Performance of OFDM Systems With Best-m Feedback, Scheduling, and Delays for Uniformly Correlated Subchannels

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***Abstract*—Contemporary cellular standards, such as Long Term Evolution (LTE) and LTE-Advanced, employ orthogonal frequency-division multiplexing (OFDM) and use frequency-domain scheduling and rate adaptation. In conjunction with feedback reduction schemes, high downlink spectral efficiencies are achieved while limiting the uplink feedback overhead. One such important scheme that has been adopted by these standards is best-m feedback, in which every user feeds back its m largest subchannel (SC) power gains and their corresponding indices. We analyze the single cell average throughput of an OFDM system with uniformly correlated SC gains that employs best-m feedback and discrete rate adaptation. Our model incorporates three schedulers that cover a wide range of the throughput versus fairness tradeoff and feedback delay. We show that, for small m, correlation significantly reduces average throughput with best-m feedback. This result is pertinent as even in typical dispersive channels, correlation is high. We observe that the schedulers exhibit varied sensitivities to correlation and feedback delay. The analysis also leads to insightful expressions for the average throughput in the asymptotic regime of a large number of users.**

***Index Terms*—OFDM, correlation, feedback, best-m, scheduling, adaptation, delay, order statistics.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is the preferred downlink access scheme in next generation wireless systems such as Long Term Evolution (LTE) and LTE-Advanced (LTE-A). It divides the available system bandwidth into several narrow-band orthogonal subcarriers. In LTE and LTE-A, contiguous subcarriers are grouped into subchannels (SCs). OFDM achieves high spectral efficiency through frequency-domain scheduling, in which an SC is opportunistically allocated to a user based on the SC gains of all the users, and rate adaptation, in which data is transmitted using a suitable modulation and coding scheme (MCS) [1]. For example, in LTE and LTE-A, the equivalent of an SC is a physical resource block (PRB), which consists of 12 contiguous subcarriers and has a bandwidth of 180 kHz. The channel is typically assumed to be frequency flat within an SC [2].

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In order to schedule users and adapt rates, the base station (BS) needs to be aware of all the SC gains of all users. However, in practice, the BS does not have a priori access to this channel state information (CSI). Requiring every user to feed back the gains of all its SCs to the BS significantly decreases the effective bandwidth available for data transmission on the uplink. Several feedback reduction schemes have been proposed to circumvent this problem, e.g., thresholding in [3], [4], and the references therein, and best-m feedback [5]–[10]. Specifically, in best-m feedback, which is the focus of this paper, the users report to the BS those SC power gains that are among the m largest along with their corresponding indices. Given its good performance, variants of best-m feedback are used in standards such as LTE [11].

A. Literature on Performance Analysis of Best-m Feedback

Given the vast literature on OFDM and feedback schemes, we focus below on papers related to best-m feedback, which has attracted considerable interest in the literature. In [5], [6], the system throughput with best-m feedback and the greedy scheduler, in which the user that reports the largest SC power gain among all users that report that SC is scheduled, is analyzed. In [7], the average throughput with best-m feedback for the round-robin (RR) scheduler is studied. The throughputs with best-m feedback, thresholding scheme, and a hybrid scheme are compared in [8]. The average throughput with best-m feedback, greedy scheduler, and feedback delay is analyzed in [9]. A joint optimization of m and the number of signal-to-noise ratio (SNR) quantization bits is done in [10] for best-m feedback with the greedy scheduler. While [5], [8]–[10] assume continuous rate adaptation, [6], [7] analyze for discrete rate adaptation [12]. The Monte Carlo simulations in [13] compare best-m feedback with other feedback schemes for the typical urban (TU) channel. Simulations are also used to study the performance with best-m feedback of two proportional fair (PF) schedulers in [14], and for the greedy scheduler in [15].

The above papers that analyze best-m feedback in OFDM assume that the SC gains are identical and independently distributed (i.i.d.). Although this assumption is not made in [13], [15], only simulation results are presented in them. In practice, however, the SC gains are highly correlated. Consider, for example, the TU and rural area (RA) reference channel models [16], [17], which are widely used in the performance evaluation of cellular systems. The corresponding correlation coefficients between two subcarriers that are 180 kHz apart are 0.86 and 0.95. A similar observation also holds for International Telecommunication Union (ITU) defined channel models such as Pedestrian A (PedA) and Pedestrian B (PedB), and extended ITU channel models [2, Chap. 21].

B. Focus and Contributions

In this paper, we analyze the downlink average throughput of a single cell OFDM system with best-m feedback when the SC gains are uniformly correlated. Discrete rate adaptation, which is inevitably used in practice [2], is integrated into our model. This is done both without and with feedback delay. We first analyze the case where the SC gains of different users are i.i.d. In the asymptotic regime of a large number of users per cell and a small correlation coefficient, our analysis leads to an elegant and insightful expression that brings out how correlation affects the average throughput. The study is then extended to the general non-i.i.d. case, in which the SC gains of different users are not statistically identical. This occurs when the users are located at different distances from the BS.

Another important aspect of our analysis is that it incorporates the greedy, modified PF (MPF), and RR schedulers, which together span a wide range of the throughput versus (vs.) fairness trade-off and significantly influence the cell average throughput with best-m feedback. Lastly, we generalize the model and analysis to account for feedback delay, due to which the SC gains at the time of feedback and at the time of data transmission are not the same. We show in all these cases that when a user selectively reports only a subset of the SC power gains, correlation degrades the average throughput. Intuitively, this occurs due to less frequency diversity. However, this does not happen when every user reports all its SC power gains to the BS.[1]

In order to make the order statistics based analysis of correlated random variables (RVs) tractable and to gain valuable insights, we focus on the uniform correlation model, in which the SC gains are correlated with each other by the same correlation coefficient. The analysis with the general correlation model involves handling the joint cumulative distribution function (CDF) of the RVs, the expression for which requires us to deal with as many integrals as the number of RVs being ordered [18]–[20].[2] We note that a similar problem arises in multi-antenna systems with correlated antennas that use generalized selection combining, in which the signals from the L antennas with the largest SNRs are combined [23], [24]. However, in these works, the main goal is to characterize the statistics of the sum of the L largest RVs and not the statistics of the pth largest RV, for each value of p between 1 and L. While the latter has been derived in [19], [25], the resultant expressions are not amenable to further analysis. We also note that important issues such as the effect of the scheduler and the impact of feedback delay are not considered in these works.

Our analysis is novel and relevant for the following reasons. To the best of our knowledge, such a unified performance analysis is not available in the literature – both without and with feedback delay.[3] The characterization of the interaction between the scheduler and best-m feedback for correlated SC gains is also a contribution of this paper. This combination of correlation, scheduling, and best-m feedback requires analytical techniques that are more involved than with complete feedback or when the SC gains of a user are assumed to be i.i.d. Such an analysis is valuable even for implementing Monte Carlo simulations because it provides an independent verification of the results. It also helps mathematically discern the effect of various system parameters. This can be seen, for example, from our asymptotic expressions, which cannot be obtained from simulations.

C. Organization and Notation

The system model and assumptions are discussed in Section II. The average throughput analysis is developed in Section III and Section IV for the cases without and with feedback delay, respectively. Simulation results are presented in Section V, and are followed by our conclusions in Section VI.

We shall use the following notation. The probability of an event is denoted by $P[\cdot]$. The conditional probability of an event A given B is denoted by $P[A|B]$. The CDF of an RV is denoted by $F(\cdot)$, the probability density function (PDF) by $f(\cdot)$, the conditional PDF of RV X given $Y=y$ by $f_{X|Y}(x|y)$, and the expectation by $E[\cdot]$. The notation $p[X=x,A]$ involving RV X and event A is defined as $p[X=x,A]=\lim_{\delta\to 0} P[x\leq X\leq x+\delta, A]/\delta$. The multinomial coefficient $\binom{s}{l_1,\ldots,l_p}$ is equal to $\frac{s!}{l_1!\ldots l_p!}$. The symbol $|\cdot|$ represents modulus. The complex conjugate is denoted by $(\cdot)^*$. The zeroth order modified Bessel function of the first kind is denoted by $I_0(\cdot)$, the Marcum-Q function by $Q_M(\cdot,\cdot)$, the zeroth-order Bessel function of the first kind by $J_0(\cdot)$, and the lower incomplete Gamma function by $L(x,k)=\frac{1}{(k-1)!}\int_0^x e^{-t}t^{k-1}\,dt$ [27].

II. System Model

We first discuss the system model without feedback delay, which is extended in Section IV to incorporate feedback delay.

We consider the downlink of a single-cell OFDM system with N flat-fading SCs. There are K users in the cell, each equipped with a single receive antenna. The gain of SC n from the BS to user k is denoted by $H_{k,n}$. It is modeled as a circularly symmetric complex Gaussian RV [1]. The corresponding SC power gains, denoted by $\gamma_{k,n}=|H_{k,n}|^2$, are exponential RVs with mean $\bar{\gamma}_k$. The SC power gains of different users are independent of each other. While the gains of different SCs of a user are statistically identical [1, Chap. 3], they are not mutually independent. We assume that the SC gains are uniformly correlated with correlation coefficient ρ [19], [20],

[1]Performance measures such as outage probability with best-m feedback, which has been defined as the probability that no user reports an SC gain [8], also do not capture the effect of correlation.

[2]Further simplifications are possible if the number of RVs being ordered is four or less [21], [22].

[3]In [13], [26], no special structure for the correlation across subcarriers is assumed since the channel is directly modeled in the time domain. However, in [26], the correlation does not matter other than affecting a Lagrangian constant (cf. [26, Sec. III]) because full CSI is assumed to be fed back. In [13], the effect of correlation on best-m feedback is not investigated.

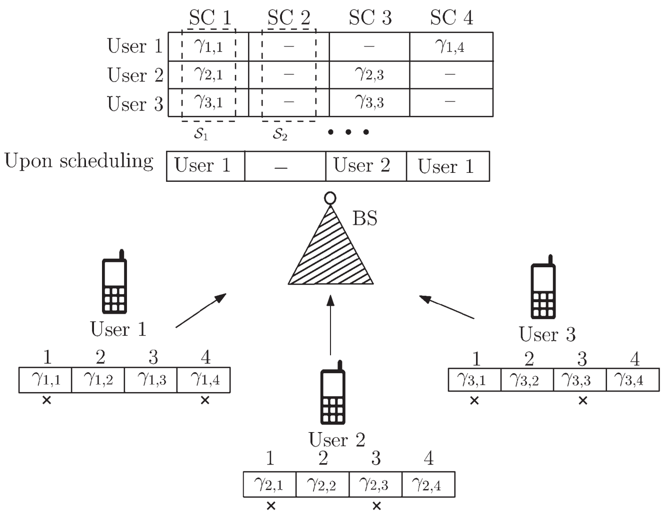

Fig. 1. Illustration of best-m feedback by 3 users and scheduling at the BS ($N = 4$ and $m = 2$). Cross marks ($\times$) denote the SCs that are fed back. The marker '$-$' indicates user did not report the SC.

[24], [28], i.e., for $k = 1, 2, \ldots, K$ and $n_1, n_2 = 1, 2, \ldots, N$, $E[H_{k,n_1} H^*_{k,n_2}]/\bar{\gamma}_k = \rho, n_1 \neq n_2$.

A. Feedback, Scheduling, and Discrete Rate Adaptation

Let the ordered SC power gains of a user k be denoted as $\gamma_{k,1:N} \leq \gamma_{k,2:N} \leq \cdots \leq \gamma_{k,N:N}$, where $q : N$, for $q = 1, 2, \ldots, N$, denotes the index of the SC with the qth largest power gain [29]. Thus, $\gamma_{k,N:N}$ denotes the largest SC power gain among the N SCs. The user then feeds back the m largest SC power gains $\gamma_{k,N-m+1:N}, \ldots, \gamma_{k,N:N}$ along with their indices to the BS.

The BS selects the user for each SC according to the scheduler used and the SC power gains reported by all the users. Let the set of users that report SC n be $\mathcal{S}_n$. For example, in Fig. 1, $\mathcal{S}_1$ consists of three users, while $\mathcal{S}_2$ is empty. We study the following three schedulers, which specify the user assigned to SC n, for $1 \leq n \leq N$:

1) *Greedy scheduler*: Among the users that report SC n, the one that reports the largest SC power gain is chosen to transmit on it [3], [5], [6],

$$i_n^\star = \arg\max_{i \in \mathcal{S}_n} \gamma_{i,n}. \tag{1}$$

2) *MPF scheduler*: Among the users that report SC n, the one that reports the largest SC power gain normalized with respect to its mean, is chosen to transmit on it,

$$i_n^\star = \arg\max_{i \in \mathcal{S}_n} \frac{\gamma_{i,n}}{\bar{\gamma}_i}. \tag{2}$$

The MPF scheduler is a variant of the PF scheduler proposed in [30], which is based on time-window averaging. The former is widely used in the literature as it is analytically tractable and provides similar trade-offs [31], [32]. It is fair because it can be shown that the probability of being scheduled on SC n is the same for all users regardless of their mean channel power gains.

3) *RR scheduler*: Users are assigned to SC n in a predetermined order [7], which depends on neither the set $\mathcal{S}_n$ nor the power gains reported for it. Note that feedback is still required for discrete rate adaptation.

If SC n is not reported by any user, then $i_n^\star = 0$ and $\gamma_{i_n^\star,n} = 0$; The BS transmits no data on that SC since it has no CSI to determine the rate [12].

The BS assigns one among M rates $0 = R_1 < R_2 < \cdots < R_M$ to the user scheduled on an SC as follows. The range of SC power gains is divided into M disjoint intervals by $M + 1$ thresholds $0 = \Gamma_1 < \Gamma_2 < \cdots < \Gamma_{M+1} = \infty$. The thresholds are determined to ensure a target packet error rate [12]. If the reported SC power gain lies in the interval $[\Gamma_r, \Gamma_{r+1})$, an MCS corresponding to the rate R_r is assigned to the selected user $i_n^\star$. Note that $R_1 = 0$ implies that the channel is too weak to support a reliable transmission with any of the rates $R_2, \ldots, R_M$. These rates are pre-specified; see, for example, [2, Tab. 10.1] in LTE.

B. Assumptions, Prevalence, and Limitations

In order to develop a tractable model that characterizes the effect of SC gain correlation on the average throughput, in combination with the feedback scheme, feedback delay, scheduler, and discrete rate adaptation, we assume the following:

1) We assume uniformly correlated SC gains [19], [20], [24], [28]. We note that such special correlation models have been often used to make the problem tractable; see, for example, the literature on antenna selection (AS) with spatially correlated antennas [24], [28], [33]. The exponential correlation model is another special model that has been studied in the literature [19], [20], [34]. However, in our problem, the various subsets of the RVs being ordered do not retain this structure. While a general correlation structure is analyzed in [23] and the references therein, the number of RVs being ordered is limited to at most three. In general, the correlation structure is a function of the power delay profile (PDP) of the channel. It can be different from the uniform and exponential correlation models. Nevertheless, we shall see in Section V that the trends for all the schedulers with reference channel models, mirror those obtained from our analysis.
2) The users know the SC power gains without error [3]–[8], and feed them back as per best-m feedback. The analysis can be extended to the scenario in which the index of the rate that the SC can support, is fed back [6], [11]. However, this yields limited additional insights.
3) We focus on a single-cell system [4]–[7], [9].

III. Average Throughput Without Feedback Delay

As the SC gains of each user are statistically identical, it is sufficient to focus on a single SC n. For notational simplicity, the subscript n is dropped in $i_n^\star$, which is hereafter referred to by $i^\star$. The average downlink throughput $\bar{R}$ for SC n is

$$\begin{aligned}\bar{R} &= \sum_{r=1}^{M} R_r P[\Gamma_r \leq \gamma_{i^\star,n} < \Gamma_{r+1}], \\ &= \sum_{r=1}^{M} R_r \left(P[\gamma_{i^\star,n} < \Gamma_{r+1}] - P[\gamma_{i^\star,n} < \Gamma_r]\right).\end{aligned} \tag{3}$$

To gain intuition, we first derive expressions for the CDF $P[\gamma_{i^\star,n} < x]$ for the scenario in which the SC gains of different users are i.i.d. We shall henceforth refer to this as the *i.i.d. case*. Thereafter, the more general case in which the SC gains of different users are not i.i.d. is analyzed. This shall be referred to as the *non-i.i.d. case*.

A. I.I.D. Case

As the SC gains of different users are statistically identical, $\bar{\gamma}_k = \bar{\gamma}$, for $k = 1, 2, \ldots, K$. Hence, the greedy and MPF schedulers are equivalent for $\bar{\gamma} > 0$. Let $F_{k,q:N}(x)$ denote the CDF of $\gamma_{k,q:N}$, for $k = 1, 2, \ldots, K$.

Result 1: The CDF of the SC power gain of the selected user $i^\star$ for the greedy scheduler is given by

$$P[\gamma_{i^\star,n} < x] = \left(1 - \frac{m}{N}\right)^K + \sum_{a=1}^{K} \binom{K}{a} \frac{1}{N^a} \left(1 - \frac{m}{N}\right)^{K-a} \times \left(\sum_{j=N-m+1}^{N} (-1)^{j+N-m-1} \times \binom{j-2}{N-m-1} \binom{N}{j} F_{1,j:j}(x) \right)^a, \quad (4)$$

where $F_{1,j:j}(x)$ is the CDF of the maximum of j uniformly correlated exponential RVs and is given by [20]

$$F_{1,j:j}(x) = \frac{1-\rho}{1+(j-1)\rho} \sum_{s=0}^{\infty} \left(\frac{\rho}{1+(j-1)\rho}\right)^s \times \sum_{\substack{l_1,\ldots,l_j \geq 0 \\ l_1+\ldots+l_j = s}} \binom{s}{l_1, \ldots, l_j} \times \prod_{i=1}^{j} L\left(\frac{x}{\bar{\gamma}(1-\rho)}, l_i + 1\right). \quad (5)$$

Proof: The proof is given in Appendix A. ■

Substituting (4) in (3) yields the final average throughput expression. Recall that $L(x, k)$ is the lower incomplete Gamma function and is defined in Section I-C. For $m = 1$, Result 1 simplifies considerably. For example, for $N = 2$ and $m = 1$, we get $P[\gamma_{i^\star,n} < x] = (1 + F_{1,2:2}(x))^K / 2^K$, where

$$F_{1,2:2}(x) = \frac{1-\rho}{1+\rho} \sum_{s=0}^{\infty} \left(\frac{\rho}{1+\rho}\right)^s \sum_{l_1=0}^{s} \binom{s}{l_1} \times L\left(\frac{x}{\bar{\gamma}(1-\rho)}, l_1+1\right) L\left(\frac{x}{\bar{\gamma}(1-\rho)}, s-l_1+1\right). \quad (6)$$

For the RR scheduler, the average throughput is obtained by replacing K with one because the SC gains of the users are statistically identical [35]. In this case, (4) simplifies to

$$P[\gamma_{i^\star,n} < x] = 1 - \frac{m}{N} + \sum_{j=N-m-1}^{N} (-1)^{j+N-m+1} \times \binom{j-2}{N-m-1} \binom{N}{j} \frac{F_{1,j:j}(x)}{N}. \quad (7)$$

1) Asymptotic Insights: While the above analysis is exact, the final expressions are quite involved. To gain insights, we now analyze the asymptotic regime in which $K \to \infty$.

Result 2: For a large enough K, the difference Δ between the maximum rate R_M and the average throughput decreases exponentially in K, and is given by

$$\Delta = R_M - \bar{R} = (R_M - R_{M-1}) \left(1 - \frac{m}{N} + \frac{m\varphi}{N}\right)^K, \quad (8)$$

where $\varphi = P[\gamma_{1,n} \leq \Gamma_M |$ User 1 reports SC $n]$. For $\rho \ll 1$ and $(\Gamma_M/\bar{\gamma}) > 1$, φ is given by

$$\varphi = \varphi_{\text{iid}} \left[1 + \frac{\rho\Gamma_M}{\bar{\gamma}\left(e^{\Gamma_M/\bar{\gamma}} - 1\right)}\right]^{N-m} \left[1 - \frac{(N-m)\rho\Gamma_M}{\bar{\gamma}\left(e^{\Gamma_M/\bar{\gamma}} - 1\right)}\right] + \mathcal{O}(\rho^2), \quad (9)$$

where $\varphi_{\text{iid}} = \frac{1}{m} \sum_{j=N-m+1}^{N} (-1)^{(j+N-m-1)} \binom{j-2}{N-m-1} \binom{N}{j} (1 - e^{-\Gamma_M/\bar{\gamma}})^j$ and the notation $\mathcal{O}(\cdot)$ is defined in [36].

Proof: The proof is given in Appendix B. ■

Equation (8) shows that the difference between $\bar{R}$ and maximum rate R_M decreases exponentially with the number of users. The first term φ_{iid} in (9) corresponds to φ for i.i.d. SC gains. The remaining two terms capture the effect of correlation on φ. The second term $\left(1 + \frac{\rho\Gamma_M}{\bar{\gamma}(e^{\Gamma_M/\bar{\gamma}}-1)}\right)^{N-m}$ is greater than one. It increases exponentially with N and makes the product of the second and third terms exceed unity for larger N. Hence, φ is greater than or equal to φ_{iid}. Consequently, the difference Δ between the maximum rate and the average throughput decreases at a slower rate in the presence of correlation. Note that the accuracy of (9) also depends on the system parameters. For example, for $N = 10$, $m = 3$, $\Gamma_M = 20.6$ dB, and $\bar{\gamma} = 16$ dB, ignoring the $\mathcal{O}(\rho^2)$ term in (9) gives an error of 1.2% and 13.3% for $\rho = 0.1$ and 0.35, respectively. For $\bar{\gamma} = 12$ dB, the corresponding errors are just 0.02% and 0.04%.

2) Comparison With Complete Feedback: With complete feedback, i.e., $m = N$, we can show that (4) reduces to

$$P[\gamma_{i^\star,n} < x] = \left[1 - e^{-x/\bar{\gamma}}\right]^K. \quad (10)$$

Substituting (10) in (3), we see that correlation does not affect $\bar{R}$ with complete feedback.

B. Non-I.I.D. Case

We now have to track the specific subset of users that report the SC and not just the number of users that do so. For this, let A_l^a denote the lth subset of K users with a elements. There are $\binom{K}{a}$ such subsets. Now, $P[\gamma_{i^\star,n} < x]$ is given as follows.

Result 3: The CDF of the SC power gain of the selected user $i^\star$ for the greedy scheduler is

$$P[\gamma_{i^\star,n} < x] = \left(1 - \frac{m}{N}\right)^K + \sum_{a=1}^{K} \left(\frac{1}{N}\right)^a \left(1 - \frac{m}{N}\right)^{K-a} \times \sum_{l=1}^{\binom{K}{a}} \left(\prod_{u \in A_l^a} \sum_{j=N-m+1}^{N} (-1)^{j+N-m-1} \times \binom{j-2}{N-m-1} \binom{N}{j} F_{u,j:j}(x) \right). \quad (11)$$

Proof: The proof is given in Appendix C. ■

For the RR scheduler, (11) reduces to

$$P[\gamma_{i^\star,n} < x] = 1 - \frac{m}{N} + \frac{1}{NK} \sum_{j=N-m+1}^{N} (-1)^{j+N-m-1} \times \binom{j-2}{N-m-1} \binom{N}{j} \sum_{l=1}^{K} F_{l,j:j}(x). \quad (12)$$

Result 4: The CDF of the SC power gain of the selected user $i^\star$ for the MPF scheduler is

$$P[\gamma_{i^\star,n} < x] = \left(1 - \frac{m}{N}\right)^K + \sum_{a=1}^{K} \frac{1}{a} \left(\frac{m}{N}\right)^a \left(1 - \frac{m}{N}\right)^{K-a} \times \sum_{l=1}^{\binom{K}{a}} \sum_{u \in A_l^a} \left(\sum_{j=N-m+1}^{N} (-1)^{j+N-m-1} \times \binom{j-2}{N-m-1} \binom{N}{j} \tilde{F}_{1,j:j}\left(\frac{x}{\bar{\gamma}_u}\right) \right)^a, \quad (13)$$

where $\tilde{F}_{1,j:j}$ is given by (5) but $\bar{\gamma}$ in it is replaced with unity.

Proof: The proof is given in Appendix D. ■

Substituting (13), (12), and (11) in (3) yields the corresponding expressions for $\bar{R}$.

IV. Average Throughput With Feedback Delay

We now analyze the impact of feedback delay on best-m feedback. For this, we first extend the model in Section II. For user k, let $H_{k,n}^d$ and $H_{k,n}$ denote the gains of SC n, at the time of feedback from the user and at the time of data transmission from the BS, respectively. The corresponding SC power gains are denoted by $\gamma_{k,n}^d$ and $\gamma_{k,n}$.

The baseband SC gains are assumed to follow a wide sense stationary Gaussian random process [12]. Therefore, $H_{k,n}$ is also a circular symmetric complex normal RV with zero mean and variance $\bar{\gamma}_k$. From the Jakes' model [37], the correlation coefficient ρ_d between $H_{k,n}^d$ and $H_{k,n}$, which are jointly Gaussian, is given by $\rho_d = J_0(2\pi\phi_d\tau)$, where ϕ_d is the Doppler spread in Hertz and τ is the delay in seconds. The RVs $H_{k,n}^d$, for $n = 1, 2, \ldots, N$, are still uniformly correlated with correlation coefficient ρ, as described in Section II.

The scheduling and rate assignment performed by the BS are as described in Section II, except that they are now based on $\gamma_{k,n}^d$. This leads to a sub-optimal user selection and rate assignment. For SC n, a rate R_r is chosen when the SC power gain $\gamma_{i^\star,n}^d$ (at the time of feedback) lies in the interval $[\Gamma_r, \Gamma_{r+1})$. The selected user $i^\star$ can successfully receive at this rate only if $\gamma_{i^\star,n}$ (at the time of data transmission) also exceeds Γ_r. Therefore, $\bar{R}$ for SC n is now

$$\bar{R} = \sum_{r=1}^{M} R_r P\left[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r\right]. \quad (14)$$

Due to space constraints, we directly proceed to the general non-i.i.d. case.

Result 5: The probability $P[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r]$ of the selected user $i^\star$ for the greedy scheduler with feedback delay for the non-i.i.d. case is given by

$$P\left[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r\right] = \sum_{a=1}^{K} \frac{1}{N^a} \left(1 - \frac{m}{N}\right)^{K-a} \sum_{l=1}^{\binom{K}{a}} \sum_{u_i \in A_l^a} \int_{\Gamma_r}^{\Gamma_{r+1}} \left[\prod_{\substack{u_p \in A_l^a \\ u_p \neq u_i}} \times \sum_{j_1=N-m+1}^{N} (-1)^{j_1+N-m-1} \binom{j_1-2}{N-m-1} \binom{N}{j_1} F_{u_p,j_1:j_1}(x) \right] \times \sum_{j_2=N-m+1}^{N} (-1)^{j_2+N-m-1} \binom{j_2-2}{N-m-1} \binom{N}{j_2} f_{u_i,j_2:j_2}(x) \times Q_1\left(\sqrt{\frac{2\rho_d^2 x}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}, \sqrt{\frac{2\Gamma_r}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}\right) \mathrm{d}x, \quad (15)$$

where $F_{u_p,j_1:j_1}(x)$ is given by (5) with $\bar{\gamma}$ in it replaced by $\bar{\gamma}_{u_p}$, and $f_{u_i,j_2:j_2}(x)$ is the PDF of $\gamma_{u_i,j_2:j_2}$, which is equal to

$$f_{u_i,j_2:j_2}(x) = \frac{1-\rho}{1+(j_2-1)\rho} \sum_{s=0}^{\infty} \left(\frac{\rho}{1+(j_2-1)\rho}\right)^s \times \sum_{\substack{l_1,\ldots,l_{j_2} \ge 0 \\ l_1+\ldots+l_{j_2}=s}} \binom{s}{l_1,\ldots,l_{j_2}} \sum_{p=1}^{j_2} \frac{e^{-\frac{x}{(\bar{\gamma}_{u_i}(1-\rho))}} x^{l_p}}{(\bar{\gamma}_{u_i}(1-\rho))^{l_p+1} l_p!} \times \left[\prod_{\substack{k=1 \\ k \neq p}}^{j_2} L\left(\frac{x}{\bar{\gamma}_{u_i}(1-\rho)}, l_k+1\right)\right], \quad x \ge 0. \quad (16)$$

Proof: The proof is given in Appendix E. ■

The integral in the above result is evaluated numerically given the involved form of its integrand, which is due to correlations across both frequency and time. The feedback delay manifests itself in the Marcum-Q function, which contains ρ_d. As before, the corresponding result for the RR scheduler can be derived from (15); it is not shown due to space constraints.

Result 6: The probability $P[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r]$ of the selected user $i^\star$ for the MPF scheduler is given by

$$P\left[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r\right] = \sum_{a=1}^{K} \frac{1}{N^a} \left(1 - \frac{m}{N}\right)^{K-a} \sum_{l=1}^{\binom{K}{a}} \sum_{u_i \in A_l^a} \int_{\frac{\Gamma_r}{\bar{\gamma}_{u_i}}}^{\frac{\Gamma_{r+1}}{\bar{\gamma}_{u_i}}} \left(\sum_{j_1=N-m+1}^{N} \times (-1)^{j_1+N-m-1} \binom{j_1-2}{N-m-1} \binom{N}{j_1} \tilde{F}_{1,j_1:j_1}(x) \right)^{a-1} \times \sum_{j_2=N-m+1}^{N} (-1)^{j_2+N-m-1} \binom{j_2-2}{N-m-1} \binom{N}{j_2} \tilde{f}_{1,j_2:j_2}(x) \times Q_1\left(\sqrt{\frac{2\rho_d^2 x}{(1-\rho_d^2)}}, \sqrt{\frac{2\Gamma_r}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}\right) \mathrm{d}x, \quad (17)$$

where $\tilde{f}_{1,j_2:j_2}(x)$ is given by (16) with $\bar{\gamma}_{u_i}$ replaced by unity.

Proof: The proof is given in Appendix F. ■

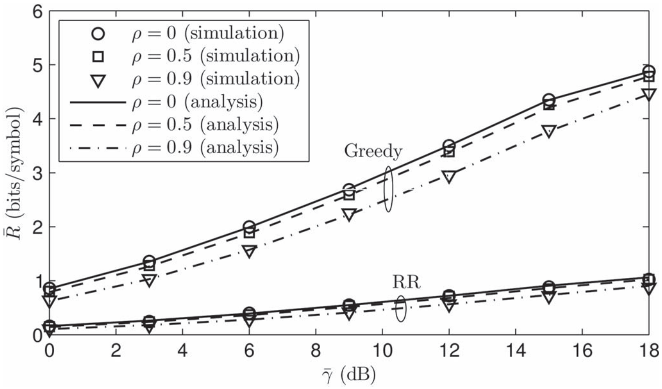

Fig. 2. I.I.D. case: Average throughput as a function of mean SC power gain $\bar{\gamma}$ for different ρ for the greedy and RR schedulers ($K = 10$, $N = 10$, and $m = 2$).

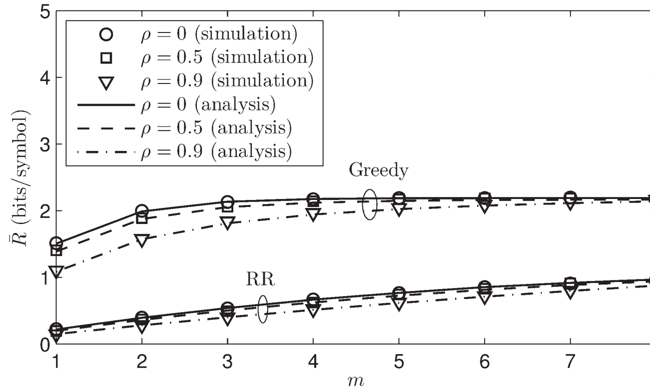

Fig. 3. I.I.D. case: Average throughput as a function of m for different ρ for the greedy and RR schedulers ($K = 10$, $N = 10$, and $\bar{\gamma} = 6$ dB).

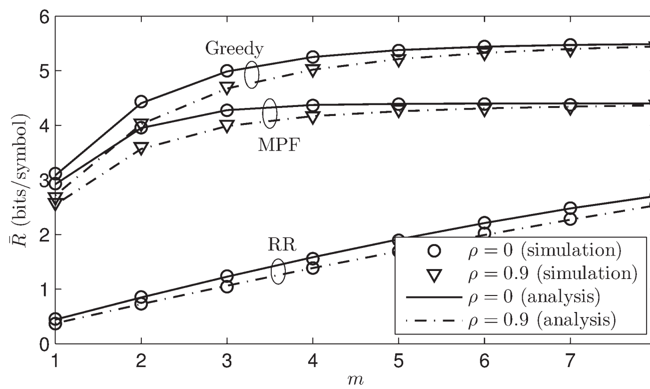

Fig. 4. Non-I.I.D. case: Average throughput as a function of m for the greedy, MPF, and RR schedulers ($K = 10$, $N = 10$, $\alpha = 1.6$, and $\bar{\gamma} = 6$ dB).

V. Simulation Results

We now present Monte Carlo simulation results averaged over 10^4 samples to verify the analysis and quantitatively understand the role of various system parameters. The $M = 16$ rates are as specified in LTE [2, Tab. 10.1]. These range from $R_2 = 0.15$ bits/symbol to $R_{16} = 5.55$ bits/symbol. The thresholds for discrete rate adaptation are calculated using the formula [38]: $R_r = \log_2(1 + \zeta\Gamma_r)$, where $\zeta = 0.398$ accounts for the coding loss of a practical code [38]. The number of users is $K = 10$ and number of SCs is $N = 10$. In all figures, the simulation results are shown by markers and the analytical results by lines. The CDF $F_{i,j:j}(x)$ and the PDF $f_{i,j:j}(x)$ are evaluated by truncating the infinite series in (5) and (16). The number of terms summed over increases with ρ to ensure numerical accuracy because of the presence of the $1 - \rho$ term in the denominator of the first argument of $L\left(\frac{x}{\bar{\gamma}_i(1-\rho)}, l_i + 1\right)$ in (5) and (16). The number of terms is determined by checking when the variation in the truncated series summation falls below a pre-specified threshold. For our results, we have found 11 terms for $\rho = 0.5$ and 60 terms for $\rho = 0.9$ to be sufficient to ensure numerical accuracy.

A. Without Feedback Delay

1) I.I.D. Case: For the greedy and RR schedulers with best-m feedback, Fig. 2 plots the average throughput as a function of the mean SC power gain $\bar{\gamma}$ with $m = 2$. Recall that the MPF scheduler reduces to the greedy scheduler for the i.i.d. case. The average throughput increases with $\bar{\gamma}$ for all values of ρ. We see that there is a perceptible decrease in the average throughput for large ρ. For example, for $\bar{\gamma} = 6$ dB, when ρ is increased from 0 to 0.9, $\bar{R}$ decreases by 23% and 29% for the greedy and RR schedulers, respectively. The analysis and the simulation results match well with each other. A marginal mismatch between them occurs at higher values of ρ due to the aforementioned truncation of the infinite series in (5).

Fig. 3 plots the average throughput as a function of m, for $\bar{\gamma} = 6$ dB for the greedy and RR schedulers. The average throughput increases as m increases since more CSI is fed back. More importantly, we see that it is more sensitive to correlation for smaller values of m. This is true for both schedulers. However, the difference between the average throughput with uncorrelated and correlated SC gains decreases faster for the greedy scheduler as m increases as compared to the RR scheduler. Also, for $m = N$, in which all SC power gains are fed back, the correlation does not affect $\bar{R}$ (cf. Section III-A2).

2) Non-I.I.D. Case: In order to model the non-i.i.d. SC gains of different users, we set $\bar{\gamma}_k = \bar{\gamma}\alpha^{k-1}$, where $\alpha > 1$, for $k = 1, 2, \ldots, K$. The larger the α, the more asymmetric are the users. For $\alpha = 1.6$, $K = 10$, and $\bar{\gamma} = 6$ dB, the average throughput as a function of m is shown in Fig. 4 for the three schedulers. Notice that the analysis and simulation results match each other well. For $m = 1$, the average throughput again decreases as ρ increases for all schedulers. For $m = 4$, the greedy scheduler assigns User 1 to an SC 0.7% of the time and User 10 for 30% of the time, whereas the MPF scheduler assigns each user to a specific SC 10% of the time.

3) Monte Carlo Simulations for TU and RA Channels: Fig. 5 compares the cell average throughput per SC when the SC gains of each user are i.i.d. with that of the RA channel model for a system bandwidth of 5 MHz, a PRB bandwidth of 180 KHz, and $\alpha = 1.6$. The result for the TU channel lies in between, and is not shown here to avoid clutter. For $m = 1$, the average throughput decreases by 17%, 19%, and 9% for the greedy, MPF, and RR schedulers, respectively. For $m = 3$, the corresponding reduction is 6%, 9%, and 9%. Thus, just as in the uniformly correlated SC gains model, the reduction is significant for smaller m, and the RR scheduler is the least affected by correlation or m.

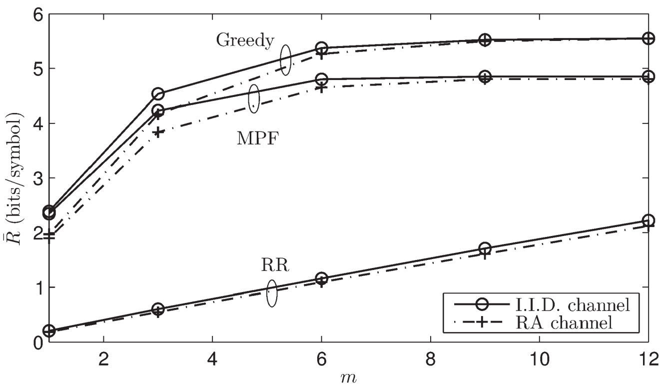

Fig. 5. Non-I.I.D. case and RA channel model: Average throughput as a function of m for different ρ for the greedy, MPF, and RR schedulers ($K = 15$, $N = 24$, $\alpha = 1.6$, and $\bar{\gamma} = 6$ dB).

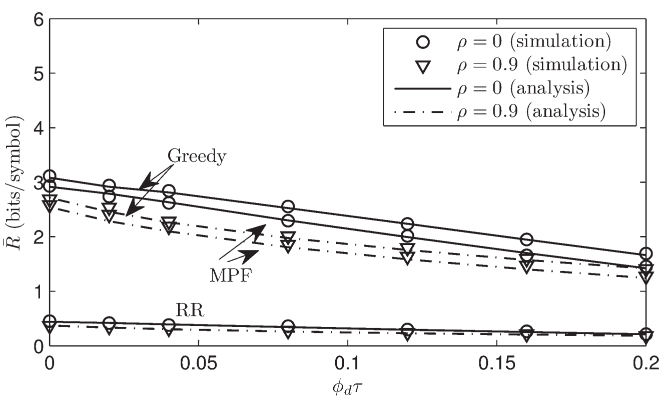

Fig. 6. Non-I.I.D. case with feedback delay: Average throughput as a function of $\phi_d\tau$ for different ρ for the greedy, MPF, and RR schedulers ($K = 10$, $N = 10$, $\alpha = 1.6$, $\bar{\gamma} = 6$ dB, and $m = 1$).

B. With Feedback Delay

Fig. 6 plots $\bar{R}$ as a function of $\phi_d\tau$ for different ρ for the three schedulers for non-i.i.d. SC gains of different users. We observe that $\bar{R}$ decreases significantly as $\phi_d\tau$ increases. Further, even with feedback delay, $\bar{R}$ is sensitive to ρ. For example, for $\phi_d\tau = 0.08$, it decreases by 21%, 19%, and 23% for the greedy, MPF, and RR schedulers, respectively, when compared to the uncorrelated SC gains case. The analysis and simulation results match each other well.

Fig. 7 plots $\bar{R}$ as a function of m for various ρ for the three schedulers. We observe that $\bar{R}$ increases with m, as was the case without feedback delay. While the average throughput for the greedy and MPF schedulers saturates for larger m, this is not so for the RR scheduler, which does not exploit multiuser diversity. With feedback delay, the greedy scheduler is marginally more sensitive to ρ than the MPF scheduler.

VI. Conclusion

The SC gains are highly correlated even for channel models that are considered dispersive. In order to characterize the effect of correlation in such OFDM systems, we derived closed-form expressions for the cell average throughput of the practically important best-m feedback scheme for uniformly correlated SC gains with frequency-domain scheduling and discrete rate adaptation. Thereafter, we incorporated feedback delay into the model, which in a time-varying channel also leads to suboptimal scheduling and rate adaptation. In all these cases, we saw that SC gain correlation degrades the average throughput achieved by best-m feedback, for $m < N$. We observed that the trends predicted by our analysis of the uniform SC gain correlation model mirrored those observed for reference channel models.

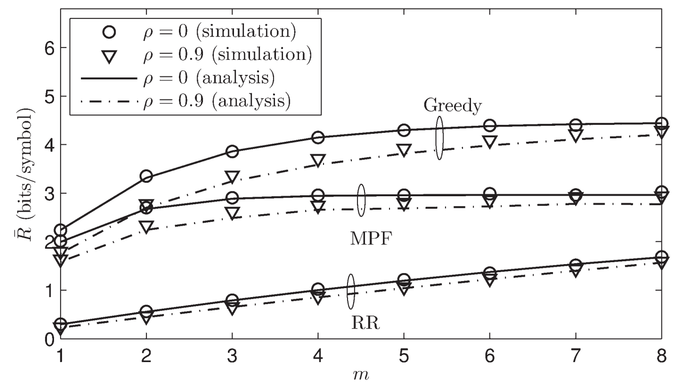

Fig. 7. Non-I.I.D. case with feedback delay: Average throughput as a function of m for different ρ for the greedy, MPF, and RR schedulers ($K = 10$, $N = 10$, $\alpha = 1.6$, $\bar{\gamma} = 6$ dB, and $\phi_d\tau = 0.12$).

Appendix

A. Proof of Result 1

From the law of total probability, the CDF of SC power gain $\gamma_{i^\star,n}$ of the selected user $i^\star$ is

$$P[\gamma_{i^\star,n} < x] = \sum_{a=0}^{K} P[\gamma_{i^\star,n} < x | a \text{ users report SC } n] \times P[a \text{ users report SC } n]. \quad (18)$$

A user reports an SC to the BS when the SC power gain is at least the mth best among its N SCs. The probability of this event is m/N because the SC gains are statistically identical. Since the SC power gains of different users are i.i.d., the probability that exactly a of them report SC n is given by

$$P[a \text{ users report SC } n] = \binom{K}{a} \left(\frac{m}{N}\right)^a \left(1 - \frac{m}{N}\right)^{K-a}. \quad (19)$$

When no user reports the SC, i.e., $a = 0$, we have $\gamma_{i^\star,n} = 0$ by definition. Therefore, in this case, $P[\gamma_{i^\star,n} < x | a \text{ users report SC } n] = 1$, for all $x > 0$. Since the SC gains of different users are i.i.d., the conditional CDF in (18), for $a \geq 1$, equals

$$\begin{aligned} &P[\gamma_{i^\star,n} < x | a \text{ users report SC } n] \\ &= P[\gamma_{1,n} < x, \ldots, \gamma_{a,n} < x | \text{Users } 1, \ldots, a \text{ report SC } n], \\ &= (P[\gamma_{1,n} < x | \text{User 1 reports SC } n])^a. \quad (20) \end{aligned}$$

Using Bayes' rule and the law of total probability, we get

$$\begin{aligned} &P[\gamma_{1,n} < x | \text{User 1 reports SC } n] \\ &= \frac{\sum_{q=N-m+1}^{N} P[\gamma_{1,n} < x, \text{SC } n \text{ is } q^{\text{th}} \text{ best SC}]}{P[\text{User 1 reports SC } n]}, \\ &= \frac{N}{m} \sum_{q=N-m+1}^{N} P[\gamma_{1,q:N} < x, \text{SC } n \text{ is } q^{\text{th}} \text{ best SC}]. \quad (21) \end{aligned}$$

Averaging over all the N SCs, $F_{1,q:N}(x) = P[\gamma_{1,q:N} < x] = \sum_{n=1}^{N} P[\gamma_{1,q:N} < x, \text{SC } n \text{ is } q^{\text{th}} \text{ best SC}]$. Since the N SC gains are statistically identical, any one of them can be the qth best with equal probability. Therefore,

$$F_{1,q:N}(x) = NP[\gamma_{1,q:N} < x, \text{SC } n \text{ is } q^{\text{th}} \text{ best SC}]. \quad (22)$$

Therefore, (21) simplifies to

$$P[\gamma_{1,n} < x|\text{User 1 reports SC } n] = \frac{1}{m} \sum_{q=N-m+1}^{N} F_{1,q:N}(x). \quad (23)$$

From [29], the CDF $F_{1,q:N}(x)$ is given by

$$F_{1,q:N}(x) = \sum_{j=q}^{N} (-1)^{j-q} \binom{j-1}{q-1} \binom{N}{j} F_{1,j:j}(x), \quad (24)$$

where $F_{1,j:j}(x)$ is the CDF of the maximum of j uniformly correlated SC power gains of User 1. Its expression in (5) is obtained by integrating the expression for the joint PDF in [20, (91)] and using the fact that $F_{1,j:j}(x) = P[\gamma_{1,j:j} < x] = P[\gamma_{1,1} < x, \ldots, \gamma_{1,j} < x]$. Substituting (24) in (23) and changing the order of summation, we get

$$P[\gamma_{1,n} < x|\text{User 1 reports SC } n] = \sum_{j=N-m+1}^{N} \frac{F_{1,j:j}(x)}{m} \binom{N}{j} \sum_{q=N-m+1}^{j} (-1)^{j-q} \binom{j-1}{q-1}. \quad (25)$$

It can be shown that the inner summation $\sum_{q=N-m+1}^{j} (-1)^{(j-q)} \binom{j-1}{q-1}$ in (25) is identically equal to $(-1)^{(j+N-m-1)} \binom{j-2}{N-m-1}$. Hence, we get

$$P[\gamma_{1,n} < x|\text{User 1 reports SC } n] = \sum_{j=N-m+1}^{N} (-1)^{j+N-m-1} \binom{j-2}{N-m-1} \binom{N}{j} \frac{F_{1,j:j}(x)}{m}. \quad (26)$$

Finally, substituting (26) in (20) and the resulting expression in (18) yields (4).

B. Proof of Result 2

Expanding (3) and rearranging, we get $R_M - \bar{R} = R_1 P[\gamma_{i^\star,n} < \Gamma_1] + (R_2 - R_1) P[\gamma_{i^\star,n} < \Gamma_2] + \cdots + (R_M - R_{M-1}) P[\gamma_{i^\star,n} < \Gamma_M]$. For large K, the last term in the above equation is the dominant term. This is because $P[\gamma_{i^\star,n} < \Gamma_1] \le P[\gamma_{i^\star,n} < \Gamma_2] \le \cdots \le P[\gamma_{i^\star,n} < \Gamma_M]$. As K increases, $P[\gamma_{i^\star,n} < \Gamma_i] \to 0$, for $1 \le i \le M-1$. Hence, for large K, we have

$$R_M - \bar{R} = (R_M - R_{M-1}) P[\gamma_{i^\star,n} < \Gamma_M]. \quad (27)$$

From (18), (19), and (20), we have $P[\gamma_{i^\star,n} < \Gamma_M] = \sum_{a=0}^{K} \varphi^a \binom{K}{a} (m/N)^a (1 - (m/N))^{K-a}$, where $\varphi = P[\gamma_{1,n} < \Gamma_M|\text{User 1 reports SC } n]$. On simplification, we get

$$P[\gamma_{i^\star,n} < \Gamma_M] = \left(1 - \frac{m}{N} + \frac{m\varphi}{N}\right)^K. \quad (28)$$

Substituting (28) in (27) yields (8). From (26), φ is given by

$$\varphi = \sum_{j=N-m+1}^{N} (-1)^{(j+N-m-1)} \binom{j-2}{N-m-1} \binom{N}{j} \frac{F_{1,j:j}(\Gamma_M)}{m}. \quad (29)$$

$\rho \ll 1$ scenario: The expression for the CDF $F_{1,j:j}(\Gamma_M)$ is given in (5). By splitting it into the $s = 0$ term and other terms and simplifying, we get

$$\begin{aligned} F_{1,j:j}(\Gamma_M) = {} & \frac{1-\rho}{1+(j-1)\rho} \left[L\left(\frac{\Gamma_M}{\bar{\gamma}(1-\rho)}, 1\right)\right]^j \\ & + \frac{(1-\rho)j\rho}{(1+(j-1)\rho)^2} \left[L\left(\frac{\Gamma_M}{\bar{\gamma}(1-\rho)}, 1\right)\right]^{j-1} \\ & \times L\left(\frac{\Gamma_M}{\bar{\gamma}(1-\rho)}, 2\right) + \mathcal{O}(\rho^2). \end{aligned} \quad (30)$$

Expanding using Taylor series around $\rho = 0$, we get $L(\Gamma_M/(\bar{\gamma}(1-\rho)), 1) = 1 - e^{-\Gamma_M/\bar{\gamma}} + (\rho\Gamma_M/\bar{\gamma}) e^{-\Gamma_M/\bar{\gamma}} + \mathcal{O}(\rho^2)$ and $L(\Gamma_M/(\bar{\gamma}(1-\rho)), 2) = 1 - e^{-\Gamma_M/\bar{\gamma}} - \frac{\Gamma_M}{\bar{\gamma}} \left(1 - \frac{\rho\Gamma_M}{\bar{\gamma}}\right) e^{-\Gamma_M/\bar{\gamma}} + \mathcal{O}(\rho^2)$. Substituting these in (30) and simplifying, we obtain

$$\begin{aligned} F_{1,j:j}(\Gamma_M) = {} & \left[1 - e^{-\Gamma_M/\bar{\gamma}}\right]^j \left[1 + \frac{\rho\Gamma_M}{\bar{\gamma}\left(e^{\Gamma_M/\bar{\gamma}} - 1\right)}\right]^{j-1} \\ & \times \left[1 - \frac{(j-1)\rho\Gamma_M}{\bar{\gamma}\left(e^{\Gamma_M/\bar{\gamma}} - 1\right)}\right] + \mathcal{O}(\rho^2). \end{aligned} \quad (31)$$

Substituting (31) in (29), and rearranging terms yields (9).

C. Proof of Result 3

The probability that only the users from subset A_l^a report SC n to the BS is $P[\text{Users from } A_l^a \text{ report SC } n] = (m/N)^a (1 - (m/N))^{K-a}$. Therefore, averaging over the number of users that report SC n and various subsets of users of a given cardinality, we get

$$\begin{aligned} P[\gamma_{i^\star,n} < x] = {} & \left(1 - \frac{m}{N}\right)^K + \sum_{a=1}^{K} \sum_{l=1}^{\binom{K}{a}} \left(\frac{m}{N}\right)^a \left(1 - \frac{m}{N}\right)^{K-a} \\ & \times P\left[\gamma_{i^\star,n} < x|\text{Users from } A_l^a \text{ report SC } n\right], \end{aligned} \quad (32)$$

where, as defined earlier, $i^\star$ denotes the user scheduled on SC n. The last term $(1 - (m/N))^K$ in the above equation is the probability that no user reports SC n.

The conditional CDF in (32) for the non-i.i.d. case equals

$$P\left[\gamma_{i^\star,n} < x|\text{Users from } A_l^a \text{ report SC } n\right] = \prod_{u \in A_l^a} P\left[\gamma_{u,n} < x|\text{User } u \text{ from } A_l^a \text{ reports SC } n\right], \quad (33)$$

which follows as the SC gains of different users are mutually independent. The expression for the conditional CDF $P[\gamma_{u,n} < x|\text{ User } u \text{ from } A_l^a \text{ reports SC } n]$ is the same as that in (26) except that $F_{1,j:j}(x)$ is replaced with $F_{u,j:j}(x)$. Substituting it in (33) and the resulting expression in (32) yields (11).

D. Proof of Result 4

The CDF $P[\gamma_{i^\star,n} < x]$ for the MPF scheduler is also given by (32). For SC n, let $\tilde{\gamma}_{k,n} \triangleq \gamma_{k,n}/\bar{\gamma}_k$, denote the normalized SC power gain of user k. The MPF scheduler selects the user with largest $\tilde{\gamma}_{k,n}$ among those users that report that SC (cf. (2)). Using the law of total probability, we get

$$\begin{aligned} &P\left[\gamma_{i^\star,n} < x | \text{Users from } A_l^a \text{ report SC } n\right] \\ &\quad = \sum_{u \in A_l^a} P\left[\gamma_{u,n} < x, \text{User } u \text{ selected} | \right. \\ &\qquad\qquad \left. \text{Users from } A_l^a \text{ report SC } n\right]. \end{aligned} \tag{34}$$

Writing (34) in terms of the RVs $\tilde{\gamma}_{u,n}$, for $u = 1, 2, \ldots, K$, which are i.i.d. with unit mean, we get $P[\gamma_{u,n} < x, \text{User } u \text{ selected}|\text{Users from } A_l^a \text{ report SC } n] = P[\tilde{\gamma}_{1,n} < x/\bar{\gamma}_u, \text{ User 1 selected}|\text{Users from } A_l^a \text{ report SC } n]$. Using the same method as that to obtain (22) and using the fact that the RVs $\tilde{\gamma}_{u,n}$ are i.i.d., we get

$$\begin{aligned} &P\left[\tilde{\gamma}_{1,n} < \frac{x}{\bar{\gamma}_u}, \text{User 1 selected}\middle| \text{Users from } A_l^a \text{ report SC } n\right] \\ &\quad = \frac{1}{a} P\left[\tilde{\gamma}_{i^\star,n} < \frac{x}{\bar{\gamma}_u}\middle| \text{Users from } A_l^a \text{ report SC } n\right], \\ &\quad = \frac{1}{a}\left(P\left[\tilde{\gamma}_{1,n} < \frac{x}{\bar{\gamma}_u}\middle| \text{User 1 from } A_l^a \text{ reports SC } n\right]\right)^a. \end{aligned} \tag{35}$$

The CDF $P[\tilde{\gamma}_{1,n} < x/\bar{\gamma}_u|$ User 1 from A_l^a reports SC $n]$ is the same as that in (26) except that the mean $\bar{\gamma}$ is unity. Substituting it in (35), the resulting expression in (34) and then in (32), yields the desired result.

E. Proof of Result 5

For the non-i.i.d. case, the probability $P[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r]$ is equal to

$$\begin{aligned} &P[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r] \\ &\quad = \sum_{a=1}^{K} \sum_{l=1}^{\binom{K}{a}} \left(1 - \frac{m}{N}\right)^{K-a} P\left[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r,\right. \\ &\qquad\qquad \left.\text{users from } A_l^a \text{ report SC } n\right]. \end{aligned} \tag{36}$$

Let $u_1, u_2, \ldots, u_a$ denote the users in the set A_l^a. Summing over which user gets selected yields

$$\begin{aligned} &P\Big[\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r, \text{users from } A_l^a \\ &\qquad \text{report SC } n\Big] = \sum_{u_i \in A_l^a} \Upsilon_a(i), \end{aligned} \tag{37}$$

where

$$\begin{aligned} \Upsilon_a(i) \triangleq P\Big[&\Gamma_r \le \gamma_{i^\star,n}^d < \Gamma_{r+1}, \gamma_{i^\star,n} \ge \Gamma_r, \text{Users } u_1, u_2, \\ &\ldots, u_a \text{ report SC } n, \text{User } u_i \text{ is selected}\Big]. \end{aligned} \tag{38}$$

The greedy scheduler selects User u_i if and only if $\gamma_{u_p,n}^d < \gamma_{u_i,n}^d$, for $u_p \in A_l^a$ and $p \neq i$. Averaging over all values of $\gamma_{u_i,n}^d$ and $\gamma_{u_i,n}$, $\Upsilon_a(i)$ can be written as

$$\begin{aligned} \Upsilon_a(i) = \int_{\Gamma_r}^{\Gamma_{r+1}} \int_{\Gamma_r}^{\infty} p\,&\big[\gamma_{u_i,n}^d = x, \gamma_{u_i,n} = y, \text{Users } u_1, u_2, \\ &\ldots, u_a \text{ report SC } n, \gamma_{u_1,n}^d < x, \gamma_{u_2,n}^d < x, \\ &\ldots, \gamma_{u_a,n}^d < x\big]\, dy\, dx. \end{aligned} \tag{39}$$

Since the SC gains of different users are mutually independent, $\Upsilon_a(i)$ simplifies to

$$\begin{aligned} \Upsilon_a(i) = &\left(\frac{m}{N}\right)^{a-1} \int_{\Gamma_r}^{\Gamma_{r+1}} \int_{\Gamma_r}^{\infty} \\ &\times p\left[\gamma_{u_i,n}^d = x, \gamma_{u_i,n} = y, \text{User } u_i \text{ reports SC } n\right] \\ &\times \prod_{\substack{u_p \in A_l^a \\ u_p \neq u_i}} P\left[\gamma_{u_p,n}^d < x | \text{User } u_p \text{ reports SC } n\right] dy\, dx. \end{aligned} \tag{40}$$

Evaluating $p[\gamma_{u_i,n}^d = x, \gamma_{u_i,n} = y, \mathit{User}\ u_i\ \mathit{reports}\ \mathit{SC}\ n]$: From the law of total probability, we know that

$$\begin{aligned} &p\left[\gamma_{u_i,n}^d = x, \gamma_{u_i,n} = y, \text{User } u_i \text{ reports SC } n\right] \\ &\quad = \sum_{q=N-m+1}^{N} p\left[\gamma_{u_i,n}^d = x, \gamma_{u_i,n} = y, \text{SC } n \text{ is } q^{\text{th}} \text{ best SC}\right]. \end{aligned} \tag{41}$$

To simplify further, we use the theory of concomitants or induced order statistics [29].[4] In our problem, the selection is done on the basis of the RV $\gamma_{u_i,n}^d$, which is one among the best-m SCs of user u_i. Therefore, based on the order statistics notation defined in Section II, $\gamma_{u_i,q:N}^d$ denotes the SC power gain of the qth best SC, and its concomitant, which is the SC power gain at the time of data transmission, is denoted by $\gamma_{u_i,[q:N]}$. Since the SC gains are statistically identical, it follows that

$$\begin{aligned} &p\left[\gamma_{u_i,q:N}^d = x, \gamma_{u_i,[q:N]} = y, \text{SC } n \text{ is } q^{\text{th}} \text{ best SC}\right] \\ &\qquad = \frac{1}{N} f_{\gamma_{u_i,q:N}^d, \gamma_{u_i,[q:N]}}(x, y). \end{aligned} \tag{42}$$

Therefore, (41) simplifies to

$$\begin{aligned} &p\left[\gamma_{u_i,n}^d = x, \gamma_{u_i,n} = y, \text{User } u_i \text{ reports SC } n\right] \\ &\qquad = \frac{1}{N} \sum_{q=N-m+1}^{N} f_{\gamma_{u_i,[q:N]}|\gamma_{u_i,q:N}^d}(y|x) f_{\gamma_{u_i,q:N}^d}(x). \end{aligned}$$

Substituting this in (40), $\Upsilon_a(i)$ simplifies to

$$\begin{aligned} &\Upsilon_a(i) \\ &\quad = \frac{m^{a-1}}{N^a} \int_{\Gamma_r}^{\Gamma_{r+1}} \prod_{\substack{u_p \in A_l^a \\ u_p \neq u_i}} P\left[\gamma_{u_p,n}^d < x | \text{User } u_p \text{ reports SC } n\right] \\ &\qquad \times \sum_{q=N-m+1}^{N} f_{\gamma_{u_i,q:N}^d}(x) \left[\int_{\Gamma_r}^{\infty} f_{\gamma_{u_i,[q:N]}|\gamma_{u_i,q:N}^d}(y|x)\, dy\right] dx. \end{aligned} \tag{43}$$

The PDF of $\gamma_{u_i,q:N}^d$ is given by [29] $f_{\gamma_{u_i,q:N}^d}(x) = \sum_{j=q}^{N} (-1)^{j-q} \binom{j-1}{q-1} \binom{N}{j} f_{\gamma_{u_i,j:j}^d}(x)$. Since $H_{u_i,q:N}^d$ and

[4] A concomitant is formally defined as follows [29, Chap. 6.8]. Let (X_i, Y_i), for $i = 1, 2, \ldots, N$, be a random sample from a bivariate distribution. If the sample is ordered on the basis of X_i, then the Y-variate associated with the qth largest RV $X_{q:N}$ is denoted by $Y_{[q:N]}$ and is called the concomitant of the qth order statistic.

$H_{u_i,[q:N]}$ are jointly Gaussian, $|H_{u_i,[q:N]}|$ conditioned on $|H^d_{u_i,q:N}|$ is a Rician RV [12]. The conditional PDF $f_{\gamma_{u_i,[q:N]}|\gamma^d_{u_i,q:N}}(y|x)$ can then be shown to be [39]

$$f_{\gamma_{u_i,[q:N]}|\gamma^d_{u_i,q:N}}(y|x) = \frac{1}{(1-\rho_d^2)\bar{\gamma}_{u_i}} \exp\left(-\frac{y+\rho_d^2 x}{(1-\rho_d^2)\,\bar{\gamma}_{u_i}}\right) \times I_0\left(\frac{2\rho_d\sqrt{xy}}{\bar{\gamma}_{u_i}}\right), \quad x, y \geq 0. \quad (44)$$

From the definition of the Marcum-Q function [40], we get

$$\int_{\Gamma_r}^{\infty} f_{\gamma_{u_i,[q:N]}|\gamma^d_{u_i,q:N}}(y|x)\,\mathrm{d}y = Q_1\left(\sqrt{\frac{2\rho_d^2 x}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}, \sqrt{\frac{2\Gamma_r}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}\right). \quad (45)$$

Substituting the above results in (43), $\Upsilon_a(i)$ simplifies to

$$\Upsilon_a(i) = \frac{m^{a-1}}{N^a}\int_{\Gamma_r}^{\Gamma_{r+1}} \prod_{\substack{u_p\in A_l^a \\ u_p\neq u_i}} P\left[\gamma^d_{u_p,n} < x|\text{User } u_p \text{ reports SC } n\right] \times \sum_{q=N-m+1}^{N}\sum_{j=q}^{N}(-1)^{j-q}\binom{j-1}{q-1}\binom{N}{j} f_{\gamma^d_{u_i,j:j}}(x) \times Q_1\left(\sqrt{\frac{2\rho_d^2 x}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}, \sqrt{\frac{2\Gamma_r}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}\right)\mathrm{d}x. \quad (46)$$

Following steps similar to those used to derive (26), the summation in the above equation is simplified. Finally, $P[\gamma^d_{u_p,n} < y|$ User u_p reports SC $n]$ is obtained by replacing $\bar{\gamma}$ in (26) with $\bar{\gamma}_{u_p}$. Substituting it in (46), the resulting expression for $\Upsilon_a(i)$ in (37), and then in (36), yields (15).

F. Brief Proof of Result 6

Due to space constraints, only the key steps in the derivation are discussed below. For the MPF scheduler too, (37) holds, where $\Upsilon_a(i)$ is given in (38). Writing $\Upsilon_a(i)$ in terms of the unit mean RVs $\tilde{\gamma}^d_{i,n}$, for $i = 1, 2, \ldots, K$, which the MPF scheduler uses to select users, we get

$$\Upsilon_a(i) = P\left[\frac{\Gamma_r}{\bar{\gamma}_{u_i}} \leq \tilde{\gamma}^d_{u_i,n} < \frac{\Gamma_{r+1}}{\bar{\gamma}_{u_i}}, \tilde{\gamma}_{u_i,n} \geq \frac{\Gamma_r}{\bar{\gamma}_{u_i}}, \tilde{\gamma}^d_{u_1,n} < \tilde{\gamma}^d_{u_i,n}, \ldots, \tilde{\gamma}^d_{u_a,n} < \tilde{\gamma}^d_{u_i,n}, \text{Users } u_1, u_2, \ldots, u_a \text{ report SC } n\right]. \quad (47)$$

Proceeding along lines similar to Appendix E, (47) simplifies to $\Upsilon_a(i) = \int_{\frac{\Gamma_r}{\bar{\gamma}_{u_i}}}^{\frac{\Gamma_{r+1}}{\bar{\gamma}_{u_i}}} (P[\tilde{\gamma}^d_{2,n} < x,\ \text{User 2 reports SC } n])^{a-1} \times \int_{\frac{\Gamma_r}{\bar{\gamma}_{u_i}}}^{\infty} p[\tilde{\gamma}^d_{u_i,n} = x, \tilde{\gamma}_{u_i,n} = y,\ \text{User } u_i \text{ reports SC } n]\ \mathrm{d}y\ \mathrm{d}x$. As in Appendix E, simplifying the integral $\int_{\frac{\Gamma_r}{\bar{\gamma}_{u_i}}}^{\infty} p[\tilde{\gamma}^d_{u_i,n} = x, \tilde{\gamma}_{u_i,n} = y,\ \text{User } u_i \text{ reports SC } n]\,\mathrm{d}y$, we obtain

$$\Upsilon_a(i) = \frac{m^{a-1}}{N^a}\int_{\frac{\Gamma_r}{\bar{\gamma}_{u_i}}}^{\frac{\Gamma_{r+1}}{\bar{\gamma}_{u_i}}} \left(P\left[\tilde{\gamma}^d_{2,n} < x|\text{User 2 reports SC } n\right]\right)^{a-1} \times \sum_{j=N-m+1}^{N}(-1)^{j+N-m-1}\binom{j-2}{N-m-1}\binom{N}{j} f_{\tilde{\gamma}^d_{u_i,j:j}}(x) \times Q_1\left(\sqrt{\frac{2\rho_d^2 x}{1-\rho_d^2}}, \sqrt{\frac{2\Gamma_r}{\bar{\gamma}_{u_i}(1-\rho_d^2)}}\right)\mathrm{d}x. \quad (48)$$

The probability $P[\tilde{\gamma}^d_{2,n} < x|$ User 2 reports SC $n]$ is the same as that in (26) but with $\bar{\gamma}$ replaced by unity. Substituting (48) in (37) and finally in (36) yields (17).

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