LETTER

Analysis of serpentine folded-waveguide slow-wave structure using elliptical conformal transformation

M. Sumathy [a,*], K.J. Vinoy [b], Subrata Kumar Datta [a]

[a] Microwave Tube Research and Development Centre, Ministry of Defence, Government of India, Bharat Electronics Complex, Jalahalli Post, Bangalore 560013, India
[b] Department of Electronics and Communication Engineering, Indian Institute of Science, Bangalore 560054, India

ARTICLE INFO

ABSTRACT

Elliptical conformal transformation was used to derive closed form expressions for the equivalent circuit series inductance and shunt capacitance per period of a serpentine folded-waveguide slow-wave structure including the effects of the beam-hole. The lumped parameters were subsequently interpreted for the dispersion and interaction impedance characteristics of the structure. The analysis was benchmarked for two typical millimeter-wave structures operating in Ka- and W-bands, against measurement, 3D electromagnetic modeling using CST Microwave Studio, parametric analysis and equivalent circuit analysis.

1. Introduction

Serpentine folded-waveguide slow-wave structure (SFW-SWS) has become popular for use in broadband high-power millimeter-wave traveling-wave tubes (TWTs) [1,2]. The SFW-SWS is a periodic structure with each constitutive period (of periodicity $=p$) consisting of two waveguide elements: a straight rectangular waveguide section (having broad-wall dimension $=a$ and narrow-wall dimension $=b$) accommodating the beam-hole as apertures (radius $=r_c$) at the broad-walls, and a serpentine E-plane bend-section (Fig. 1). Design of such a slow-wave structure (SWS) for a traveling-wave tube requires knowledge of the slow-wave characteristics (dispersion and interaction impedance characteristics) of the structure, which were analyzed in the past by parametric approach [3–6], equivalent circuit analysis [5–7], and 3D electro-magnetic simulation [5–7]. However, to the best of our knowledge, analysis of an SFW-SWS using conformal transformation has not been reported so far, except for a preliminary study by us [8]. Incidentally, in an earlier work we used Schwarz's polygon conformal transformation for analyzing a rectangular folded-waveguide SWS [9].

In the present work, we used elliptical conformal transformation for deriving closed form expressions for the lumped circuit parameters of the SFW-SWS, in terms of the physical dimensions of the structure. The effect of beam-hole has also been included in the analysis. The lumped circuit parameters have been subsequently interpreted to obtain the dispersion and interaction impedance characteristics of the structure. The analysis has been benchmarked for two typical structures operating in Ka- and W-bands.

2. Analysis

The analysis has been carried out in two steps considering only the fundamental TE_{10} mode of propagation in the structure. First, the longitudinal-section of a period of the SWS has been approximately modeled in Z-plane ($Z(x, y)$) as a set of two con-focal semi-ellipses without considering its depth (Fig. 2). Next, the con-focal semi-ellipses in Z-plane have been transformed into a set of concentric semi-circles in W-plane ($W(\rho, \theta)$) such that $0 \leq \theta \leq \pi$. Finally, the effect of the presence of beam-hole has been included following Sumathy et al. [10,11].

Now, conserving the continuity and analyticity, the transformation functions in the Z-plane and W-plane are related as [12,13]

$$Z(a_n + jb_n) = k_n \cosh\{W(\rho_n \exp(j\theta))\} \tag{1}$$

with

$$k_n = \sqrt{a_n^2 - b_n^2} \quad \text{and} \quad \rho_n = \cosh^{-1}\left\{\frac{a_n}{\sqrt{a_n^2 - b_n^2}}\right\}$$

Here, the subscript $n = 1$ refers to the inner ellipse (circle) and the subscript $n = 2$ refers to the outer ellipse (circle). The variable $2k_n$ defines the focal distances of the con-focal ellipses and the variable ρ_n defines the radii of the concentric circles. The parameters a_n and b_n are the major and minor radii, respectively,

* Corresponding author. Tel.: +91 80 28386810; fax: +91 80 28381750.
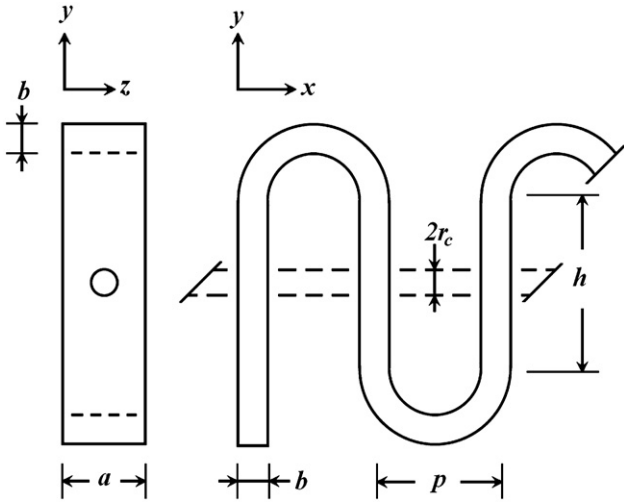E-mail address: msumti@yahoo.co.in (M. Sumathy).

Fig. 1. Schematic of a typical serpentine folded-waveguide slow-wave structure showing the relevant dimensions.
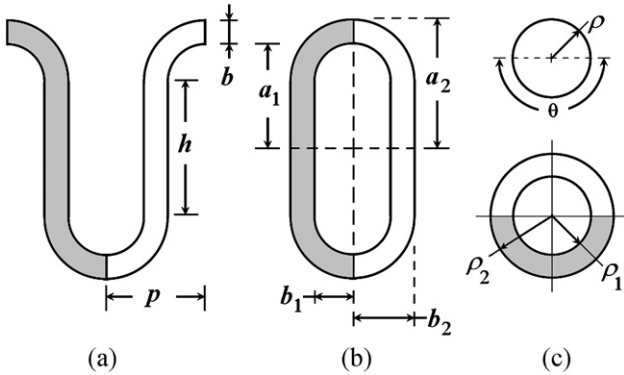

Fig. 2. (a) One period of the serpentine folded-waveguide SWS, (b) elliptical transformation in Z-plane, (c) transformation from Z-plane to W-plane.

of the ellipses in the Z-plane, approximately related to the transverse and longitudinal dimensions of the structure as follows:

$$a_1 = h/2 + b_1$$

$$b_1 = (p - b)/2$$

$$a_2 = a_1 + b$$

$$b_2 = b_1 + b \qquad (2)$$

Using the transformed parameters from (1)–(2), we can now easily express the lumped circuit capacitance (C_W) and inductance (L_W) per period of the structure (without beam-hole) having depth a as [10,14]

$$C_W = \varepsilon_0 a \left(\ln\left\{ \frac{\rho_2}{\rho_1} \right\} \right)^{-1}$$

$$L_W = \left(\frac{\mu_0 a}{\pi^2} \right) \ln\left\{ \frac{\rho_2}{\rho_1} \right\} \qquad (3)$$

The beam-hole has been treated as a circular aperture at the broad-wall of the rectangular waveguide at the location of electric field maximum. The effect of the beam-hole is included through an equivalent circuit, for which the expressions for the series

inductance and the shunt capacitance are expressed as [10,11]

$$L_H = \frac{1280\pi^2 r_c^3}{\omega \lambda_0 ab}$$

$$C_H = \frac{r_c^3}{45\omega \lambda_0 ab} \qquad (4)$$

Here, ω is the radian frequency and λ_0 is the free space wavelength. After cascading the equivalent circuit of the beam-hole with the equivalent circuit of the folded-waveguide, one can easily express the dispersion relation for the forward space-harmonic propagating mode as [8,9]

$$\beta_0 = \left(\frac{\omega^2 - (LC)^{-1}}{c^2} \right)^{1/2} \left(\frac{l_{eff}}{p} \right) + \frac{\pi}{p} \qquad (5)$$

Here, $L(=L_W + L_H)$ and $C(=C_W + C_H)$ are the lumped circuit series inductance and shunt capacitance of the SWS per period, respectively, and $l_{eff}(=h + \pi(p-b)/2)$ is the effective length of the structure per period.

Subsequently, using the dispersion relation (5), one can express the on-axis forward space-harmonic interaction impedance (K_c) of the slow-wave structure as [15]

$$K_c = \left(\frac{L}{C} \right)^{1/2} \left(\frac{\sin\{\beta_0 b/2\}}{(\beta_0 b/2)} \right)^2 \left(\frac{1}{\beta_0 p} \right)^2 \qquad (6)$$

Eqs. (5) and (6) can be now used for computing the dispersion and interaction impedance characteristics, respectively, with the knowledge of the dimensions of the structure under consideration.

3. Results and discussion

For numerical appreciation of the problem, two typical slow-wave structures were considered, one operating at Ka-band and other at W-band, for which the dimensional details as well as the measured and 3D analysis results of dispersion and interaction impedance characteristics are available in the literature [7]. The dispersion and interaction impedance characteristics of the structure were computed using the present analysis and compared to those from 3D eigen-mode analysis using CST Microwave Studio, parametric analysis and equivalent circuit analysis. The working expressions for the equivalent circuit analysis and the parametric analysis are given in Appendix A for ready reference. The benchmarking results are presented in Figs. 3 and 4 that show the efficacy of the present approach.
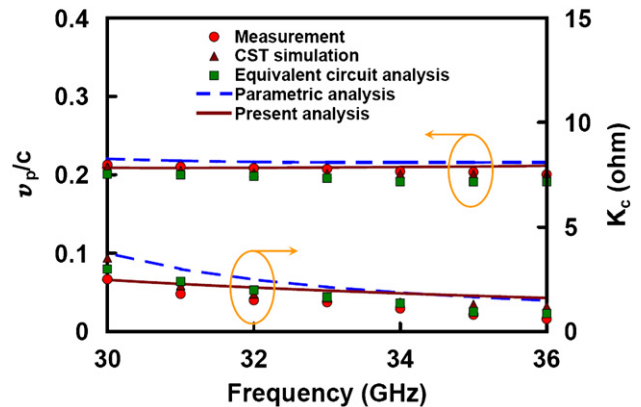

Fig. 3. Dispersion and interaction impedance characteristics of the Ka-band structure.
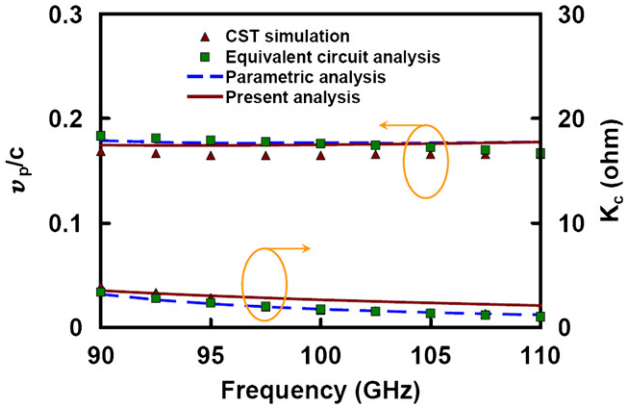
Fig. 4. Dispersion and interaction impedance characteristics of the W-band structure.

4. Conclusion

Closed form expressions have been developed for the computation of dispersion and interaction impedance characteristics of a serpentine folded-waveguide slow-wave structure following the method of elliptical conformal transformation. The analysis has been extensively validated against measurement, 3D electromagnetic modeling using CST Microwave Studio, parametric analysis and equivalent circuit analysis. The analysis is simple and is expected to be of ample use to the TWT community.

Acknowledgment

Authors are thankful to Dr. Lalit Kumar for his constant encouragement and the many valuable suggestions to improve the readability of the manuscript.

Appendix A

Working expressions for the equivalent circuit analysis and the parametric analysis are given here for ready reference.

Equivalent circuit analysis: An equivalent circuit analysis was introduced by the present authors in [7]. The expressions for the dispersion and interaction impedance characteristics are given as follows:

$$\beta_0 = \frac{1}{p}\left[2\pi - \cos^{-1}\left\{1 - (1/k)(1 - (f/f_W)^2)F_H\right\}\right] \tag{A.1}$$

$$K_c = 120\pi\left(\frac{\lambda_g}{\lambda_0}\right)\left(\frac{b}{a}\right)\frac{1}{(\beta_0 p)^2}\left(\frac{\sin\{\beta_0 b/2\}}{(\beta_0 b/2)}\right)^2 \tag{A.2}$$

with

$$F_H = 2 + a_k - a_k(f/f_H)^2$$

$$f_W = \frac{1}{2\pi\sqrt{L_W C_W}}, \quad f_H = \frac{1}{2\pi\sqrt{L_H C_H}}, \quad L_W = \frac{\mu_0 l_{eff} b}{\pi^2(l_{eff}+b)}, \quad C_W = \varepsilon_0 l_{eff}\frac{(l_{eff}+b)}{b}$$

$$L_H = \frac{1280\pi^2 r_c^3}{\omega\lambda_0 ab}, \quad C_H = \frac{r_c^3}{45\omega\lambda_0 ab}, \quad k \approx 0.3\exp\{20 r_c/a\}, \quad a_k = \frac{L_H}{k L_W},$$

Here, λ_g is the guided-wavelength of the rectangular waveguide. The above formulae work within the parametric regime of $0 \le r_c/a \le 0.1$

Parametric analysis: Following the parametric approach of Ha et al. [4] and Booske et al. [6], the dispersion relation may be expressed as

$$\beta_0 = \left(\frac{2\pi}{\lambda_g}\right)\left(1 + \frac{\lambda_g}{2l_{eff}}\right)\left(\frac{l_{eff}}{p}\right) \tag{A.3}$$

For the estimation of the interaction impedance, one has to use the expression (A.2), however with the value of β_0 from (A.3). One might note that the parametric analysis does not consider the effect of the beam-hole.

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M. Sumathy received her Master of Engineering degree from Department of Electronics and Communication Engineering, Madurai Kamaraj University, India in 1999. She is currently pursuing her doctoral work in Indian Institute of Science, Bangalore, India. She is currently working as a Scientist at Microwave Tube Research and Development Centre (MTRDC), Bangalore, India. Her research area includes slow-wave structures for traveling-wave tubes, high power microwave sources (HPM), and effects of HPM on electronic circuits.

K.J. Vinoy received the Bachelors degree from the University of Kerala, India, Masters degree from Cochin University of Science and Technology, India and Ph.D. degree from the Pennsylvania State University, USA, in 1990, 1993, and 2002, respectively. He is currently working as an Assistant Professor in the department of Electrical Communication Engineering, Indian Institute of Science, Bangalore, India. He was a research assistant at the Center for the Engineering of Electronic and Acoustic Materials and Devices (CEEAMD) at the Pennsylvania State University from 1999 to 2003. He has published over 50 papers in technical journals and conferences. He is author of books: *Radar Absorbing Materials: From Theory to Design and Characterization*, (Boston: Kluwer, 1996) and *RF MEMS and their Applications* (London: John Wiley, 2002). He also holds one US patent.

S.K. Datta received his M. Tech and Ph. D. degrees in Microwave Engineering in 1991 and 2000, respectively, from the Institute of Technology, Banaras Hindu University, Varanasi, India. He is currently working as a Scientist at Microwave Tube Research and Development Centre (MTRDC), Defence Research and Development Organization (DRDO), Bangalore, India. He received Sir C. Ambashankaran Award of Indian Vacuum Society for Best Paper (1998), INAE Young Engineer Award (2001), Sir C. V. Raman Young Scientist Award (2002), DRDO Agni Award for Excellence in Self Reliance (2003) and DRDO Laboratory Scientist of the Year Award (2009). His current areas of research include computer-aided design and development of helix and coupled-cavity traveling-wave tubes, Eulerian and Lagrangian analysis of the nonlinear effects in traveling-wave tubes, and the studies on electromagnetic wave propagation in slow-wave structures. He is a Fellow of the IETE of India and a member of Magnetics Society of India. He is the founder General Secretary and a Fellow of the Vacuum Electronic Devices and Applications Society (VEDAS), India.