



LETTER

A simple equivalent circuit analysis of rectangular folded-waveguide slow-wave structure

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ABSTRACT

A simple yet accurate equivalent circuit model was developed for the analysis of slow-wave properties (dispersion and interaction impedance characteristics) of a rectangular folded-waveguide slow-wave structure. Present formulation includes the effects of the presence of beam-hole in the circuit, which were ignored in existing approaches. The analysis was benchmarked against measurement as well as with 3D electromagnetic modeling using MAFIA for two typical slow-wave structures operating in Ka- and Q-bands, and close agreements were observed. The analysis was extended for demonstrating the effect of the variation of beam-hole radius on the RF interaction efficiency of the device.

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1. Introduction

Rectangular folded-waveguide (RFW) slow-wave structures (SWS) became popular for millimeter-wave traveling-wave tubes (TWTs) for its potential for reasonable bandwidth along with its capability of high power operation and ease of fabrication [1,2]. However, analytical approaches available for the computation of slow-wave characteristics of a rectangular folded-waveguide slow-wave structure are limited: Hyun et al. [3], Carter [4] and Seong et al. [5] proposed approximate parametric analytical approaches that ignored the effects of the beam-hole, and thus have limited capabilities in analyzing the exact configuration of the structure. The present authors very recently proposed a rigorous and exact analysis of rectangular folded-waveguide SWS using conformal transformations [6]. Though rigorous, the analytical model based on conformal transformations is somewhat difficult to implement because of the complexity involved in it.

In this paper we propose a complete equivalent circuit model for a rectangular folded-waveguide slow-wave structure (RFW-SWS) for the analysis of the slow-wave characteristics including the effects of the presence of the beam-hole in the structure. The analysis was carried out following a recently proposed equivalent circuit approach by the authors used for analyzing a serpentine folded-waveguide slow-wave structure [7]. The analysis has been benchmarked against measurement and 3D electromagnetic model using MAFIA with close agreement for two typical RFW-

SWSs, one operating at Ka-band and the other operating at Q-band. Subsequently, the analysis has been used for demonstrating the effect of the variation of the radius of the beam-hole on the dispersion and interaction impedance characteristics, and also the beam-wave interaction efficiency.

2. Analysis

The analytical model considers each period of the RFW-SWS (Fig. 1) to be consisting of two parts: a straight rectangular waveguide section accommodating the beam-holes as apertures on the broad-wall, and rectangular *E*-plane waveguide bends. The structure has been considered to be loss free and the equivalent circuits of straight waveguide section, waveguide bends and beam-holes (Fig. 1) are arrived at following existing approaches [7–10], and are subsequently cascaded following Curnow's approach [11] to arrive at the dispersion relation of the structure.

The equivalent circuit parameters for the effective waveguide length per periodicity (Fig. 1) are expressed following Carter and Liu [8] as

$$L_W = \frac{\mu_0 l_{eff}}{\pi^2 \left(1 + \frac{l_{eff}}{b}\right)}$$

$$C_W = \varepsilon_0 l_{eff} \left(1 + \frac{l_{eff}}{b}\right) \quad (1)$$

Here, ε_0 and μ_0 are permittivity and permeability of free space, respectively, $l_{eff}(=2h+p-b/2)$ is the effective length of the waveguide per period, $2h$ is the straight waveguide length, b is

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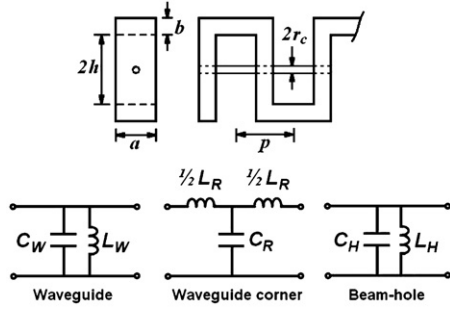


Fig. 1. Schematic of a typical rectangular folded-waveguide slow-wave structure (RFS-SWS) and the equivalent circuit representations of its different segments.

narrow dimension of the waveguide and p is the periodicity of the RFS-SWS.

The discontinuity due to the E-plane corner-bend also introduces additional inductance and capacitance that are expressed following Marcuvitz [9] as

$$L_R = Y_0 \frac{\lambda_g}{2\pi b \omega} \left(1 - 0.114 \left(\frac{2b}{\lambda_g} \right)^2 \right)$$

$$C_R = Y_0 \frac{4b}{\omega \lambda_g} \left(0.878 + 0.498 \left(\frac{2b}{\lambda_g} \right)^2 \right) \quad (2)$$

Here, ω is the radian frequency, $Y_0 (= \lambda_0 / (120\pi \lambda_g))$ is the wave-admittance of the rectangular waveguide, λ_0 is the free space wavelength, and λ_g is the guided-wavelength of the rectangular waveguide.

The total effective inductance and capacitance per period of the structure including the corner correction can be now expressed as

$$L_{WR} = L_W + L_R$$

$$C_{WR} = C_W + C_R \quad (3)$$

The equivalent circuit parameters of the beam-hole are expressed as [9,10]

$$L_H = \frac{1280\pi^2 r_c^3}{\omega \lambda_0 ab}$$

$$C_H = \frac{r_c^3}{45\omega \lambda_0 ab} \quad (4)$$

Following Curnow [11], one can now cascade the equivalent circuits of the waveguide per period and the beam-holes using the equivalent circuit parameters obtained in (1)–(4) and express the dispersion relation for the cascaded network as

$$\beta = \frac{1}{p} [2\pi - \cos^{-1} \{ 1 - (1/k)(1 - \omega^2 L_{WR} C_{WR}) F_H \}]$$

with

$$F_H = 2 + a_k - a_k \omega^2 L_H C_H$$

$$a_k = \frac{L_H}{k L_{WR}}$$

and

$$k = 0.3 \exp(20r_c/a) \quad (5)$$

For the case of the structure without beam hole, a_k vanishes and the factor incorporating the effects of the beam-holes (F_H) assumes the value $F_H = 2$. These formulas work within the parametric regime of $0 \leq r_c/a \leq 10\%$. Subsequently, using the dispersion relation (5), one can express the on-axis

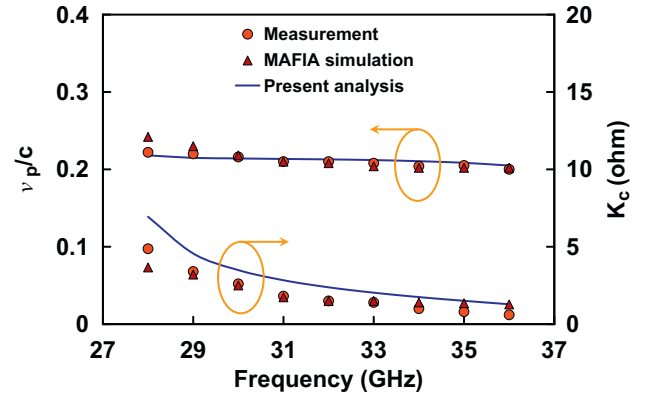


Fig. 2. Dispersion and interaction impedance characteristics of the Ka-band structure ($a/p = 3.68$, $b/p = 0.36$, $h/p = 0.92$ and $r_c/p = 0.17$).

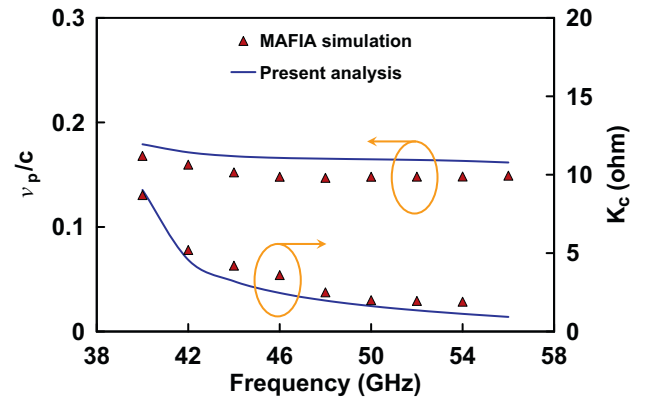


Fig. 3. Dispersion and interaction impedance characteristics of the Q-band structure ($a/p = 5.43$, $b/p = 0.54$, $h/p = 1.36$ and $r_c/p = 0.24$).

forward space-harmonic interaction impedance (K_c) of the slow-wave structure following Sumathy et al. [7] and Booske et al. [12] as

$$K_c = 120\pi \left(\frac{b}{a} \right) \left(\frac{\lambda_g}{\lambda_0} \right) \left(\frac{1}{\beta p} \right)^2 \left(\frac{\sin\{\beta b/2\}}{\beta b/2} \right)^2 \quad (6)$$

Eqs. (5) and (6) can be now used for computing the dispersion and interaction impedance characteristics, respectively, with the knowledge of the dimensions of the structure under consideration.

Having the interaction impedance characteristics available, we now consider a figure of merit (Pierce's normalized circuit gain CN) that will demonstrate the device performance, expressed as [1]

$$CN = \left(\frac{K_c I_0}{4V_0} \right)^{\frac{1}{3}} \left(\frac{\omega l_{ckt}}{2\pi u_0} \right)$$

Here, $C (= (K_c I_0 / 4V_0)^{1/3})$ is the Pierce's gain parameter [1], I_0 is the beam current, V_0 is the beam voltage, l_{ckt} is the interaction length and u_0 is the electronic velocity.

3. Results and discussion

For the purpose of benchmarking and numerical appreciation of the analysis, we considered two typical millimeter-wave RFS-SWSs operating in forward space-harmonic mode: one

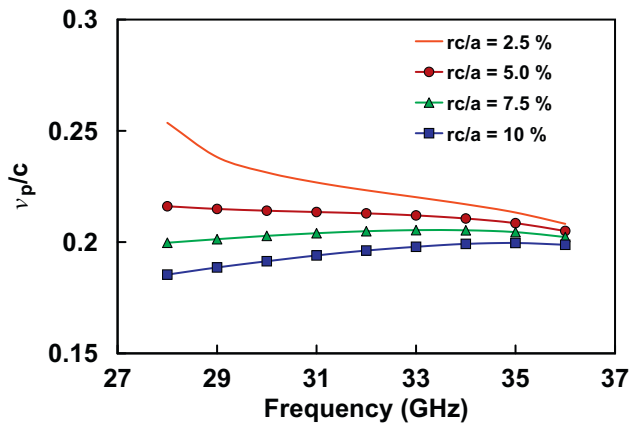


Fig. 4. Variation of normalized phase velocity versus frequency for the Ka-band SWS with beam-hole radius as the parameter.

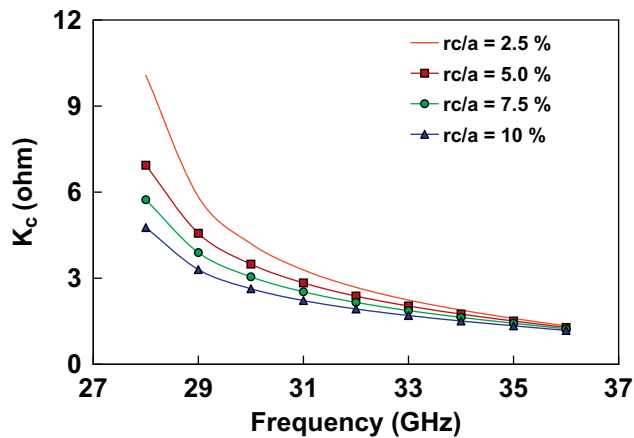


Fig. 5. Variation of interaction impedance versus frequency for the Ka-band SWS with beam-hole radius as the parameter.

operating in Ka-band and the other in Q-band for which the dimensional details and slow-wave characteristics are published in the literature [6]. The dispersion and interaction impedance characteristics of these structures were evaluated using present approach and compared to those from 3D electromagnetic modeling in MAFIA for both the structures and against measurement for the Ka-band structure alone. The benchmarking results are presented in Figs. 2 and 3 that show the efficacy of the present approach.

An increase in the beam-hole radius causes a decrease in both phase velocity and interaction impedance which is demonstrated for a typical Ka-band structure in Figs. 4 and 5, respectively. The normalized circuit gain (CN) has been computed for the Ka-band structure for the beam voltage of 12kV and beam current of 120 mA. The value of CN was found to be reduced significantly with the increase in the beam-hole radius (Fig. 6). Thus, one would need to design the electron beam diameter and the corresponding beam-hole diameter suitably for achieving desired RF efficiency of the structure.

4. Conclusion

A complete equivalent circuit model has been developed for the analysis of slow-wave characteristics of a rectangular folded-

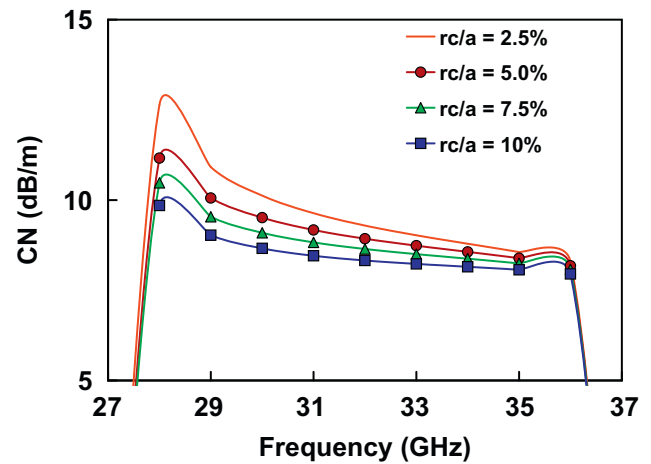


Fig. 6. Variation of normalized circuit gain versus frequency for the Ka-band SWS with beam-hole radius as the parameter.

waveguide slow-wave structure and thoroughly benchmarked against measurement and 3D electromagnetic modeling. Harmful effects of increasing the beam-hole radius are also demonstrated. With the simplicity and the demonstrated reasonable accuracy, the approach is expected to be of ample use for the travelling-wave tube community.

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References

- [1] Basu BN. Electromagnetic theory and applications in beam-wave electronics. Singapore: World Scientific; 1996.
- [2] Wang W, Wei Y, Yu G, Gong Y, Huang M, Zhao G. Review of the novel slow-wave structures for high-power traveling-wave tube. *International Journal of Infrared and Millimeter Waves* 2003;24:1469–84.
- [3] Hyun JH, Jung SS, Park GS. Linear theory of folded-waveguide traveling-wave tubes. *Journal of Korean Physics Society* 1999;34:297–300.
- [4] Carter RG. An improved equivalent circuit model of folded-waveguide slow-wave structures. In: *Proceedings of the conference on displays and vacuum electronics (ITG-VDE)*. 2001.
- [5] Han S-T, Kim J-II, Park GS. Design of folded waveguide traveling-wave tube. *Microwave and Optical Technology Letters* 2003;38:161–5.
- [6] Sumathy M, Vinoy KJ, Datta SK. Analysis of rectangular folded-waveguide millimeter-wave slow-wave structures using conformal transformations. *International Journal of Infrared, Millimeter, and Terahertz Waves* 2009;30:294–301.
- [7] Sumathy M, Vinoy KJ, Datta SK. Equivalent circuit analysis of serpentine folded waveguide slow-wave structure for millimeter-wave TWTs. *International Journal of Infrared, Millimeter, and Terahertz Waves* 2009;30:151–8.
- [8] Carter RG, Liu S. Method for calculating the properties of coupled-cavity slow-wave structures from their dimensions. *IEE Proceedings-H, Microwaves, Antennas and Propagation* 1986;133:330–4.
- [9] Marcuvitz N. *Waveguide handbook*. London: Peter Peregrinus; 1986.
- [10] Collin RE. *Foundations for microwave engineering*. New York: IEEE Press; 2001.
- [11] Curnow HJ. A general equivalent circuit for couple-cavity slow-wave structures. *IEEE Transaction on Microwave Theory and Techniques* 1965;13:671–5.
- [12] Booske JH, Converse MC, Kory CL, Chevalier CT, Gallagher DA, Kreisler KE, et al. Accurate parametric modeling of folded waveguide circuits for millimeter-wave traveling wave tubes. *IEEE Transactions on Electron Devices* 2005;52:685–93.



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