Approximate Synthesis Formulas for Microstrip Line with Aperture in Ground Plane

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ABSTRACT: Approximate closed-form expressions for the propagation characteristics of a microstrip line with a symmetrical aperture in its ground plane are reported in this article. Well-known expressions for the characteristic impedance of a regular microstrip line have been modified to incorporate the effect of this aperture. The accuracy of these expressions for various values of substrate thickness, permittivity and line width has been studied in detail by fullwave simulations. This has been further verified by measurements. These expressions are easier to compute and find immense use in the design of broadband filters, tight couplers, power dividers, transformers, delay lines, and matching circuits. A broadband filter with aperture in ground plane is demonstrated in this article. © 2011 Wiley Periodicals, Inc. Int J RF and Microwave CAE 22:124–130, 2012.

Keywords: microstrip line; defected ground; characteristic impedance; effective perimittivity; approximate expressions

I. INTRODUCTION

Planar transmission lines such as microstrip line, stripline, and coplanar waveguide (CPW) are used in microwave-integrated circuits and interconnects [1]. Propagation characteristics such as characteristic impedance and effective dielectric constant depend on physical parameters. Microstripline with defected ground is found to be very useful in the design of broadband filters, tight couplers, transformers, impedance matching circuits, high impedance line, and RF interconnections [2–9].

There are many well-established approaches to evaluate the electrical parameters of the various microstrip line structures. Closed-form formulae for the design and analysis of microstrip lines were originally derived by Wheeler [10]. Conformal mapping is one of the most popular methods in analyzing planar transmission lines and this approach has been used to find the capacitance per unit length for microstrip line and stripline structures [11]. However, this approach becomes restrictive, if layers of materials have finite dimensions as the analytical derivation of complex integrals in this case is not straight forward.

To overcome this, approaches based on calculating transmission line properties by various numerical methods have been followed in [12–15]. Even though these approaches yield accurate results, they are fairly complex and do not necessarily give a direct physical insight into their operational behavior. Several models are reported for simple analytical expressions for evaluating electrical parameters of microstrip lines [16–18]. Although microstrip lines with perturbed ground plane are found to be very useful, closed-form analytical expressions are not available for their synthesis. Simple empirical relations for the characteristic impedance of microstrip line with apertures in ground plane using data extracted from full wave simulations are presented in [19] and [20].

The present article proposes an approach to derive expressions for propagation parameters of a microstrip line with a longitudinal rectangular aperture (slot) in the ground conductor (shown in Fig. 1) by suitably modifying the commonly used expressions for regular microstrip lines [1]. Section II explains the formulation of these expressions. For the present perturbed microstrip line case, the distributed capacitance is affected by the transverse distance ‘‘h_{eff}’’ (mean effective length of field lines in the dielectric medium). Hence the challenge is to approximate this effective height ‘‘h_{eff}’’ of the substrate for various scenarios. Section III presents the results obtained through parametric study using the derived expressions. Results obtained using proposed expressions are compared against the results from full wave simulations and experiments.

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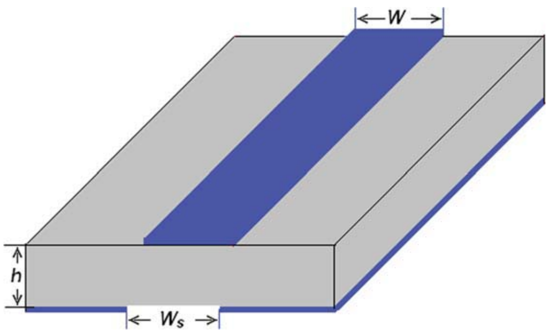

Figure 1 Microstrip line with aperture in ground plane. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com].

Possible potential use of these expressions is discussed in section IV by the method of a broadband filter design. Section IV concludes this article.

II. FORMULATIONS

It is clear from the geometry in Figure 1 that the aperture underneath the transmission line affects the electric field distribution between the line and the ground. As the effective transverse distance (between the line and the ground conductors) increases with an increase in aperture width "W_s," the capacitance per unit length reduces, which in turn increases characteristic impedance of the line. The equivalent conventional microstrip arrangement (with h_{eff}) is shown in Figure 2. The proposed formulation aims at evaluating the characteristic impedance of the defected microstrip line by evaluating "h_{eff}" through parallel plate capacitance per unit length. The following assumptions are used for this analysis:

i. Conductors are perfectly conducting so that their thickness need not be considered.

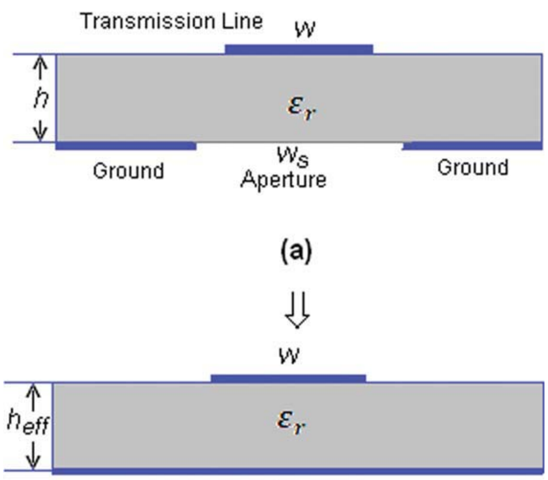

Figure 2 (a) Microstripline with aperture in ground plane, (b) equivalent conventional microstrip line. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com].

ii. Dielectric slab and the ground conductor are infinitely wide.
iii. Aperture or slot is symmetrical with respect to the center of the line (signal trace).
iv. Slot width (W_s) is much less than the guided wavelength at the desired frequency.

A. Effective Height for $0 \leq W_s \leq W$

As the geometry is symmetrical, this can be divided into two halves. Each of these halves is further divided into three regions as shown in Figure 3 and marked as I, II, and III to simplify the analysis. Region III does not have MIM structure and does not contribute much to the capacitance of the structure. Therefore, capacitance per unit length for regions I and II needs to be calculated. The distance between the parallel plates $h(x)$ in region I varies along with the strip width. For region I, the MIM capacitance is calculated by integrating the capacitance for an infinitesimal width dx separated by a height $h(x)$, using the following integral:

$$C_{pI} = \varepsilon_o \varepsilon_r \int_0^{\frac{W_s}{2}} \frac{dx}{h(x)}. \tag{1}$$

Where $h(x)$, the transverse length of field line is a function of x and is given by:

$$h(x) = \sqrt{h^2 + \left(\frac{W_s}{2} - x\right)^2}. \tag{2}$$

After substituting Eq. (2) in Eq. (1) and evaluating the integral, the authors get

$$C_{pI} = \varepsilon_o \varepsilon_r \sinh^{-1}\left(\frac{W_s}{2h}\right). \tag{3}$$

But for the region II, the distance between the plates remains a constant ($=h$) and its capacitance is evaluated as:

$$C_{pII} = \frac{\varepsilon_o \varepsilon_r}{h}\left(\frac{W - W_s}{2}\right). \tag{4}$$

The overall parallel plate capacitor C_p is the twice the sum of capacitors of region I and region II and is given by:

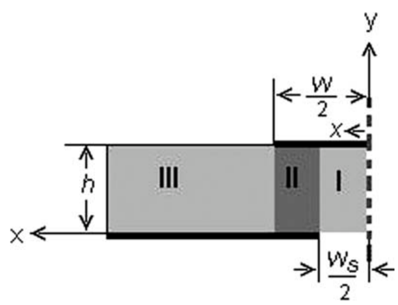

Figure 3 Case: $0 \leq W_s \leq W$.

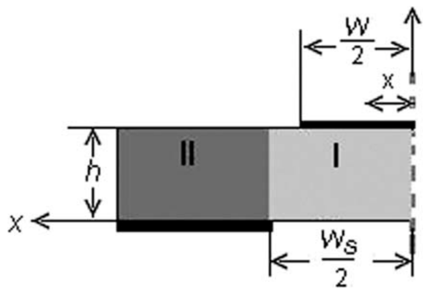

Figure 4 Case: $W_s > W$.

$$C_p = 2\varepsilon_o\varepsilon_r \sinh^{-1}\left(\frac{W_s}{2h}\right) + \frac{\varepsilon_o\varepsilon_r(W - W_s)}{h}. \tag{5}$$

The parallel plate capacitance for the equivalent conventional microstrip line structure with the same capacitance per unit length as the line with a slot in the ground plane shown in Figure 2b is:

$$C_p = \frac{\varepsilon_o\varepsilon_r W}{h_{eff}}. \tag{6}$$

Equating (5) and (6), the modified height "h_{eff}" for $0 \le W_s \le W$ is given by:

$$h_{eff} = \frac{W}{\left[2\sinh^{-1}\left(\frac{W_s}{2h}\right) + \frac{(W-W_s)}{h}\right]}. \tag{7}$$

B. Effective Height for $W_s > W$

As shown in Figure 4, the average transverse distance (in dielectric medium) "h_{eff}" traveled by the field lines from the line to ground for the case of $W_s > W$ can be found using the parallel plate capacitor "C_{pI}." Region II need not be considered in this case as it does not have MIM structure. As explained in the earlier case, overall parallel plate capacitance is given by:

$$C_p = 2C_{pI}. \tag{8}$$

The capacitor C_{p1} is evaluated using the following integral.

$$C_{pI} = \varepsilon_o\varepsilon_r \int_0^{\frac{W}{2}} \frac{dx}{h(x)}. \tag{9}$$

Separation between the plates $h(x)$ is given by:

$$h(x) = \sqrt{h^2 + \left(\frac{W_s}{2} - x\right)^2}. \tag{10}$$

After substituting Eq. (10) in Eq. (9) and evaluating the integral, the authors get:

$$C_p = 2C_{pI} = \varepsilon_o\varepsilon_r\left(\sinh^{-1}\left(\frac{W_s}{2h}\right) - \sinh^{-1}\left(\frac{W_s - W}{2h}\right)\right). \tag{11}$$

Hence for $W_s > W$, "h_{eff}" is evaluated by eqs. (6) and (11) and is given by:

$$h_{eff} = \frac{W}{2\left[\sinh^{-1}\left(\frac{W_s}{2h}\right) - \sinh^{-1}\left(\frac{W_s - W}{2h}\right)\right]}. \tag{12}$$

C. Properties of Modified Microstrip Line

Based on the effective height derived above, closed-form expressions for the characteristic impedance "Z_c" and effective dielectric constant "ε_{eff}" of microstrip line with ground plane aperture can now be obtained using the standard expressions [1] for regular microstrip lines, wherein "h" is replaced by h_{eff}:

$$Z_c = \begin{cases} \frac{60}{\sqrt{\varepsilon_{eff}}}\ln\left(\frac{8h_{eff}}{W} + \frac{W}{4h_{eff}}\right) & \text{for } \frac{W}{h_{eff}} \le 1 \\ \frac{120\pi}{\sqrt{\varepsilon_{eff}}}\left(\frac{W}{h_{eff}} + 1.393 + 0.677\ln\left(\frac{W}{h_{eff}} + 1.444\right)\right)^{-1} & \text{for } \frac{W}{h_{eff}} \ge 1. \end{cases} \tag{13}$$

and

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}F. \tag{14}$$

where

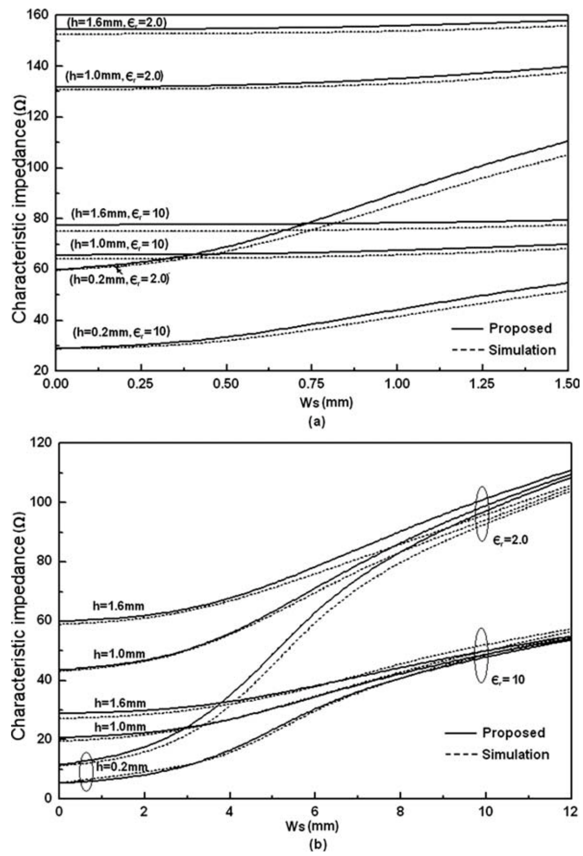

Figure 5 Comparison of characteristic impedance. (a) W = 0.5 mm, (b) W = 4 mm.

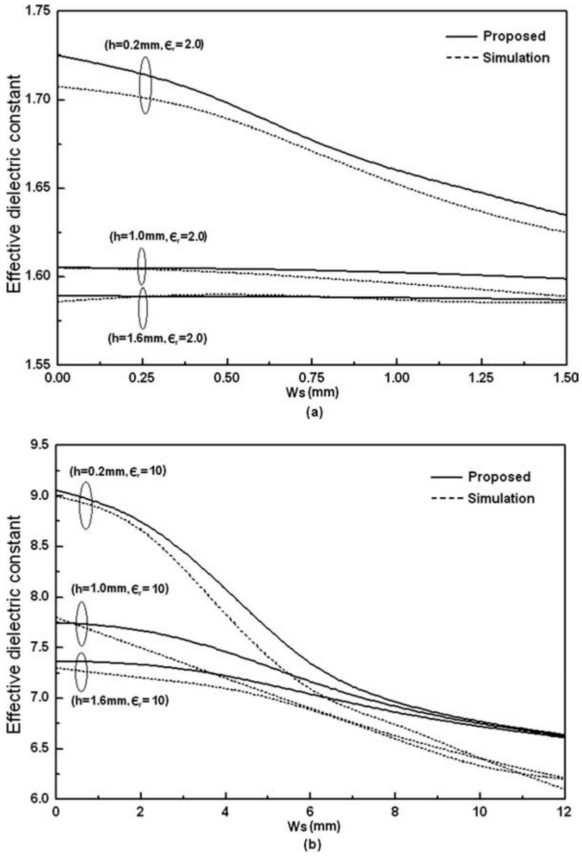

Figure 6 Comparison of effective dielectric constant. (a) $W = 0.5$ mm, (b) $W = 4$ mm.

$$F = \begin{cases} \left(1 + \frac{12h_{\text{eff}}}{W}\right)^{-0.5} + 0.04\left(1 - \frac{W}{h_{\text{eff}}}\right)^2 & \text{for } \frac{W}{h_{\text{eff}}} \leq 1 \\ \left(1 + \frac{12h_{\text{eff}}}{W}\right)^{-0.5} & \text{for } \frac{W}{h_{\text{eff}}} \geq 1 \end{cases}. \quad (15)$$

The case of $W_s = 0$ makes h_{eff} equal to h leading to a conventional microstrip line.

III. VALIDATION OF EXPRESSIONS

The modified expressions for the characteristic impedance and effective permittivity of the microstrip line is useful for the design of various microstrip components having defected ground. It may be noticed that these expressions are simple and can be useful for initial synthesis of microstrip line with aperture in ground plane. Expressions are easier to compute when compared with the expressions given in [19]. This present approach can also be used to evaluate the characteristic properties of CPW, in which the signal trace is offset from ground plane.

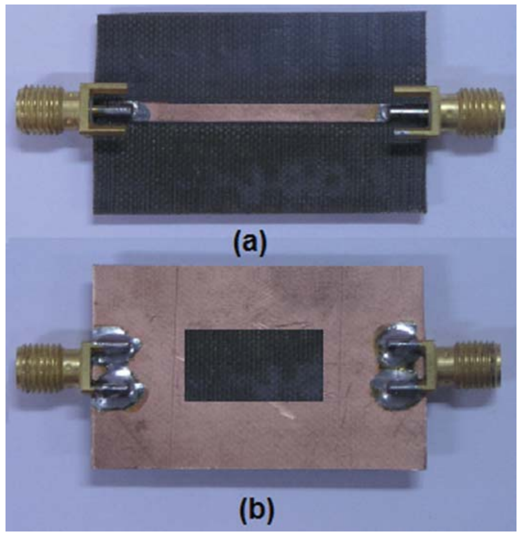

Figure 7 Photograph of a typical assembled microstrip line with aperture in ground plane. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com].

To validate these expressions, the authors present comparison with extensive full wave simulations and a few cases of experimentally realized transmission lines. Further, to demonstrate the utility of these expressions, these are used in the design of a simple filter circuit with defected ground geometry. These validations are discussed in the following paragraphs.

A. Full Wave Simulations

Different sets of characteristic impedances for various values of h, ϵ_r, W, and W_s are extracted from full wave simulation using the scattering parameters of the microstrip structures with ground plane aperture for the purpose of study. Figure 5 compares the characteristic impedances obtained from proposed method with the full wave simulation results [21] for various substrates. Results are satisfactorily matching with each other and are accurate with the tolerance of $\pm 5\Omega$. The error is only 1–2% for the aperture width less than or equal to the line width ($W_s \leq W$). The error increases (3–5%) when slot width is greater than line width ($W_s > W$). Figure 6 compares the effective dielectric constant obtained from proposed method with the full wave simulation results for various substrates.

As seen from Figure 5, characteristic impedance increases monotonically as slot width increases. Effective

TABLE I Comparison of Characteristic Impedances of Defected Ground Microstrip Lines

Defected Ground Slot width (W_s)	From [19] (Ω)	Proposed Synthesis formula (Ω)	EM Simulation (Ω)	Measurement (Ω)
2.2 mm ($h = 0.63$ mm, $\varepsilon_r = 10$, and $W = 0.7$ mm)	60.6	60.1	59.6	61.8
4.4 mm ($h = 0.78$ mm, $\varepsilon_r = 2.17$, and $W = 2.4$ mm)	80.9	81.5	80.2	83.1
7.2 mm ($h = 0.78$ mm, $\varepsilon_r = 2.17$, and $W = 2.4$ mm)	102.2	106.84	105.8	108.2

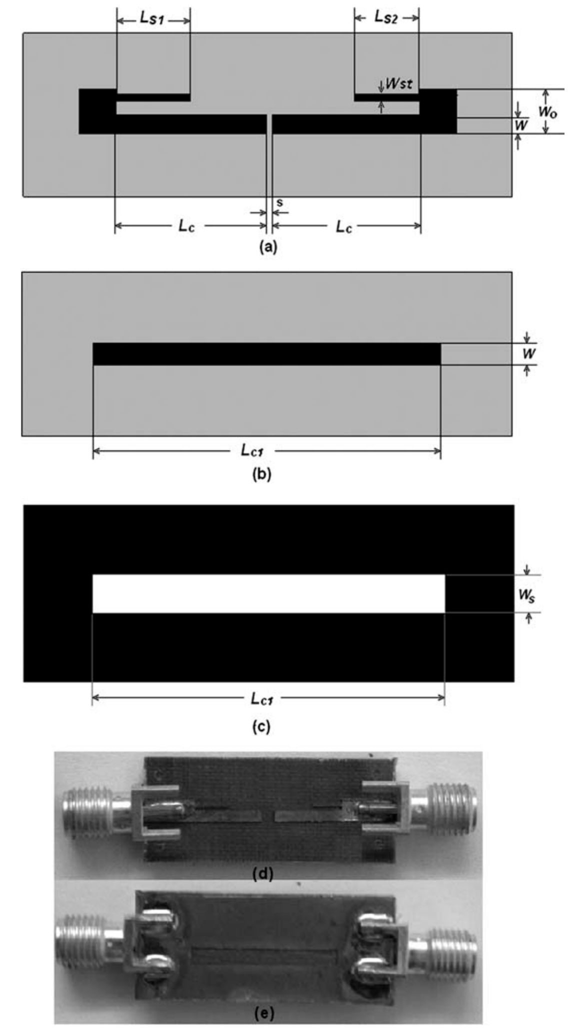

Figure 8 Layers of filter. (a) Top layer, (b) middle layer, (c) bottom layer, (d) top layer of fabricated filter, (b) bottom layer of fabricated filter.

permittivity reduces as slot width increases as seen in Figure 6. For the substrate thickness "h" of 1.6 mm, the impedance curve is almost flat and this indicates that slot has negligible effect on impedance for narrow strip widths for thicker substrates. For wider strips, impedance is affected by slot significantly as can be observed in Figure 5. These expressions estimate characteristic impedance "Z_c" over a widely used range of geometrical parameters 0.2 mm $< h <$ 1.6 mm, $2 < \epsilon_r < 10$, $0 < W_s/W < 3$, and 0.5 mm $< W <$ 4 mm.

B. Experimental

Different microstrip line structures (having line widths of 0.7 and 2.4 mm) have been made with aperture in ground plane using standard printed circuit board fabrication on various substrates ("ε_r" of 2.17 and 10) and measured using vector network analyzer. Photograph of one of fabricated microstripline structure is shown in Figure 7. Regular 50 Ω microstrip lines are attached to the line with slot to connect ports.

Characteristic impedance of the microstrip structures were calculated from the measured scattering parameters at 3 GHz. The effect of the 50 Ω microstrip lines is first removed by a de-embedding procedure. Table I gives the comparison of characteristic impedances of microstrip lines having ground plane aperture widths of 0.7, 4.4, and 7.2 mm. It is evident from the measurement that characteristic impedance value of line increases as aperture width increases. The impedance values are accurate with the measured values with 2% error.

IV. FILTER DESIGN

A broadband microstrip line filter with a defected ground is designed to demonstrate the utility of this approach in the design of such components. The filter has the following specifications.

- Frequency band: 3–8 GHz
- Each layer substrate permittivity "ϵ_r": 2.17
- Each layer substrate thickness: 0.38 mm

Different layers and photograph of the filter are shown in Figure 8. Aperture in the bottom layer enhances the coupling between the broadside coupled lines to realize broadband response. Open circuit stubs of lengths L_{s1} and L_{s2} are embedded with the coupled line sections to improve the stop band rejection characteristics of the

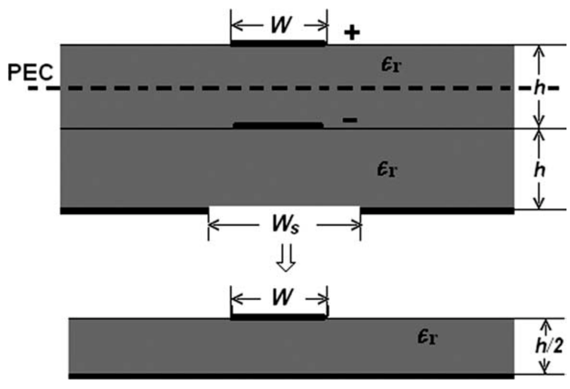

Figure 9 Odd mode analysis.

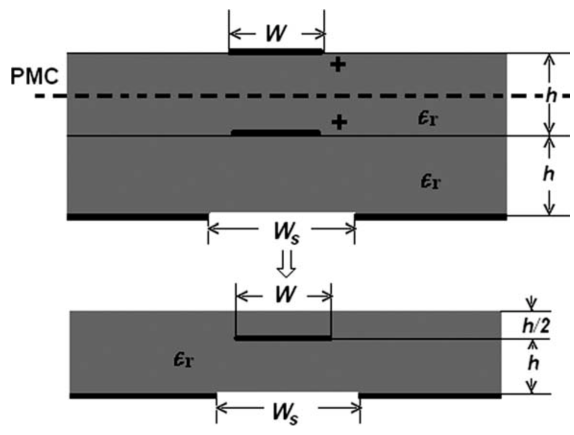

Figure 10 Even mode analysis.

TABLE II Physical Parameters of Filter

Parameters	Values (mm)
W_o	2.4
W	1.3
W_{st}	0.4
L_c	7.9
L_{s1}	4.25
L_{s2}	3.55
s	1.3
W_s	1.4

filter. The broadside coupling has been analyzed as shown in Figure 9 for the odd mode. When two conductors are excited with opposite and equal amplitudes (odd mode), a PEC boundary is included midway between them. The odd mode impedance is approximated to the impedance of microstrip structure having thickness of "$h/2$" and permittivity of "ϵ_r" and can be calculated using the following formula.

$$Z_{oo} = Z_o\left(\frac{h}{2}, \varepsilon_r, W\right). \quad (16)$$

Z_{oo} can be found using Eq. (13) with h_{eff} replaced by $h/2$.

When two conductors are excited with equal amplitudes (even mode), the fields in the structure is similar to having a PMC wall included at a plane midway between them, as shown in Figure 10 using image theory. The even mode impedance can be approximated using the following equation.

$$Z_{oe} = 2Z_o\left(h_{eff}, \frac{h}{2}, \varepsilon_r, W, W_s\right). \quad (17)$$

The closed-form expressions given in [11] can be used with "h" replaced by h_{eff} to find Z_{oe}.

Even and odd mode impedances (Z_{oe} and Z_{oo}) of the coupled line section obtained using the eqs. (16) and (17) are 115 and 32 Ω, respectively. Physical parameters of the filter are given in Table II. Calculated transmission matrices of coupled line sections and stubs are used to determine the scattering parameters of this filter. Coupled lines are modeled based on the data obtained using the closed-form expressions explained in Section II. Figure 11 shows the various sections of the filter under analysis. Measured results along with circuit modeling and full wave EM simulation results are shown in Figure 12. Comparison shows a good agreement except for slight increase in bandwidth (380 MHz), which can be attributed to the quasi-static modeling of the structure and fabrication tolerances. This

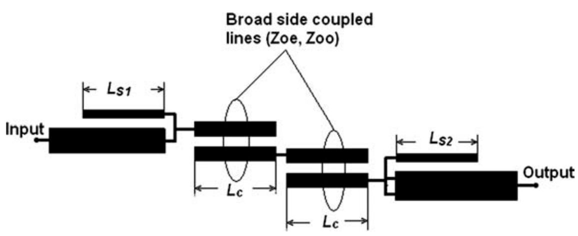

Figure 11 Filter sections.

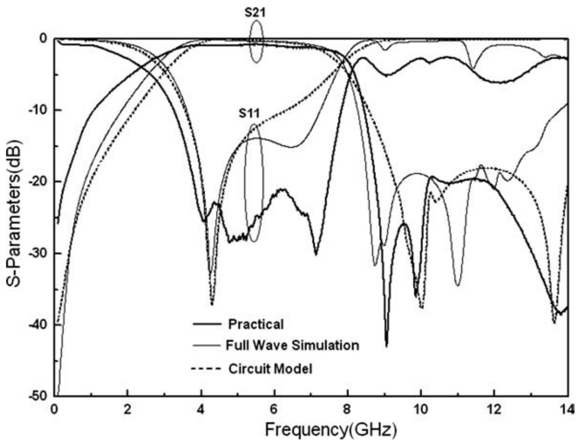

Figure 12 Measured S-parameters of broadband filter.

demonstrates that the expressions derived in this article can be effectively used for designing microstrip components such as broadband filters, antennas, tight couplers, and multi-layer interconnects with defected ground.

V. CONCLUSION

Approximate closed-form expressions for the analysis of microstrip line with an aperture in the ground plane are presented in this article. Characteristic impedance of the microstrip structure changes with varying width of the slot in the ground plane. Closed-form results were compared against the simulations and measurements to validate the accuracy. Furthermore, potential of the expressions in the synthesis of microstrip components was demonstrated by designing a broadband microstrip filter with a defected ground. These expressions can therefore be used for applications in microwave-integrated circuits and are valid over the range of design parameters 0.2 mm $< h <$ 1.6 mm, $2 < \varepsilon_r < 10$, $0 < W_s/W < 3$ and 0.5 mm < W < 4 mm.

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