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Calculation of Equivalent Circuit Parameters of a Rectangular Reentrant Cavity for Klystron

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***Abstract*- Rectangular reentrant cavities are most widely used in the high frequency compact klystrons. However, an analytical approach for calculation of cavity parameters of a rectangular re-entrant cavity with cylindrical ferrule is not available in the published literature. A simple approach for calculation of cavity parameters, namely, the resonant frequency (f_0), the unloaded quality factor (Q_0) and the characteristic impedance (R_{sh}/Q_0) of a rectangular reentrant cavity with cylindrical ferrule was developed, by way of transforming the rectangular reentrant cavity geometry to that of an equivalent cylindrical reentrant cavity. The equivalent lumped circuit parameters were then obtained using the procedure available for cylindrical reentrant cavity. Proposed analysis was benchmarked against the results obtained from the 3D electromagnetic simulation and measurement results.**

***Index Terms*- Equivalent Circuit analysis, Klystron, Reentrant cavity.**

I. INTRODUCTION

High frequency compact klystrons employ reentrant cavities having rectangular cross-sections with cylindrical ferrules (Fig.1) as their interaction structure due to ease of fabrication and assembly of the components with precision alignment. The rectangular geometry also offers the possibility of adding features to tune the resonant frequency of the cavity during assembly and even after final integration of the klystron device [1-3].

Any reentrant cavity having either cylindrical or rectangular geometries can be represented by parallel resonant circuit in equivalent circuit model analysis [4]. Single reentrant cylindrical cavity with cylindrical ferrules has been analytically treated using equivalent circuit analysis by Fujisawa [5], Carter *et al.* [6] and Barroso *et al.* [7], and Double reentrant cylindrical cavity with cylindrical ferrule has also been treated analytically for obtaining the equivalent circuit parameters of the cavity by Carter *et al.* [8] and Nguyen *et al.* [9]. However, to the best of the knowledge of the authors, an equivalent circuit analysis for rectangular reentrant cavity with cylindrical ferrule has not been available in the published literature.

In this paper, a simple approach for calculating the cavity parameters, namely, the resonant frequency (f_0), the unloaded quality factor (Q_0) and the characteristic impedance (R_{sh}/Q_0) of a rectangular reentrant cavity with cylindrical ferrule has been presented. The analysis is carried out in two steps: (i) the geometry of the rectangular reentrant cavity has been suitably transformed to an equivalent cylindrical reentrant cavity and then (ii) an analytical formulation for calculating the equivalent circuit parameters of the cavity has been proposed following the procedure given by Fujisawa [5] for single reentrant cylindrical cavity. For the purpose of benchmarking, the rectangular reentrant cavity and the transformed equivalent cylindrical reentrant cavity have been simulated using CST Microwave Studio [10] for various combinations

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of cavity dimensions. Finally, Analytical results are compared with those obtained from the simulation and measurement.

The approach for transformation of the cavity geometry and the analysis for calculating the cavity parameters have been presented in section II. Analytical, simulated and measured results are presented and discussed in section III. Finally, the conclusions are drawn in Section IV.

II. ANALYSIS

Equivalent circuit analysis for the azimuthally symmetric single reentrant cylindrical cavity is proposed by Fujisawa [5]. The analysis proposed by him provides fairly accurate estimation of the equivalent circuit parameters for a single reentrant cylindrical cavity operating in fundamental TM_{010} mode with some limitation on the cavity dimensions. A rectangular reentrant cavity geometrically differs from a cylindrical reentrant cavity only in its transverse cross section as shown in Fig.1.

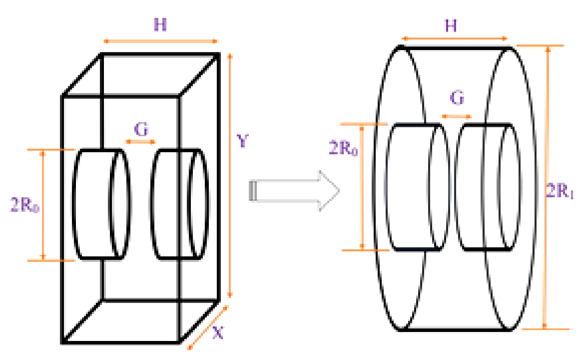
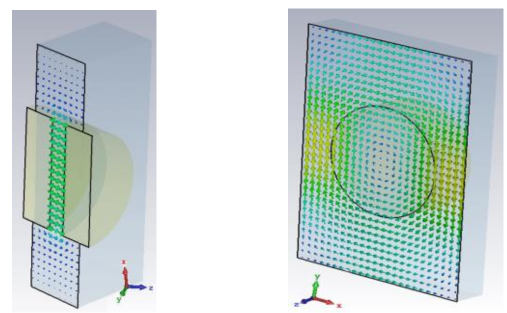
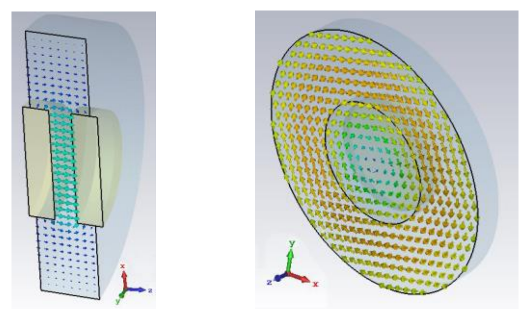


Fig.1. A rectangular reentrant cavity and its equivalent cylindrical reentrant cavity

The rectangular reentrant cavity with cylindrical ferrule operates in fundamental TM_{110} mode. The electric and magnetic field orientations and their distributions are similar in both the circular and rectangular reentrant cavities for the operating modes of TM_{010} and TM_{110}, respectively as shown in Fig.2. This similarity in the distributions of the electric and the magnetic fields allows a suitable transformation of a rectangular reentrant cavity to an equivalent cylindrical reentrant cavity. In order to maintain the field distributions, the cavity dimensions of the rectangular cavity such as the ferrule radius (R_0), the drift gap (G) and cavity height (H) are kept the same as those of the cylindrical cavity while transforming the geometry. Once the geometry is transformed, the equivalent circuit parameters of the cavity can be obtained using the procedure given in [5].



(a)



(b)

Fig.2. Electric and magnetic field distributions (a) in a rectangular reentrant cavity operating in TM_{110} mode, and (b) in a cylindrical reentrant cavity operating in TM_{010} mode.

A number of approaches have been tested for transforming the rectangular reentrant cavity cross section dimension to the radius of outer circumference (R_1) of equivalent cylindrical cavity, like, exact equivalence of cross sectional areas, arithmetic average of cavity dimensions X and Y, geometric average of cavity dimensions X and Y, etc. All these approaches does not hold good for a square cavity case where cavity aspect ratio $m = Y/X$ becomes

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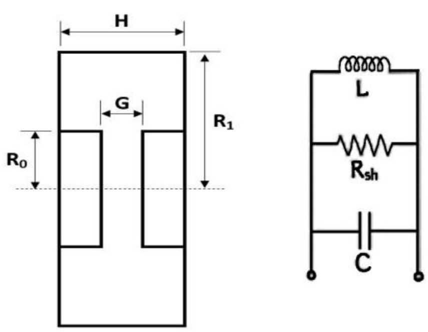
unity. Finally, area equivalence has been arrived at based on a curve-fitting exercise on the simulated performance of rectangular and equivalent cylindrical cavities that works reasonably well for the range of cavity aspect ratio used in practical devices ($m \in (1, 1.5)$). The area equivalence gives the equivalent cavity radius (R_1) as

$$R_1 = A\sqrt{XY/\pi} \tag{1}$$

Here, A is a correction function for minimizing the errors in the estimation of cavity parameters using the proposed approach. The correction function used here is a first order polynomial function of cavity aspect ratio ($m = Y/X$). The zero-order and the first order constants of the correction function are obtained empirically by analyzing parameters of square and rectangular cavity, respectively, and comparing those with respective values obtained through 3D electromagnetic simulation. The values of the constants are chosen such that the correction function provides least error in estimation of the resonant frequency using the proposed analysis with simulated results. The correction factor arrived at is given below:

$$A = 0.95 - 0.1(m-1) \tag{2}$$

Once all the physical dimensions of an equivalent cylindrical cavity are obtained, one can then get the cavity parameters using the analytical approach for a cylindrical cavity. The equivalent lumped circuit of a cylindrical reentrant cavity can be given as shown in Fig.3.



(a) (b)

Fig.3. (a) Schematic of a cylindrical reentrant cavity and (b) its lumped equivalent circuit.

The lumped circuit parameters of double reentrant cylindrical cavity have been calculated using the procedure given in [5]. Double reentrant cavity can be considered as two single reentrant cavities cascaded back to back in series, for which the lumped circuit parameters are arrived at as follows

$$C = \frac{\varepsilon_0 \pi R_0^2}{G} + 2\varepsilon_0 R_0 \ln\left(\frac{2.718\sqrt{(R_1 - R_0)^2 + 0.25H^2}}{G}\right) \tag{3}$$

$$L = \frac{\mu_0 H}{2\pi} \ln\left(\frac{R_1}{R_0}\right) \tag{4}$$

Where, L and C are the inductance and the capacitance, respectively, of the equivalent circuit of a given cavity; ε_0 is the permittivity of free space and μ_0 is the permeability of free space. Using the lumped circuit parameters given in (3) and (4) and the physical dimensions of the cavity, the cavity parameters may be calculated as follows:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{5}$$

$$R_{sh} = \frac{2\pi\sigma\delta\omega_0^2 L^2}{\left(\frac{H-G}{R_0} + \frac{H}{R_1} + 2\ln\left(\frac{R_1}{R_0}\right)\right)} \tag{6}$$

$$Q_0 = \omega R_{sh} C \tag{7}$$

Where, σ is the bulk conductivity of the cavity material; δ is the skin depth at resonant frequency; R_{sh} is the on-axis shunt resistance of the cavity; Q_0 is the unloaded quality factor and ω_0 ($= 2\pi f_0$) is the angular resonant frequency.

III. RESULTS AND DISCUSSION

An approach for calculating the parameters of a rectangular reentrant cavity with cylindrical ferrule has been discussed in the previous section. The benchmarking of the results obtained from proposed analysis has been done

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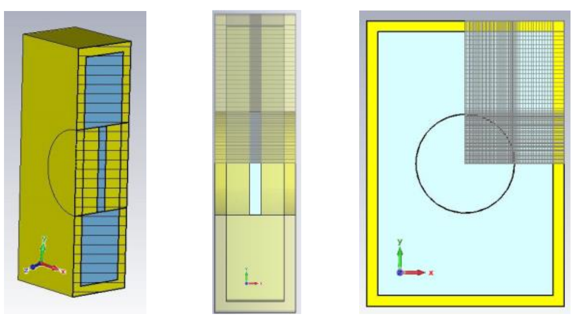
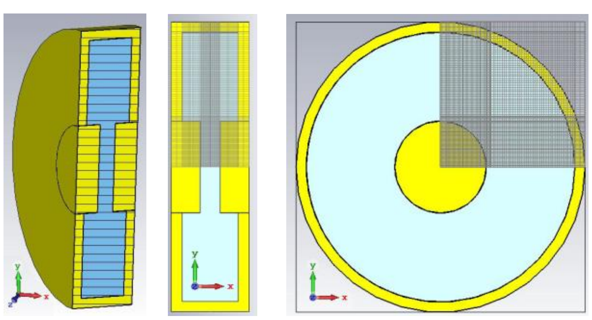
against the simulated results obtained from 3D electromagnetic field solver CST microwave studio [10]. A rectangular reentrant cavity with cylindrical ferrule has been modeled as shown in Fig.4(a) and cavity parameters have been simulated using eigen-mode solver. Hexahedral meshing has been used and spatial localized meshing has also been done where field gradient is high as shown in Fig.4(a) to obtain more accurate results. Meshing at other areas of cavity has also been optimized such that results having negligible variation when meshing is further increased. OFHC copper material (having bulk conductivity 5.87×10^7 S/m) has been assigned in simulation as well as in the analysis.

A wide range of combinations of cavity dimensions have been simulated to validate the accuracy of the analysis. Similarly, transformed equivalent cylindrical reentrant cavity has also been modeled along with optimized hexahedral mesh as shown in Fig.4(b) and cavity parameters have been computed.



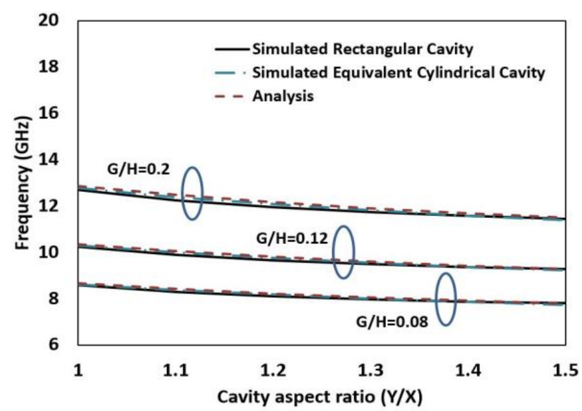
(a)



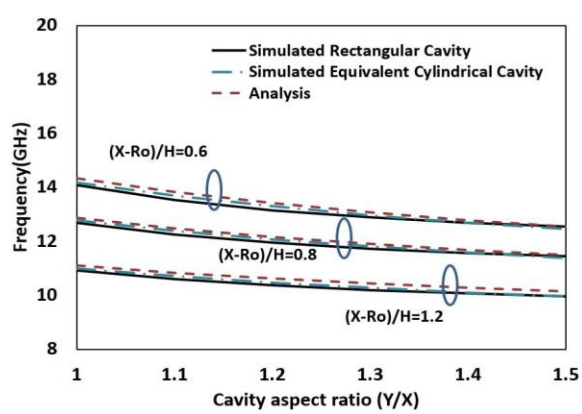
(b)

Fig.4. The sectional view and the mesh used for simulation in CST Studio for (a) rectangular reentrant cavity and (b) cylindrical reentrant cavity.

The resonant frequency of a rectangular reentrant cavity as obtained from the simulation are compared against those obtained both from simulation and analysis of the equivalent cylindrical cavity for two cases: (i) $(X-R_0)/H$ parameter is kept constant (typically at the value of 0.8) and cavity aspect ratio is varied for different values of (G/H) ratio ($H=2.5$ mm, $R_0=2.5$ mm) as shown in Fig.5(a), and (ii) (G/H) ratio is kept constant (typically at the value of 0.2) and cavity aspect ratio is varied for different values of $(X-R_0)/H$ ratio (corresponding to values of $X=8.0$ mm, 9.0 mm and 11.0 mm) as shown in Fig.5(b).



(a)



(b)

Fig.5. Resonant frequency of a rectangular cavity against cavity aspect ratio (*Y/X*) for parametric variation of the (a) normalized parameter (G/H) keeping $(X-R_0)/H=0.8$ and (b) normalized parameter $(X-R_0)/H$ keeping $(G/H)=0.2$.

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Results for unloaded quality factor and characteristic impedance are compared in Fig.6 and Fig.7, respectively, keeping the value of $(X-R_0)/H$ constant (typically at the value of 1.2) for both the cases, while the cavity aspect ratio is varied for different values of (G/H) ratio. Further, the analysis is benchmarked against measurement for two typical cavity structures (Table-1) and found to be in close agreement.

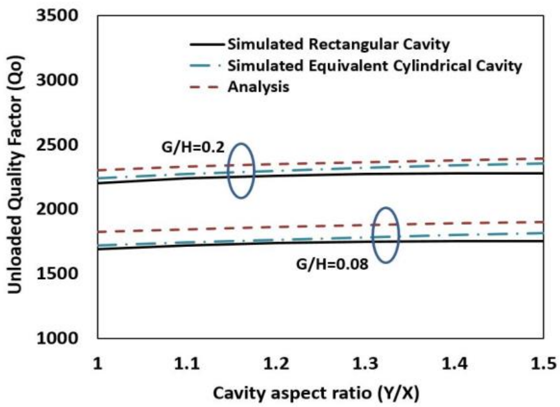

Fig.6. Unloaded quality factor parameter of a rectangular cavity against cavity aspect ratio with G/H as the parameter keeping $(X-R_0)/H=1.2$.

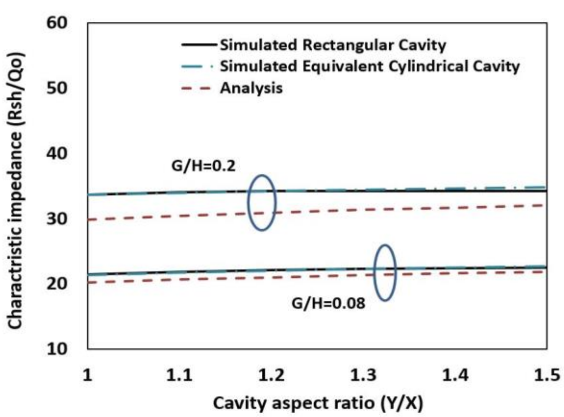

Fig.7. Characteristic impedance parameter of a rectangular cavity against cavity aspect ratio with G/H as the parameter keeping $(X-R_0)/H=1.2$.

Table-1: Benchmarking of the proposed analysis against simulation and measurement

Cavity Dimensions (mm)	Resonant frequency (GHz)		
	Analysis	Simulation	Measurement
S-band Cavity-1: *X*=28.5, *Y*=41.5, *H*=11.0 *G*=1.75, R_0=8.5	3.214	3.227	3.188
X-band Cavity-2: *X*=9.5, *Y*=13.3, *H*=4.0, *G*=0.72, R_0=3	9.975	10.003	9.892

The simulated cavity parameters for the rectangular cavity and the equivalent transformed cylindrical cavity are also found to be in close agreement. The deviations in the resonant frequency (f_0), unloaded quality factor (Q_0) and characteristic impedance (R_{sh}/Q_0) are around 2%, 6% and 3%, respectively. The cavity resonant frequency obtained from the proposed analysis departs from that obtained from the simulation of the rectangular cavity by around 3%. However, the values of the unloaded quality factor and the characteristic impedance obtained from the analysis show a deviation with those from the simulation of the rectangular cavity well within 20%. The deviation in cavity parameters obtained through analysis is attributed to the inherent inaccuracy of the formulation as mentioned by Fujisawa [5]. Notwithstanding, this inaccuracy in the analysis is within acceptable limits and the analysis can be used for initial design of a rectangular reentrant cavity for high frequency klystrons. The analysis has no limitation on frequency band of operation of the cavity as long as the mode of operation is preserved to dominant TM_{110} mode. Limitation on the cavity dimensions remain same as proposed by Fujisawa [5] as analysis is based on same approach.

IV. CONCLUSION

A simple approach for analysis of cavity parameters of a rectangular reentrant cavity with cylindrical ferrules has been presented. The analysis is benchmarked against results obtained from the 3D electromagnetic simulation using CST Microwave Studio and also against

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measurement. The proposed analysis provides reasonably good estimation of the parameters of the given rectangular reentrant cavity over practical parametric regimes: cavity aspect ratio $Y/X \in (1.0,1.5)$; and normalized cavity parameters $G/H \in (0.08,0.2)$ and $(X - R_0)/H \in (0.6,1.2)$. These parameters being normalized, the proposed formulation is valid for any frequency or wavelength.

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