A New Method for Segmentation using Fractal Properties of Images

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Abstract
We propose a new algorithm to evaluate fractal dimension and lacunarity of images at various box sizes by modifying the traditional sliding box counting method using Brownian motion expansions, and demonstrate the effectiveness of these quantities in image segmentation. In this approach, the mass variation (total pixel value) in each scale is used during the conventional sliding box approach, and then the power law formula is applied to determine both fractal dimension and lacunarity for the respective box sizes. Exploiting the synergy between fractal properties, the ratio of lacunarity to fractal dimension is used to determine an optimal threshold for edge detection in Canny method. The proposed segmentation results have been compared to Otsu method for Brodatz, UIUC, Berkeley University images, and show improved capabilities.

Keywords
Fractal Dimension, Lacunarity, Mass and Moment, Edge detection, Segmentation.

I. Introduction

Approaches to fix an optimal threshold value for image segmentation remains a difficult challenge and hence hot research topic in image analysis [1,2]. There are dozens of methods to select this threshold [3] but the Canny method remains the most popular. This method may be summarized in the following steps: (i) convert the image to grayscale; (ii) reduce noise using Gauss filter mask; (iii) compute the magnitude and angle of gradient; (iv) suppress regions where the gradient is below maximum to remove pixels from regions other than edges and (v) determine a hysteresis threshold value to eliminate regions whose gradient magnitude falls below this threshold. The quality of segmentation is decided by the optimal threshold determined in the last step and Otsu method proposed in 1979 is one of the successful approaches to do this automatically [4, 5]. In this paper we compute the threshold exploiting fractal properties such as lacunarity and dimension.

Fractal concepts for Image analysis have constantly gained importance during the past few decades [6]. Fractal geometry has found wide application in various disciplines since the term fractal was coined by Mandelbrot [7-12]. This concept has provided a theoretical framework to describe and analysis several natural phenomena and complex structures through properties such as fractal dimension, and lacunarity [13-15]. Arguably, the most widely used fractal property is the fractal dimension. It characterizes how much space the pattern fills [14, 10]. However its application in image analysis is limited as most natural images are characterized by different values of fractal dimensions for different scales [13, 16-18]. Most images are therefore considered multifractal. Similarly, fractal lacunarity is another scale-dependent property of fractals which could be used for image analysis. Lacunarity describes how the pattern fills the space and allows to quantify the degree of translational invariance of the analyzed objects [19].

Several mathematical methods have been developed to compute fractal dimension and Lacunarity of images [20-24]. But the most popular used is box counting method due to the simplicity in the algorithm, compared to the original Hausdorff definition [20]. The main issue with these approaches is that these use a least square best fit method to determine a unique value for the fractal dimension of a given image. While this approach gives a global value and is highly suited for analyzing truly fractal (self-similar) geometries [25, 26], it is not a convenient measure for textural images. We therefore propose a new method of computation of fractal properties based on a Brownian motion expansion concept. This approach can be used to analyze both fractal and non-fractal patterns. As demonstration of its practical application, the proposed approach has been used to set the optimal threshold for image segmentation. The rest of the paper is organized as follows. In the next section, a brief description of approaches to compute fractal dimension and lacunarity based on traditional sliding box counting method is presented. Section III focuses on the proposed method and its demonstration while simulations of image analysis using the proposed approach and a discussion is provided in section IV.

II. Theorical development

Lacunarity is interpreted as a measure of the lack of rotational or translational invariance of an image. In a broader sense, it is a measure of the degree of non-homogeneity within an object, image or pattern. Methods for calculating lacunarity were given in general terms by Mandelbrot [11, 12] and these were refined later to use the sliding box algorithm. [27-30]. This method for determining the lacunarity of a digitized image can be described as the follows.

A small box of size r is placed over the top left-hand corner of an image, and the grayscale values of pixels covered by the box are added together to determine the mass of this box m_r. Then this box slides in fixed steps (this value may differ, but boxes should overlap) to the right until it reaches the extreme right. In this paper we fix step is equal to one which is mostly used of an efficient analysis of the image.After this, the original box at the left is slid down by a step, and the scan continues until the entire image has been covered. With this process the mass m_r of the box at each position (i, j) is determined for the scale r. Next, the procedure is repeated with a slightly bigger box size or scale value.

Based on the above computation one can get the frequency distribution $N(r)$, which can be converted to a probability distribution $Q(m_r, r)$ by normalizing with the number of boxes $N(r)$ of size r. The lacunarity is determined using the first and second moments as [15]:

$$\ddot{E}(r)=\frac{Z^{(2)}(r)}{\left[Z^{(1)}(r)\right]^2} \tag{1}$$

Where

$$Z^{(1)}(r)=\sum\nolimits_M mQ(m_r,r) \tag{2}$$

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$$Z^{(2)}(r)=\sum_M m^2 Q(m_r,r) \tag{3}$$

Using the power law distribution of mass for multi-fractals, eq. (1) can be written as [8]:

$$\Lambda(r)=\lambda_r r^{\tau_r} \tag{4}$$

Where, $\tau_r=D(r)-E$ for, λ_r and $D(r)$ being the prefactor, the Euclidean dimension and the fractal dimension respectively. The value of λ_r depends on the scale value chosen.

Several algorithms and theories are available for the computation of fractal dimension of images. Many of these are based on the probability distribution of mass given by:

$$M(r)=k_r r^{D(r)} \tag{5}$$

Where $M(r), k_r, r, D(r)$ represent the total mass, the prefactor of the mass, the box size and the Minkowski or mass dimension, respectively. In eq. (5), k_r depends on the box size chosen. Furthermore, if $M(r)$ is mass $D(r)$ is called the mass dimension; and if $M(r)$ is moments $D(r)$ is called the fractal dimension.

In practice, fractal dimension for images is determined by plotting $\log(M(r))$ vs. $\log(r)$ in eq. (5). Usually the slope m of this log-log curve is related to the fractal dimension $D(r)$ by the expression: $D(r)=2-s_1$. However this approach is limited by the estimation of s_1 for this curve. In practice, we find that s_1 is different for different box sizes, and one uses least square approximation to find a unique value for $D(r)$.

Similarly, the curve representing lacunarity for various box sizes is obtained by $\log(M(r))$ vs. $\log(r)$ plot of eqn. (4). A modified unique measure of lacunarity of truly fractal images has been obtained in [8]. However, in the proposed approach below, we demonstrate the use of scale-dependent fractal dimension and lacunarity in estimating the threshold values required in image segmentation.

III. Proposed Method

Determination of Lacunarity and Dimension for a specified scale

Let I be a pattern or any image with size $height \times width$ pixels, and $p(i,j)$ the pixel value in the i^{th} row and the j^{th} column. In the sliding box counting method the expanded image size depends on the size of the sliding box (scale). For example at scale r the total number of boxes required to cover the entire image is $(height-r+1)\times(width-r+1)$ and the total number of pixels of the expanded image will be $(height-r+1)\times(width-r+1)\times r^2$. In this paper the box is a square box of size $r \times r$, and the minimum box size r is two the maximum is $height-r+1$ for the height, or $width-r+1$ for the width.

Let I_r, $M(r)$ be the expanded image and the total mass of the original I at scale r. These may be obtained as:

$$I_r=\begin{bmatrix} I_r(1,1), I_r(1,2), \cdots, I_r(1,w) \\ \vdots \qquad \ddots \qquad \vdots M \\ I_r(h,1), I_r(h,2), \cdots, I_r(h,w) \end{bmatrix} \tag{6}$$

And

$$M_r=\begin{bmatrix} m_r(1,1), m_r(1,2), \cdots, m_r(1,w) \\ \vdots \qquad \ddots \qquad \ddots \\ m_r(h,1), m_r(h,2), \cdots, m_r(h,w) \end{bmatrix} \tag{7}$$

Where $h=height-r+1, w=width-r+1$, $I_r(i,j)$ is the sub-image of mass $m_r(l,k)$ for the position (l,k) of the box of scale r and $m_r(l,k)$ is the sum of the total pixel value covered by the box calculated using:

$$m_r(l,k)=\sum_{l=i}^{i+r}\sum_{k=j}^{j+r} p(l,k) \tag{8}$$

The total mass of the expanded image I_r at scale r is the sum of sub- image mass as:

$$M(r)=\sum_{i=1}^{h}\sum_{j=1}^{w} m_r(i,j) \tag{9}$$

Based on the above concepts, we analyze the mass variation between the consecutive boxes within I_r using the fractal Brownian motion [31].

As introduced by Mandelbrot in [12] fractal Brownian motion is a non-stationary self-affine random process that can describe the random fractals in nature. According to the fractal Brownian motion, the variation between consecutive boxes inside the image for a fixed r can be obtained from

$$E|\Delta I_r| = (\Delta k_r) \times r^{D(r)} \tag{10}$$

This assumes that the mass distribution of boxes obeys the normal power law distribution $S(0,\sigma^2)$. In eq. (10) $E(\bullet)$ is the expectation, ΔI_r is the variation of mass (Δm_r) between consecutive sub-images. To compute Δk_r the following two directions should be considered:

The horizontal variation $\Delta k_{r,horit}$ is given as,

$$\begin{aligned} \Delta k_{r,horit} &= \sum_{i=1}^{h}\sum_{j=1}^{w-1} |m_r(i,j)-m_r(i,j+1)| \\ &= \sum_{i=1}^{h}\sum_{j=1}^{w-1} \left|\sum_{i=1}^{r} p(i,j)-\sum_{i=1}^{r} p(i,r+j)\right| \end{aligned} \tag{11}$$

Similarly, the vertical variation $\Delta k_{r,vent}$ is

$$\Delta k_{r,vert}=\sum_{i=1}^{h-1}\sum_{j=1}^{w} |m_r(i,j)-m_r(i+1,j)| = \sum_{i=1}^{h-1}\sum_{j=1}^{w}\left|\sum_{i=1}^{r} p(i,j)-\sum_{i=1}^{r} p(i+r,j)\right| \tag{12}$$

Finally the total variation is computed as:

$$\Delta k_r = |\Delta k_{r,horit}| + |\Delta k_{r,vert}| \tag{13}$$

Therefore, for a given scale value, $\Delta m_r = \Delta k_r$. The total variation of the first moments of consecutive boxes (horizontal and vertical) is:

$$\begin{aligned} \Delta k_{rfm} = \frac{1}{m(r)} \Bigg(&\left|\sum_{i=1}^{h}\sum_{j=1}^{w-1} m_{rfm}(i,j)\right| - \left|\sum_{i=1}^{h}\sum_{j=1}^{w-1} m_{rfm}(i+1,j)\right| + \\ &\left|\sum_{i=1}^{h-1}\sum_{j=1}^{w} m_{rfm}(i,j)\right| - \left|\sum_{i=1}^{h-1}\sum_{j=1}^{w} m_{rfm}(i,j+1)\right| \Bigg) \end{aligned} \tag{14}$$

Where $m_{rfm}(i,j) = m_r(i,j) \times Q(m_r(i,j),r)$. From eq. (2) we obtain the total first moment as,

$$Z^{(1)}(r)=\frac{1}{M(r)}\sum_{i=1}^{h}\sum_{j=1}^{h} m_{rfm}(i,j) \tag{15}$$

By substituting eqs. (15) and (14) to eq. (10) we get the fractal dimension $D_{fm}(r)$ for a specified scale r as:

$$D_{fm}(r)=\frac{\log\left(Z^{(1)}(r)\right)-\log\left(\Delta k_{rfm}\right)}{\log(r)} \tag{16}$$

Similarly, if $m_{rsm}(i,j)$ and Δk_{rsm} are the second moment of the box (i,j) and the variation of second moment of two consecutive boxes at scale r ; the total second moment of the entire image is.

$$Z^{(2)}(r)=\frac{1}{M(r)}\sum_{i=1}^{h}\sum_{j=1}^{w} m_{rsm}(i,j) \tag{17}$$

Since we have assumed that the mass follows the power law distribution, the second moment can be written as a of the fractal dimension [8]:

$$z^{(2)}(r) = \Delta k_{rsm}\, r^{D(r)} \tag{18}$$

The variation of second moment can be calculated as

$$\Delta k_{rsm}=\frac{1}{M(r)}\left(\left|\sum_{i=1}^{h}\sum_{j=1}^{w-1} m_{rsm}(i,j)-\sum_{i=1}^{h}\sum_{j=1}^{w-1} m_{rsm}(i+1,j)\right|+\left|\sum_{i=1}^{h-1}\sum_{j=1}^{w} m_{rsm}(i,j)-\sum_{i=1}^{h-1}\sum_{j=1}^{w} m_{rsm}(i,j+1)\right|\right) \tag{19}$$

Using eq. (18) and (19) in eq. (10) we can get an alternate expression for the fractal dimension $D_{sm}(r)$ from the second moment as:

$$D_{sm}(r)=\frac{\log\left(Z^{(2)}(r)\right)-\log\left(\Delta k_{rsm}\right)}{\log(r)} \tag{20}$$

It may be verified that eq(16) and (20) result in exactly the same values for the dimension at a given scale.

The lacunarity for a specified box size $\Lambda(r)$ can be computed by applying the first and second moments in eq. (15) and (17) to eq. (1). Alternately, one may use the fractal dimension computed by either of the above approaches in eq. (4), to obtain the prefactor λ as

$$\lambda_r = (\Delta k_{rsm})/(\Delta k_{rfm})^2 \tag{22}$$

As in the case of dimension, both these approaches result in exactly the same values for scale-dependent lacunarity.

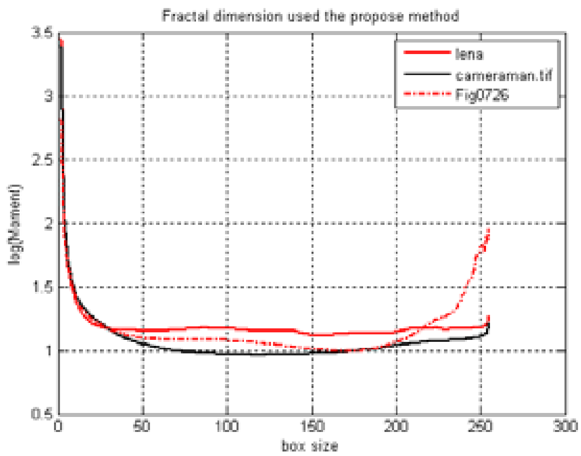

Fig 1 : Represent the fractal dimension curve for the following images: Lena, Pirate Fig0726 images used the proposed method. From fig 1 all the curve of fractal dimension for the very small box size are greater than two. So this method allows to resolve limitation of box size. From fig 1, we can see the method allow getting the fractal dimension at any scale (box size) value. Using the best fit method the fractal dimension is 1.4075, 1.4274 and 1.3751 for Lena, Cameraman and Fig. 0726 image, respectively. These values of fractal dimension correspond to the fractal value of the proposed method for the maximum box size (255) for Lena and Pirate image and box size 230 for Fig. 0726 image. Fig 2 represents the lacunarity using the propose method.

Fig. 1: Scale-dependent dimension of images, Lena, Cameraman and Fig0726 computed using the proposed method (eq. 4).

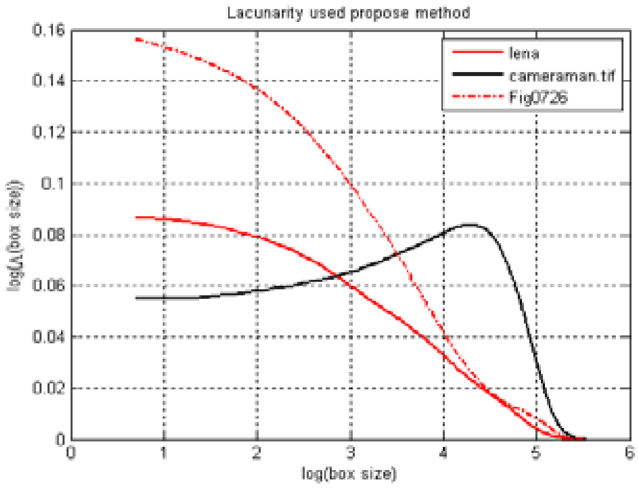

Fig. 2: Lacunaritycurve Lena, Cameraman and Fig. 0726 computed using the proposed method (eq. 21).

B. Determination of the Segmentation Threshold

Image segmentation is a critical step towards visual pattern recognition and image understanding. Many segmentation techniques have been motivated by specific application purposes. Canny algorithm is used for extracting the contour of edges of object by setting appropriate parameters for σ, T_{max} and T_{min} are the Gauss function distribution, the high and low threshold values respectively. The quality of segmentation depends on the threshold chosen [32, 33]. High value of the maximum threshold reduces the number of edges to be detected, leaving only the most obvious edges; a low value increases the number of edges produced, and can result in a large number of undesirable edge pixels. High value of low threshold reduces the number of edges which are detected. Setting the low threshold lower increases the extent of the edges, but may produce edge lines where edges are not required.

Otsu [3, 4] algorithm was introduced to improve the quality of edges in Canny method by efficiently computing the high threshold value. The basic principle of this approach is as follows. The gray pixels $[0,L]$ of the original image is first split into two classes $[0,t]$ and $[t,L]$, where L is the maximum gray pixel value. Then the best threshold T_{max} is computed using the criterion function defined as the variance between the two parts, which is expressed as:

$$\eta^2(t) = \alpha_0\alpha_1(\mu_0-\mu)^2 \tag{21}$$

Where $\alpha_0=\sum_{v=0}^{t}p_v, \alpha_1=1-\alpha_0, \mu_0=\sum_{v=0}^{t}\frac{vp_v}{\alpha_0}$ and $\mu_1=\sum_{v=t+1}^{L-1}\frac{vp_v}{\alpha_1}$ are the probability of the first and second parts, and their average gray value respectively, and p_v is probability of the pixel value. The maximum variance corresponds to the maximum threshold T_{max} of eq. (21) and the lower threshold $T_{min}=0.5\times T_{max}$. T_{max} is critical to the quality of efficient detection edges. Yuyand Zhou et al, shown that threshold obtained by maximum between-class variance method (i.e. Otsu method) is biased when the area of object and background differs significantly and may lead to failure segmentation. In this paper we proposed fractal theory to improve the quality of segmentation.

Based on the discussion above, fractal dimension measures the

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geometrical complexity of images and lacunarity characterizes the spatial heterogeneity of the texture in the image. We therefore define the ratio of Lacunarity to fractal dimension ρ to characterize the homogeneity to the fullness of the image of each scale. Fig. 3. Represents curves of the coefficient $\tilde{n}$ for images Lena, Cameraman, and Fig. 0726 images.

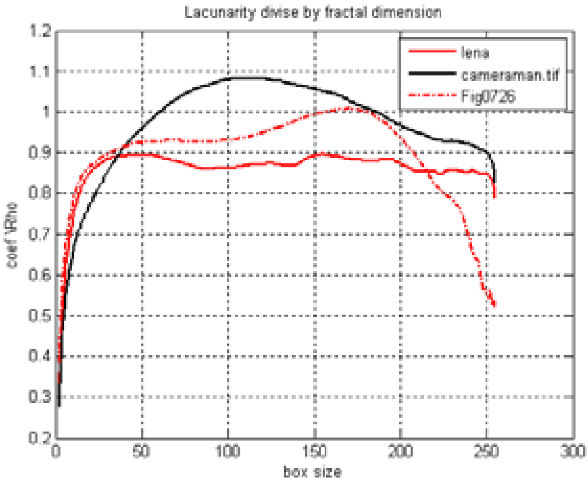

Fig. 3: Coefficient ρ for images Lena, Cameraman, and Fig 0726 .

The box size corresponding to the maximum value of the coefficient ρ is taken as the optimal box size (scale value r_{opt}) for a given image. In the proposed method we compute the vertical and horizontal variances ($\Delta k_{r,horit}$, $\Delta k_{r,vert}$ and for this optimal box size. These are used to find the new upper threshold T_{max} as

$$T_{max} = \sqrt{\left(\Delta k_{r,horit}/r_{opt}\right)^2 + \left(\Delta k_{r,vert}/r_{opt}\right)^2} \qquad (23)$$

As in Otsu method, we also have a lower threshold $T_{low} = 0.5 \times T_{max}$. These values are used to replace the threshold required in the step (v) of the Canny method. Table 1 shows the computation of optimal threshold for various images.

Table 1: Thresholds coefficients and box size

Optimal Threshold Value				
Image	Proposed Method			Otsu Method
	Coeffi-cient ρ	Box size	Optimal Threshold	Optimal Threshold
Lena.tif	0.89	50.00	50.05	127.00
Cameraman. tif	1.09	120.00	123.66	89.00
Fig0726.tif	1.01	170.00	104.54	65.00

The above table contain the threshold parameters, with which we have get the following simulation results of Lena, Cameraman, and fig 0726 images (fig 4, 5, and 6).

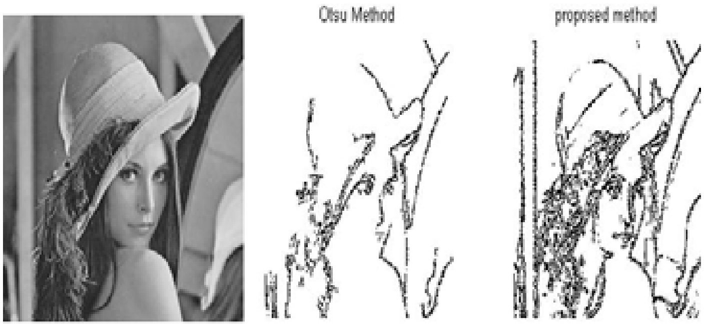

Fig. 4: Lena original image and the segmentation results of the two methods

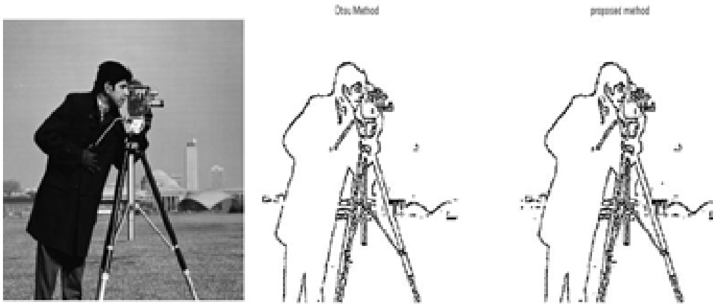

Fig. 5: Cameraman original image and the segmentation results of the two methods

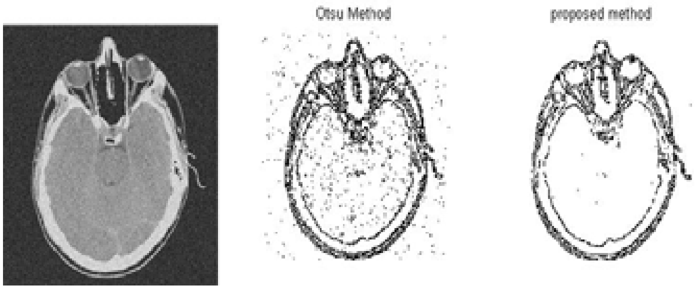

Fig. 6: fig. 0726 original image and the segmentation results of the two methods

The performance of edge segmentation of Otsu and fractal method are analyzed on above three images.We Use the synergy of Fractal dimension and Lacunarity to compute and efficient threshold for hysteresis step in Canny edge extraction process. The simulation result shown that for a fix Gauss function value (in this paper $d = 0.75$) fractal theory can give better in segmentation than Otsu Method. For example in fig 4, 5 and 6 Otsu method fails to eliminated and non-essential edge.

V. Conclusion

The proposed algorithm is akin to gliding box-counting algorithms used to estimating the both parameters (lacunarity and fractal Dimension). Then exploiting the synergy between fractal dimension and lacunarity, the two parameters have been used to calculate the optimal threshold for edge segmentation. Simulation shown that the propose method comparing to Otsu method provide an efficient segmentation results.

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References

[1] Y. Ireaneus Anna Rejani, Dr. S. Thamarai Selvi, "Breast Cancer Detection Using Multilevel Thresholding" (IJCSIS) International Journal of Computer Science and Information Security, 2009, Vol. 6, No.1.

[2] Xinming Zhang ,Quan dang Sun, "Improved Image Thresholding Based on Chaos Optimization and 2-D Tsallis Entropy", Journal of Information and Communication Technology, 2010, 3(2), pp. 88-94.

[3] Wang Xiang-ke, Zhen Zhi-qiang, "An Application of Multi-Thresholding Ostu Algorithm on Chromatic Image", Computer Application.vol.26, 2008, pp.14-15.

[4] Ostu N A, "Threshold Selection Method from Gray-Level Histograms", IEEE Trans. on System, Man, and Cybernetics, 1979 vol.9, no.1, pp.62-66.

[5] Mei Fand et al., "The study on an application of Otsu Method in Canny Operator", Proceeding of the 2009 International Symposium on Information Processing (ISIP'09) Huang Shan P.R. China, 21-23, 2009, pp 109-112.

[6] Karim S. Komati, Evandro O. T. Salles, Mario Sarcinelli Filho., "Fractal-JSEG: JSEG Using an Homogeneity Measurement Based on Local Fractal Descriptor", XXII Brazilian Symposium on Computer Graphics and Image processing. IEEE computer society 2009, pp. 253-260.

[7] N. Richard, M. Léard, V. Legeay, "Classification of roadway asphalt by use of fractal Signature" Proc. QCAV99, Quebec, Canada, 1999, pp. 57–62.

[8] K. Sengupta, K. J. Vinoy, "A new measure of lacunarity for generalized fractals and its impact in the electromagnetic behavior of Koch Dipole antennas", Fractals, 2006, Vol. 14, No.4, pp. 271-282.

[9] Daniel Gimenez, Janice L. Karmon, adolfo Posadas, Richard K. Shaw, "Fractal dimension of mass estimated from intact and eroded soil aggregates", Soil & Tillage Research 64 (2002), pp. 165-172.

[10] J. M. Blackledge, D. A. Dubovitski:, "Texture Classification using Fractal Geometry for Diagnosis of Skin Cancers, "EG UK Theory and Practice of Computer Graphics (2009), pp. 1-8Wen Tang, John Collomosse (Editors).

[11] B. B. Mandelbrot, "Fractals form, Chance, and Dimension" W. H. Freeman, New York, 1977, pp. 31-34.

[12] Mandelbrot, B.B., 1982, "The Fractal Geometry of Nature", New York: W.H. Freeman.

[13] Agterberg, F. P., 1994, "Fractals, multifractals, and change of support", in Dimitrakopoulus, R.,ed., Geostatistics for the next century: Kluwer Acad. Publ., Dordrecht, pp. 223-234.

[14] K. Falconer, "Fractal Geometry, Mathematical Foundations and Applications", JohnWiley & Sons, New York, NY, USA, 1990.

[15] Qiuming Cheng., "Multifractal Modeling and Lacunarity Analysis", Mathematical Geology, vol. 29. No 7, 1997.

[16] Klinkenberg, B., 1994, "A review of methods used to determine the fractal dimension of linear features". Mathematical Geology, 26, pp. 23–46.

[17] Goochild , M.F. and Mark, D.M., 1987, "The fractal nature of geographic phenomena". Annals of the Association of American Geographers, 77, pp. 265–278.

[18] F. Murtagh, A. Alexander, A. Bouridane, D. Crookes, J. Campbell, J. Starck, F. Bonnarel, Z. Geradts, J. G. Campbell, F. D. Murtagh, Eds., "Fractal and multiscale method for content-based image retrieval" in Proc. Irish Mach. Vis. Image Process. Conf., Belfast, Ireland, Aug. 2000, pp. 175–183.

[19] Y. –K. Wu, "Lacunarity and universality" J. Phys. A Math. Gen. 21 (1988), pp. 4251-4254.

[20] K. Falconer "Fractal Geometry, Mathematical Foundations and Applications" JohnWiley & Sons, New York, NY, USA, 1990.

[21] D. A Fabio, Aarao Reis, R Riera., "Lacunarity calculation in the true fractal limit". J. Phys. A. Math. Gen. 27 (1994) pp. 1827-1835, Printed in the UK.

[22] B. S. Raghavendra, D. Narayana Dutt, "Computing Fractal Dimension of Signal usingMultiresolution Box-counting Method", International Journal of Information and Mathematical Science 6: 1 2010

[23] Charles R. Tolle, Timothy R. McJunkin, David J. Gorsich. "An efficient implementation of the gliding box lacunarity algorithm", Physica D 237 (2008), pp. 306-315.

[24] W. Sun, G. Xu, P. Gong, S. Liang., "Fractal analysis of remotely sensed images: A review of methods and applications", International Journal of remote sending. Vol. 27, No. 22, 2006, pp. 4963-4990.

[25] XianghuaXie., " A Review of Recent Advances in Surface Defect Detection using Texture analysis Techniques". Electronic Letters on Computer Vision and Image Analysis 7(3):1-22, 2008.

[26] G. N. Srinivasan, Shobha G., "Statistical Texture Analysis". Proceeding of world academy of science, engineering and technology vol. 36, 2008, pp. 2070-3740.

[27] Y. Gefen, Y. Meir, B. B. Mandelbrot, A. Aharony, Phys. Rev. Lett. 50, 145 (1983); Y. Gefen, Y. Aharony, and B. B. Mandelbrot.J. Phys. A 17, 1277 (1984).

[28] B. Lin, Z. R. Yang, "A suggested lacunarity expression for Sierpinski carpets", J. Phys. A math, Gen. 19 (1986) pp. 4952.

[29] R. E. Plotnick, R. H. Garner, W. W. Hargrove, K. Prestegaard, M. Perlmutter., "Lacunarity analysis: a general technique for the analysis of spatial patterns" Physical Review E, vol. 53, no. 5, 1996, pp. 5461–5468.

[30] C. Allain, M. Cloitre, "Characterizing the lacunarity of random and deterministic fractal sets", Physical Review A, vol. 44, no. 6, 1991, pp. 3552–3558.

[31] B.B. Chaudhuri, N. Sarkar, P. Kundu. "Improved fractal geometry based texture segmentation technique", IEE Proceedings-E, 1993, Vol. 140, No.5.

[32] T. Esch, M. Thiel, M. Bock, A. Roth, S. Dech., "Improvement of Image Segmentation Accuracy Based on Multiscale Optimization Procedure", IEEE Geoscience and remote sensing, 2008, vol. 5, no. 3, 463.

[33] Xinming Zhang, Quandang Sun. "Improved Image Thresholding Based on Chaos Optimization and 2-D Tsallis Entropy", Journal of Information and Communication Technology, 2010, 3(2), pp. 88-94.

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