Resource and Computationally Efficient Subchannel Allocation for D2D in Multi-Cell Scenarios With Partial and Asymmetric CSI

Bala Venkata Ramulu Gorantla, *Student Member, IEEE*, and Neelesh B. Mehta, *Fellow, IEEE*

***Abstract*— In underlay device-to-device (D2D) communication, assigning more D2D pairs to a subchannel can increase the spectral efficiency but it also increases the inter-D2D interference and causes interference to the cellular users (CUs). We consider the assignment of at most K D2D pairs per subchannel in a multi-cell scenario with multiple uplink subchannels. We propose a q-bit quantized feedback and resource allocation model that provides a quality-of-service guarantee to the CUs and ensures that the rates assigned by the base station (BS) to the D2D pairs can be decoded with a pre-specified outage probability even with unknown inter-cell and inter-D2D interferences. We propose a novel, polynomial-time, cardinality-constrained subchannel assignment algorithm (CCSAA) that applies for any K and achieves at least $1/2$ and $1/3$ of the optimal D2D sum throughput for $q = 1$ and $q \geq 2$ bits, respectively. We also propose an alternate cardinality-constrained locally greedy algorithm (CCLGA) that has an even lower complexity and is just as effective in practice. We present a rate upgradation step that exploits the inherent asymmetry in the channel state information at the BS and D2D users to improve spectral efficiency. Our approach also addresses a novel extension to dynamic two-way D2D communications.**

***Index Terms*— D2D, subchannel allocation, partial channel state information, feedback, multi-cell, interference.**

I. Introduction

DEVICE-TO-DEVICE (D2D) communication is a key technology for next generation cellular communication systems. It enables users to directly communicate with each other without routing their data through the base station (BS) [2]. In the overlay mode [3]–[5], the D2D users and the cellular users (CUs) use orthogonal subchannels, while in the underlay mode, they use the same subchannels. While the underlay mode can improve spectral efficiency, it also results in the CUs and D2D users interfering with each other. Therefore, interference-aware allocation of subchannels to D2D users is crucial to achieve a high spectral efficiency while ensuring that excessive interference is not caused to the CUs.

In the underlay mode, considerable research has been done on D2D resource allocation. Its literature differs in three key aspects [6]–[21]: 1) whether the BS is assumed to have full channel state information (CSI) or partial CSI, 2) whether the subchannel assigned to a CU is shared by only one D2D pair or multiple D2D pairs, and 3) whether a single-cell or a multi-cell scenario is considered.

A. Literature Survey

In the following, we categorize and summarize the most pertinent literature on the underlay D2D mode for the single-cell and multi-cell scenarios.

Single-Cell Scenario: This is the most studied scenario, as our summary below shows.

1) *Full-CSI:* In this model, the BS has the CSI of all CU to BS links, CU to D2D receiver (DRx) links, D2D transmitter (DTx) to BS links, and DTx to DRx links.

a) *Only One D2D Pair per Subchannel:* In [6], transmit powers of peak power constrained CU and DTx are optimized while guaranteeing a minimum rate for the CU. In [7], a three-step subchannel and power allocation scheme for the CUs and D2D pairs is proposed, which adheres to constraints on the minimum signal-to-interference-plus-noise ratio (SINR) and on the peak transmit power for the CUs and D2D pairs. In [8], an iterative rounding algorithm is proposed for allocating subchannels to the CUs and D2D pairs while satisfying minimum rate constraints for them.

b) *Multiple D2D Pairs per Subchannel:* In [9], a greedy algorithm is proposed for allocating subchannels to D2D pairs subject to a minimum rate constraint for the CUs. In it, only D2D pairs that cause negligible interference to each other are scheduled on the same subchannel. In [10], a heuristic two-phase resource allocation algorithm is proposed for allocating subchannels to the D2D pairs and determining their power while adhering to minimum rate constraints for the CUs and D2D pairs. In [11], downlink subchannels are allocated to the CUs and D2D pairs using an interference graph based algorithm.

2) *Partial-CSI:* In this model, the BS has partial or statistical knowledge about the CU to DRx and DTx to DRx channel gains.

a) *Only One D2D Pair per Subchannel:* In [12], two CSI models are considered for allocating subchannels to the CUs

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The authors are with the Department of Electrical Communication Engineering, Indian Institute of Science (IISc), Bengaluru 560012, India (e-mail: balavenkataramulu@gmail.com; neeleshbmehta@gmail.com).

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and D2D pairs and determining their transmit powers. The first one is a statistical CSI model in which the BS only knows the probability distribution function of the CU to DRx channel power gains. In the second one, each DRx feeds back channel power gains of the interference links from the K farthest CUs. In [13], for the statistical CSI model, the allocation of subchannels and transmit powers to the D2D pairs, subject to constraints on the outage probability for the transmission rates of the CUs and D2D pairs, is modeled as a maximum bipartite matching problem. In [14], a joint user scheduling, D2D mode selection, and discrete rate adaptation algorithm is proposed for the partial CSI model.

b) *Multiple D2D Pairs per Subchannel:* In [15], for the statistical CSI model, a branch-reduce-and-bound method is used to obtain the optimal D2D powers while guaranteeing each CU a minimum rate with a pre-specified probability of outage. In [16], for the statistical CSI model, a sub-optimal algorithm that clusters the CUs and D2D pairs is proposed for allocating subchannels to them, while guaranteeing their minimum rate with a pre-specified probability of outage. In [17], [18], game-theoretic approaches are employed to allocate subchannels and determine the transmit powers of the D2D pairs.

Multi-Cell Scenario: This scenario has attracted attention rather recently. In [19], a game-theoretic approach is proposed for a model in which one D2D pair is present in the overlapping area of two cells with only one CU in each cell. In [20], a heuristic two-step process is proposed to determine the transmit powers of a CU and a D2D transmitter subject to constraints on their SINR and the inter-cell interference they cause to the BSs. In [21], a distributed admission control method is proposed when each cell has only one CU and multiple D2D pairs, all of which reuse the same subchannel.

B. Focus and Contributions

We consider subchannel allocation to the D2D pairs in the underlay mode in a multi-cell scenario with multiple uplink subchannels and limited CSI. The goal is to maximize the D2D sum throughput by allocating multiple D2D pairs per subchannel while providing a quality-of-service (QoS) guarantee for the CUs scheduled on those subchannels.

Our model captures the following fundamental CSI issues that practically arise in such a scenario. First, the BS only knows the channel power gains of the CU to BS and DTx to BS links; it does not a priori know the channel power gains of the DTx to DRx and CU to DRx links as it is not a receiver in these links. The DRx has to feed back the CSI of these links to the BS. Second, the D2D pair does not know the inter-D2D interference because it does not know a priori which other D2D pairs will share the same subchannel with it. Third, the BS and the D2D pairs do not know the inter-cell interference from the neighboring cells. This is because it requires sharing of the schedules between the BSs before every slot, which entails a significant control overhead. It also requires the BS and the D2D users to know the channel gains from the scheduled users in the neighboring cells to them. In our model, they only know the statistics of the inter-cell interference.

Contributions: We make the following contributions:

Feedback With Reliability Guarantees for D2D Users: We propose a feedback scheme in which a D2D pair feeds back only q bits to the BS about its SINR on a subchannel. It provides the BS with a conservative estimate of the rate that the D2D pair can communicate with on that subchannel. The q-bit feedback is designed to ensure that the probability of outage for the D2D SINR is less than or equal to ϵ_D. We also ensure that each CU transmits at a rate greater than or equal to its target minimum rate with a probability of outage that is at most ϵ_C. Here, ϵ_D and ϵ_C are pre-specified system parameters that depend on the nature of the data traffic of the D2D and cellular users.

Novel Low-Complexity Algorithm With Theoretical Performance Guarantees: We consider a new problem formulation in which at most K D2D pairs can be assigned to a subchannel. This is motivated by the following trade-off associated with K. Increasing K can increase the number of D2D pairs assigned to a subchannel and improves the frequency reuse. However, it also potentially increases the inter-D2D interference since more D2D pairs can share the same subchannel, which can decrease the rate of each D2D pair. For this model, we propose a novel cardinality-constrained subchannel assignment algorithm (CCSAA) for assigning the subchannels to the D2D pairs. CCSAA is a combination of a greedy algorithm for the cardinality-constrained knapsack problem [22] and the Goundan-Schultz algorithm [23]. It is appealing because it is a polynomial-time algorithm and gives much sought-after theoretical guarantees about its performance. Specifically, it provably achieves a D2D sum throughput that is at least $1/2$ and $1/3$ of the optimal D2D sum throughput for $q = 1$ and $q \geq 2$ feedback bits, respectively.

Reducing Complexity Further: For the case when K is equal to the total number of D2D pairs, we propose an alternate novel algorithm called locally greedy algorithm (LGA), which has a lower complexity than CCSAA but provides the same theoretical guarantees. Based on LGA, we present a cardinality-constrained locally greedy algorithm (CCLGA) that can assign D2D pairs to subchannels for any K.

Rate Upgradation to Exploit CSI Asymmetry: Next, we propose a novel rate upgradation (RU) step, which exploits the asymmetric nature of the CSI at the BS and D2D pairs that arises due to the limited feedback. While the BS has only q-bit feedback about the D2D SINR, the D2D pair knows its SINR with more precision. In RU, the D2D pairs exploit this to improve their transmission rates with the same reliability guarantees. Our performance benchmarking shows that for small q, the above algorithms along with RU achieve a D2D sum throughput that is close to that with full intra-cell CSI.

Extension to Dynamic Two-Way D2D: We show that adaptations of CCSAA and CCLGA can lead to an efficient subchannel allocation in a novel and practically motivated dynamic two-way D2D communication model in which the DTx and the DRx can dynamically interchange their roles. Unlike the literature, the BS does not need to rerun the D2D resource allocation algorithm every time the DTx and DRx interchange their roles.

Comments: Our approach differs from the literature in the model considered as well as the algorithms developed. We do not assume full CSI at the BS, which is assumed in [6]–[11], since the heavy signaling overhead it entails makes it impractical. In [3]–[5], the focus is on the overlay D2D mode and only one subchannel is considered for allocation. The effect of cumulative inter-D2D interference from multiple D2D pairs is not taken into account during the allocation. Furthermore, the randomness of the inter-cell interference is not modeled.

Unlike the heuristic approaches in [9]–[11], [13], [16], our algorithms provide theoretical guarantees about their performance. To the best of our knowledge, a polynomial-time algorithm with a theoretical guarantee for D2D systems is available only in [8]. However, it assumes full CSI at the BS and allows only one D2D pair per subchannel. In the game-theoretic approaches studied in [17]–[19], the focus is primarily on showing that an equilibrium exists. However, limited insights are available about how close the sum throughput of this equilibrium is to that with full CSI. Furthermore, the multi-step interaction required between the users is challenging to implement in time-varying environments. Only one CU and one subchannel per cell is considered in [19]–[21], while we consider multiple CUs and multiple subchannels per cell.

C. Outline

The paper is organized as follows. Section II discusses the system model. Section III presents the subchannel allocation algorithms and the rate upgradation technique. The extension to dynamic two-way D2D is presented in Section IV. Section V presents numerical results. Our conclusions follow in Section VI.

II. System Model

We consider the underlay D2D mode in a multi-cell scenario in which the D2D pairs share the subchannels with CUs that are operating in the uplink mode [6]–[10]. In a cell, there are N orthogonal uplink subchannels, indexed $1, 2, \ldots, N$, and M D2D pairs, indexed $1, 2, \ldots, M$. Let $\mathcal{S} = \{1, 2, \ldots, N\}$ and $\mathcal{D} = \{1, 2, \ldots, M\}$. Each subchannel has been assigned to one CU by the scheduler at the BS. Therefore, without loss of generality, let CU i be assigned to subchannel i. The cell is surrounded by neighboring cells that reuse the same subchannels.

The transmit power of the CU is P^c and that of the DTx of a D2D pair is P^d. Let $g_{ji}(i)$ be the channel power gain from CU i to the DRx of D2D pair j on subchannel i. The uplink channel power gain from CU i to the BS on subchannel i is $h_{Bi}(i)$. The channel power gain of the DTx to DRx link of D2D pair j on subchannel i is $h_{jj}(i)$, and from the DTx of D2D pair j to the BS is $g_{Bj}(i)$. The channel power gain from the DTx of D2D pair k to the DRx of D2D pair j on subchannel i is $g^d_{jk}(i)$. We assume the block fading model in which the channel gains remain constant during the transmission of a packet. The system model is shown in Fig. 1 and the notation is summarized in Table I.

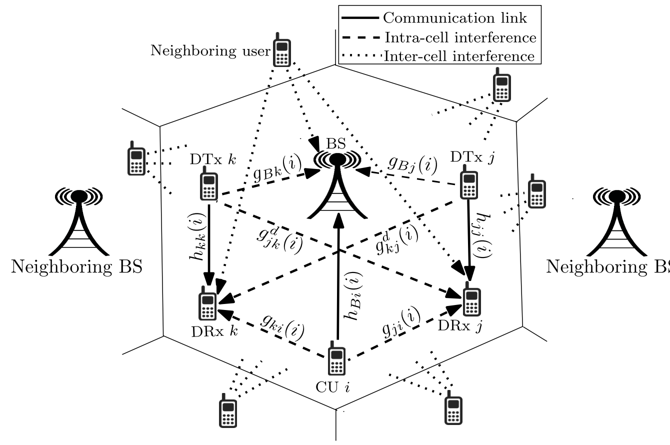

Fig. 1. Multi-cell system model that illustrates multiple D2D pairs sharing a subchannel with a CU. Also shown are the DTx to DRx, CU to BS, CU to DRx, DTx to BS links, and the inter-cell interference at the BS and DRxs.

TABLE I
NOTATION

Symbol	*Description*
P^c	Transmit power of a CU
P^d	Transmit power of the DTx of a D2D pair
$h_{Bi}(i)$	Channel power gain from CU i to BS on subchannel i
$h_{jj}(i)$	Channel power gain between DTx and DRx of D2D pair j on subchannel i
$g_{ji}(i)$	Channel power gain from CU i to DRx of D2D pair j on subchannel i
$g_{Bj}(i)$	Channel power gain from DTx of D2D pair j to BS on subchannel i
$g^d_{jk}(i)$	Channel power gain from DTx of D2D pair k to DRx of D2D pair j on subchannel i
$\Psi_1, \ldots, \Psi_L$	SINR thresholds for D2D links
δ_{ij}	Feedback about SINR of D2D pair j on subchannel i
x_{ij}	1 if subchannel i is assigned to D2D pair j, and is 0 otherwise
C_{ij}	Throughput of D2D pair j on subchannel i

A. CSI and Limited Feedback Model

1) CSI Model: The CSI available at the BS and D2D pairs is different and is as follows:

At BS: The BS knows $h_{Bi}(i)$ and $g_{Bj}(i)$, $\forall i \in \mathcal{S}, j \in \mathcal{D}$. The BS is the receiver in these links and can estimate them using the reference signals transmitted by the users.

At D2D Pairs: The DRx of D2D pair j knows $h_{jj}(i)$ and $g_{ji}(i), \forall i \in \mathcal{S}$. It is the receiver in these links and can estimate them using, for example, sounding reference signals [24].

This model is applicable to both frequency division duplexing (FDD) and time division duplexing (TDD) modes of operation. We note that the CSI model changes if the CUs and D2D pairs were to transmit on the downlink.

Inter-D2D Interference: Since multiple D2D pairs can share a subchannel, the D2D pairs can experience inter-D2D interference. The inter-D2D interference I_{jk} from the DTx of D2D pair k to the DRx of D2D pair j is $I_{jk} = P^d g^d_{jk}(i)$. Knowing the inter-D2D interference is challenging because it requires coordination among all the D2D pairs and entails a significant control overhead. Also, a D2D pair does not know a priori the other D2D pairs that will share a subchannel with it. Therefore, we assume that only the statistics of I_{jk} are known to the D2D pair j.

Inter-Cell Interference: The BS and the D2D pairs also do not know the inter-cell interference from the users in the neighboring cells as this requires a priori information about the users that are scheduled in these cells and the channel gains from those users. Let I_B and I_j^D denote the inter-cell interference at the BS and D2D pair j, $\forall j \in \mathcal{D}$, respectively. As above, only the statistics of I_B and I_j^D are known to the BS and D2D pair j.

SINR of CU: Let x_{ij} be a binary variable that is 1 if subchannel i is assigned to D2D pair j, and is 0 otherwise. Given x_{ij}, $\forall j \in \mathcal{D}$, the SINR $\xi_i^C(i)$ of CU i on subchannel i is given by

$$\xi_i^C(i) = \frac{P^c h_{Bi}(i)}{\sum_{j=1}^{M} x_{ij} P^d g_{Bj}(i) + I_B + \sigma^2}, \tag{1}$$

where σ^2 is the additive white Gaussian noise power.

SINR of D2D Pair: The SINR $\xi_j^D(i)$ of D2D pair j on subchannel i is given by

$$\xi_j^D(i) = \frac{P^d h_{jj}(i)}{P^c g_{ji}(i) + I_j + \sigma^2}, \tag{2}$$

where I_j is the sum of the inter-D2D and inter-cell interferences at user j and is given by

$$I_j = \sum_{k=1, k\neq j}^{M} x_{ik} I_{jk} + I_j^D. \tag{3}$$

2) Limited Feedback Model: The BS does not know $h_{jj}(i)$ and $g_{ji}(i)$. Therefore, it does not know the SINR of D2D pair j. Hence, the DRx sends a q-bit feedback δ_{ij} about its SINR to the BS. However, even the D2D pair does not know its instantaneous SINR since the inter-D2D and inter-cell interferences are not known to it. Even so, it can guarantee that its SINR on subchannel i exceeds a value $T_{ij}(\epsilon_D)$ with a probability $1 - \epsilon_D$, where $T_{ij}(\epsilon_D)$ is given as follows:

$$\Pr\left\{\frac{P^d h_{jj}(i)}{P^c g_{ji}(i) + I_j + \sigma^2} \geq T_{ij}(\epsilon_D)\right\} = 1 - \epsilon_D. \tag{4}$$

Rearranging and writing in terms of the cumulative distribution function (CDF) $F_j(\cdot)$ of I_j, we get

$$F_j\left(\frac{P^d h_{jj}(i)}{T_{ij}(\epsilon_D)} - P^c g_{ji}(i) - \sigma^2\right) = 1 - \epsilon_D. \tag{5}$$

Rearranging terms again, we get

$$T_{ij}(\epsilon_D) = \frac{P^d h_{jj}(i)}{P^c g_{ji}(i) + F_j^{-1}(1 - \epsilon_D) + \sigma^2}, \tag{6}$$

where $F_j^{-1}(\cdot)$ is the inverse of the CDF of I_j.

We note that at the time of generating feedback, a D2D pair also does not know which D2D pairs will interfere with it. Therefore, we proceed as follows. First, we allow at most K D2D pairs to be allocated to a subchannel. This implies that

$$\sum_{j=1}^{M} x_{ij} \leq K, \quad \forall i \in \mathcal{S}. \tag{7}$$

We refer to K as the *D2D cardinality limit*, and shall optimize this system parameter numerically in Section V. Second, to evaluate $F_j^{-1}(\cdot)$ in (6), we conservatively assume that the $(K-1)$ closest D2D pairs interfere with the DRx of D2D pair j as they cause the highest average interference. Hence, in the expression for I_j in (3), we replace $\sum_{k=1,k\neq j}^{M} x_{ik} I_{jk}$ with $\sum_{k=1}^{K-1} I_{j(k)}$, where (k) denotes the k^{th} closest D2D pair to DRx of D2D pair j. This ensures that $T_{ij}(\epsilon_D)$ thus determined is achievable with an outage probability that is at most ϵ_D. We shall see in Section V that significant gains accrue even with this conservative approach.

Feedback: The DRx sends a q-bit feedback δ_{ij} to the BS by quantizing $T_{ij}(\epsilon_D)$. The feedback δ_{ij} is given by

$$\delta_{ij} = l, \quad \text{if } \Psi_l \leq T_{ij}(\epsilon_D) < \Psi_{l+1}, \tag{8}$$

where $0 = \Psi_0 < \Psi_1 < \cdots < \Psi_{L-1} < \infty$ are the $L = 2^q$ quantization thresholds.

Implications: Given δ_{ij}, the BS only knows that $T_{ij}(\epsilon_D)$ exceeds $\Psi_{\delta_{ij}}$. Hence, using Shannon's formula, the BS assigns a rate of $\log_2(1 + \Psi_{\delta_{ij}})$ to the D2D pair j. This framework accounts for adaptive modulation and coding because $\Psi_{\delta_{ij}}$ varies with the instantaneous channel gains $h_{jj}(i)$ and $g_{ji}(i)$. Since the outage probability is ϵ_D, the *throughput* C_{ij} achieved by the D2D pair j on subchannel i is

$$C_{ij} = (1 - \epsilon_D) \log_2(1 + \Psi_{\delta_{ij}}). \tag{9}$$

B. Minimum Rate Guarantee for CUs

We require that CU i, whose SINR is affected by the interference from the D2D pairs that will be scheduled on subchannel i and the unknown inter-cell interference I_B, must be able to transmit at a minimum rate of $R_{\min}^{(i)}$ with a probability of outage at most ϵ_C. Therefore,

$$\Pr\left\{\log_2\left(1 + \xi_i^C(i)\right) \geq R_{\min}^{(i)}\right\} \geq 1 - \epsilon_C. \tag{10}$$

Substituting (1) and rearranging terms, we get

$$\sum_{j=1}^{M} x_{ij} w_{ij} \leq b_i, \tag{11}$$

where $w_{ij} = P^d g_{Bj}(i)$ is the interference from the DTx of D2D pair j to the BS on subchannel i, $b_i = (P^c h_{Bi}(i)) / \left(2^{R_{\min}^{(i)}} - 1\right) - \sigma^2 - F_B^{-1}(1 - \epsilon_C)$, and $F_B^{-1}(\cdot)$ is the inverse of the CDF of I_B. Thus, the sum of interferences at the BS from the D2D pairs assigned to subchannel i should not exceed b_i.

C. Comments About System Model

The above formulation is general because it applies to any CDF of I_j and I_B. For example, with Rayleigh fading and lognormal shadowing, I_j and I_B are sums of composite Rayleigh-lognormal random variables (RVs). Therefore, they can be well approximated as lognormal RVs [25, Ch. 3]. In this case, let the dB-mean and dB-standard deviation of I_j be μ_j and σ_j, respectively, and those of I_B be μ_B and σ_B. Then, the CDF $F_j(\cdot)$ of I_j can be shown to be $F_j(x) = 1 - Q\left((10\log_{10}(x) - \mu_j)/\sigma_j\right)$, for $x \geq 0$, where $Q(\cdot)$ is the Q-function [25, Ch. 3]. Therefore, in this case, $F_j^{-1}(x) = 10^{0.1(\mu_j + \sigma_j Q^{-1}(1-x))}$, for $x \in [0, 1]$, where $Q^{-1}(\cdot)$ is the inverse Q-function. Similarly, the inverse of the CDF of I_B is $F_B^{-1}(x) = 10^{0.1(\mu_B + \sigma_B Q^{-1}(1-x))}$, for $x \in [0,1]$.

In our model, we fix the transmit powers and focus on developing algorithms with provable performance guarantees for assigning subchannels to D2D pairs. A more general formulation, which jointly optimizes the subchannel allocation and transmit power, is also NP-hard [2]. Typically, this has been addressed by first employing heuristic methods to allocate the subchannels to D2D pairs while assuming pre-specified transmit powers, which are then updated based on the allocation so obtained [9]–[11]. Developing joint algorithms for power control and subchannel allocation with provable performance guarantees is an interesting avenue for future work. In our model, we allocate resources based on the statistics of the inter-D2D and inter-cell interferences. This is advantageous because the rate at which these statistics change is several orders of magnitude slower than the rate at which the instantaneous CSI changes. The statistics can be acquired as follows. A D2D user j measures the interference it sees from the other D2D users when it is idle. It can estimate the CDF $F_j(\cdot)$ using either parametric [26, Ch. 4] or non-parametric techniques [27]. Similarly, the statistics of the inter-cell interference can be acquired by the BS.

D. Cardinality-Constrained Subchannel Allocation Problem

Our problem of allocating multiple D2D pairs to subchannels to maximize the sum of the D2D throughputs is as follows:

$$\mathcal{P}: \max_{x_{ij}, \forall i \in \mathcal{S}, j \in \mathcal{D}} \left\{ \sum_{i=1}^{N} \sum_{j=1}^{M} x_{ij} C_{ij} \right\}, \tag{12}$$

$$\text{subject to } \sum_{i=1}^{N} x_{ij} \leq 1, \quad \forall j \in \mathcal{D}, \tag{13}$$

$$\sum_{j=1}^{M} x_{ij} w_{ij} \leq b_i, \quad \forall i \in \mathcal{S}, \tag{14}$$

$$\sum_{j=1}^{M} x_{ij} \leq K, \quad \forall i \in \mathcal{S}, \tag{15}$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in \mathcal{S},\ j \in \mathcal{D}. \tag{16}$$

Constraint (13) mandates that at most one subchannel can be assigned to a D2D pair, Constraint (14) arises due to the minimum rate constraint of CUs, and Constraint (15) ensures that at most K D2D pairs are assigned to a subchannel. The above binary integer programming problem is known to be NP-hard [22], [23].

We note that the feedback overhead can also be accounted for. For example, in 3GPP, D2D feedback is likely to be carried over the uplink physical sidelink shared channel [28]. Accounting for the feedback, the objective function in (12) changes to $BT(\sum_{i=1}^{N}\sum_{j=1}^{M} x_{ij}C_{ij}) - MNq$ or equivalently to $\sum_{i=1}^{N}\sum_{j=1}^{M} x_{ij}C_{ij} - (MNq/BT)$, where B is the transmission bandwidth and T is the slot duration. Since the difference is a constant, our algorithms apply as is.

III. Subchannel Allocation Algorithms and Rate Upgradation

We now propose polynomial-time subchannel allocation algorithms.

A. Cardinality-Constrained Subchannel Assignment Algorithm (CCSAA)

In order to describe CCSAA, we first define the following terminology. A set of D2D pairs $s_i \subseteq \mathcal{D}$ is called a *feasible* D2D set for subchannel i if its cumulative interference at the BS on that subchannel is less than or equal to b_i (cf. (14)) and its cardinality is at most K (cf. (15)). Let $\mathcal{F}_i$ denote the family of feasible D2D sets for subchannel i. The tuple (i, s_i) refers to the subchannel i and its associated feasible D2D set $s_i \in \mathcal{F}_i$.[1] Let $\mathcal{B}_i = \{(1,s_1),(2,s_2),\ldots,(i,s_i)\}$. Note that $s_1, s_2, \ldots, s_i$ need not be mutually exclusive.

We define a set function f on $\mathcal{B}_i$ as follows:

$$f(\mathcal{B}_i) \triangleq \sum_{j=1}^{M} \max_{l \in \{1,2,\ldots,i\}} \{C_{lj} : \exists\,(l, s_l) \in \mathcal{B}_i, s_l \in \mathcal{F}_l, j \in s_l\}. \tag{17}$$

Here, $f(\mathcal{B}_i)$ is the D2D sum throughput of the set $\mathcal{B}_i$. It is the sum of throughputs of the D2D pairs present in $s_1, s_2, \ldots, s_i$, such that if a D2D pair appears in the feasible D2D sets of multiple subchannels then the maximum of its throughputs on those subchannels is considered.

It can be shown that f is non-negative, i.e., $f(\mathcal{X}) \geq 0$ for all sets $\mathcal{X}$. It is also non-decreasing, which means that if there are two sets $\mathcal{X}$ and $\mathcal{Z}$ such that $\mathcal{X} \subseteq \mathcal{Z}$, then $f(\mathcal{X}) \leq f(\mathcal{Z})$. Lastly, f is a submodular function that can be shown to satisfy the following diminishing returns property. Let $\rho_e(\mathcal{A})$ be the incremental gain in f when an element e is included in the set $\mathcal{A}$:

$$\rho_e(\mathcal{A}) \triangleq f(\mathcal{A} \cup \{e\}) - f(\mathcal{A}). \tag{18}$$

For two sets $\mathcal{X}$ and $\mathcal{Z}$ such that $\mathcal{X} \subseteq \mathcal{Z}$, the incremental gain in f when an element e is included in the smaller set $\mathcal{X}$ is greater than or equal to that for the bigger set $\mathcal{Z}$ [29]:

$$\rho_e(\mathcal{X}) \geq \rho_e(\mathcal{Z}), \quad \forall \mathcal{X} \subseteq \mathcal{Z},\ e \notin \mathcal{Z}. \tag{19}$$

For brevity, we define p_{ij} as the *incremental gain* in f when $(i, \{j\})$ is included in $\mathcal{B}_{i-1}$:

$$p_{ij} \triangleq \rho_{(i,\{j\})}(\mathcal{B}_{i-1}). \tag{20}$$

We shall refer to it as the incremental gain of D2D pair j for subchannel i. Similarly, $\rho_{(i,s_i)}(\mathcal{B}_{i-1})$ is the incremental gain in f when (i, s_i) is included in $\mathcal{B}_{i-1}$. It is equal to the sum of the incremental gains of the D2D pairs in s_i, i.e., $\rho_{(i,s_i)}(\mathcal{B}_{i-1}) = \sum_{j \in s_i} p_{ij}$.

Algorithm Description: The CCSAA algorithm finds $\mathcal{B}_N$ as follows. Initially, $\mathcal{B}_0 = \emptyset$ and $f(\emptyset) = 0$. We start from subchannel $i = 1$. Given $\mathcal{B}_{i-1}$, for subchannel i, a feasible D2D set s_i that has an incremental gain of $\rho_{(i,s_i)}(\mathcal{B}_{i-1})$ is

[1] If no D2D pair satisfies the feasibility condition for subchannel i, then $s_i = \emptyset$, where $\emptyset$ is the null set.

obtained as follows. Compute p_{ij} for each D2D pair j whose interference w_{ij} at the BS on subchannel i is less than or equal to b_i. Only these D2D pairs are considered for allocation in the following steps. The process of obtaining the feasible D2D set s_i for subchannel i is different for $q = 1$ and $q \geq 2$, and is as follows.

1) When $q = 1$: In this case, from (9), the throughput of a D2D pair is either 0 or $(1-\epsilon_D)\log_2(1+\Psi_1)$. Hence, from (17) and (20), the incremental gain of a D2D pair is either 0 or $(1-\epsilon_D)\log_2(1+\Psi_1)$. Only D2D pairs with non-zero incremental gains for subchannel i are considered for assignment. These are arranged in the non-decreasing order of their interferences w_{ij}. Using order statistics notation, let $[k]$ denote the D2D pair with the k^{th} smallest interference. Thus, $w_{i[1]} \leq w_{i[2]} \leq \cdots \leq w_{i[M]}$. Find d such that $\sum_{j=1}^{d} w_{i[j]} \leq b_i$ and $\sum_{j=1}^{d+1} w_{i[j]} > b_i$. If $d \leq K$, then $s_i = \{[1],[2],\ldots,[d]\}$, otherwise $s_i = \{[1],[2],\ldots,[K]\}$.

2) When $q \geq 2$: For each subchannel i, we find s_i by solving the following cardinality-constrained knapsack problem $\mathcal{P}'$ that aims to maximize the sum of the incremental gains:

$$\mathcal{P}' : \max_{\{z_j, \forall j \in \mathcal{D}\}} \sum_{j=1}^{M} z_j p_{ij}, \tag{21}$$

$$\text{subject to } \sum_{j=1}^{M} z_j w_{ij} \leq b_i, \tag{22}$$

$$\sum_{j=1}^{M} z_j \leq K, \quad z_j \in \{0,1\}, \ \forall j \in \mathcal{D}. \tag{23}$$

$\mathcal{P}'$ is known to be NP-hard [22]. We, therefore, use the following algorithm, which gives a solution that provably achieves at least half of the optimal objective value of $\mathcal{P}'$. First, we relax the integer constraint in (23) to $0 \leq z_j \leq 1$, $\forall j \in \mathcal{D}$, which makes the problem a linear program. For this linear program, we invoke the following theorem about its optimal solution $z_j^*, \forall j \in \mathcal{D}$. In the following, if $0 < z_j^* < 1$, then we shall call it a *fractional component*.

Theorem *1 ([30, Ch. 9]):* For a linear program with $M \geq 2$ variables and two linear constraints, there exists an optimal solution that has at most two fractional components.

We obtain the optimal solution with the above structure using the dual-simplex algorithm [31, Ch. 4]. For different number of fractional components, the feasible D2D set s_i is obtained from the optimal solution as follows. Let $I = \{j : z_j^* = 1\}$ be the set of D2D pairs for which $z_j^* = 1$.

1) *Two Fractional Components z_q^* and z_r^*:* Without loss of generality, let $w_{ir} > w_{iq}$. In this case, $s_i = I \cup \{q\}$ if $\sum_{j \in I} p_{ij} + p_{iq} > p_{ir}$, otherwise $s_i = \{r\}$.
2) *One Fractional Component z_r^*:* In this case, $s_i = I$ if $\sum_{j \in I} p_{ij} > p_{ir}$, otherwise $s_i = \{r\}$.
3) *No Fractional Component:* In this case, $s_i = I$.

Having determined s_i, we update the set $\mathcal{B}_i$ as follows: $\mathcal{B}_i = \mathcal{B}_{i-1} \cup \{(i, s_i)\}$. This procedure is repeated until $i = N$.

Once $\mathcal{B}_N$ is obtained, the D2D pairs in s_i are allocated to subchannel i. However, if a D2D pair is present in the feasible D2D set of more than one subchannel, then it is allocated only to the subchannel in which it has the maximum throughput. The final D2D sum throughput $f(\mathcal{B}_N)$ obtained by CCSAA is

$$f(\mathcal{B}_N) = (f(\mathcal{B}_N) - f(\mathcal{B}_{N-1})) + (f(\mathcal{B}_{N-1}) - f(\mathcal{B}_{N-2})) + \cdots + (f(\mathcal{B}_1) - f(\mathcal{B}_0)), \tag{24}$$

$$= \sum_{i=1}^{N} \rho_{(i,s_i)}\left(\mathcal{B}_{i-1}\right). \tag{25}$$

The last equality follows from the definition in (18), as per which $\rho_{(i,s_i)}\left(\mathcal{B}_{i-1}\right) = f(\mathcal{B}_i) - f(\mathcal{B}_{i-1})$.

The pseudocode of CCSAA is given in Algorithm 1.

Algorithm 1 Cardinality-Constrained Subchannel Assignment Algorithm (CCSAA)

Initialization: Set $\mathcal{B}_0 = \emptyset$.
for subchannel $i = 1$ to N **do**
Given $\mathcal{B}_{i-1}$, compute p_{ij} for all D2D pairs j that satisfy $w_{ij} \leq b_i$.
If $\mathbf{q = 1}$:
- Arrange the D2D pairs in the non-decreasing order of w_{ij}: $w_{i[1]} \leq w_{i[2]} \leq \cdots \leq w_{i[M]}$.
- Find d such that $\sum_{j=1}^{d} w_{i[j]} \leq b_i$ and $\sum_{j=1}^{d+1} w_{i[j]} > b_i$.
- If $d \leq K$ then set $s_i = \{[1],\ldots,[d]\}$, otherwise $s_i = \{[1],\ldots,[K]\}$.

Else, if $\mathbf{q \geq 2}$:
- Given p_{ij}, compute the optimal solution for the integer-relaxed linear program. Let $I = \{j : z_j^* = 1\}$.
- If the optimal solution has two fractional components z_r^* and z_q^* and $w_{ir} > w_{iq}$, then
  - Set $s_i = I \cup \{q\}$ if $\sum_{j \in I} p_{ij} + p_{iq} > p_{ir}$, otherwise $s_i = \{r\}$.
- Else, if the optimal solution has one fractional component z_r^*, then
  - Set $s_i = I$ if $\sum_{j \in I} p_{ij} > p_{ir}$, otherwise $s_i = \{r\}$.
- Else, set $s_i = I$.

$\mathcal{B}_i \leftarrow \mathcal{B}_{i-1} \cup \{(i, s_i)\}$.
end for
Allocate D2D pairs to subchannels to maximize throughput.

We now state the theoretical guarantee provided by CCSAA.

Result *1:* CCSAA guarantees a D2D sum throughput that is at least $1/2$ of the optimal D2D sum throughput for $q = 1$ and at least $1/3$ of the optimal D2D sum throughput for $q \geq 2$.

Proof: The proof is given in Appendix A. ■

Computational Complexity: For a subchannel, finding the feasible D2D set involves solving a linear program in M variables, which entails a complexity of $\mathcal{O}\left(M^2 \log M\right)$ [32]. For N subchannels, the complexity is, therefore, $\mathcal{O}\left(NM^2 \log M\right)$.

B. Special Case of $K = M$: Locally Greedy Algorithm (LGA)

The problem $\mathcal{P}$ simplifies for the case when $K = M$, i.e., all the D2D pairs can share a subchannel. This is because

Constraint (15) in $\mathcal{P}$ is always satisfied and can, therefore, be dropped. This enables us to propose an alternate algorithm called the locally greedy algorithm (LGA), which has a lower complexity than CCSAA and yet provides the same theoretical guarantees. We describe LGA below using the same terminology as CCSAA.

In LGA, the definition of a feasible D2D set simplifies as follows. A set of D2D pairs is *feasible* for subchannel i if its cumulative interference at the BS on that subchannel is less than or equal to b_i (cf. (14)). As in CCSAA, let $\mathcal{F}_i$ denote the family of feasible D2D sets and $\mathcal{B}_i = \{(1, s_1), (2, s_2), \ldots, (i, s_i)\}$. The definition of the set function $f(\mathcal{B}_i)$ is the same as in (17) and is interpreted as the D2D sum throughput of the set $\mathcal{B}_i$. It can again be shown that the function f is non-negative, non-decreasing, and submodular. The incremental gain p_{ij}, as defined in (20), is $p_{ij} = \rho_{(i,\{j\})}(\mathcal{B}_{i-1})$.

Algorithm Description: LGA finds $\mathcal{B}_N$ as follows. Initially, we set $\mathcal{B}_0 = \emptyset$ and $f(\emptyset) = 0$, and start from subchannel $i = 1$. Given $\mathcal{B}_{i-1}$, a feasible D2D set s_i is selected for subchannel i as follows. We compute p_{ij} only for the D2D pairs j whose interference w_{ij} at the BS is less than or equal to b_i. These D2D pairs are arranged in the non-increasing order of the incremental gain to interference ratio $R_{ij} \triangleq p_{ij}/w_{ij}$. Let $[k]$ denote the D2D pair with the k^{th} largest ratio. Thus, $R_{i[1]} \geq R_{i[2]} \geq \cdots \geq R_{i[M]}$.

Let d be such that the D2D set $\{[1], [2], \ldots, [d]\}$ is feasible, but $\{[1], \ldots, [d], [d+1]\}$ is not, i.e., $\sum_{j=1}^{d} w_{i[j]} \leq b_i$ and $\sum_{j=1}^{d+1} w_{i[j]} > b_i$. If the incremental gain $\sum_{j=1}^{d} p_{i[j]}$ of the tuple $(i, \{[1], \ldots, [d]\})$ is greater than the incremental gain $p_{i[d+1]}$ of the tuple $(i, \{[d+1]\})$, then $s_i = \{[1], \ldots, [d]\}$; otherwise, $s_i = \{[d+1]\}$.[2] Then, $\mathcal{B}_i = \mathcal{B}_{i-1} \cup \{(i, s_i)\}$. This procedure is repeated until $i = N$. Once the set $\mathcal{B}_N$ is obtained, the D2D pairs are allocated to subchannels as described in CCSAA. The pseudocode of LGA is given in Algorithm 2.

The theoretical guarantees provided by LGA are the same as CCSAA.

Result *2:* LGA guarantees a D2D sum throughput that is at least $1/2$ of the optimal D2D sum throughput for $q = 1$ and at least $1/3$ of the optimal D2D sum throughput for $q \geq 2$.

Proof: The proof is given in Appendix B. ∎

Computational Complexity: For a subchannel, finding the feasible D2D set involves sorting the M D2D pairs, which entails a complexity of $\mathcal{O}(M \log M)$. For N subchannels, the complexity is, therefore, $\mathcal{O}(NM \log M)$ [23]. Compared to CCSAA, this is less by a factor of M.

C. Cardinality-Constrained LGA (CCLGA)

LGA is applicable only when $K = M$. Given its lower complexity, we modify it as follows to make it applicable for any $K \leq M$. We shall refer to it as cardinality-constrained LGA (CCLGA). Our modification changes the feasible D2D set s_i that is obtained in LGA to another set s_i' that satisfies the cardinality constraint in (15). While CCLGA does not come with any theoretical guarantees, we shall see in Section V that it is as effective as CCSAA in practice.

Algorithm 2 Locally Greedy Algorithm (LGA)

1: **Initialization:** Set $\mathcal{B}_0 = \emptyset$.
2: **for** subchannel $i = 1$ to N **do**
- Given $\mathcal{B}_{i-1}$, compute p_{ij} for all D2D pairs j that satisfy $w_{ij} \leq b_i$.
- Arrange the D2D pairs in the non-increasing order of R_{ij}: $R_{i[1]} \geq R_{i[2]} \geq \cdots \geq R_{i[M]}$.
- Find D2D pair $[d]$ such that $\sum_{j=1}^{d} w_{i[j]} \leq b_i$ and $\sum_{j=1}^{d+1} w_{i[j]} > b_i$.
- Set $s_i = \{[1], \ldots, [d]\}$ if $\sum_{j=1}^{d} p_{i[j]} > p_{i[d+1]}$, otherwise $s_i = \{[d+1]\}$.
- $\mathcal{B}_i \leftarrow \mathcal{B}_{i-1} \cup \{(i, s_i)\}$.

3: **end for**
4: Allocate D2D pairs to subchannels to maximize throughput.

Using Step 2 of LGA (cf. Algorithm 2), for each subchannel i, a D2D set s_i is obtained. Let $|s_i|$ denote the cardinality of s_i. If $|s_i| \leq K$, i.e., s_i also satisfies (15), then $s_i' = s_i$. However, if $|s_i| > K$, then the D2D pairs in the set s_i are sorted in the non-increasing order of their incremental gains for subchannel i. Only the first K of them are included in s_i'. Mathematically, this can be written as follows. Let $[k]$ denote the D2D pair in s_i with the k^{th} largest incremental gain among the D2D pairs in s_i.[3] Thus, $p_{i[1]} \geq p_{i[2]} \geq \cdots \geq p_{i[|s_i|]}$. Then, $s_i' = \{[1], \ldots, [K]\}$. Once the feasible D2D set s_i' is determined for each subchannel i, $\mathcal{B}_i$ is updated as $\mathcal{B}_i = \mathcal{B}_{i-1} \cup \{(i, s_i')\}$. Once $\mathcal{B}_N$ is obtained, the final allocation is done in a manner similar to CCSAA.

Computational Complexity: CCLGA has the same complexity as LGA, which is $\mathcal{O}(NM \log M)$.

D. Rate Upgradation (RU) to Exploit CSI Asymmetry

As mentioned, the CSI at the BS and the D2D pairs is asymmetric. Specifically, the BS only knows the q-bit quantized SINR $\Psi_{\delta_{ij}}$ of the D2D link. However, the DRx of the D2D pair j already knows $T_{ij}(\epsilon_D)$, which, by the design of the feedback scheme, is greater than or equal to $\Psi_{\delta_{ij}}$. After subchannel allocation, the DRx feeds back the upgraded rate $(1 - \epsilon_D) \log_2 (1 + T_{ij}(\epsilon_D))$ corresponding to $T_{ij}(\epsilon_D)$ to the DTx, which then transmits at that rate. Therefore, the D2D pair j can increase its rate from $(1 - \epsilon_D) \log_2 \left(1 + \Psi_{\delta_{ij}}\right)$, as assigned by the BS during subchannel allocation, to $(1 - \epsilon_D) \log_2 (1 + T_{ij}(\epsilon_D))$ while ensuring that its outage probability does not exceed ϵ_D. We shall henceforth refer to CCSAA with RU as "CCSAA + RU" and CCLGA with RU as "CCLGA + RU".

[2]For subchannel i, if no feasible D2D set is possible, then $s_i = \emptyset$. On the other hand, if the entire D2D set $\{[1], [2], \ldots, [M]\}$ is feasible, then $s_i = \{[1], [2], \ldots, [M]\}$.

[3]Note that the order statistics notation $[k]$ is different in CCSAA, LGA, and CCLGA.

IV. Extension: Dynamic Two-Way D2D Communication

Let j_1 and j_2 be the two nodes of D2D pair j that engage in an exchange of information with each other. In the dynamic two-way D2D communication model, j_1 and j_2 can interchange their roles as DTx and DRx. Our goal is to assign D2D pairs to subchannels without having to recompute the allocation every time this interchange happens.

When node j_1 is the DTx and node j_2 is the DRx, j_2 experiences an interference of $P^c g_{j_2 i}(i)$ from CU i and j_1 causes an interference of $P^d g_{Bj_1}(i)$ at the BS on subchannel i. Hence, as per (6), the SINR $T_{ij_2}(\epsilon_D)$ of the D2D pair j that ensures a probability of outage that is at most ϵ_D is

$$T_{ij_2}(\epsilon_D) = \frac{P^d h_{jj}(i)}{P^c g_{j_2 i}(i) + F_{j_2}^{-1}(1-\epsilon_D) + \sigma^2}, \tag{26}$$

where $F_{j_2}^{-1}(\cdot)$ is the inverse CDF of the sum of the inter-D2D interference and inter-cell interference experienced at j_2. Similarly, when j_2 is the DTx and j_1 is the DRx, j_1 experiences an interference of $P^c g_{j_1 i}(i)$ from CU i and j_2 causes an interference of $P^d g_{Bj_2}(i)$ at the BS on subchannel i. Hence, the SINR value $T_{ij_1}(\epsilon_D)$ of the D2D pair j that ensures a probability of outage that is at most ϵ_D is

$$T_{ij_1}(\epsilon_D) = \frac{P^d h_{jj}(i)}{P^c g_{j_1 i}(i) + F_{j_1}^{-1}(1-\epsilon_D) + \sigma^2}, \tag{27}$$

where $F_{j_1}^{-1}(\cdot)$ is the inverse CDF of the sum of the inter-D2D interference and inter-cell interference experienced at j_1. We assume that $T_{ij_1}(\epsilon_D)$ and $T_{ij_2}(\epsilon_D)$ are known to both nodes.

Revised Feedback Model: As before, we propose that a D2D pair j sends a q-bit feedback δ_{ij} to the BS, but with one difference. Now, δ_{ij} is set to l as follows:

$$\delta_{ij} = l, \quad \text{if } \Psi_l \le \min\{T_{ij_1}(\epsilon_D), T_{ij_2}(\epsilon_D)\} < \Psi_{l+1}. \tag{28}$$

Consequently, the rate assigned to the D2D pair j is $\log_2\left(1+\Psi_{\delta_{ij}}\right)$ and it achieves a throughput of $C_{ij} = (1-\epsilon_D)\log_2(1+\Psi_{\delta_{ij}})$. Furthermore, $\max\{P^d g_{Bj_1}(i), P^d g_{Bj_2}(i)\}$ is considered as the interference from D2D pair j to the BS on subchannel i. Hence, the problem $\mathcal{P}$ changes to

$$\mathcal{P}'': \max_{\{x_{ij},\forall i\in\mathcal{S}, j\in\mathcal{D}\}} \left\{\sum_{i=1}^{N}\sum_{j=1}^{M} x_{ij} C_{ij}\right\}, \tag{29}$$

$$\text{subject to } \sum_{i=1}^{N} x_{ij} \le 1, \quad \forall j \in \mathcal{D}, \tag{30}$$

$$\sum_{j=1}^{M} x_{ij} \max\left\{P^d g_{Bj_1}(i), P^d g_{Bj_2}(i)\right\} \le b_i, \quad \forall i \in \mathcal{S}, \tag{31}$$

$$\sum_{j=1}^{M} x_{ij} \le K, \quad \forall i \in \mathcal{S}, \tag{32}$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in \mathcal{S},\ j \in \mathcal{D}. \tag{33}$$

$\mathcal{P}''$ has the same structure as $\mathcal{P}$. Hence, both CCSAA and CCLGA can be applied, and the former comes with its corresponding theoretical performance guarantees. Note that the resultant solution is different from that of $\mathcal{P}$ because C_{ij} in (29) is different from that in (12) and the feasibility criterion (31) in $\mathcal{P}''$ is different from that in (14) in $\mathcal{P}$. With RU, the D2D rate can be increased from $(1-\epsilon_D)\log_2\left(1+\Psi_{\delta_{ij}}\right)$ to $(1-\epsilon_D)\log_2\left(1+\min\{T_{ij_1}(\epsilon_D), T_{ij_2}(\epsilon_D)\}\right)$.

V. Numerical Results and Performance Benchmarking

We present Monte Carlo simulation results to assess the efficacy of the algorithms and benchmark their performance. We drop N CUs and the DRxs of M D2D pairs randomly with uniform probability in a cell of radius 500 m. To model different DTx-DRx distances, the location of the DTx is chosen so that it lies randomly with uniform probability within a circle of radius 50 m around the DRx. Since the subchannel allocation is done based on the intra-cell channel gains and the statistics of the inter-cell interference, the schedulers in the different cells operate independently of each other. Consequently, to implement the algorithms, we can model the statistics of the inter-cell interference terms I_B and $I_j^D, \forall j \in \mathcal{D}$, as follows.

Modeling Statistics of Inter-Cell and Inter-D2D Interferences: For a subchannel, one CU and K D2D pairs are dropped randomly in each of the neighboring cells. Short-term fading, lognormal shadowing, and path-loss are generated for all the links from the CUs and D2D pairs in these cells to the BS and the DRxs. The cumulative interference on the subchannel from these neighboring cell users is measured at the BS and the DRxs. This is repeated 10,000 times. From these, the CDFs of I_B and I_j^D, $\forall j \in \mathcal{D}$, are determined. Given the path-loss between the D2D pairs j and k, the inter-D2D interference I_{jk} is a composite Rayleigh-lognormal RV. It is approximated as a lognormal RV, whose statistics are determined as per [25, Ch. 2]. Lastly, the D2D pair-specific conservative estimate $I_j = \sum_{k=1}^{K-1} I_{j(k)} + I_j^D$ is a sum of composite Rayleigh-lognormal RVs. It is approximated as a lognormal RV using the Fenton-Wilkinson method [25, Ch. 3].

Simulation Details: The Monte Carlo simulations average over 10,000 drops and channel realizations. The path-loss in dB for the DTx to DRx and CU to DRx links is $148 + 40\log_{10}(d)$, and for the CU to BS and DTx to BS links is $128.1 + 37.6\log_{10}(d)$, where d is the distance in km [33]. We illustrate the results for Rayleigh fading, lognormal shadowing with dB-standard deviation of 6, and set $P^c = 10$ dBm, $P^d = -10$ dBm, $\sigma^2 = -121$ dBm, $\epsilon_C = \epsilon_D = 0.1$, and $R_{\text{min}}^{(i)} = 1$ bps/Hz, $\forall i \in \mathcal{S}$.[4] The rate assigned to the D2D pair j is $\log_2\left(1+\Psi_{\delta_{ij}}\right)$. Therefore, in the simulations, the throughput achieved by the D2D pair without RU is taken to be $(1-\epsilon_D)\log_2\left(1+\Psi_{\delta_{ij}}\right)$. In a multi-cell simulation, the throughput without RU will lie between $(1-\epsilon_D)\log_2\left(1+\Psi_{\delta_{ij}}\right)$ and $\log_2\left(1+\Psi_{\delta_{ij}}\right)$ since

[4]The noise variance of -121 dBm corresponds to a bandwidth of 180 kHz, which is that of a resource block in the long term evolution (LTE) standard, and room temperature. The transmit powers of the CU and DTx are chosen such that the fading-averaged SNRs of a CU at the cell edge (14.2 dB) and a D2D pair at the distance of 50 m (15.0 dB) are within the range of -5.6 dB to 20.6 dB that rate adaptation in LTE is designed for [34, Ch. 10].

the interference at the time of transmission is overestimated in our formulation. However, the difference between these two is small when ϵ_D is small. Similarly, the corresponding throughput with RU is $(1-\epsilon_D)\log_2(1+T_{ij}(\epsilon_D))$.

A. Benchmarking Schemes

We compare with the following schemes:

- *Exhaustive Search (ES):* In this, we search over all possible 2^{MN} assignments of D2D pairs to subchannels to find the optimal solution. However, this is computationally infeasible except for small values of N and M.
- *Semi-orthogonal Sharing Assignment (SSA) [6]–[8], [12]–[14]:* In this, at most one D2D pair can be assigned to a subchannel. It has the advantage of having no inter-D2D interference. To ensure a fair comparison, we consider the same CSI model in which the BS only knows $\Psi_{\delta_{ij}}$ and D2D pair j knows $T_{ij}(\epsilon_D)$. It can be shown that the optimal subchannel allocation that maximizes the D2D sum throughput is a solution of the maximum weighted bipartite matching problem and can be found using the Kuhn-Munkres algorithm [35, Ch. 3].
- *Full Intra-cell CSI ($q=\infty$):* In this, the BS also has CSI of the links from the CU to DRx and the DTx to DRx of a D2D pair but only the statistics of the inter-cell and inter-D2D interferences. This is equivalent to setting q to ∞. Consequently, RU is not required. Since ES is computationally infeasible, the subchannels are allocated using CCLGA or CCSAA.

We note that a comparison with the schemes proposed in [3]–[5], [9]–[11], [15]–[18], which aim to allocate multiple D2D pairs to a subchannel, is not possible due to differences in the CSI and feedback models, and QoS guarantees. For example, [3]–[5], [9]–[11] require the BS to have full CSI of all the links. In [3]–[5], the overlay D2D mode with only one subchannel is considered. In [15], [16], no feedback is considered and the objective functions are different. The game-theoretic models in [17]–[21] assume a multi-step interaction between users over time; while [17], [18] consider only the single-cell scenario, [19]–[21] consider only one subchannel.

B. Numerical Results

We first benchmark CCSAA and CCLGA with ES, for different q. For $q \geq 2$, it becomes computationally cumbersome to numerically optimize the $L = 2^q$ threshold levels. Therefore, we use the threshold values specified in Table II. These are centered around 4 dB and span a 8 dB range for $q = 2$ and a 14 dB range for $q = 4$.

Fig. 2 compares the D2D sum throughput per subchannel of SSA, CCSAA, CCLGA, and ES as a function of the D2D cardinality limit K for different q. Given the computational complexity of ES, it shows results for a toy model with $N = 4$ subchannels and $M = 6$ D2D pairs. Even for this, ES has a complexity of $\mathcal{O}(2^{24})$, which requires considerable computational effort. The performance with RU is not shown in Fig. 2 to avoid clutter; it is explored in the subsequent figures. The D2D sum throughputs of CCSAA and CCLGA are close to each other and to that of ES for all q and K. Thus, the two algorithms are near-optimal in practice. They significantly outperform SSA, whose performance is insensitive to K. The reason for this is that SSA assigns at most one D2D pair per subchannel, whereas the proposed algorithms can assign multiple D2D pairs per subchannel. Note that these gains occur despite the fact that there is no inter-D2D interference in SSA.

A more realistic scenario with $N = 8$ subchannels and $M = 12$ D2D pairs is studied in Fig. 3. It plots the D2D sum throughput per subchannel of CCSAA + RU and CCLGA + RU as a function of the D2D cardinality limit K for different q. We do not show the performance without RU to avoid clutter. ES is no longer computationally feasible, and is not shown. We observe trends that are similar to Fig. 2. The D2D sum throughputs of CCSAA + RU and CCLGA + RU are close

TABLE II

D2D SINR QUANTIZATION THRESHOLDS FOR DIFFERENT q

q	*D2D SINR Thresholds in dB*
1	$\Psi_1 = 4$
2	$\Psi_1 = 0$, $\Psi_2 = 4$, $\Psi_3 = 8$
4	$\Psi_1 = -3$, $\Psi_2 = -2$, $\Psi_3 = -1$, $\Psi_4 = 0$, $\Psi_5 = 1$, $\Psi_6 = 2$, $\Psi_7 = 3$, $\Psi_8 = 4$, $\Psi_9 = 5$, $\Psi_{10} = 6$, $\Psi_{11} = 7$, $\Psi_{12} = 8$, $\Psi_{13} = 9$, $\Psi_{14} = 10$, $\Psi_{15} = 11$

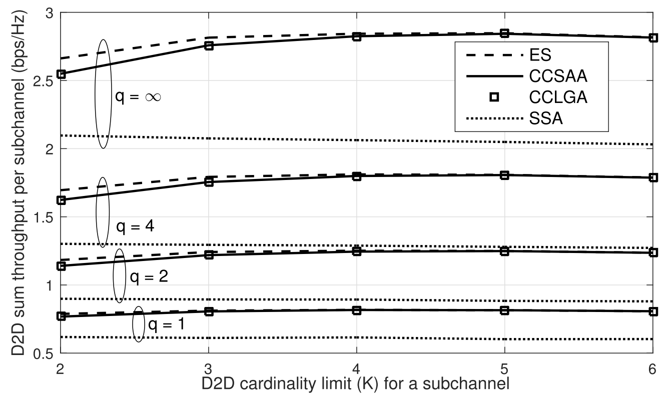

Fig. 2. Toy example ($N = 4$ and $M = 6$): Comparison of D2D sum throughputs of CCSAA and CCLGA with SSA and ES, for different q.

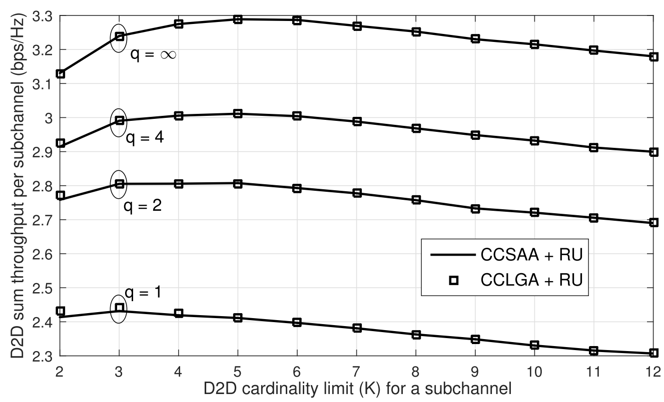

Fig. 3. Zoomed-in comparison of the D2D sum throughputs of CCSAA + RU and CCLGA + RU as a function of K, for different q ($N = 8$ and $M = 12$).

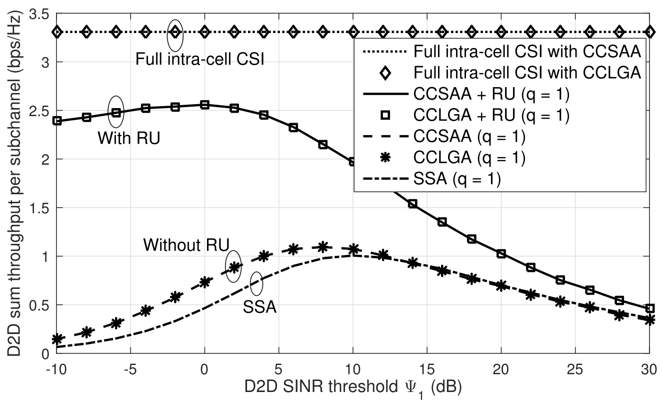

Fig. 4. Comparison of D2D sum throughputs as a function of the SINR threshold Ψ_1 for CCSAA and CCLGA with and without RU, for $q = 1$ and $q = \infty$ ($N = 8$, $M = 12$, and $K = 5$).

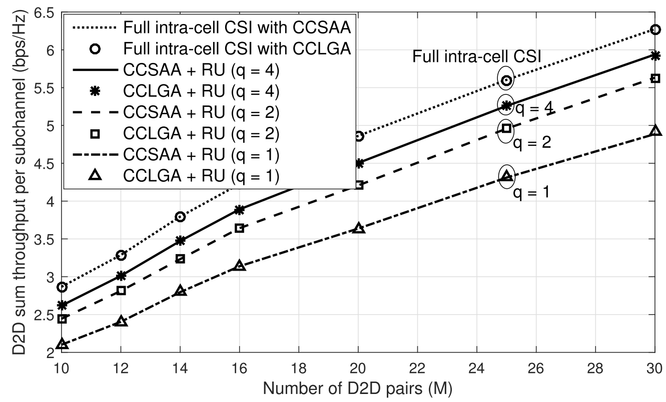

Fig. 5. Variation of D2D sum throughputs of CCSAA + RU and CCLGA + RU with M for different q ($N = 8$ and $K = 5$).

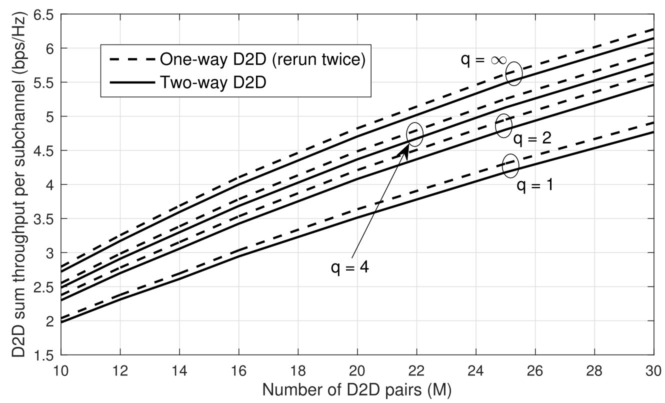

Fig. 6. Comparison of two-way D2D and one-way D2D models using CCSAA + RU for different q ($N = 8$ and $K = 5$).

to each other. As K increases, they first increase and then decrease. The increase occurs because a D2D pair can be present in the feasible D2D sets of more subchannels as K increases. This implies that it has a better chance of getting assigned to a subchannel with a high throughput. However, for larger K, the increased inter-D2D and inter-cell interferences decrease the D2D throughput. The optimal value of K is 3 for $q = 1$ and is 5 for $q \geq 2$.

Optimizing Feedback SINR Threshold for $q = 1$ and Effect of RU: Fig. 4 focuses on 1-bit feedback that informs the BS whether the SINR of a D2D pair is greater than or equal to the SINR threshold Ψ_1. It plots the D2D sum throughput per subchannel as a function of Ψ_1 for CCSAA, CCLGA, and SSA for $q = 1$ and $q = \infty$. Results with and without RU are shown. Without RU, the D2D sum throughputs of CCSAA and CCLGA are indistinguishable. With RU, the D2D sum throughputs of both CCSAA and CCLGA increase significantly and are again close to each other. As Ψ_1 increases, the D2D sum throughputs of these algorithms first increase, reach a maximum value, and then decrease. The optimal threshold for both algorithms is 8 dB without RU and is 0 dB with RU; it turns out to be insensitive to M (figure not shown). For small values of Ψ_1, the number of D2D pairs whose SINR exceeds Ψ_1 is large while the throughput per D2D pair is small. On the other hand, for large Ψ_1, the number of D2D pairs whose SINRs exceed Ψ_1 decreases even though the throughput per D2D pair increases. The maximum D2D sum throughputs of the two algorithms with RU are 154% more than that of SSA.

Effect of M: Fig. 5 plots the D2D sum throughputs of CCSAA + RU and CCLGA + RU as a function of the number of D2D pairs M for different values of q, when $K = 5$. We see that they are indistinguishable and increase as M increases because of multi-user diversity. Thus, the algorithms are scalable. The D2D sum throughputs increase as q increases due to the better resolution of the feedback. At $M = 20$, the D2D sum throughputs of CCSAA + RU and CCLGA + RU for $q = 1, 2$, and 4 are within 24%, 14%, and 7%, respectively, of that for $q = \infty$. Thus, performance close to that for $q = \infty$ is achievable with much less feedback.

Two-way D2D: To present and compare the results of the two-way D2D model with the one-way D2D model, we proceed as follows. In the one-way D2D model, for a given drop of D2D users and a given choice of DTx and DRx, the D2D sum throughput is determined. Then, the DTx and DRx are swapped, the algorithms are rerun, the D2D sum throughput is recalculated, and the average of the two sum throughputs so obtained is shown.

Given the similar performance of CCSAA + RU and CCLGA + RU, we show results only for CCSAA + RU. Fig. 6 plots the D2D sum throughput per subchannel as a function of M for the two models for different q. We see that the D2D sum throughput increases as q or M increases. The two-way D2D model, given its conservative feedback as well as its tighter interference constraint, has a D2D sum throughput that is lower than that of the one-way D2D model for $q = 1, 2, 4$, and ∞. However, the decrease is marginal despite no reassignment being required.

VI. Conclusion

We presented two interference-aware subchannel allocation algorithms CCSAA and CCLGA, which ensured that at most K D2D pairs were allocated to a subchannel. They guaranteed a minimum QoS for the CUs, and did so with limited CSI. The q-bit feedback scheme that we considered ensured that the D2D users could communicate at the assigned rates with a pre-specified probability of outage. For any K, CCSAA achieved a sum throughput that was at least $1/2$ and $1/3$ of the optimal achievable throughput for $q = 1$ and $q \geq 2$, respectively. While CCLGA did not provide any theoretical guarantees, we saw empirically that it was as effective as CCSAA. In conjunction with RU, CCSAA and CCLGA achieved sum throughputs that were substantially higher than SSA and $q = 4$ bits were sufficient to ensure performance close to that with $q = \infty$ in both one-way and dynamic two-way D2D scenarios. An interesting avenue for future work is developing resource allocation algorithms when each D2D pair can simultaneously transmit on multiple subchannels.

Appendix

A. Proof of Result 1

Let $x^*_{ij}, \forall i \in \mathcal{S}, j \in \mathcal{D}$, be the optimal solution for $\mathcal{P}$. Let $s^*_i = \{j : x^*_{ij} = 1, j \in \mathcal{D}\}$ be the feasible D2D set associated with subchannel i in the optimal solution. Define the set of tuples $\mathcal{B}^*_N \triangleq \{(i, s^*_i) \mid 1 \leq i \leq N, s^*_i \in \mathcal{F}_i\}$. The optimal D2D sum throughput for $\mathcal{P}$ is then $f(\mathcal{B}^*_N) = \sum_{i=1}^N \sum_{j=1}^M x^*_{ij} C_{ij}$. The solution obtained by CCSAA is $\mathcal{B}_N = \{(i, s_i) \mid 1 \leq i \leq N, s_i \in \mathcal{F}_i\}$. Let $x_{ij}, \forall i \in \mathcal{S}, j \in \mathcal{D}$, be the allocation derived from $\mathcal{B}_N$. Its sum rate is $f(\mathcal{B}_N) = \sum_{i=1}^N \sum_{j=1}^M x_{ij} C_{ij}$.

Since f is a non-decreasing submodular function, it satisfies the following inequality [29, Prop. 2.2.(iv′)]:

$$f(\mathcal{B}^*_N) \leq f(\mathcal{B}_N) + \sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_N)\, I_{\{s^*_i \neq s_i\}}, \quad (34)$$

$$\leq f(\mathcal{B}_N) + \sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_N). \quad (35)$$

Since $\mathcal{B}_{i-1} \subseteq \mathcal{B}_N$, invoking the diminishing returns property of the submodular function, we get

$$\sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_N) \leq \sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_{i-1}). \quad (36)$$

To evaluate $\sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_{i-1})$, we consider $q = 1$ and $q \geq 2$ separately below.

1) When $q = 1$: The feasible D2D set s_i obtained is the largest feasible D2D set possible because the D2D pairs are selected in the non-decreasing order of their interference w_{ij} until either of the feasibility conditions (14) and (15) are violated. Since every D2D pair has the same incremental gain for subchannel i, this implies that the sum of the incremental gains of the D2D pairs in s_i is the largest among all the feasible D2D sets. Hence, $\rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq \rho_{(i,s_i)}(\mathcal{B}_{i-1})$. Summing over all the subchannels, we get

$$\sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq \sum_{i=1}^N \rho_{(i,s_i)}(\mathcal{B}_{i-1}) = f(\mathcal{B}_N), \quad (37)$$

where the equality follows from (25). Substituting (36) and (37) in (35), we get $f(\mathcal{B}^*_N) \leq 2\ f(\mathcal{B}_N)$.

2) When $q \geq 2$: Since the linear program is an integer-relaxed version of $\mathcal{P}'$, the optimal incremental gain $\sum_{j=1}^M z^*_j p_{ij}$ obtained by it is greater than or equal to that of $\mathcal{P}'$, i.e., $\sum_{j=1}^M z^*_j p_{ij} \geq \sum_{j=1}^M z_j p_{ij}$.

When there are two fractional components z^*_q and z^*_r, $w_{ir} > w_{iq}$ implies that $p_{ir} > p_{iq}$ as otherwise $z^*_q = 1$ and $z^*_r = 0$ would be the optimal solution. When $\sum_{j\in I} p_{ij} + p_{iq} > p_{ir}$, CCSAA sets $s_i = I \cup \{q\}$. This implies that

$$\sum_{j\in I} p_{ij} + p_{iq} > \frac{\sum_{j\in I} p_{ij} + p_{iq}}{2} + \frac{p_{ir}}{2} \geq \frac{\sum_{j=1}^M z^*_j p_{ij}}{2}. \quad (38)$$

Else, when $p_{ir} \geq \sum_{j\in I} p_{ij} + p_{iq}$, CCSAA sets $s_i = \{r\}$. It can be shown that $p_{ir} \geq \left(\sum_{j=1}^M z^*_j p_{ij}\right)/2$. When there is only one fractional component z^*_r, it can again be shown that $\sum_{j\in s_i} p_{ij} \geq \left(\sum_{j=1}^M z^*_j p_{ij}\right)/2$. When there is no fractional component, we have $\sum_{j\in s_i} p_{ij} = \sum_{j=1}^M z^*_j p_{ij}$. Therefore, in all cases, the sum of the incremental gains is at least half of the corresponding optimal value.

Hence, $\rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq 2\rho_{(i,s_i)}(\mathcal{B}_{i-1})$. Summing over all the subchannels, we get

$$\sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq 2\sum_{i=1}^N \rho_{(i,s_i)}(\mathcal{B}_{i-1}) = 2\ f(\mathcal{B}_N). \quad (39)$$

Substituting (36) and (39) in (35), we get $f(\mathcal{B}^*_N) \leq 3\ f(\mathcal{B}_N)$.

B. Proof of Result 2

The proof follows along lines similar to Appendix A. In LGA also, f is a non-decreasing submodular function. Hence, (35) and (36) hold true. However, the process of obtaining the feasible D2D set s_i for subchannel i is different as we shall see below. To evaluate $\sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_{i-1})$ in (36), we consider $q = 1$ and $q \geq 2$ separately.

1) When $q = 1$: In Step 2 of LGA (cf. Algorithm 2), if $p_{ij} = 0$, then the D2D pair j will not be included in s_i. For the other D2D pairs, s_i is formed by taking the D2D pairs in the non-increasing order of the ratio $\frac{p_{ij}}{w_{ij}}$ until the set s_i is no longer feasible. As seen before, these D2D pairs have the same incremental gain for $q = 1$. Hence, this is equivalent to selecting the D2D pairs in the increasing order of their interference w_{ij} until the set s_i is no longer feasible. Thus, s_i is the largest feasible D2D set possible. Therefore, Step 2 ensures that the sum of the incremental gains of the D2D pairs in s_i is the largest among all the feasible sets. Hence, $\rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq \rho_{(i,s_i)}(\mathcal{B}_{i-1})$. Summing over all subchannels, we get $\sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq \sum_{i=1}^N \rho_{(i,s_i)}(\mathcal{B}_{i-1}) = f(\mathcal{B}_N)$. Therefore, from (35) and (36), we get $f(\mathcal{B}^*_N) \leq 2\ f(\mathcal{B}_N)$.

2) When $q \geq 2$: It can be shown using the result in [30, Ch. 2] about the greedy algorithm for the knapsack problem that Step 2 of LGA selects a feasible D2D set s_i for subchannel i such that the incremental gain of adding the tuple (i, s_i) to $\mathcal{B}_{i-1}$ is at least half of the optimal incremental gain. The proof is involved and is not repeated here. Therefore, $\rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq 2\rho_{(i,s_i)}(\mathcal{B}_{i-1})$. Summing over all subchannels, we get $\sum_{i=1}^N \rho_{(i,s^*_i)}(\mathcal{B}_{i-1}) \leq 2\sum_{i=1}^N \rho_{(i,s_i)}(\mathcal{B}_{i-1}) = 2\ f(\mathcal{B}_N)$. Therefore, from (35) and (36), we get $f(\mathcal{B}^*_N) \leq 3\ f(\mathcal{B}_N)$.

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Bala Venkata Ramulu Gorantla (S'19) received the B.Tech. degree in electronics and communication engineering from Jawaharlal Nehru Technological University, Hyderabad, in 2006, and the M.Eng. degree in communication systems from the PSG College of Technology, Coimbatore, in 2009. He is currently pursuing the Ph.D. degree with the Department of Electrical Communication Engineering, Indian Institute of Science, Bengaluru. His research interests include wireless communications, device-to-device communications, UAV communications, and next-generation wireless technologies and optimization.

Neelesh B. Mehta (S'98–M'01–SM'06–F'19) received the B.Tech. degree in electronics and communications engineering from IIT Madras in 1996 and the M.S. and Ph.D. degrees in electrical engineering from the California Institute of Technology, Pasadena, USA, in 1997 and 2001, respectively. He is currently a Professor with the Department of Electrical Communication Engineering, Indian Institute of Science, Bengaluru. He is also a fellow of the Indian National Science Academy (INSA), the Indian National Academy of Engineering (INAE), and the National Academy of Sciences India (NASI). He was a recipient of the Shanti Swarup Bhatnagar Award 2017 and the Swarnjayanti Fellowship. He served on the Board of Governors of the IEEE ComSoc from 2012 to 2015. He served on the Executive Editorial Committee of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS from 2014 to 2017, and as its Chair from 2017 to 2018.