(x)) \omega(z | x')}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \mu(dz) \\ &\leq 2^{-b-k} \int_{\mathcal{Z}} \sum_{x \in \mathcal{X}} \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}} |\mathcal{S}|^{-1} \sum_{s \in \mathcal{S}} \mathbb{1}(f_s(x') = f_s(x)) \omega(z | x')}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \mu(dz), \end{aligned} \quad (9)$$

where the last inequality is by Jensen's inequality applied to the log function. Furthermore, using the UHF property (1) for the UHF  $\{f_s, s \in \mathcal{S}\}$  we have

$$\frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \mathbb{1}(f_s(x') = f_s(x)) \leq 2^{-k} \mathbb{1}(x' \neq x) + \mathbb{1}(x' = x),$$

which along with (9) gives

$$\begin{aligned} I(M \wedge Z, S) &\leq 2^{-b-k} \int_{\mathcal{Z}} \sum_{x \in \mathcal{X}} \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}} (2^{-k} \mathbb{1}(x' \neq x) + \mathbb{1}(x' = x)) \omega(z | x')}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \mu(dz) \\ &\leq 2^{-b-k} \int_{\mathcal{Z}} \sum_{x \in \mathcal{X}} \omega(z | x) \log \frac{\sum_{x' \in \mathcal{X}} \omega(z | x') + 2^k \omega(z | x)}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \mu(dz) \\ &= 2^{-b-k} \int_{\mathcal{Z}} \sum_{x \in \mathcal{X}} \omega(z | x) \log \left( 1 + \frac{2^k \omega(z | x)}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \right) \mu(dz) \\ &\leq \frac{2^{-b}}{\ln 2} \int_{\mathcal{Z}} \frac{\sum_{x \in \mathcal{X}} \omega(z | x)^2}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \mu(dz), \end{aligned}$$

where the previous inequality uses  $\ln(1+x) \leq x$  for all  $x \geq 0$ . Therefore, on observing that

$$\frac{\sum_{x \in \mathcal{X}} \omega(z | x)^2}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \leq \max_x \omega(z | x) = 2^{I_{\max}(W)},$$

we get

$$I(M \wedge Z, S) \leq \frac{1}{\ln 2} \cdot 2^{-(b-I_{\max}(W))},$$

which completes the proof for the case  $\epsilon = 0$ .

Moving to the case  $\epsilon > 0$ , consider a set  $\mathcal{T} \subset \mathcal{X} \times \mathcal{Z}$  satisfying (3). Note that by log-sum inequality

$$\begin{aligned} & \sum_{x \in \mathcal{X}} \mathbb{1}(f_s(x) = m) \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}} \mathbb{1}(f_s(x') = m) \omega(z | x')}{\sum_{x'' \in \mathcal{X}} \omega(z | x'')} \\ & \leq \sum_{x \in \mathcal{X}: (x, z) \in \mathcal{T}} \mathbb{1}(f_s(x) = m) \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}: (x', z) \in \mathcal{T}} \mathbb{1}(f_s(x') = m) \omega(z | x')}{\sum_{x'' \in \mathcal{X}: (x'', z) \in \mathcal{T}} \omega(z | x'')} + \\ & \quad \sum_{x \in \mathcal{X}: (x, z) \in \mathcal{T}^c} \mathbb{1}(f_s(x) = m) \omega(z | x) \log \frac{2^k \sum_{x' \in \mathcal{X}: (x', z) \in \mathcal{T}^c} \mathbb{1}(f_s(x') = m) \omega(z | x')}{\sum_{x'' \in \mathcal{X}: (x'', z) \in \mathcal{T}^c} \omega(z | x'')} \end{aligned} \quad (10)$$

Thus, upon denoting the right-side of (8) by  $g(V)$ , (8) and (10) give

$$I(M \wedge Z, S) \leq g(V_{\mathcal{T}}) + g(V_{\mathcal{T}^c}),$$

where  $V_{\mathcal{T}}$  and  $V_{\mathcal{T}^c}$  are defined in (2). Proceeding as in the  $\epsilon = 0$  case, we get

$$g(V_{\mathcal{T}}) \leq \frac{1}{\ln 2} \cdot 2^{-b+I_{\max}(V_{\mathcal{T}})}.$$

Furthermore, using the simple bound

$$\log \frac{2^k \sum_{x' \in \mathcal{X}: (x', z) \in \mathcal{T}^c} \mathbb{1}(f_s(x') = m) \omega(z | x')}{\sum_{x'' \in \mathcal{X}: (x'', z) \in \mathcal{T}^c} \omega(z | x'')} \leq k$$

we get

$$g(V_{\mathcal{T}^c}) \leq P((X, Z) \in \mathcal{T}^c) k \leq \epsilon k,$$

where the previous inequality uses the assumption that  $\mathcal{T}$  satisfies (3). It follows upon combining the inequalities above that

$$I(M \wedge Z, S) \leq \frac{1}{\ln 2} \cdot 2^{-(b-I_{\max}(V_{\mathcal{T}}))} + \epsilon k.$$

The proof is completed using the definition of  $\epsilon$ -smooth max-information upon optimizing  $I_{\max}(V_{\mathcal{T}})$  over sets  $\mathcal{T}$  that satisfy (3).  $\blacksquare$

Thus, the leakage  $I(M \wedge Z, S)$  is small as long as  $b$  is much smaller than  $I_{\max}^\epsilon(V)$ . As in the case of source model, here, too, it is of interest to determine the optimal leakage exponent. Furthermore, it is of interest to derive bounds on leakage for other measures such as the total variation distance measure<sup>8</sup>; one instance of such bound is available in [46] for the special case when the channel  $V$  is given by a

<sup>8</sup>In applying this bound to the case of wiretap channel, the channel  $V$  will be chosen to be the concatenation of the legitimate transmission channel and an error correcting code for it.

concatenation of a random code and another transmission channel.

### C. Implementations

An efficient implementation of a  $k$ -bit UHF for an  $l$ -bit input can be obtained as follows [17], [60]: Let  $\{0, 1\}^l$  correspond to the elements of  $GF(2^l)$  and let  $\mathcal{S} = \{0, 1\}^l \setminus \{\mathbf{0}\}$ . For  $k \leq l$ , define a mapping  $f : \mathcal{S} \times \{0, 1\}^l \rightarrow \{0, 1\}^k$  as follows:

$$f(s, x) = (s \cdot x)_k,$$

where  $(x)_k$  selects the  $k$  most significant bits of  $x$ . It is easy to see that the family of mappings  $\{f_s(x) := f(s, x), s \in \mathcal{S}\}$  constitutes a UHF. In fact, it is easy to see that this UHF is a  $(l - k)$ -balanced UHF. Furthermore, for  $m \in \mathcal{M} = \{0, 1\}^k$ , a uniform distribution on the inverse-image set  $f_s^{-1}(m)$  (required in the channel version of the leftover hash lemma) can be computed efficiently, too, using the mapping  $\phi(s, m, R) = s^{-1} \cdot (m, R)$ , where  $R$  denotes  $(l - k)$  uniform random bits and  $(m, R)$  denotes the concatenation of  $m$  and  $R$ . Note that  $\phi$  is indeed the inverse of  $f$  since  $f(s, \phi(s, m, r)) = m$  for every  $s, m, r$ .

Note that in order to implement the aforementioned UHF  $f_s$  (and its inverse  $\phi$ ) efficiently, we require an efficient implementation of multiplication and inversion in  $GF(2^l)$ . One such efficient implementation was given in [88] for special values of  $l$ . Specifically, since the polynomial

$$\Phi(X) = X^l + X^{l-1} + \dots + X + 1$$

is irreducible in  $GF(2)[X]$  if and only if

- 1)  $l + 1$  is prime, and
- 2) 2 is a primitive root modulo  $l + 1$ , i.e., the powers  $1, 2, 2^2, \dots, 2^l$  are distinct modulo  $l + 1$ ,

for the values of  $l$  satisfying the two conditions above,  $GF(2^l)$  can be embedded as a subring of polynomials modulo  $X^{l+1} - 1$ . In this case, the multiplication of two elements in  $GF(2^l)$  is tantamount to multiplying the corresponding polynomials modulo  $X^{l+1} - 1$ , which in turn corresponds to the convolution of the two binary vectors of length  $l$ . As is well-known, this convolution can be realized using  $O(l \log l)$  computations using FFT, and also on hardware using a linear finite shift register (LFSR) of length  $l$ . Also, the inverse of elements of  $GF(2^l)$ , too, can be computed efficiently following the algorithm outlined in [88, Section 2.5].

The main limitation of the construction above is that it is feasible only for selected values of  $l$  satisfying the two conditions above. However, this is perhaps not a severe limitation since, if Artin's conjecture

holds, the number of such  $l$ s is infinite and one can identify such an  $l$  of a practically relevant order by running a simple computer code <sup>9</sup>.

An alternative construction, which circumvents the aforementioned limitation on the input length  $l$ , entails using a randomly chosen Toeplitz matrix. Specifically, for a random seed  $S$  consisting of  $(l+k-1)$  bits, the hash function  $f_S : \{0, 1\}^l \rightarrow \{0, 1\}^k$  is given by a  $k \times l$  matrix  $A$  with the first row and the first column consisting of elements of  $S$  and  $A_{i,j} = A_{i-1,j-1}$  for  $1 < i \leq k$  and  $1 < j \leq l$ . It was shown in [67] that the family of mappings  $f_s(x) = Ax, s \in \{0, 1\}^{l+k-1}$ , constitutes a  $k$ -bit UHF for inputs of length  $l$ . Note that we can view the multiplication of an  $l$ -length vector  $x$  with a Toeplitz matrix  $A$  as multiplying the extended  $(l+k-1)$ -length vector  $\bar{x} = (x_1, \dots, x_l, 0, 0, 0, \dots, 0)$  with the circulant extension of  $A$  and taking the first  $k$  entries [60], [49]. Thus, we can efficiently implement this UHF since multiplication with a circulant matrix is the same as convolution, which in turn can be computed efficiently using FFT.

A simple modification of the Toeplitz matrix based UHF above was given in [40] for which the inverse-image set can be efficiently computed as well. In this modified version, the random seed  $S$  consisting of  $(l-1)$  bits is used first to form a  $k \times (l-k)$  Toeplitz matrix  $A$  as before, but  $f_S(x)$  is given by  $[A, I]x$ , where  $I$  is the  $k$ -dimensional identity matrix. Clearly, the corresponding family of mappings constitutes a  $k$ -bit UHF with input length  $l$ . Furthermore, for  $m \in \{0, 1\}^l$ , a uniform distribution on the inverse-image set  $f_S^{-1}(m)$  can be computed efficiently, too, using the mapping  $\phi(S, m, R) = (R, m - AR)$  where  $R$  denotes  $(l-k)$  uniform random bits. Note that  $\phi$  is indeed the inverse of  $f$  since

$$f_S(\phi(S, m, r)) = [A, I](r, m - Ar) = Ar + m - Ar = m,$$

for every  $s, m, r$ . However, this Toeplitz matrix based construction does not satisfy the conditions for a balanced UHF and, therefore, cannot be used in Lemma 4. To wit, for a nonzero vector  $x \in \{0, 1\}^l$  with the first  $l-k$  entries 0,  $f_s(x) = m$  holds for every  $s$  if  $m = -(x_{l-k+1}, \dots, x_l)$  and for no  $s$  otherwise, thereby violating condition (b) in the definition of a balanced UHF. Nevertheless, it satisfies condition (a) and, by [42, Section V], will satisfy Lemma 4 when we restrict to a uniform random variable  $M$ .

It is also of interest to implement a UHF with as little shared randomness  $S$  as possible. See [49] for constructions based on finite field arithmetic requiring the best known lengths of the shared seed  $S$ . In particular, see [49, Table I] for a comparison of seed length required by various implementations available in the literature. Another concern in hardware implementation of UHF is the power consumption. To this end, a variant of the finite field arithmetic UHF proposed in [13] has been implemented as a low power

<sup>9</sup>A list of first 110 such  $l$ 's is available on <http://oeis.org/A001122>.

CMOS circuit in [107].

For the remainder of this article, we shall assume that the required UHF or balanced UHF is implemented using the finite field arithmetic based construction described above and depicted in Figure 4.



Fig. 4: An efficiently implementable UHF based on finite field arithmetic

#### IV. PRACTICAL SECRET KEY AGREEMENT SCHEMES USING UHF

Extracting secret keys from correlated observations  $X$  and  $Y$  has two obstacles. First, although  $X$  and  $Y$  are correlated they may not give rise to any shared randomness for the two parties. In fact, a seminal result of Gács and Körner [29] says that, in general, correlation cannot be converted into shared bits without communication. Second, the shared bits that the parties can generate by communicating may not be uniform or may not be concealed from the eavesdropper with access to the communication. All known secret key agreement schemes circumvent these obstacles separately by first communicating to agree on a shared randomness, a step referred to as *information reconciliation*, and then, extracting secret keys from the generated shared randomness in the *privacy amplification* step. The choice of shared randomness to generate and the tools for privacy amplification vary across the literature. For instance, the schemes in [70], [2], [22], [71], [84], [28], [45] recover  $X$  as shared randomness at both parties while that in [25], [26] recovers both  $X$  and  $Y$ . Also, [93] explores the role of the choice of shared randomness established in the information reconciliation step in reducing the amount of communication for secret key agreement. For privacy amplification, [22], [3], [25], [26] rely on the *balanced coloring lemma* which was introduced in [3]. On the other hand, [11], [71], [84], [82], [43], [45] among several other works rely on the leftover hash lemma.

A general construction in the context of biometric security is given in [28]. This construction is an efficient implementation of the secret key agreement scheme suggested in [11] and [84], and many special cases have appeared in implementation of PUFs; see, for instance, [30]. Also, constructions based on low density parity check (LDPC) codes are given in [14] for a weaker notion of security, and the ones on polar codes are given in [81], [19]; extensions to specific multiterminal models is considered in [105].

We now describe a generic secret key agreement scheme that can be implemented efficiently. For simplicity, assume that  $X = (X_1, \dots, X_n)$  consists of  $n$  independent, unbiased, random bits and  $Y = (Y_1, \dots, Y_n)$  is such that  $(X_i, Y_i)$  are mutually independent and each  $Y_i$  is a possibly flipped version of

$X_i$ , where flip occurs with probability  $\epsilon$ . Therefore, for large  $n$ , the Hamming distance between  $X^n$  and  $Y^n$  will be roughly  $\tau = n\epsilon$ . In fact, this scenario is typical, and it is common to process and quantize the raw physical observations to extract independent bits (*cf.* [106], [66]). The extracted independent bits can be tested for independence using standardized tests such as NIST SP-800-22-rev1a. For the purpose of this article, we shall assume that  $n$  independent correlated bits  $(X_i, Y_i)_{i=1}^n$  have been extracted and have been distributed between the two parties.

The first component of our secret key agreement scheme is an *error-correcting code* (ECC) that will facilitate a compressed transmission of  $X^n$  to  $Y^n$ . This classical problem in distributed data compression was introduced by Slepian and Wolf in [89], and several efficient coding schemes accomplishing this are known. For instance, [64] gives an implementation based on LDPC codes and [59] gives an implementation based on polar codes. In fact, a simple implementation based on linear ECC was suggested in [102] and was used for secret key agreement in [105]; we review this scheme here. Let  $\mathcal{C}$  be a linear ECC of length  $n$  that can be efficiently decoded and can correct up to  $\tau$  errors. On observing  $\mathbf{x}$ , the first party finds the coset leader for  $\mathbf{x}$  in the standard array for the code  $\mathcal{C}$ . This can be implemented efficiently by using  $\mathbf{x}$  as the input to an efficient decoder for  $\mathcal{C}$ , noting the decoded codeword  $c_{\mathbf{x}}$  and evaluating  $e_{\mathbf{x}} = \mathbf{x} \oplus c_{\mathbf{x}}$ . This coset leader  $e_{\mathbf{x}}$  is communicated to the second party over the public channel. The second party knows  $\mathbf{y}$  and computes  $\mathbf{y} \oplus e_{\mathbf{x}} = \mathbf{x} \oplus e \oplus e_{\mathbf{x}} = c_{\mathbf{x}} \oplus e$ . Recall that  $e$  has weight less than  $\tau$  with large probability, and therefore,  $c_{\mathbf{x}}$  can be recovered using the decoding algorithm for  $\mathcal{C}$ . The second party can recover  $\mathbf{x}$  as  $c_{\mathbf{x}} \oplus e_{\mathbf{x}}$ , completing the information reconciliation step.

At this point, both parties agree on an  $n$ -bit vector  $X^n$ , with a small probability of disagreement, *i.e.*, the second party has an estimate  $\widehat{X}^n$  of  $X^n$  which differs from  $X^n$  with small probability of error. Furthermore, a communication of, say,  $r$  bits has been revealed to the eavesdropper via the public channel. In the privacy amplification step, the parties will use a UHF to extract a secret key from shared bits  $X^n$ . In particular, to use the UHF of Figure 4, which can be implemented efficiently for input lengths  $l$  such that  $l+1$  is an odd prime and 2 is a primitive root modulo  $l+1$ , we find the largest such  $l \leq n$  and use just the first  $l$  bits  $(X_1, \dots, X_l)$ . In order to select the range-size  $k$ , we first need to select a security criterion and fix the desired security level under that criteria. For instance, to attain a security of  $\delta$  under the total variation distance, it follows from<sup>10</sup> Lemma 2 that  $k = \lfloor H_{\min}^{\delta/2}(P_{X^l Z^l} | Z^l) - r - 2 \log(2/\delta) \rfloor$  suffices, where  $Z$  denotes the side-information of the eavesdropper. Note that the security parameter  $\delta$  is predecided and  $r$  corresponds to maximum number of bits that may be communicated in the information reconciliation step. Thus, to determine  $k$ , we only need to form an estimate of the quantity  $H_{\min}^{\delta/2}(P_{X^l Z^l} | Z^l)$ . For the case

<sup>10</sup>We apply Lemma 2 with eavesdropper's side-information in the role of  $Z_1$  and public communication in the role of  $Z_2$ .

of IID random variables  $(X^l, Z^l)$  considered here, the smooth conditional min-entropy  $H_{\min}^{\delta/2}(P_{X^l Z^l} | Z^l)$  can be approximated by  $lH(X|Z)$  (see [51, Theorem 1] for bounds on approximation error at a fixed  $l$ ). The Shannon entropy  $H(X|Z)$  itself can be estimated by using  $N = \Theta(|\mathcal{X}||\mathcal{Z}|/\log|\mathcal{X}||\mathcal{Z}|)$  independent samples from  $(X, Z)$  [97]. If getting samples is expensive, we can take recourse to an alternative form of the leftover hash lemma where the threshold is determined by Rényi entropy of order 2 (cf. [54], [11], [82]). Specifically, using this form for the special case of constant  $Z$ , we can find an appropriate value of  $k$  by estimating the Rényi entropy of order 2 of  $X$ , which requires only  $\Theta(\sqrt{|\mathcal{X}|})$  samples [1].

Once the value  $k$  is determined, a secret key is extracted by applying a  $k$ -bit UHF to  $X^l$  and  $\widehat{X}^l$  at the first and the second party, respectively. The overall scheme discussed here is illustrated in Figure 5. The resulting secret key agreement is capacity achieving if we use an optimal rate Slepian-Wolf code in the information reconciliation step. Note that the proposed scheme uses one-side communication between the two parties, which can be strictly suboptimal at finite blocklengths if interactive communication is allowed [45].

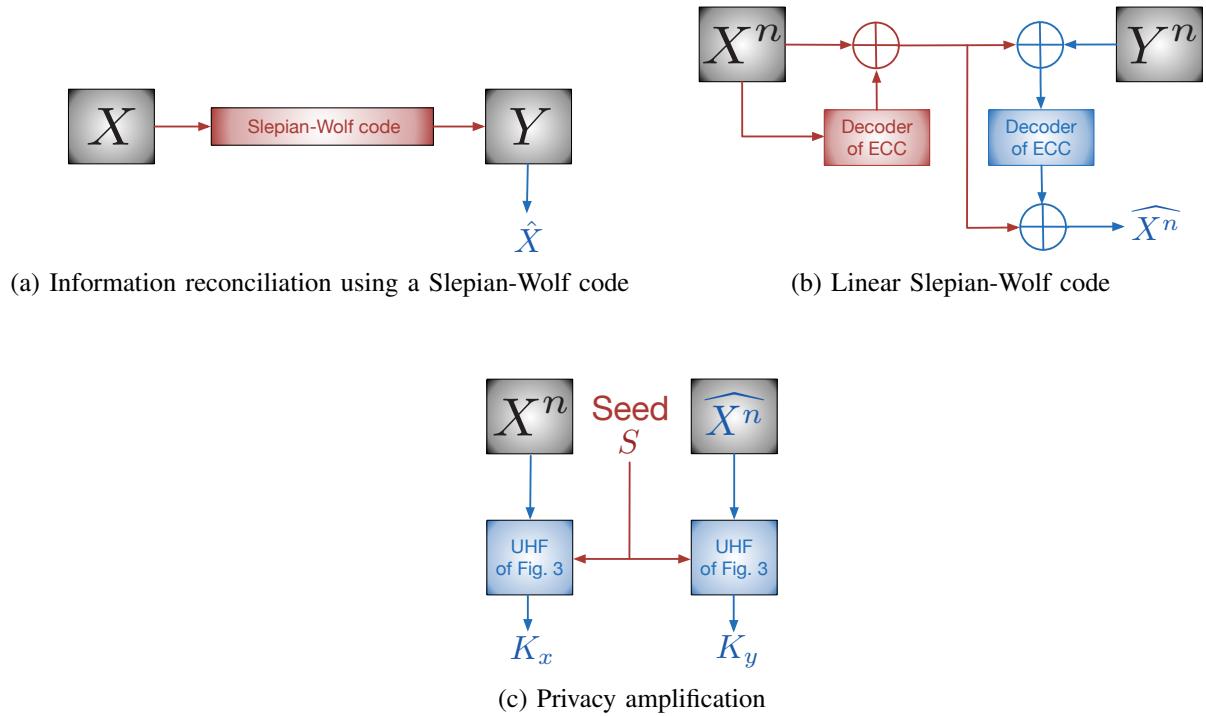


Fig. 5: An efficient scheme for secret key agreement

## V. PRACTICAL (AND MODULAR) WIRETAP CODES USING UHF

In order to present the main ideas underlying the constructive wiretap coding schemes, we review briefly the classical capacity-achieving, information-theoretic coding schemes.

To construct an  $(n, k)$  code for a wiretap channel (of rate  $k/n$ ), Wyner [103] suggested to start with a code  $\mathcal{C}$  of length  $n$  and consider its partition  $\mathcal{C} = \sqcup_{i=1}^{2^k} \mathcal{C}_i$  such that

- 1) Each element of  $\mathcal{C}$  lies in the typical set  $\mathcal{T}_{[\mathcal{P}]}^n$  (for a definition of typical set, see [24]);
- 2)  $\mathcal{C}$  is a “good channel code” for  $T$  with small average probability of error;
- 3) each  $\mathcal{C}_i$  is a channel code for  $T$  with average probability of error  $\epsilon_i$ , and the average of  $\epsilon_i$  with respect to  $i$  is small.

To encode a uniformly distributed message  $M$ , the channel input  $X^n$  is chosen uniformly over  $\mathcal{C}_M$ . It was shown in [103] that this scheme constitutes a valid wiretap code. In fact, by selecting  $\mathcal{C}$  and its partition randomly, we can attain the capacity of a degraded wiretap channel.

Interestingly, while [103] identified the general properties that an “ad-hoc” channel code  $\mathcal{C}$  and the corresponding partition  $\sqcup_i \mathcal{C}_i$  must satisfy to yield a good wiretap code, the actual code construction in [103] entailed a joint selection of the code  $\mathcal{C}$  as well as the corresponding partition. The construction in [23] is of similar form and here, too, the wiretap code is obtained by a joint selection of the random channel code and its partition. The construction in [22] (see, also, [24]), which attains the wiretap capacity for a discrete, memoryless channel under strong security, also starts with a random code  $\mathcal{C}$  and partitions it using random binning<sup>11</sup>. The same holds for the scheme in [37] which relates a randomly generated wiretap code to a channel resolvability code [34].

The information theoretic schemes above raise the following question: Is it possible to obtain good wiretap codes by starting with *any* good channel code for  $T$  and partitioning it appropriately? Or is the combined design suggested in the schemes above necessary? In fact, most of the constructive coding schemes proposed for a wiretap channel follow the general template outlined above and design wiretap codes by jointly selecting  $\mathcal{C}$  and its partition, *i.e.*, the partition is selected, intrinsically, based on the underlying code itself. For instance, the LDPC codes based schemes in [92] extend the *coset coding* scheme of [74] and select both the partition and the overall code  $\mathcal{C}$  based on a specifically designed parity check matrix (see [92, eqn. (21)]); the polar codes based scheme in [65] obtains the aforementioned partitioning, in effect, by partitioning the polarized bits – the polarized bits that are “good” for the legitimate receiver yield the overall code  $\mathcal{C}$  and the partition is obtained by fixing the bits that are good only for the legitimate receiver, one part for each fixed value of these bits (see, for instance, [65, eqn. (25)]); other polar coding schemes in [86], [81], [32] have a similar form except that the partitioning of polarized bits is more involved – a clear depiction of the partitioning of polarized bits in these schemes is given in [32, Figures 1-4]; the same is true for the lattice codes based scheme suggested for the Gaussian

<sup>11</sup>Strong security is shown by taking recourse to the *balanced coloring lemma*; see [24] for a detailed account.

wiretap channel in [7], [63], [62] where the partition corresponds to appropriately selected cosets in the transmission lattice. These schemes, while quite important, will not be covered here in further detail. An interested reader can see [35] for a review.

Thus, deployment of any of these schemes in place of existing insecure channel codes will require a complete redesign of the encoder and the decoder, which may not be feasible. Recently, [42], [8] proposed a *modular scheme* that starts with a good channel code for  $T$  and converts it into a good wiretap code by adding a pre-processing layer based on UHFs<sup>12</sup>. In fact, this modular scheme appeared first in [40] for the special case when the underlying channel code for  $T$  is linear, and was shown to achieve the capacity of a wiretap channel when both  $T$  and  $W$  are additive, with strong security (based on the mutual information criterion<sup>13</sup>). The pre-processing layer of the proposed modular scheme is based on UHFs and is shown to achieve the capacity of any symmetric, degraded, discrete wiretap channel in [42], [8], [91] as well as that of a Gaussian wiretap channel in [95] (see, also, [47, Appendix D]), both under strong security. In fact, when the underlying channel code for  $T$  has a certain linear structure, [8] showed that this scheme achieves the capacity of a symmetric, degraded, discrete wiretap channel even under semantic security (see, also, [47] for capacity results for the modular scheme under different restrictions on the wiretap channel and the underlying channel code for  $T$ ). It remains unclear if such schemes can attain the capacities of more general (including nondegraded) wiretap channels, as do the schemes of [81], [32], or how does their overall performance compare with that of the schemes mentioned above. Nevertheless, their ease of implementation makes them a leading contender for deployment in practical applications such as protection against side-channel attack [16].

In the remainder of this section, we review this modular scheme. In the first subsection below, we begin by presenting a *seeded wiretap coding scheme* where the encoder and the decoder, additionally, have access to a uniformly distributed random seed  $S$ . In the subsequent subsection, this assumption of shared random seed will be relaxed using the *seed recycling scheme* of [9], [10]. Specifically, a seed  $S$  is transmitted to the legitimate receiver over the first few channel uses, and the same seed is re-used for multiple instances of the seeded wiretap code. The security of this combined scheme relying on seed recycling was established in [9], [10] using a hybrid argument.

<sup>12</sup>For a different, model of a wiretap channel, a coding scheme based on invertible extractors was given in [18].

<sup>13</sup>It was extended to the total variation distance based security in [41].

### A. Seeded wiretap codes

To motivate the scheme, suppose that we transmit a message  $U$  by first encoding it using an ECC for  $T$  and then the legitimate receiver decodes  $U$  as  $\hat{U}$ . Then, we are in a similar situation as that in the secret key agreement of Figure 5 with  $U$  and  $\hat{U}$  corresponding to the estimates of the reconciled information after the first part of the scheme. We can extract a secret key  $M$  from  $U$  that remains concealed from the eavesdropper's observations using a UHF, as in the privacy amplification step of the scheme in Figure 5. However, in the wiretap coding problem we are given a message  $M$ , and we must generate  $U$  from  $M$  rather than the other way around. The main observation that leads to a wiretap coding scheme is that if the extractor  $F$  obtained by uniformly choosing a mapping from a UHF is invertible, then we can apply its inverse to the message  $M$  to obtain  $U$  and apply the extractor itself to the decoded message  $\hat{U}$ , thereby simulating the privacy amplification step in Figure 5 and ensuring security.

The key technical component required for formalizing this idea is Lemma 4, the channel version of the leftover hash lemma. Specifically, Lemma 4 shows that a balanced UHF constitutes a stochastic transformation  $\Gamma$  which converts a given channel  $V$  into a channel  $V' = V \circ \Gamma$  with a different input alphabet  $\mathcal{M}$  but the same output alphabet such that the input  $m$  of  $V'$  remains secure from an observer of the output of  $V'$  and an observer of the random input of  $V$  (output of  $\Gamma$ ) can determine  $m$ .

Suppose that we are given an ECC  $\mathcal{C}$  for the transmission channel  $T$  with encoder  $e_0 : \{0, 1\}^l \rightarrow \mathcal{X}^n$ , where  $\mathcal{X}$  denotes the input of the wiretap channel. The code  $\mathcal{C}$  is assumed to facilitate a reliable transmission of  $l$ -bit messages over  $T^n$  with the maximum probability of error less than  $p_e$ . To convert this code into an  $(n, k)$  wiretap code, we add a pre-processing layer to it consisting of a  $b$ -balanced  $k$ -bit UHF  $\{f_s, s \in \mathcal{S}\}$  with input length  $l$ . In order to send a message  $m \in \mathcal{M}$ , the pre-processing layer generates a seed  $S$  uniformly over  $\mathcal{S}$  and outputs a random binary vector  $U$  of length  $l$  distributed uniformly over  $f_S^{-1}(m)$ . This vector  $U$  is then encoded using  $e_0$  and transmitted over  $W$ . In particular, we use the efficiently invertible  $(l - k)$ -balanced UHF of Figure 4 for  $\mathcal{M} = \{0, 1\}^k$ . By our assumptions for the code  $\mathcal{C}$ , the random vector  $U$  can be decoded at the output of the transmission channel  $T$  with probability of error less than  $p_e$ . Thus, if the random seed  $S$  is available to the legitimate receiver, the transmitted message  $m \in \mathcal{M}$ , too, can be recovered with probability of error less than  $p_e$  by applying  $f_S$  to the decoded vector  $\hat{U}$ . For the security of this scheme, it follows from Lemma 4, applied with the augmented channel  $W_{n,C} = W^n \circ e_o$  in the role of  $V$ , that for a uniformly distributed message  $M$

$$I(M \wedge Z, S) \leq \frac{1}{\ln 2} \cdot 2^{-(l-k-I_{\max}^\epsilon(W_{n,C}))} + \epsilon k.$$

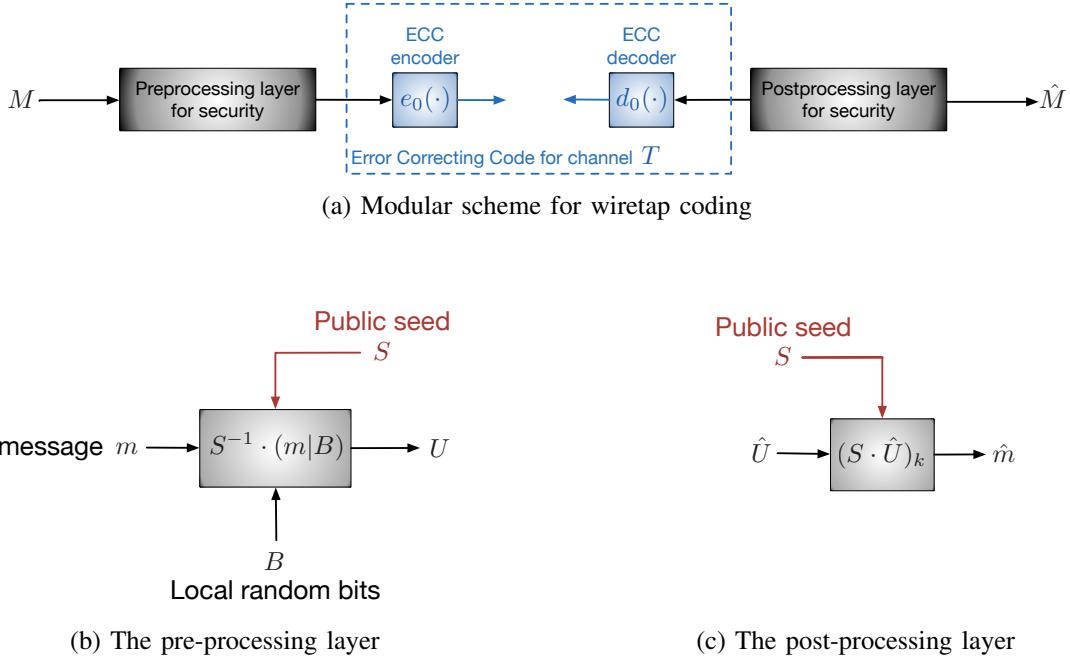


Fig. 6: A seeded coding scheme for the wiretap channel

Therefore, the overall modular scheme, depicted in Figure 6, constitutes a good  $(n, k)$  wiretap code<sup>14</sup> provided that  $k$  is selected appropriately to ensure small leakage  $I(M \wedge Z, S)$ . Specifically, suppose that the code  $\mathcal{C}$  is of rate  $R$ , i.e.,  $l = nR$ . We show in the Appendix that there exists a  $c > 0$  such that, for  $\epsilon = 2^{-nc}$ ,  $I_{\max}^\epsilon(W_{n,\mathcal{C}})$  is asymptotically less than  $nC_W$ , where  $C_W$  denotes the capacity of the channel  $W$ , both in the case of a discrete memoryless channel (DMC)  $W$  and in the case of an additive white Gaussian noise (AWGN) channel  $W$  with average input power constraints. Thus, upon choosing

$$\frac{k}{n} = R' < R - C_W,$$

for a uniformly distributed  $M$ ,  $I(M \wedge Z^n, S)$  vanishes to zero exponentially rapidly in  $n$ .

In fact, for a symmetric, discrete channel  $W$ , if the underlying ECC is linear and the balanced  $(l - k)$ -balanced UHF of Figure 4 is used, it was shown in [9], [10] that strong security shown above implies semantic security as well.

### B. Modular wiretap coding scheme based on seed recycling

In the previous section, we established the security of our scheme assuming that a random seed  $S$  was shared publically. We now show that this assumption is not required, even for semantic security, using a

<sup>14</sup>To be precise, the proposed code with the choice of UHF in Figure 4 can send  $2^k - 1$  messages because the all 0 message is excluded from the message set  $\mathcal{M}$ .

seed recycling trick from [9], [10]. We first use the legitimate channel  $T$  to transmit the seed  $S$  reliably to the receiver in  $nc$  channel uses, where the constant  $c$  is chosen to ensure the recovery of  $S$  at the receiver with probability of error less than  $p_e$ . Next, to compensate for the rate loss due to the transmission of  $S$ , we use the same shared seed  $S$  to send  $t_n$  messages  $M_1, \dots, M_{t_n}$  using  $t_n$  independent implementations of the seeded wiretap coding scheme of the previous subsection. The resulting probability of error in transmitting the concatenated message  $M^{t_n} = (M_1, \dots, M_{t_n})$  in overall  $N$  channel uses is bounded above by  $(t_n + 1)p_e$ . Also, by combining<sup>15</sup> [9, Theorem 4.5, 4.9], [8, Lemma 4.2], and the fact that  $I(M \wedge Z^n, S)$  vanishes to 0 exponentially rapidly in  $n$ , it follows that  $\max_{M^{t_n}} I(M^{t_n} \wedge Z^N) \leq t_n 2^{-nc'}$  for some constant  $c' > 0$ . The rate of the overall scheme is given by

$$\lim_{n \rightarrow \infty} \frac{t_n k}{(t_n + c)n},$$

which equals  $R'$  as long as  $t_n \rightarrow \infty$  as  $n \rightarrow \infty$ . Therefore, if we choose  $t_n$  such that this condition is satisfied and both<sup>16</sup>  $(t_n + 1)p_{e,n}$  and  $t_n 2^{-nc'}$  vanish to 0, we get a wiretap coding scheme satisfying semantic security of any rate  $R' < R - C_W$ . Furthermore, if the underlying ECC  $\mathcal{C}$  can be implemented efficiently, so can the combined scheme above. Note that the argument above is required to reduce the semantic security of an unseeded scheme to that of a seeded scheme. For the strong security criterion, a much simpler argument based on chain rule for mutual information suffices. Specifically, consider a uniformly distributed message  $(M_1, \dots, M_{t_n})$ . Note that  $M_i$ ,  $1 \leq i \leq t_n$  are IID uniform, which further implies that for each  $i$  the random variables  $(M_i, Z^n(i))$  are conditionally independent of  $(M_j, Z^n(j))_{j \neq i}$  given  $S$ . Therefore,

$$\begin{aligned} I(M_1, \dots, M_{t_n} \wedge Z^N) &\leq I(M_1, \dots, M_{t_n} \wedge Z^N, S) \\ &= I(M_1, \dots, M_{t_n} \wedge Z^n(1), \dots, Z^n(t_n)|S) \\ &\leq \sum_{i=1}^{t_n} I(M_i \wedge Z^n(1), \dots, Z^n(t_n)|S) \\ &= \sum_{i=1}^{t_n} I(M_i \wedge Z^n(i)|S) \\ &= t_n I(M_1 \wedge Z^n(1), S). \end{aligned} \tag{11}$$

<sup>15</sup>As mentioned before, our security requirement is even stronger than the original semantic security requirement of [8], which can be shown for the combined scheme simply by using [8, Lemma 4.2]; [9, Theorem 4.5, 4.9] are required to move between the two notions of security.

<sup>16</sup>The probability of error  $p_e = p_{e,n}$  for the transmission code  $\mathcal{C}$  depends on  $n$  and, in principle, can vanish to 0 exponentially rapidly in  $n$ .

The security proof is completed by appropriately choosing  $t_n \rightarrow \infty$  as above.

To summarize, the argument above allows us to convert any efficiently implementable transmission code for  $T$  of rate  $R$  into a code of rate  $R - C_W$  for the wiretap channel, with a vanishing probability of error and under strong security. Furthermore, the conversion is done simply by including an efficiently implementable pre-processing layer based on a balanced UHF. Note that for the special case of a Gaussian wiretap channel or a symmetric, degraded, discrete wiretap channel, the modular scheme described above attains the wiretap capacity if the underlying ECC  $\mathcal{C}$  achieves the capacity  $C_T$  of the transmission channel  $T$  since, for these cases, the wiretap capacity is given by  $C_T - C_W$  [61]. In fact, for a discrete symmetric wiretap channel, if the underlying capacity achieving ECC is linear and capacity achieving for  $T$ , the modular scheme achieves the wiretap capacity even under semantic security.

Recall that the balanced UHF of Figure 6 can be implemented efficiently only for selected values of input length  $l$ . Thus, given an ECC  $\mathcal{C}$  for  $T$ , we simply use the largest  $l$  less than the input length of  $\mathcal{C}$  (in bits). Also, the analysis above was asymptotic and cannot be applied for a fixed  $n$ . For a fixed  $n$ , the output length  $k$  of the balanced UHF, and consequently the message length, must be chosen to be appropriately smaller than  $l - I_{\max}^\epsilon(W_{n,\mathcal{C}})$  to get the desired security level. For this purpose, it is required to estimate the quantity  $I_{\max}^\epsilon(W_{n,\mathcal{C}})$  for a given ECC  $\mathcal{C}$  and for a sufficiently small  $\epsilon$ ; however, there are no results to report in this context yet. Furthermore, one might also wish to compare the finite blocklength performance of this scheme, for different choices of ECC  $\mathcal{C}$ , with the fundamental lower bounds similar to those derived for the channel coding problem in [39], [76]. However, no such bounds are available. In fact, even the strong converse for a degraded wiretap channel was proved only recently in [44].

The coding scheme for the basic wiretap model above is a stepping-stone for deriving schemes for more complicated wiretap channel models such as the MIMO wiretap channel considered in [58]. It can be expected that, based on the simple scheme above, schemes for other more complicated physical-layer channel models will emerge. One such extension, with a rather wide scope, appears in [47].

## APPENDIX

Consider a channel  $W : \mathcal{X} \rightarrow \mathcal{Z}$  and an encoder (for ECCs)  $e_0 : \{0,1\}^l \rightarrow \{0,1\}^n$ . Denote by  $W_{e_0} : \{0,1\}^l \rightarrow \mathcal{Z}^n$  the augmented channel  $W^n \circ e_0$  where given an input  $v \in \{0,1\}^n$ , with  $x_i$  denoting the  $i$ th coordinate of  $e_0(v)$ , the outputs  $Z_i$  are independent and distributed as  $W(\cdot|x_i)$ ,  $1 \leq i \leq n$ . In this section, we shall derive an asymptotic bound for  $I_{\max}^{\epsilon_n}(W_{e_0})$  for an exponentially small  $\epsilon_n$  and for a DMC  $W$  with any encoder  $e_0$  as well as for an AWGN channel  $W$  with an encoder  $e_0$  satisfying the average power constraint  $P$  with probability 1.

First, consider a DMC  $W : \mathcal{X} \rightarrow \mathcal{Z}$ .

**Lemma 5.** *For any encoder  $e_0$  and a DMC  $W : \mathcal{X} \rightarrow \mathcal{Z}$  with finite input and output alphabets  $\mathcal{X}$  and  $\mathcal{Y}$ , respectively, there exists a constant  $c > 0$  such that for  $\epsilon_n = e^{-nc}$*

$$I_{\max}^{\epsilon_n}(W_{e_0}) \leq n \max_{P_X} I(X \wedge Z) + o(n).$$

*Proof:* We first prove the result for a constant composition code where each codeword  $e_0(v)$  is of a fixed type  $P$ , i.e., for  $e_0$  such that each element  $x \in \mathcal{X}$  appears  $nP(x)$  times in every codeword  $x^n = e_0(v)$ .

Denote by  $\mathcal{T}_{[W]}$  the set of sequences  $(x^n, z^n)$  such that  $z^n$  is  $W$ -conditionally typical given  $x^n$ , by  $\mathcal{T}_{[P,W]}$  the set of sequences  $(x^n, z^n) \in \mathcal{T}_{[W]}$  such that  $x^n$  has type  $P$ , and by  $\mathcal{T}_{[PW]}$  the projection of  $\mathcal{T}_{[P,W]}$  on  $\mathcal{Z}^n$  (these notations are a slight deviation from those used in [24]). Then, using basic results from the method of types (see [24, Chapter 2]) for each  $(x^n, z^n) \in \mathcal{T}_{[P,W]}$ , it holds that

$$\log W^n(z^n | x^n) \leq -nH(W | P) + o(n).$$

Furthermore,

$$\log |\mathcal{T}_{[PW]}| \leq nH(PW) + o(n),$$

where  $PW$  denotes the output distribution for channel  $W$  when the input distribution is  $P$ . Since there exists a  $c > 0$  such that for all  $v$

$$\sum_{z^n : (e_0(v), z^n) \notin \mathcal{T}_{[W]}} W^n(z^n | e_0(v)) \leq 2^{-nc},$$

for the subnormalized channel  $W_{e_0, \mathcal{T}_{[W]}}$  defined by (2), we have

$$\begin{aligned} I_{\max}^{\epsilon}(W_{e_0}) &\leq I_{\max}(W_{e_0, \mathcal{T}_{[W]}}) \\ &= \log \sum_z \max_v W^n(z^n | e_0(v)) \mathbb{1}((e_0(v), z^n) \in \mathcal{T}_{[W]}) \\ &\leq -nH(W | P) + \log \sum_z \max_v \mathbb{1}((e_0(v), z^n) \in \mathcal{T}_{[W]}) + o(n) \\ &\leq -nH(W | P) + \log |\mathcal{T}_{[PW]}| + o(n) \\ &\leq nI(P; W) + o(n), \end{aligned} \tag{12}$$

where the last-but-one inequality uses the fact that  $(e_0(v), z^n) \in \mathcal{T}_{[W]}$  implies  $z^n \in \mathcal{T}_{[PW]}$  and so

$$\max_v \mathbb{1}((e_0(v), z^n) \in \mathcal{T}_{[W]}) \leq \mathbb{1}(z^n \in \mathcal{T}_{[PW]}) = |\mathcal{T}_{[PW]}|.$$

This completes the proof for a constant composition code.

Proceeding to the case of a general code, denote by  $\mathcal{C}_P$  the set of codewords  $e_0(v)$  of type  $P$ . As before, we have

$$\begin{aligned} I_{\max}^{\epsilon}(W_{e_0}) &\leq I_{\max}(W_{e_0, \mathcal{T}_{[W]}}) \\ &= \log \sum_z \max_v W^n(z^n | e_0(v)) \mathbb{1}((e_0(v), z^n) \in \mathcal{T}_{[W]}) \\ &\leq \log \sum_P \sum_z \max_{x^n \in \mathcal{C}_P} W^n(z^n | x^n) \mathbb{1}((x^n, z^n) \in \mathcal{T}_{[W]}) \\ &\leq n \max_{P_X} I(P_X; W) + o(n), \end{aligned}$$

where the final inequality is obtained in the manner of (12) upon using the fact that the number of types is polynomial in  $n$  (*cf.* [24, Lemma 2.1]).  $\blacksquare$

Next, consider an AWGN channel  $W : \mathbb{R} \rightarrow \mathbb{R}$ , *i.e.*, a channel such that for an input  $x \in \mathbb{R}$  the output  $Z$  is distributed as  $W(\cdot|x) = \mathcal{N}(0, \sigma_W^2)$ . Let  $e_0 : \{0, 1\}^l \rightarrow \{0, 1\}^n$  be an encoder satisfying the average power constraint

$$\frac{1}{n} \|e_0(v)\|_2^2 \leq P, \quad \forall v \in \{0, 1\}^l. \quad (13)$$

The next result shows that the  $\epsilon_n$ -smooth max-information for  $W_{e_0}$  is bounded above by, roughly,  $n$  times the capacity of the AWGN  $W$  with average input power constraint  $P$ , for an exponentially small  $\epsilon_n$ .

**Lemma 6.** *Let  $W : \mathbb{R} \rightarrow \mathbb{R}$  be an AWGN channel with noise variance  $\sigma_W^2$ , and let  $e_0 : \{0, 1\}^l \rightarrow \mathbb{R}^n$  be an encoder satisfying (13). Then, denoting  $\epsilon_n = e^{-n\delta^2/8}$ , it holds for the combined channel  $W_{e_0}$  for every  $0 < \delta < 1/2$  that*

$$I_{\max}^{\epsilon_n}(W_{e_0}) \leq \frac{n}{2} \log \left( 1 + \delta + \frac{P}{\sigma_W^2} \right) + \frac{n\delta \log e}{2} + o(n).$$

*Proof:* Denote by  $g(z)$  the standard normal density on  $\mathbb{R}^n$ . For the set

$$\mathcal{Z}_0 := \left\{ z \in \mathbb{R}^n \mid \left| \frac{1}{n} \|z\|_2^2 - 1 \right| \leq \delta \right\},$$

the standard measure concentration results for chi-squared RVs (cf. [5, Exercise 2.1.30]) yield

$$\int_{\mathcal{Z}_0} g(z) dz \geq 1 - 2e^{-n\delta^2/8}.$$

Define  $\mathcal{T}$  as the set of sequences  $(x^n, z^n)$  such that the sequence  $z^n - x^n$  belongs to  $\mathcal{Z}_0$ . Thus, for each  $x^n \in \mathbb{R}^n$ ,

$$W^n(\{z^n \mid (x^n, z^n) \in \mathcal{T}\} \mid x^n) = \int_{\mathcal{Z}_0} g(z) dz \geq 1 - 2e^{-n\delta^2/8}.$$

Further, let  $\mathcal{T}_{P, \mathcal{Z}_0}$  denote the set  $\bigcup_{x^n: \|x^n\|_2 \leq nP} \{\mathcal{Z}_0 + x^n\}$ . We get the following inequalities:

$$\begin{aligned} I_{\max}^{\epsilon_n}(W_{e_0}) &\leq I_{\max}(W_{e_0 n, \mathcal{T}}) \\ &= \log \int_{\mathbb{R}^n} \max_v g\left(\frac{z - e_0(v)}{\sigma_W}\right) \mathbb{1}((e_0(v), z^n) \in \mathcal{T}) dz \\ &\leq \log \frac{e^{-\frac{n(1-\delta)}{2}}}{(2\pi\sigma_W^2)^{\frac{n}{2}}} \int_{\mathbb{R}^n} \max_v \mathbb{1}((e_0(v), z^n) \in \mathcal{T}) dz \\ &\leq \log \frac{e^{-\frac{n(1-\delta)}{2}}}{(2\pi\sigma_W^2)^{\frac{n}{2}}} \text{vol}(\mathcal{T}_{P, \mathcal{Z}_0}), \end{aligned}$$

where the second inequality holds since  $z - e_0(v) \in \mathcal{Z}_0$  and the last since  $e_0(v)$  satisfy (13) for all  $v$ . Denote by  $\mathcal{B}_n(\rho)$  the sphere of radius  $\rho$  in  $\mathbb{R}^n$  and by  $\nu_n(\rho)$  its volume, which can be approximated as (cf. [99])

$$\nu_n(\rho) = \frac{1}{\sqrt{n\pi}} \left(\frac{2\pi e}{n}\right)^{\frac{n}{2}} \rho^n (1 + O(n^{-1})) \quad (14)$$

Note that for  $\rho_n = \sqrt{n(\sigma_W^2(1+\delta) + P)}$ ,

$$\mathcal{T}_{P, \mathcal{Z}_0} \subseteq \mathcal{B}_n(\rho_n).$$

Therefore, by the inequalities above we get

$$\begin{aligned} I_{\max}^{\epsilon_n}(W_{e_0}) &\leq \log \frac{e^{-\frac{n(1-\delta)}{2}}}{(2\pi\sigma_W^2)^{\frac{n}{2}}} \nu_n(\rho_n), \\ &= \log \left[ \frac{e^{\frac{n\delta}{2}}}{(\sigma_W^2)^{\frac{n}{2}} \sqrt{n\pi} n^{\frac{n}{2}}} \rho^n (1 + O(n^{-1})) \right] \\ &= \log \left[ \frac{e^{\frac{n\delta}{2}}}{\sqrt{n\pi}} \left(1 + \delta + \frac{P}{\sigma_W^2}\right)^{\frac{n}{2}} (1 + O(n^{-1})) \right] \\ &= \frac{n}{2} \log \left(1 + \delta + \frac{P}{\sigma_W^2}\right) + \frac{n\delta \log e}{2} + \frac{1}{2} \log n\pi, \end{aligned}$$

which yields the claimed inequality. ■

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