Accurate Performance Analysis of Single and Opportunistic AF Relay Cooperation with Imperfect Cascaded Channel Estimates

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***Abstract*—Given the significant gains that relay-based cooperation promises, the practical problems of acquisition of channel state information (CSI) and the characterization and optimization of performance with imperfect CSI are receiving increasing attention. We develop novel and accurate expressions for the symbol error probability (SEP) for fixed-gain amplify-and-forward relaying when the destination acquires CSI using the time-efficient cascaded channel estimation (CCE) protocol. The CCE protocol saves time by making the destination directly estimate the product of the source-relay and relay-destination channel gains. For a single relay system, we first develop a novel SEP expression and a tight SEP upper bound. We then similarly analyze an opportunistic multi-relay system, in which both selection and coherent demodulation use imperfect estimates. A distinctive aspect of our approach is the use of as few simplifying approximations as possible, which results in new results that are accurate at signal-to-noise-ratios as low as 1 dB for single and multi-relay systems. Using insights gleaned from an asymptotic analysis, we also present a simple, closed-form, nearly-optimal solution for allocation of energy between pilot and data symbols at the source and relay(s).**

***Index Terms*—Amplify-and-forward (AF) relay, cascaded channel estimation, fading channels, power allocation, symbol error probability (SEP), training, diversity.**

I. Introduction

COOPERATIVE wireless relay systems exploit spatial diversity using simple single antenna nodes. Several relay cooperation strategies such as amplify-and-forward (AF), decode-and-forward (DF), selection relaying, and incremental relaying have been proposed in the literature [1]–[4]. Among the different relaying techniques, AF relaying is considered to be easy to implement since the relay does not decode its received signal.

While the performance of AF relaying with perfect channel state information (CSI) is well characterized [1]–[10], its performance with imperfect or noisy CSI is still an active area of research and is the focus of this paper. A performance analysis of AF is challenging because the received signal at the destination contains the product (or cascade) of source-relay (SR) and relay-destination (RD) channel gains. Further, it also contains the product of the RD channel gain and the noise at the relay. These products arise in both fixed-gain [1], [11] and variable-gain [2], [5], [12] relaying. With imperfect channel estimates, which inevitably occur in practice, the analysis becomes even more challenging because extra error terms arise and modulate the above products. This has led to the use of several simplifying assumptions – with different degrees of accuracy – in the literature as discussed below.

A. Training Protocols

The imperfection in CSI and its impact on AF performance inherently depend upon the training protocol used. We, therefore, first discuss various relevant training protocols below. In the three-phase *disintegrated channel estimation (DCE)* protocol [13], the source transmits a pilot, which helps the relay and destination estimate the source-destination (SD) and SR channel gains, respectively. The relay then quantizes and forwards the SR channel estimate to the destination. The relay also transmits a pilot symbol to the destination to enable it to estimate the RD channel gain for coherent demodulation. In [14], a variant of DCE is proposed wherein the relay transmits pilot symbols to the destination in the first phase. In the second phase, the source transmits pilots and the relay then amplifies and forwards its received signal to the destination.

On the other hand, in the two-phase *cascaded channel estimation (CCE)* protocol [13], [15], the relay simply forwards the signal it receives from the source to the destination, without estimating the SR gain or inserting another pilot. As a result, the destination only estimates the product of SR and RD channel gains, and not the SR and RD channel gains separately. In [11], the relay precodes its received signal before forwarding it to the destination, while [16] exploits reciprocity by making the relay forward to the source the noisy training signal it received from the destination.

B. Analysis of Single Relay with Imperfect CSI

In the aforementioned papers on training protocols [11], [13], [14], [16], the focus is on the mean square estimation error and – at best – Monte Carlo simulations are used to determine the symbol error probability (SEP). In [15], the diversity order is characterized and the performance of CCE and DCE is compared using simulations. An expression for

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the SEP of MPSK and MQAM over Rayleigh fading channels is derived in the form of a double integral for variable-gain AF in [12]. However, the destination is assumed to know the individual channel estimates of the SR, RD, and SD links, which is not feasible in CCE. In addition, in order to implement variable-gain relaying, the relay is assumed to know the SR channel gain. In [17], an approximate SEP expression for fixed-gain AF relaying with CCE is derived, but only for BPSK. Further, in the analysis, the probability density function (PDF) of the estimated channel gain is replaced by that of the true cascaded channel gain while averaging the conditional SEP. The resulting approximate SEP expression is not accurate for large signal-to-noise-ratios (SNRs).

In [18], the diversity order for orthogonal and non-orthogonal AF protocols is derived by considering the pairwise error probability (PEP) and assuming that the estimate of the cascaded channel gain is the same as the product of the individual estimates of the SR and RD channel gains. However, the analysis is accurate only for asymptotically large SNRs. Outage probability and optimal power allocation for pilot and data symbols for fixed and variable-gain relaying are derived in [19]. However, to facilitate analysis, the channel estimation error term is replaced with its mean power. The outage probability expression is further simplified by replacing the estimated channel gains with the true channel gains and the noise terms with their mean powers. Consequently, the resultant outage probability expression is inaccurate at small and large SNRs for fixed-gain relaying, with the extent of mismatch depending upon the location of the relay.

C. Multiple Relays with Imperfect CSI

For variable-gain opportunistic relaying, in which one relay is selected to forward data, outage probability and SEP bounds are derived for imperfect CSI in [20]. Such opportunistic selection is practically relevant because only one relay transmits, which eliminates the difficult problem of ensuring symbol-level synchronization among multiple transmitting relays. The estimation error in each channel gain is modeled as additive Gaussian noise with a pre-specified variance. However, the error model is simplistic because it does not model the dependence of the channel estimation error on the training protocol, channel state, and noise at the time of estimation. Consequently, the model predicts an error floor, which is not the case in our system. A similar error model is also used in [21], but for a Rician channel. However, its results are accurate only in the asymptotic regime of large SNR. Further, in [20], [21], the destination is assumed to know not only the estimates of RD and SD links, but also each link between the source and the multiple relays, which is not feasible with CCE. Similar assumptions are also made in [22], [23]. Furthermore, multiple pilot symbols are used in the training phase in [23].

D. Contributions

In this paper, we develop an accurate SEP analysis of fixed-gain AF relaying when the channel estimates are acquired using the CCE training protocol for both single relay and opportunistic multi-relay systems. The training protocol is an integral part of our model and analysis. We focus on the CCE protocol because it is time-efficient and the relay does not estimate the SR channel gain, does not forward its estimate to the destination, and does not insert pilot symbols. It also outperforms the DCE protocol when the number of bits available for quantization is limited [15]. As explained below, our analysis differs in several ways from the analyses with imperfect CSI in the literature, and leads to several novel and extremely accurate results for the SEP of MPSK with CCE.

1) Single Relay with Imperfect CSI: First, we derive a new expression for the SEP of a single relay system. While approximations are necessary in order to facilitate analysis, a notable feature of our approach is the use of as few simplifying approximations as possible. This yields novel expressions for the SEP that are very accurate at SNRs as low as 1 dB, and begets an analysis that differs significantly from those in [17]–[19]. For instance, we do not replace the estimate of the product of the SR and RD channel gains with the product of the estimates of SR and RD channel gains, as was done in [18], [19], [22] for variable-gain relaying. Second, we develop a new, simpler closed-form SEP upper bound that is tight even at low SNRs. Third, we determine the optimal energy allocation between the training and data symbols at the source and the relay, when each of them operate under separate total energy constraints. Despite the involved form of the SEP expressions, we show that considerable insight about the optimal allocation can be gleaned from an asymptotic analysis. Using it, we propose a simple closed-form solution for the energy allocation and show that it is indistinguishable from the optimal solution even at low SNRs.[1] Extensions to include peak power constraints and a constraint on the total energy of the source and the relay are also considered.

2) Opportunistic Multi-Relay Systems with Imperfect CSI: First, we derive a novel expression for the SEP with imperfect channel estimates, which again is accurate even at low SNRs. Our analysis accounts for the fact that relay selection and coherent demodulation are based on imperfect channel estimates. Further, our system model covers the general case where the various SR and RD channels are statistically non-identical. The analysis and the resultant SEP expressions turn out to be considerably more involved than the single relay case. Therefore, a simpler approximate expression for the SEP is also derived. Our analysis differs from [20], [21], which assume that the instantaneous received SNR at the destination is bounded and that separate SR, SD, and RD channel estimates are available at the destination. As in the single relay case, we also determine the optimal energy allocation between the training and data symbols.

For both single and multi-relay systems, we present an extensive set of numerical results to validate our analysis and to benchmark its accuracy.

The paper is organized as follows. Section II develops the system model. The single relay and multiple relay cases are analyzed in Sec. III and Sec. IV, respectively. Numerical

[1]Optimal energy allocation was also considered in [24]. However, the focus was on maximizing a lower bound on the mutual information. Further, the optimal energy allocation derived in [24] is applicable only when the SD link is absent, and numerical optimization techniques are required when the SD link is present. In [25], a gradient search is used for numerically optimizing the pilot power for a variable-gain AF system. Our solution is, thus, simpler and more direct than those in [24], [25].

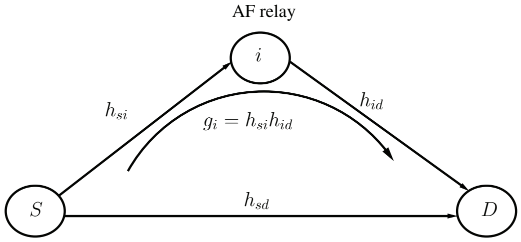

Fig. 1. Single AF relay system model showing SR, RD, and SD channel gains, and the cascaded channel gain

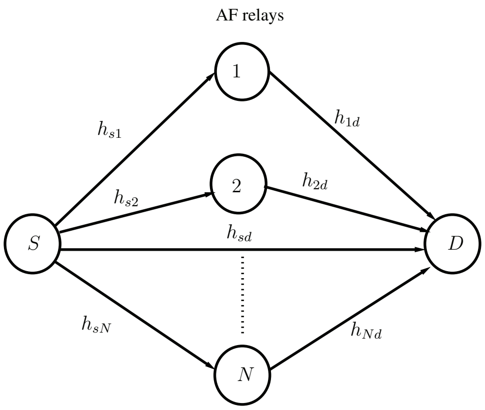

Fig. 2. Multi-relay system model showing a source, N AF relays, and a destination

results in Sec. V are followed by our conclusions in Sec. VI. Mathematical derivations are relegated to the Appendix.

II. System Model

We use the following notation henceforth. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. For an RV Z, its PDF is denoted by $p_Z(z)$, and $\mathbb{E}[Z]$ and $\mathbf{var}[Z]$ denote its expectation and variance, respectively. Similarly, $p_{Z|A}(z)$, $\mathbb{E}[Z|A]$, and $\mathbf{var}[Z|A]$ denote the conditional PDF, expectation, and variance, respectively, of Z given A. The notation $X \sim \mathcal{CN}(\sigma^2)$ implies that X is a circular symmetric zero-mean complex Gaussian RV with variance σ^2. Furthermore, $m \triangleq \sin\left(\frac{\pi}{M}\right)$.

Consider a cooperative system in which a source node S transmits d data symbols to a destination node D over a frequency-flat Rayleigh fading channel with the help of one or more AF relays. In this paper, we consider a single relay system, which is shown in Fig. 1, as well as a multi-relay system, which is shown in Fig. 2. All the nodes are equipped with a single transmit and receive antenna, and operate in the half-duplex mode. Let $h_{sd} \sim \mathcal{CN}(\sigma_{sd}^2)$ denote the SD channel gain. For relay i, let $h_{si} \sim \mathcal{CN}(\sigma_{si}^2)$ and $h_{id} \sim \mathcal{CN}(\sigma_{id}^2)$ denote the SR and RD channel gains, respectively. All channel gains are mutually independent and remain constant during the training and data communication phases, which are described below. For clarity, we first describe the system when only one relay i is present in the system. Thereafter, we describe the model when N relays are present in the system. Our notation is summarized in Table I.

TABLE I
Key notations

Variable	Description
N	Number of relays
E_s^{tot}	Total energy at source for pilot and data symbols
E_i^{tot}	Total energy at relay i for pilot and data symbols
E_s	Energy per data symbol at source
E_i	Energy per data symbol at relay i
d	Number of data symbols
h_{si}, h_{id}	SR and RD link channel gains for relay i
h_{sd}	SD link channel gain
ε_s	Pilot energy boosting factor at source
ε_i	Pilot energy boosting factor at relay i
g_i	Cascaded channel gain for relay i, equals $h_{si}h_{id}$
M	Constellation size
	Training phase
w_{si}	Noise at relay i in training phase, with variance σ_n^2
w_{id}	Noise at destination in training for relay i, with variance σ_n^2
w_{sd}	Noise at destination in training phase when source transmits
α_i	Relay i gain in training phase
w_{sid}	Total noise at destination in training phase, with variance ϖ_{sid}^2
$\hat{h}_{sd}$	Estimate of SD link channel gain
$\hat{g}_i$	Estimate of cascaded channel gain of relay i
	Data transmission phase
n_{si}	Noise at relay i in data transmission phase, with variance σ_n^2
n_{id}	Noise at destination in data transmission phase for relay i, with variance σ_n^2
n_{sd}	Noise at destination in data transmission phase when source transmits, with variance σ_n^2
β_i	Relay i gain in data transmission phase
n_{sid}	Total noise at destination in data transmission phase, with variance σ_{sid}^2

A. Cascaded Channel Estimation for Relay i

Training happens once in a coherence interval and precedes the data transmission phase. The source first transmits a pilot symbol p. The signals r_{sd} and r_{si} received by the destination and relay i, respectively, are given by

$$r_{sd} = \sqrt{\varepsilon_s E_s} h_{sd} p + w_{sd}, \tag{1}$$

$$r_{si} = \sqrt{\varepsilon_s E_s} h_{si} p + w_{si}, \tag{2}$$

where E_s is the symbol energy transmitted by the source, $|p|^2 = 1$, and $w_{sd} \sim \mathcal{CN}(\sigma_n^2)$ and $w_{si} \sim \mathcal{CN}(\sigma_n^2)$ are additive white Gaussian noise terms.

Then, relay i amplifies the signal it received from the source by a factor α_i, and transmits it to the destination. The source boosts its transmit energy by a factor $\varepsilon_s \geq 1$. Similarly, relay i boosts its transmit energy by a factor $\varepsilon_i \geq 1$. *Pilot energy boosting factors* ε_s and ε_i are system parameters, which shall be optimized. The signal r_{id} received by the destination from relay i is

$$r_{id} = \sqrt{\varepsilon_s E_s} \alpha_i g_i p + \alpha_i h_{id} w_{si} + w_{id}, \tag{3}$$

where $g_i = h_{si}h_{id}$ is called the *cascaded channel gain* and $w_{id} \sim \mathcal{CN}(\sigma_n^2)$ is noise. In a fixed-gain AF relay, the relay gain α_i is set to ensure that the average relay transmit energy is $\varepsilon_i E_i$: $\alpha_i = \sqrt{\frac{\varepsilon_i E_i}{\sigma_{si}^2 \varepsilon_s E_s + \sigma_n^2}}$.[2] The PDF $p_{|g_i|^2}(\cdot)$ of $|g_i|^2$ can

[2]Note that fixed-gain relaying requires the relay to know the average SR channel power gain in order to ensure that the relay meets the average energy constraint. However, no knowledge of instantaneous channel gains is required. Note also that the relay gain is different in the training and data transmission phases.

be derived using [19, (33)] and is given by

$$p_{|g_i|^2}(z) = \frac{2}{\sigma_{si}^2\sigma_{id}^2} K_0\left(\frac{\sqrt{z}}{\sigma_{si}^2\sigma_{id}^2}\right), \quad z \geq 0, \tag{4}$$

where K_0 is the modified Bessel function of second kind of order 0 [26, (8.407.1)].

From the observations r_{sd} and r_{id}, the destination computes the estimates of the SD channel gain $\hat{h}_{sd}$ and the cascaded channel gain $\hat{g}_i$ using a linear minimum mean square (LMMSE) estimator.[3] It can be shown that $\hat{h}_{sd}$ and $\hat{g}_i$ are given by [27, (12.6)]

$$\hat{h}_{sd} = \frac{L_{sd}}{p} r_{sd} \quad \text{and} \quad \hat{g}_i = \frac{L_{g_i}}{p} r_{id}, \tag{5}$$

where

$$L_{sd} = \frac{\sigma_{sd}^2\sqrt{\varepsilon_s E_s}}{\sigma_{sd}^2\varepsilon_s E_s + \sigma_n^2},\ L_{g_i} = \frac{\sigma_{si}^2\sigma_{id}^2\alpha_i\sqrt{\varepsilon_s E_s}}{\sigma_{si}^2\sigma_{id}^2\alpha_i^2\varepsilon_s E_s + \varpi_{sid}^2}, \tag{6}$$

and $\varpi_{sid}^2 = \left(\alpha_i^2\sigma_{id}^2 + 1\right)\sigma_n^2$.

B. Data Transmission Using Relay i

In the first phase of data transmission, the source transmits a block of d data symbols. Each symbol x is drawn with equal probability from an MPSK constellation of size M. The signals y_{sd} received by the destination and y_{si} received by relay i are given by

$$y_{sd} = \sqrt{E_s} h_{sd} x + n_{sd}, \tag{7}$$

$$y_{si} = \sqrt{E_s} h_{si} x + n_{si}, \tag{8}$$

where $|x|^2 = 1$ and $n_{sd}, n_{id} \sim \mathcal{CN}(\sigma_n^2)$. In the second phase, relay i amplifies y_{si} by a factor β_i and forwards it to the destination. Therefore, the signal y_{id} received by the destination is

$$y_{id} = \sqrt{E_s}\beta_i g_i x + \beta_i h_{id} n_{si} + n_{id}, \tag{9}$$

where $n_{id} \sim \mathcal{CN}(\sigma_n^2)$. All the noise terms w_{sd}, w_{si}, w_{id}, n_{sd}, n_{si}, and n_{id} are mutually independent. In order to keep the notation simple, we do not show the time index of the data symbols, unless required otherwise to avoid confusion. The relay gain β_i during data transmission is set to ensure an average relay transmit energy of E_i, and is $\beta_i = \sqrt{\frac{E_i}{\sigma_{si}^2 E_s + \sigma_n^2}}$.

C. Opportunistic Relaying

Training is done for each relay separately so that the destination estimates the cascaded channel gain for each relay. Selection of the best relay is done on the basis of the cascaded channel estimates and is described in Sec. IV. The selected relay then forwards data to the destination in the manner described in Sec. II-B.

[3]The minimum mean square error (MMSE) estimator is not considered for g_i in this paper and in the literature on AF relays [12], [17]–[19], [25] because neither g_i nor the effective noise $w_{sid} \triangleq \alpha_i h_{id} w_{si} + w_{id}$ in (3) are Gaussian. For the same reason, even a simulation study of the cooperative system with an MMSE estimator is computationally challenging.

III. SEP Analysis: Single Relay

We now analyze the SEP with imperfect CSI for MPSK when a single relay i is present in the system. The maximum likelihood (ML) decision variable $\mathcal{D}_i$ for detecting a data symbol x is based on the observables y_{sd}, y_{id}, $\hat{h}_{sd}$, and $\hat{g}_i$. Its statistics determine the SEP. We, therefore, evaluate them first.

From (1) and (5), we see that the SD channel estimate $\hat{h}_{sd}$ is perturbed by the noise term w_{sd}. Similarly, from (3) and (5), we see that the noise term $w_{sid} \triangleq \alpha_i h_{id} w_{si} + w_{id}$ perturbs the cascaded channel estimate $\hat{g}_i$. Note that $\hat{g}_i$ is not a Gaussian RV because of the presence of a product of two Gaussian RVs in (3). To ensure analytical tractability, we make the following three approximations and explain their specific roles:

A1) The noise term w_{sid} is assumed to be complex additive white Gaussian noise (CAWGN) with variance $\varpi_{sid}^2 = \mathbb{E}\left[|\alpha_i h_{id} w_{si} + w_{id}|^2\right] = \left(\alpha_i^2\sigma_{id}^2 + 1\right)\sigma_n^2$. Such an approximation has been effectively used earlier in analyzing AF relaying [11], [17] and double differential modulation [28], and corresponds to a worst noise model [24]. Similarly, the noise term during data transmission $n_{sid} \triangleq \beta_i h_{id} n_{si} + n_{id}$ is assumed to be CAWGN with variance $\sigma_{sid}^2 = \left(\beta_i^2\sigma_{id}^2 + 1\right)\sigma_n^2$.

A2) Conditioned on $\hat{g}_i$, w_{sid} is assumed to be Gaussian.

From A1 and A2, and since the SR and RD channel gains are independent of the SD channel gain, we get

$$\mathcal{D}_i = \frac{\left(\mathbb{E}\left[y_{sd}|\hat{h}_{sd}, x\right]\right)^*}{\mathbf{var}\left[y_{sd}|\hat{h}_{sd}, x\right]} y_{sd} + \frac{\left(\mathbb{E}\left[y_{id}|\hat{g}_i, x\right]\right)^*}{\mathbf{var}\left[y_{id}|\hat{g}_i, x\right]} y_{id}. \tag{10}$$

It can then be shown using (1), (5), and (6) that the conditional mean and variance of y_{sd} are $\mathbb{E}\left[y_{sd}|\hat{h}_{sd}, x\right] = \sqrt{E_s}\hat{h}_{sd}x$ and $\mathbf{var}\left[y_{sd}|\hat{h}_{sd}, x\right] = \left(\frac{E_s\sigma_{sd}^2}{\varepsilon_s E_s\sigma_{sd}^2 + \sigma_n^2} + 1\right)\sigma_n^2$.

A3) Conditioned on $\hat{g}_i$ and x, it can be seen from (9) that y_{id} is not a complex Gaussian RV. However, for the purpose of evaluating the conditional moments of y_{id}, the RVs g_i and $\hat{g}_i$ are assumed to be jointly Gaussian. This problem is otherwise intractable because both g_i and $\hat{g}_i$ are products of two complex Gaussian RVs.

Therefore, using A3, (3), (5), and (6), it can be shown that $\mathbb{E}\left[y_{id}|\hat{g}_i, x\right] = \beta_i\sqrt{E_s}\hat{g}_i$ and $\mathbf{var}\left[y_{id}|\hat{g}_i, x\right] = \frac{\beta_i^2\sigma_{si}^2\sigma_{id}^2\varpi_{sid}^2 E_s}{\alpha_i^2\sigma_{si}^2\sigma_{id}^2\varepsilon_s E_s + \varpi_{sid}^2} + \sigma_{sid}^2$.

From the above results, the conditional mean and variance of $\mathcal{D}_i$ can be shown to be equal to each other, and are given by

$$\mathbf{var}\left[\mathcal{D}_i|\hat{h}_{sd}, \hat{g}_i, x\right] = \mathbb{E}\left[\mathcal{D}_i|\hat{h}_{sd}, \hat{g}_i, x\right]$$
$$= \frac{E_s\left|\hat{h}_{sd}\right|^2}{\left(\frac{E_s\sigma_{sd}^2}{\varepsilon_s E_s\sigma_{sd}^2 + \sigma_n^2} + 1\right)\sigma_n^2} + \frac{\beta_i^2 E_s\left|\hat{g}_i\right|^2}{\frac{\beta_i^2\sigma_{si}^2\sigma_{id}^2 E_s\varpi_{sid}^2}{\alpha_i^2\sigma_{si}^2\sigma_{id}^2\varepsilon_s E_s + \varpi_{sid}^2} + \sigma_{sid}^2}. \tag{11}$$

Having computed the moments of $\mathcal{D}_i$, the SEP $P\left(\mathrm{Err}|\hat{h}_{sd}, \hat{g}_i, x\right)$ conditioned on the channel estimates

$\hat{h}_{sd}$ and $\hat{g}$, and the data symbol x is given by [3, (40)]

$$P\left(\mathrm{Err}|\hat{h}_{sd},\hat{g}_i,x\right)=\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\exp\left(-\frac{\left|\mathbb{E}\left[\mathcal{D}_i|\hat{h}_{sd},\hat{g}_i,x\right]\right|^2 m^2}{\mathbf{var}\left[\mathcal{D}_i|\hat{h}_{sd},\hat{g}_i,x\right]\sin^2\theta}\right)d\theta. \tag{12}$$

We now state our first result about the SEP with CCE.

Result *1:* With noisy channel estimates obtained from CCE, the SEP $P(\mathrm{Err})$ of MPSK is given by

$$P(\mathrm{Err})=\frac{\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})}{\pi m^2}\int_0^{\frac{M-1}{M}\pi}\frac{\sin^2\theta}{1+\frac{\varepsilon_s m^2\bar{\gamma}_{sd}^2}{(1+(1+\varepsilon_s)\bar{\gamma}_{sd})\sin^2\theta}}$$
$$\times U\left(1,1;\frac{1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}}{\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id}}+\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})\frac{\sin^2\theta}{m^2}\right)d\theta, \tag{13}$$

where

$$\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})\triangleq\frac{1}{\varepsilon_s^2\varepsilon_i^2\bar{\gamma}_{si}^3\bar{\gamma}_{id}^3}(1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id})$$
$$\times(1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}+\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id})$$
$$\times\left(\bar{\gamma}_{si}\bar{\gamma}_{id}+(1+\bar{\gamma}_{si}+\bar{\gamma}_{id})\left[1+\frac{\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id}}{1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}}\right]\right), \tag{14}$$

$\bar{\gamma}_{sd}=\frac{\sigma_{sd}^2E_s}{\sigma_n^2}$, $\bar{\gamma}_{si}=\frac{\sigma_{si}^2E_s}{\sigma_n^2}$, $\bar{\gamma}_{id}=\frac{\sigma_{id}^2E_i}{\sigma_n^2}$, and $U(.,.;.)$ is the confluent hyper-geometric function of the second kind [26, (9.210)].

Proof: The proof is relegated to Appendix A. ∎

A closed-form upper bound for the SEP is then as follows.

Corollary *1:* With noisy channel estimates obtained from CCE, the SEP upper bound, denoted by $B(\varepsilon_s,\varepsilon_i)$, is given by

$$P(\mathrm{Err})\le B(\varepsilon_s,\varepsilon_i)\triangleq\frac{1}{\pi m^2}\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})f(M,\varphi)$$
$$\times U\left(1,1;\frac{1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}}{\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id}}+\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})\frac{1}{m^2}\right), \tag{15}$$

where $\varphi\triangleq\frac{\varepsilon_s m^2\bar{\gamma}_{sd}^2}{1+(1+\varepsilon_s)\bar{\gamma}_{sd}}$ and $f(M,\varphi)\triangleq\pi\left(1-\frac{1}{M}\right)-\frac{\pi}{2}\sqrt{\frac{\varphi}{\varphi+1}}-\sqrt{\frac{\varphi}{\varphi+1}}\arctan\left(\sqrt{\frac{\varphi}{\varphi+1}}\cot\left(\frac{\pi}{M}\right)\right)$.

Proof: The proof is relegated to Appendix B. ∎

We obtain further insights into the SEP by analyzing the asymptotic regime in which $\bar{\gamma}_{si}$, $\bar{\gamma}_{id}$, and $\bar{\gamma}_{sd}$ are large.

Corollary *2:* In the asymptotic regime of $\bar{\gamma}_{si}\to\infty$, $\bar{\gamma}_{id}\to\infty$, and $\bar{\gamma}_{sd}\to\infty$, the SEP upper bound, after neglecting higher order terms, simplifies to

$$B(\varepsilon_s,\varepsilon_i)=\frac{(1+\varepsilon_s)(1+\max(\varepsilon_s,\varepsilon_i))}{4m^4}\left(\frac{1}{\varepsilon_s\bar{\gamma}_{si}}+\frac{1}{\varepsilon_i\bar{\gamma}_{id}}\right)$$
$$\times\frac{1}{\varepsilon_s\bar{\gamma}_{sd}}\left(-\log\left[\left(\frac{1}{\varepsilon_s\bar{\gamma}_{si}}+\frac{1}{\varepsilon_i\bar{\gamma}_{id}}\right)\left(1+\frac{1+\max(\varepsilon_s,\varepsilon_i)}{m^2}\right)\right]-\zeta\right), \tag{16}$$

where ζ is the Euler-Mascheroni constant [26, (9.73)].

Proof: The proof is relegated to Appendix C. ∎

Comments: The SEP expressions in (13) and (16) are different from those in [2], [3], [12], [17]. This is because [2], [3] assume perfect CSI; furthermore [2] focuses on the asymptotic regime of large SNRs. While the SR and RD channel estimates are assumed to be separately available at the destination in [12], which is not possible in CCE, [17] derives the approximate SEP only for BPSK.

It can be easily seen from (16) that a full diversity order of two is achieved for an AF single-relay system with Rayleigh fading. However, the convergence to the diversity order of two is slow and requires large SNRs because of the presence of the $\log$ term in (16). Note, however, that this slow convergence occurs even with perfect CSI for fixed-gain relaying [29].

A. Optimal Energy Allocation

Having analyzed the SEP, we now determine the optimal energy allocation at the source and the relay. The goal is to determine how the source and relay should each optimally apportion their energy among the pilot and d data symbols so as to minimize the SEP at the destination.[4] Let the source have a fixed total transmit energy of E_s^{tot} to transmit the data symbols and pilot. Similarly, the relay has a fixed total transmit energy of E_i^{tot} to forward the pilot and data signals that it receives from the source. Let $E_s=\frac{E_s^{\mathrm{tot}}}{d+\varepsilon_s}$ and $E_i=\frac{E_i^{\mathrm{tot}}}{d+\varepsilon_i}$ denote the energy per data symbol at the source and the relay, respectively. Further, we define the SR, RD, and SD *link SNRs* as follows:

$$\bar{\delta}_{si}\triangleq\frac{\sigma_{si}^2}{\sigma_n^2},\quad\bar{\delta}_{id}\triangleq\frac{\sigma_{id}^2}{\sigma_n^2},\quad\text{and }\bar{\delta}_{sd}\triangleq\frac{\sigma_{sd}^2}{\sigma_n^2}. \tag{17}$$

Let $\varepsilon_s^{\mathrm{opt}}$ and $\varepsilon_i^{\mathrm{opt}}$ denote the ratios of the energies of the pilot symbol and a data symbol at the source and relay, respectively, that minimize the SEP at the destination. Deriving a closed-form expression for $\varepsilon_s^{\mathrm{opt}}$ and $\varepsilon_i^{\mathrm{opt}}$ is intractable given that the SEP expression in (13) is very involved. However, considerable insights can be gained by analyzing the following SEP upper bound $\tilde{B}(\varepsilon_s,\varepsilon_i)$ in the asymptotic regime in which $\bar{\delta}_{si},\bar{\delta}_{id},\bar{\delta}_{sd}\to\infty$ such that their ratios $\nu_1\triangleq\frac{\bar{\delta}_{id}}{\bar{\delta}_{si}}$ and $\nu_2\triangleq\frac{\bar{\delta}_{sd}}{\bar{\delta}_{si}}$ are finite.

Notice in (16) that $1+\frac{1+\max(\varepsilon_s,\varepsilon_i)}{m^2}>\frac{1+\max(\varepsilon_s,\varepsilon_i)}{m^2}$ since $m^2\le 1$, for $M\ge 2$. This yields the following simpler SEP upper bound $\tilde{B}(\varepsilon_s,\varepsilon_i)$:

$$P(\mathrm{Err})<\tilde{B}(\varepsilon_s,\varepsilon_i)=\frac{(1+\varepsilon_s)(1+\max(\varepsilon_s,\varepsilon_i))}{4\varepsilon_s m^4E_s^2}$$
$$\times\left(\frac{1}{\varepsilon_s\bar{\delta}_{si}}+\frac{E_s^{\mathrm{tot}}(d+\varepsilon_i)}{\varepsilon_i\bar{\delta}_{id}E_i^{\mathrm{tot}}(d+\varepsilon_s)}\right)\frac{1}{\bar{\delta}_{sd}}$$
$$\times\log\left(\left(\frac{1}{\varepsilon_s\bar{\delta}_{si}}+\frac{E_s^{\mathrm{tot}}(d+\varepsilon_i)}{\varepsilon_i\bar{\delta}_{id}E_i^{\mathrm{tot}}(d+\varepsilon_s)}\right)\frac{(1+\max(\varepsilon_s,\varepsilon_i))}{m^2}\right)^{-1}. \tag{18}$$

The above lower bound is tight for larger M since $m^2\ll 1$. The tightness of the bound also increases when either ε_s or ε_i is large compared to 1.

Result *2:* In the asymptotic regime where $\bar{\delta}_{si}\to\infty$, the optimal ε_s and ε_i, denoted by ε_s^* and ε_i^*, respectively, that minimize the SEP upper bound $\tilde{B}(\varepsilon_s,\varepsilon_i)$, given a total energy

[4]Our optimal allocation solution also turns out to be different from that in [30]. While optimal energy allocation was also considered in [30], our formulation differs from it in several ways. While [30] aims to minimize the outage probability at the destination, we focus on minimizing the SEP. Furthermore, additional pilot symbols are inserted by the relay in [30].

constraint of E_s^{tot} at the source and a total energy constraint of E_i^{tot} at the relay, are

$$\varepsilon_s^* = \varepsilon_i^* = \sqrt{d}. \tag{19}$$

Proof: The proof is relegated to Appendix D. ■

Comment: Notice that ε_s^* and ε_i^* are independent of $\bar{\delta}_{si}$, $\bar{\delta}_{id}$, $\bar{\delta}_{sd}$, and the constellation size M. As the number of data symbols increases, more energy gets allocated to the pilot symbol in order to reduce the estimation error. We shall see in Sec. V-C that the SEP with ε_s^* and ε_i^* is indistinguishable from the SEP with $\varepsilon_s^{\text{opt}}$ and $\varepsilon_i^{\text{opt}}$, which are obtained by numerically minimizing (13), even at small SNRs.

B. Extensions

We now consider two relevant extensions to the optimal energy allocation problem.

1) With Peak Power Constraint: We now introduce a peak power constraint at both the source and the relay such that $\varepsilon E_s \leq E^{\max}$, $\varepsilon E_i \leq E^{\max}$, $E^{\max} \leq E_s^{\text{tot}}$, and $E^{\max} \leq E_i^{\text{tot}}$ in addition to the separate total energy constraints when $\varepsilon_s = \varepsilon_i = \varepsilon$. In this case, it can be shown that the optimal ε^* that minimizes the SEP upper bound $\tilde{B}(\varepsilon, \varepsilon)$ in the asymptotic regime ($\bar{\delta}_{si} \to \infty$) is given by

$$\varepsilon^* = \begin{cases} \sqrt{d}, & \text{if } E^{\max} > \frac{1}{1+\sqrt{d}} \max\left\{E_s^{\text{tot}}, E_i^{\text{tot}}\right\}, \\ \frac{d}{\frac{E_s^{\text{tot}}}{E^{\max}}-1}, & \text{if } E_s^{\text{tot}} \geq \max\left\{\left(1+\sqrt{d}\right)E^{\max}, E_i^{\text{tot}}\right\}, \\ \frac{d}{\frac{E_i^{\text{tot}}}{E^{\max}}-1}, & \text{if } E_i^{\text{tot}} \geq \max\left\{\left(1+\sqrt{d}\right)E^{\max}, E_s^{\text{tot}}\right\}. \end{cases} \tag{20}$$

The proof is omitted due to space constraints.

2) Joint Optimization Under a Sum Energy Constraint: We now consider the scenario where the total energy of the source and relay is constrained: $E_s^{\text{tot}} + E_i^{\text{tot}} = E^{\text{tot}}$. In this case, the optimal energy allocation between a source and a relay as well as between the pilot and data symbols at the source and relay for $\varepsilon_s = \varepsilon_i = \varepsilon$ is as follows. In the asymptotic regime where $\bar{\delta}_{si} \to \infty$, and $\nu_1 \triangleq \frac{\bar{\delta}_{id}}{\bar{\delta}_{si}}$ and $\nu_2 \triangleq \frac{\bar{\delta}_{sd}}{\bar{\delta}_{si}}$ are kept constant, the optimal value of $\rho = \frac{E_i^{\text{tot}}}{E_s^{\text{tot}}}$, which is denoted by ρ^*, and the optimal value of ε, which is denoted by ε^*, that minimize the SEP upper bound $\tilde{B}(\varepsilon, \varepsilon)$ are

$$\rho^* = \frac{1}{4}\sqrt{\left(\frac{\bar{\delta}_{si}}{\bar{\delta}_{id}}\right)^2 + \frac{8\bar{\delta}_{si}}{\bar{\delta}_{id}}} - \frac{\bar{\delta}_{si}}{4\bar{\delta}_{id}}, \tag{21}$$

$$\varepsilon^* = \sqrt{d}. \tag{22}$$

The proof is omitted due to space constraints. Notice that value of ε^* is the same as that in (19).

IV. SEP Analysis: Opportunistic Relaying

We now derive the SEP when multiple relays are present in the system and one of them is selected opportunistically based on the cascaded channel estimates $\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_N$. Henceforth, we shall assume $\varepsilon_s = \varepsilon_i = \varepsilon$. We do so because the analysis becomes more involved since the imperfect cascaded channel estimates affect not only demodulation but also relay selection.

The destination, which has access to only the cascaded channel estimates, selects the relay as follows:

$$\hat{s} = \underset{1 \leq i \leq N}{\operatorname{argmax}} \left|\hat{g}_i\right|^2, \tag{23}$$

where the notation $\hat{s}$ emphasizes the fact that the selection is based on noisy estimates.

Comment: The selection rules used in [8], [10], [31] assume perfect knowledge of SR and RD channel gains, while the rule in [20] requires the knowledge of the SR and RD channel estimates separately. However, these rules are not feasible in CCE, which is the focus of this paper.

We first compute the PDF $p_{|\hat{g}_q|}(y)$ of the the estimated cascaded channel gain amplitude $|\hat{g}_q|$, as it will be useful in the SEP analysis. To this end, we make the following fourth approximation.

A4) Using (3) and (5), we get $\left|\hat{g}_q\right|^2 = L_{g_q}^2\left(\varepsilon E_s \alpha_q^2 \left|g_q\right|^2 + \left|w_{sqd}\right|^2 + 2\sqrt{\varepsilon E_s}\alpha_q \left|g_q\right| \left|w_{sqd}\right| \cos\phi\right)$, where ϕ is the angle between the complex numbers g_q and w_{sqd}. For analytical tractability, we drop the $\cos\phi$ term, which yields

$$\left|\hat{g}_q\right|^2 \approx \left|L_{g_q}^2\right| \left(\varepsilon E_s \alpha_q^2 \left|g_q\right|^2 + \left|w_{sqd}\right|^2\right). \tag{24}$$

As shown in Appendix E, the PDF of the estimated cascaded channel gain $p_{|\hat{g}_q|}(y)$ is then given by

$$p_{|\hat{g}_q|}(y) = \frac{2y}{\varpi_{sqd}^2 \left|L_{g_q}\right|^2} \int_0^{\frac{y^2}{\varepsilon E_s \alpha_q^2 \left|L_{g_q}\right|^2}} p_{|g_q|^2}(z) \times \exp\left(-\frac{1}{\varpi_{sqd}^2}\left(\frac{y^2}{\left|L_{g_q}\right|^2} - \varepsilon E_s \alpha_q^2 z\right)\right) dz, \; y \geq 0. \tag{25}$$

We now present the main result about the SEP of opportunistic relaying.

Result *3:* The SEP of MPSK with opportunistic relaying with noisy estimates obtained from CCE is given by

$$P(\text{Err}) = \frac{1}{\pi}\sum_{k=1}^{N}\sum_{n=1}^{W} \frac{w_n \exp(a_n)}{\varpi_{skd}^2 \left|L_{g_k}\right|^2} \times \left[\int_0^{\frac{M-1}{M}\pi}\left(1 + \frac{\varepsilon m^2 \bar{\gamma}_{sd}^2}{\left(1+(1+\varepsilon)\bar{\gamma}_{sd}\right)\sin^2\theta}\right)^{-1} \exp\left(-\frac{\eta_k a_n}{\sin^2\theta}\right) d\theta\right] \times \left[\prod_{q=1,2,\ldots,N, q\neq k} \left(1 - \nu\sqrt{a_n} K_1\left(\nu\sqrt{a_n}\right) - I(a_n, q)\right)\right] I(a_n, k), \tag{26}$$

where

$$I(a_n, q) \triangleq \int_0^{\frac{a_n}{\varepsilon E_s \alpha_q^2 \left|L_{g_q}\right|^2}} e^{-\frac{1}{\varpi_{sqd}^2}\left(\frac{a_n}{\left|L_{g_q}\right|^2} - \varepsilon E_s \alpha_q^2 z\right)} p_{|g_q|^2}(z)\, dz,$$

$\eta_k \triangleq \frac{\beta_k^2 E_s m^2}{\frac{\beta_k^2 \sigma_{sk}^2 \sigma_{kd}^2 E_s \varpi_{skd}^2}{\alpha_k^2 \sigma_{sk}^2 \sigma_{kd}^2 \varepsilon E_s + \varpi_{skd}^2} + \sigma_{skd}^2}$, $\nu \triangleq \frac{2}{\sqrt{\varepsilon E_s}\alpha_q \sigma_{sq}\sigma_{qd}\left|L_{g_q}\right|}$, and a_n and w_n, for $1 \leq w \leq W$, are the W Gauss-Laguerre abscissas and weights, respectively.

Proof: The proof is relegated to Appendix F. ■

Comment: The expression in (26) is different from those in [8]–[10], [20], [21], [32]. While [8]–[10], [32] assume perfect CSI, [20], [21] implicitly assume that the DCE training protocol is used. Further, evaluating (26) involves computing three integrals for each value of the triplet (k, n, q). The following simplification, which is described in Appendix G, simplifies $I(a_n, q)$ considerably and writes it in the form of the standard Dawson function [33, (7.5)], which is readily available in Matlab and Mathematica:

$$I(a_n, q) \approx \frac{\exp\left(-\kappa_2 - \frac{4\kappa_1 a_n}{\varepsilon E_s \alpha_q^2 \sigma_{sq}^2 \sigma_{qd}^2 \left|L_{g_q}\right|^2}\right)}{\sqrt{\frac{\varepsilon E_s \alpha_q^2 \sigma_{sq}^2 \sigma_{qd}^2}{4\varpi_{sqd}^2} - \kappa_1}} \times D_+\left(\sqrt{\frac{a_n}{\varpi_{sqd}^2 \left|L_{g_q}\right|^2} - \frac{4\kappa_1 a_n}{\varepsilon E_s \alpha_q^2 \sigma_{sq}^2 \sigma_{qd}^2 \left|L_{g_q}\right|^2}}\right), \quad (27)$$

where $\kappa_1 = 0.1428$, $\kappa_2 = 0.8657$, and $D_+(x)$ is the Dawson function.

Notice that the SEP expression is more involved than for the single relay case.

3) Optimal Energy Allocation: As in the single relay scenario, the source has a fixed total transmit energy of E_s^{tot}, while the N relays have a transmit energy of E_i^{tot}, for $1 \leq i \leq N$, to transmit data and pilot signals. Our goal is to determine how the source and the selected relay should each optimally apportion their energy among the pilot and data symbols to minimize the SEP. An analytical characterization of ε^{opt} is intractable given the involved form of the SEP expression in (26). We, therefore, find it by numerically minimizing the simplified form of (26), which is based on the approximation developed in (27). Note that this is computationally much more efficient than finding the optimal allocation using exhaustive Monte Carlo simulations.

V. Numerical Results

We now verify and study the several analytical results derived thus far with Monte Carlo simulations that use 10^5 samples. For opportunistic relaying, we use $W = 12$ terms to accurately compute the SEP in (26) up to 10^{-3}.

A. SEP Results: Single Relay

Figure 3 plots the SEP of 8PSK and 16PSK when $\bar{\gamma}_{si} = \bar{\gamma}_{id} = \bar{\gamma}_{sd} \triangleq \gamma_0$. Notice that the analytical and simulation results are in good agreement at SNRs as low as 1 dB. Thus, the approximations A1, A2, and A3 together are accurate. Furthermore, the upper bound is tight (within 0.75 dB). Compared to perfect CSI, imperfect CSI leads to a 3.2 dB SNR loss when the SEP is 10^{-2}. Notice that no error floor occurs.

Figure 4 plots the SEP of 8PSK and 16PSK as a function of $\bar{\gamma}_{sd}$ for two different scenarios in which the SR and RD channels are not statistically identical. We again observe a good match between the analytical and simulation results, which validates the approximations even for the general scenario in which the channel gains are statistically non-identical.

Figure 5 plots the SEP of 16PSK as a function of $\bar{\gamma}_{sd}$ for different combinations of ε_s and ε_i. Scenarios where the SR and RD channels are statistically identical and non-identical are considered. Notice that the analytical and simulation results are again in good agreement at SNRs as low as 1 dB. When the SR link is 10 dB stronger than the RD and SD links, a boosting factor of $\varepsilon_i = 10$ is required to get a 1 dB improvement in the SNR at an SEP of 10^{-3} compared to the case where the pilot energy is not boosted at the source.

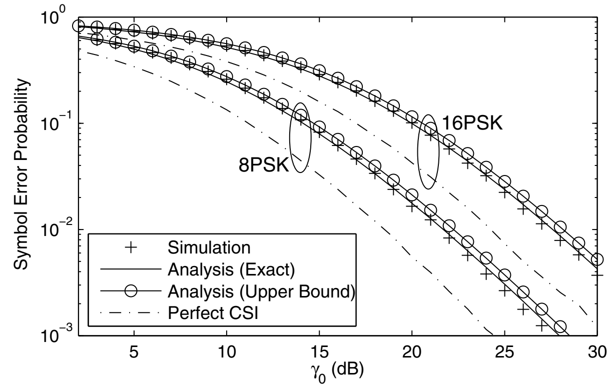

Fig. 3. Single relay case: SEP of 8PSK and 16PSK ($\bar{\gamma}_{si} = \bar{\gamma}_{id} = \bar{\gamma}_{sd} = \gamma_0$ and $\varepsilon_s = \varepsilon_i = 1$)

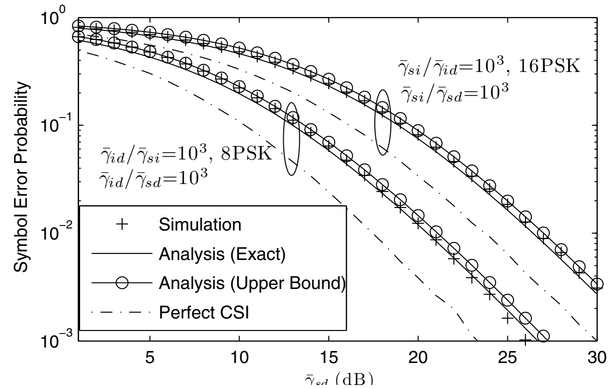

Fig. 4. Single relay case: SEPs of 8PSK and 16PSK for different SR and RD mean channel gains ($\varepsilon_s = \varepsilon_i = 1$)

B. SEP Results: Opportunistic Relaying

Figure 6 plots the SEPs of 8PSK and 16PSK for $N = 2$ relays when $\bar{\gamma}_{si} = \bar{\gamma}_{id} = \bar{\gamma}_{sd} \triangleq \gamma_0$. Figure 7 plots the

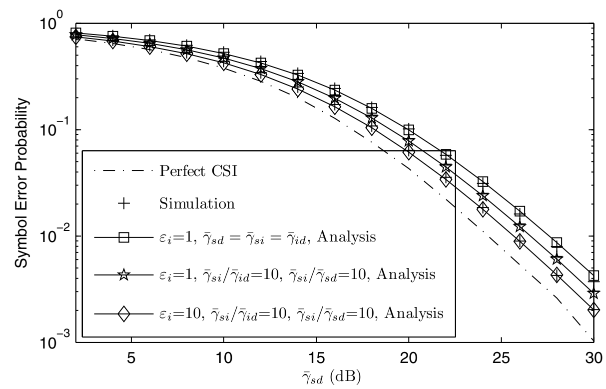

Fig. 5. Single relay case: SEP of 16PSK with different combinations of ε_i and for different SR and RD mean channel gains ($\varepsilon_s = 1$)

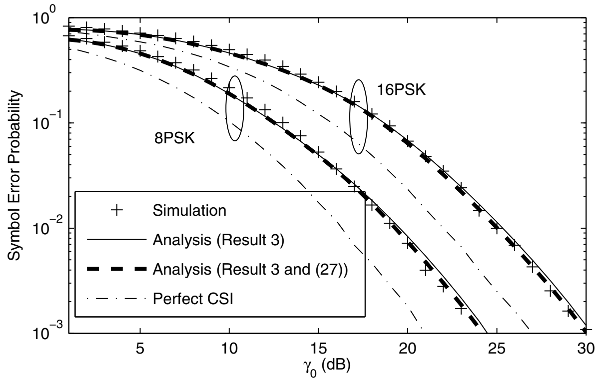

Fig. 6. Multi-relay case: SEPs of 8PSK and 16PSK ($\bar{\gamma}_{si} = \bar{\gamma}_{id} = \bar{\gamma}_{sd} = \gamma_0$, $\varepsilon_s = \varepsilon_i = 1$, and $N = 2$ relays)

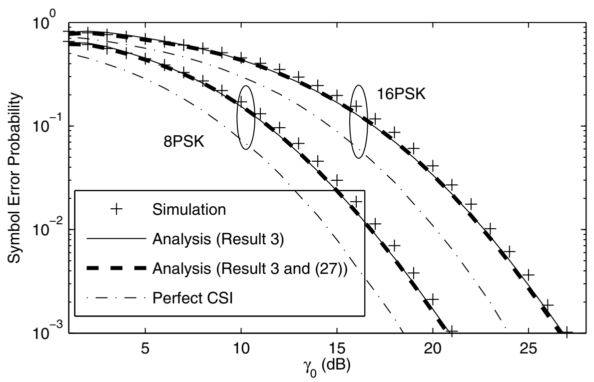

Fig. 7. Multi-relay case: SEPs of 8PSK and 16PSK ($\bar{\gamma}_{si} = \bar{\gamma}_{id} = \bar{\gamma}_{sd} = \gamma_0$, $\varepsilon_s = \varepsilon_i = 1$, and $N = 4$ relays)

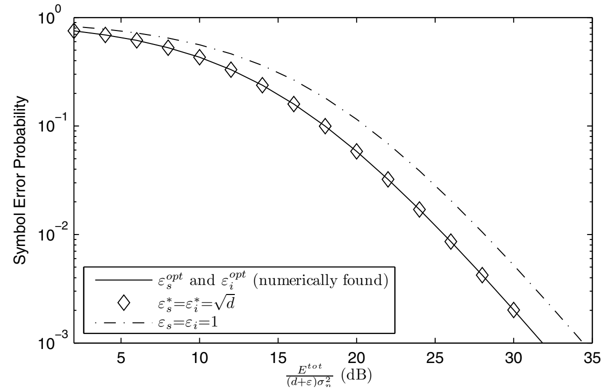

Fig. 8. Optimal energy allocation for single relay case: SEPs using ε_s^*, ε_i^*, $\varepsilon_s^{\text{opt}}$, $\varepsilon_i^{\text{opt}}$, and no pilot energy boosting for 16PSK

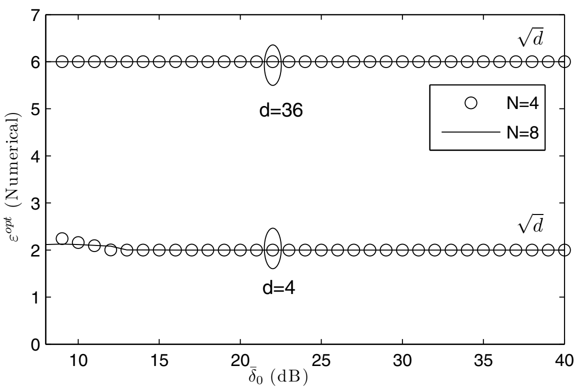

Fig. 9. Multi-relay case: Optimal ε as a function of $\bar{\delta}_0$ for 16PSK and different numbers of data symbols d and different number of relays N ($\bar{\delta}_{si} = \bar{\delta}_{id} = \bar{\delta}_{sd} = \bar{\delta}_0$)

corresponding SEP curves for $N = 4$ relays. We notice that the analytical curves drawn using Result 3 and the simulation curves, which do not make use of any approximations, are in good agreement even at SNRs as low as 1 dB. This validates the use of the approximations A1, A2, A3, and A4. The approximations together only lead to a marginal gap of 0.5 dB and 0.3 dB between the analytical and simulated curves for two and four relays, respectively. Also plotted is the SEP curve when the simplified expression of (27) is used. It also matches well with the simulation results. Compared to perfect CSI, imperfect CSI leads to a 3.0 dB SNR loss at an SEP of 10^{-2}.

C. Optimal Energy Allocation

We now study the optimal allocation of energy across pilot and data symbols at the source and relay(s) when each is subject to a total energy constraint.

1) Single Relay: We set $E_s^{\text{tot}} = E_i^{\text{tot}} = E^{\text{tot}}$ and $\bar{\gamma}_{si} = \bar{\gamma}_{id} = \bar{\gamma}_{sd} = \bar{\gamma}_0 = \frac{E^{\text{tot}}}{(d+\varepsilon)\sigma_n^2}$, which is the SNR per received symbol. Further, $\sigma_{si}^2 = \sigma_{id}^2 = \sigma_{sd}^2 = 1$. Figure 8 compares the SEPs for 16PSK with the energy allocation $\varepsilon = \varepsilon_s^* = \varepsilon_i^* = \sqrt{d}$ and with the optimal energy allocation $\varepsilon_s^{\text{opt}}$ and $\varepsilon_i^{\text{opt}}$, which is found by numerically minimizing (13). Notice that the two SEPs are indistinguishable even at low SNRs. This validates the utility of the asymptotic analysis and the insightful and simple optimal energy allocation it yields. Not using pilot energy boosting ($\varepsilon_s = \varepsilon_i = 1$) leads to a loss of 2.2 dB in SNR at an SEP of 10^{-3} for 16PSK.

2) Opportunistic Relaying: Figure 9 plots the optimal pilot energy boosting factor ε^{opt}, which is found by numerically minimizing the simplified form of (26), which is based on the approximation developed in (27), as a function of $\bar{\delta}_0$. This is done for different values of d and N. We set $E_s^{\text{tot}} = E_i^{\text{tot}} = d + \varepsilon^*$. Thus, the energy of each transmitted data symbol is 1. Further, the fading-averaged link SNRs are set as $\bar{\delta}_{sr} = \bar{\delta}_{rd} = \bar{\delta}_{sd} \triangleq \bar{\delta}_0$ (cf. (17)). We observe that as $\bar{\delta}_0$ increases, $\varepsilon^{\text{opt}} \to \sqrt{d}$, which is the same as in the single relay case.

The SEPs with pilot energy boosting factor $\varepsilon = \sqrt{d}$ and the numerically found optimal value ε^{opt} are compared in Fig. 10 for two and eight relays. We again see that the two are indistinguishable even for $\bar{\delta}_0$ as small as 1 dB for various N. Thus, the pilot energy allocation $\varepsilon = \sqrt{d}$ is accurate for the purposes of minimizing the SEP well into the non-asymptotic regime. Further, not boosting pilot energy ($\varepsilon = 1$) leads to a loss of 1.8 dB in SNR at an SEP of 10^{-3} for both two and eight relays.

D. Single Relay: Optimal Relay Placement

Let d_{si}, d_{id}, and d_{sd} denote the distances between source and relay, relay and destination, and source and destination, respectively. For simplicity, the relay is placed on the line segment connecting source and destination. Thus, $d_{sd} = d_{si} + d_{id}$.

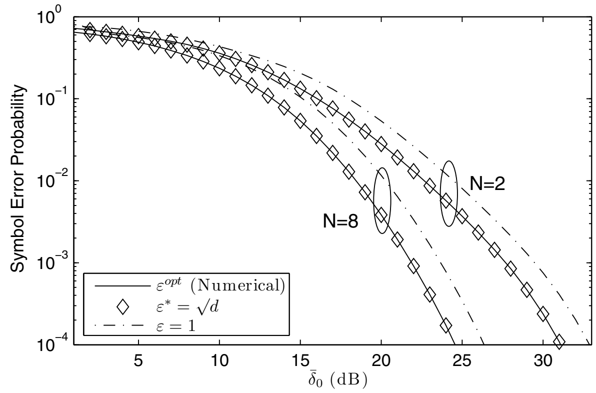

Fig. 10. Multi-relay case: SEPs using ε^*, $\varepsilon^{\mathrm{opt}}$, and no pilot energy boosting for 16PSK and different N ($\bar{\delta}_{si} = \bar{\delta}_{id} = \bar{\delta}_{sd} = \bar{\delta}_0$ and $d = 36$)

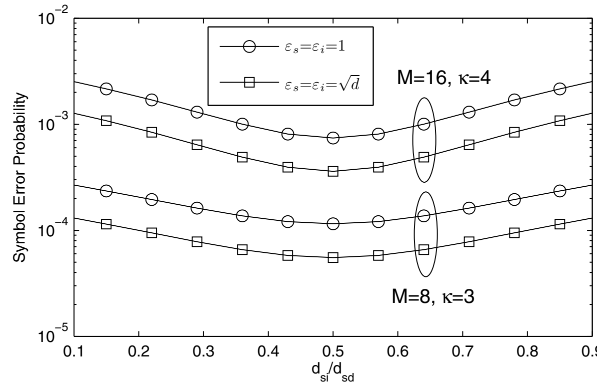

Fig. 11. Optimal relay placement: SEP as a function of d_{si}/d_{sd} ($\frac{E_s}{\sigma_n^2} = \frac{E_i}{\sigma_n^2} = 15$ dB, $\varepsilon_s = \varepsilon_i = \sqrt{d}$, and $d = 36$)

After accounting for the path loss, the average SNRs of the SR, RD, and SD links are given by $\bar{\gamma}_{si} = \frac{E_s \sigma_{si}^2}{\sigma_n^2}\left(\frac{d_0}{d_{si}}\right)^{\kappa}$, $\bar{\gamma}_{id} = \frac{E_i \sigma_{id}^2}{\sigma_n^2}\left(\frac{d_0}{d_{id}}\right)^{\kappa}$, and $\bar{\gamma}_{sd} = \frac{E_s \sigma_{sd}^2}{\sigma_n^2}\left(\frac{d_0}{d_{sd}}\right)^{\kappa}$, where κ is the path loss exponent and d_0 is the reference distance. Figure 11 plots the SEP as a function of relay location for different values of M and κ when $\frac{E_s}{\sigma_n^2} = \frac{E_i}{\sigma_n^2} = 15$ dB and $\sigma_{si}^2 = \sigma_{id}^2 = \sigma_{sd}^2 = 1$. We see that the optimal relay location is exactly in the middle between S and D. Further, the optimal relay location is insensitive to M, ε_s, ε_i, and κ.

VI. Conclusions

We analyzed the performance of fixed-gain AF relaying in the practical scenario where channel estimates required for demodulation are imperfect due to estimation noise, and are obtained using the time-efficient cascaded channel estimation protocol. The destination has an imperfect estimate of the product of the SR and RD channel gains, and does not have separate estimates of the two gains. We developed an accurate SEP analysis of a single relay system as well as an opportunistic multi-relay system. Unlike the approaches pursued in the literature, our analysis used few and specific simplifying assumptions, with the training protocol being an integral part of the system model. It led to expressions for the SEP that were accurate for SNRs as low as 1 dB for both single and multi-relay systems. It also led to a simpler, yet tight, SEP upper bound that was within 0.75 dB of the exact SEP for a single relay system. What set the multi-relay system apart from the single relay system was that in the former, not just coherent demodulation but also relay selection were based on imperfect cascaded channel estimates.

We also investigated the optimal energy allocation between the pilot and data symbols at the source and relay when both were subject to separate total energy constraints. We saw that the optimal energy allocation took a simple closed-form for a single relay system. The same form was numerically found to be optimal for the multi-relay system as well.

Several interesting avenues for research arise out of this paper. Our analysis can be extended to a frequency-selective channel by using orthogonal frequency division multiplexing, in which each of the orthogonal subcarriers sees flat-fading. SEP analysis with channel coding or under time-varying channels are other possibilities for future work. Finally, it would be interesting to extend the analysis to two-way relaying.

Appendix

A. Proof of Result 1

Substituting the conditional mean and variance of $\mathcal{D}_i$ in (12) and averaging over $\hat{h}_{sd}$ and $\hat{g}_i$, we get

$$P\left(\mathrm{Err}\right) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \mathbb{E}\left[\exp\left(-\lambda\left|\hat{h}_{sd}\right|^2 \frac{m^2}{\sin^2\theta}\right)\right] \times \mathbb{E}\left[\exp\left(-\frac{\eta_i\left|\hat{g}_i\right|^2}{\sin^2\theta}\right)\right] d\theta, \quad (28)$$

where $\eta_i \triangleq \frac{\beta_i^2 E_s m^2}{\frac{\beta_i^2 \sigma_{si}^2 \sigma_{id}^2 E_s \varpi_{sid}^2}{\alpha_i^2 \sigma_{si}^2 \sigma_{id}^2 \varepsilon_s E_s + \varpi_{sid}^2} + \sigma_{sid}^2}$ and $\lambda \triangleq \frac{E_s}{\left(\frac{E_s \sigma_{sd}^2}{\varepsilon_s E_s \sigma_{sd}^2 + \sigma_n^2} + 1\right)\sigma_n^2}$. We now evaluate the two expectation terms in the integrand above.

First term: From (1) and (5), we can see that $\hat{h}_{sd} = L_{sd}\left(\sqrt{\varepsilon_s E_s} h_{sd} + \frac{w_{sd}}{p}\right)$, where h_{sd} is independent of w_{sd}. Therefore, it follows that $\hat{h}_{sd} \sim \mathcal{CN}(\hat{\sigma}_{sd}^2)$, where $\hat{\sigma}_{sd}^2 = L_{sd}^2\left(\varepsilon_s E_s \sigma_{sd}^2 + \sigma_n^2\right)$. Hence,

$$\mathbb{E}\left[\exp\left(-\lambda\left|\hat{h}_{sd}\right|^2 \frac{m^2}{\sin^2\theta}\right)\right] = \left(1 + \frac{\varepsilon_s m^2 \bar{\gamma}_{sd}^2}{\left(1 + \left(1+\varepsilon_s\right)\bar{\gamma}_{sd}\right)\sin^2\theta}\right)^{-1}. \quad (29)$$

Second term: From (3) and (5), we get $\left|\hat{g}_i\right|^2 = L_{g_i}^2\left(\varepsilon_s E_s \alpha_i^2 \left|g_i\right|^2 + \left|w_{sid}\right|^2 + 2\sqrt{\varepsilon_s E_s}\alpha_i \left|g_i\right|\left|w_{sid}\right|\cos\phi\right)$, where ϕ is the angle between the complex numbers g_i and w_{sid}. It is uniformly distributed in $[-\pi,\pi]$, and is independent of g_i and w_{sid}. The mutual independence of g_i, w_{sid}, and ϕ implies that

$$\mathbb{E}\left[\exp\left(-\frac{\eta_i\left|\hat{g}_i\right|^2}{\sin^2\theta}\right)\right] = \mathbb{E}_{|g_i|}\left[\exp\left(-\frac{\eta_i L_{g_i}^2 \varepsilon_s E_s \alpha_i^2 \left|g_i\right|^2}{\sin^2\theta}\right) \times \mathbb{E}_{|w_{sid}|}\left(q\left(\left|w_{sid}\right|\right)\mathbb{E}_\phi\left[\Delta(\phi)\right]\right)\right], \quad (30)$$

where $\Delta(\phi) = \exp\left(-\frac{2\eta_i L_{g_i}^2 \alpha_i \sqrt{\varepsilon_s E_s}|g_i||w_{sid}|\cos\phi}{\sin^2\theta}\right)$ and $q(|w_{sid}|) = \exp\left(-\frac{\eta_i L_{g_i}^2 |w_{sid}|^2}{\sin^2\theta}\right)$. From [26, (3.339)], we get

$$\mathbb{E}_\phi[\Delta(\phi)] = I_0\left(2\frac{\eta_i L_{g_i}^2 \alpha_i \sqrt{\varepsilon_s E_s}\,|g_i|\,|w_{sid}|}{\sin^2\theta}\right), \quad (31)$$

where I_0 is the modified Bessel function of the first kind of order 0 [26, (8.406.1)].

The RV $|w_{sid}|$ is Rayleigh distributed with variance ϖ_{sid}^2. Using the above expression for $\mathbb{E}_\phi[\Delta(\phi)]$ and [26, (6.631.4)], the inner expectation in (30) simplifies to

$$\mathbb{E}_{|w_{sid}|}[q(|w_{sid}|)\mathbb{E}_\phi[\Delta(\phi)]] = \frac{1}{\frac{\eta_i L_{g_i}^2 \varpi_{sid}^2}{\sin^2\theta}+1} \times \exp\left(\frac{\eta_i^2 L_{g_i}^4 \varepsilon_s E_s \alpha_i^2 |g_i|^2 \frac{1}{\sin^4\theta}}{\frac{\eta_i L_{g_i}^2}{\sin^2\theta}+\frac{1}{\varpi_{sid}^2}}\right). \quad (32)$$

Substituting (32) in (30) and using (4), it can be shown that

$$\mathbb{E}\left[\exp\left(-\frac{\eta_i|\hat{g}_i|^2}{\sin^2\theta}\right)\right] = \frac{\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})\sin^2\theta}{m^2} \times U\left(1,1;\frac{1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}}{\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id}}+\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})\frac{\sin^2\theta}{m^2}\right). \quad (33)$$

Substituting (29) and (33) in (28) yields the desired SEP expression in (13).

B. Proof of Corollary 1

The second term $\mathbb{E}\left[\exp\left(-\frac{\eta_i|\hat{g}_i|^2}{\sin^2\theta}\right)\right]$ in the integrand in (28) can be upper bounded by replacing θ with $\pi/2$. Therefore, using (33), we get

$$\mathbb{E}\left[\exp\left(-\frac{\eta_i|\hat{g}_i|^2}{\sin^2\theta}\right)\right] \le \frac{\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})}{m^2} \times U\left(1,1;\frac{1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}}{\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id}}+\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})\frac{1}{m^2}\right). \quad (34)$$

Substituting (29) and (34) in the expression for the probability of error in (28), we get

$$P(\text{Err}) \le \frac{\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})}{\pi m^2}\int_0^{\frac{M-1}{M}\pi}\frac{\sin^2\theta}{\sin^2\theta+\frac{\varepsilon_s m^2\bar{\gamma}_{sd}^2}{1+(1+\varepsilon_s)\bar{\gamma}_{sd}}}d\theta \times U\left(1,1;\frac{1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}}{\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id}}+\frac{\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})}{m^2}\right). \quad (35)$$

Using the variable substitution $\cot\theta = x$ in in (35) and using partial fractions to simplify further yields the desired result.

C. Proof of Corollary 2

At large SNRs, we have $\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id}) = (1+\max(\varepsilon_s,\varepsilon_i))\left(\frac{1}{\varepsilon_s\bar{\gamma}_{si}}+\frac{1}{\varepsilon_i\bar{\gamma}_{id}}\right)$. Using [33, (13.5.9)], at large SNRs, we get

$$U\left(1,1;\frac{1+\varepsilon_s\bar{\gamma}_{si}+\varepsilon_i\bar{\gamma}_{id}}{\varepsilon_s\varepsilon_i\bar{\gamma}_{si}\bar{\gamma}_{id}}+\frac{\psi(\varepsilon_s,\varepsilon_i,\bar{\gamma}_{si},\bar{\gamma}_{id})}{m^2}\right) = -\zeta - \log\left(\left(\frac{1}{\varepsilon_s\bar{\gamma}_{si}}+\frac{1}{\varepsilon_i\bar{\gamma}_{id}}\right)\left(1+\frac{1+\max(\varepsilon_s,\varepsilon_i)}{m^2}\right)\right), \quad (36)$$

where ζ is the Euler-Mascheroni constant [26, (9.73)]. Similarly, at large SNRs, we have $\varphi = \frac{\varepsilon_s m^2\bar{\gamma}_{sd}}{1+\varepsilon_s}$. Therefore, for larger SNRs and, hence, for large φ, we get $f(M,\varphi) = \frac{\pi}{4\varphi}$. Substituting this expression and (36) in the SEP upper bound in (15) yields the desired result in (16).

D. Proof of Result 2

Substituting $E_s = \frac{E_s^{\text{tot}}}{d+\varepsilon_s}$ in the SEP upper bound in (18) and simplifying further, we get

$$\tilde{B}(\varepsilon_s,\varepsilon_i) = \mu_1 g(\varepsilon_s,\varepsilon_i)\log\left(\frac{\bar{\delta}_{si}}{\mu_2}\right), \quad (37)$$

where $\mu_1 = \frac{1}{4m^4 E_s^{\text{tot}}\nu_2\bar{\delta}_{si}^2}$, $\mu_2 = \left(\frac{d+\varepsilon_s}{\varepsilon_s E_s^{\text{tot}}}+\frac{d+\varepsilon_i}{\varepsilon_i E_i^{\text{tot}}\nu_1}\right)\left(\frac{1+\max(\varepsilon_s,\varepsilon_i)}{m^2}\right)$, $\nu_1 = \frac{\bar{\delta}_{id}}{\bar{\delta}_{si}}$, $\nu_2 = \frac{\bar{\delta}_{sd}}{\bar{\delta}_{si}}$, and $g(\varepsilon_s,\varepsilon_i) = \frac{(1+\varepsilon_s)(1+\max(\varepsilon_s,\varepsilon_i))(d+\varepsilon_s)}{\varepsilon_s}\left(\frac{d+\varepsilon_s}{\varepsilon_s E_s^{\text{tot}}}+\frac{d+\varepsilon_i}{\varepsilon_i E_i^{\text{tot}}\nu_1}\right)$. As $\bar{\delta}_{si}\to\infty$, we have $\log\left(\frac{\bar{\delta}_{si}}{\mu_2}\right) = \log(\bar{\delta}_{si}) + \log\left(\frac{1}{\mu_2}\right) \approx \log(\bar{\delta}_{si})$. Therefore, at large SNRs, (37) can be written as

$$\tilde{B}(\varepsilon_s,\varepsilon_i) = \mu_1 g(\varepsilon_s,\varepsilon_i)\log(\bar{\delta}_{si}). \quad (38)$$

For $\varepsilon_i \le \varepsilon_s$, $\tilde{B}(\varepsilon_s,\varepsilon_i)$ is monotonically decreasing in ε_i. Similarly, for $\varepsilon_i \ge \varepsilon_s$, $\tilde{B}(\varepsilon_s,\varepsilon_i)$ is monotonically increasing in ε_i. Therefore, $\tilde{B}(\varepsilon_s,\varepsilon_i)$ is minimized when $\varepsilon_s = \varepsilon_i$. Hence, the optimal pilot boosting factors satisfy $\varepsilon_s^* = \varepsilon_i^*$. Therefore, we only need to minimize $\tilde{B}(\varepsilon,\varepsilon)$. Using the first order condition, it can be shown that $\tilde{B}(\varepsilon,\varepsilon)$ has a unique minimum at $\varepsilon^* = \sqrt{d}$.

E. Derivation of (25)

We first evaluate the cumulative distribution function (CDF) of $|\hat{g}_q|^2$. Using (24), we get $P\left(|\hat{g}_q|^2 \le y \,\middle|\, g_q\right) = P\left(|w_{sqd}|^2 \le \frac{y}{|L_{g_q}|^2} - \varepsilon E_s\alpha_q^2|g_q|^2 \,\middle|\, g_q\right)$. Since $|w_{sqd}|^2$ is an exponential RV, we get

$$P\left(|\hat{g}_q|^2 \le y \middle| g_q\right) = \begin{cases} 1-\exp\left[-\frac{1}{\varpi_{wqd}^2}\left(\frac{y}{|L_{g_q}|^2}-\varepsilon E_s\alpha_q^2|g_q|^2\right)\right], & \text{if } |g_q|^2 \le \frac{y}{\varepsilon E_s\alpha_q^2|L_{g_q}|^2}, \\ 0, & \text{otherwise.}\end{cases} \quad (39)$$

Averaging the conditional probability in (39) over $|g_q|^2$ yields

$$P\left(|\hat{g}_q|^2 \le y\right) = \int_0^{\frac{y}{\varepsilon E_s \alpha_q^2 |L_{g_q}|^2}} p_{|g_q|^2}(z) \times \left(1 - \exp\left[-\frac{1}{\varpi_{wqd}^2}\left(\frac{y}{|L_{g_q}|^2} - \varepsilon E_s \alpha_q^2 z\right)\right]\right) dz,$$
$$= 1 - \frac{2\sqrt{y}}{\sqrt{\varepsilon E_s}\alpha_q \sigma_{sq}\sigma_{qd}|L_{g_q}|} K_1\left(\frac{2\sqrt{y}}{\sqrt{\varepsilon E_s}\alpha_q \sigma_{sq}\sigma_{qd}|L_{g_q}|}\right) - \int_0^{\frac{y}{\varepsilon E_s \alpha_q^2 |L_{g_q}|^2}} \exp\left[-\frac{1}{\varpi_{wqd}^2}\left(\frac{y}{|L_{g_q}|^2} - \varepsilon E_s \alpha_q^2 z\right)\right] p_{|g_q|^2}(z)\, dz. \quad (40)$$

Substituting (4) in (40) and then differentiating it with respect to y yields the PDF of $|\hat{g}_q|^2$. Further, using transformation of variables, we get $p_{|g_q|}(y) = 2y p_{|g_q|^2}(y^2)$, for $y \ge 0$. Simplifying further yields (25).

F. Proof of Result 3

Using the chain rule, the conditional SEP $P\left(\text{Err}\,\middle|\,\hat{h}_{sd}, \{\hat{g}_i\}_{i=1}^N, x\right)$ conditioned on x and the estimates $\hat{h}_{sd}$ and $\{\hat{g}_i\}_{i=1}^N$ can be written as

$$P\left(\text{Err}\,\middle|\,\hat{h}_{sd}, \{\hat{g}_i\}_{i=1}^N, x\right) = \sum_{k=1}^N P\left(\hat{s} = k \,\middle|\, \{\hat{g}_i\}_{i=1}^N\right) \times P\left(\text{Err}\,\middle|\,\hat{s} = k, \hat{h}_{sd}, \{\hat{g}_i\}_{i=1}^N, x\right). \quad (41)$$

From (11) and (12), the second term in (41) equals $\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \exp\left(-\frac{\lambda|\hat{h}_{sd}|^2 m^2}{\sin^2\theta} - \frac{\eta_k |\hat{g}_k|^2}{\sin^2\theta}\right) d\theta$, where η_k and λ are defined in (28).

When averaged over $\hat{h}_{sd}$ and x, the above conditional SEP simplifies to

$$P\left(\text{Err}\,\middle|\, \{\hat{g}_i\}_{i=1}^N\right) = \frac{1}{\pi}\sum_{k=1}^N P\left(\hat{s} = k \,\middle|\, \{\hat{g}_i\}_{i=1}^N\right) \times \int_0^{\frac{M-1}{M}\pi}\left(1 + \frac{\varepsilon m^2 \bar{\gamma}_{sd}^2}{(1 + (1+\varepsilon)\bar{\gamma}_{sd})\sin^2\theta}\right)^{-1} \exp\left(-\frac{\eta_k |\hat{g}_k|^2}{\sin^2\theta}\right) d\theta. \quad (42)$$

Averaging over the cascaded channel estimates, we get

$$P(\text{Err}) = \frac{1}{\pi}\sum_{k=1}^N \mathbb{E}_{|\hat{g}_k|}\left[\prod_{q=1,2,\ldots,N, q\neq k} P\left(|\hat{g}_q|^2 < |\hat{g}_k|^2 \;\middle|\; |\hat{g}_k|\right) \times \int_0^{\frac{M-1}{M}\pi}\left(1 + \frac{\varepsilon m^2 \bar{\gamma}_{sd}^2}{(1+(1+\varepsilon)\bar{\gamma}_{sd})\sin^2\theta}\right)^{-1} \exp\left(-\frac{\eta_k |\hat{g}_k|^2}{\sin^2\theta}\right) d\theta\right], \quad (43)$$

where $\mathbb{E}_{|\hat{g}_k|}$ denotes expectation over the RV $|\hat{g}_k|$. Substituting (40) in the above equation yields

$$P(\text{Err}) = \frac{1}{\pi}\sum_{k=1}^N \mathbb{E}_{|\hat{g}_k|}\left[\int_0^{\frac{M-1}{M}\pi}\left(1 + \frac{\varepsilon m^2 \bar{\gamma}_{sd}^2}{(1+(1+\varepsilon)\bar{\gamma}_{sd})\sin^2\theta}\right)^{-1} \times \exp\left(-\frac{\eta_k |\hat{g}_k|^2}{\sin^2\theta}\right) d\theta \prod_{q=1,2,\ldots,N, q\neq k}\left(1 - \nu|\hat{g}_k| K_1(\nu|\hat{g}_k|) - \int_0^{\frac{|\hat{g}_k|^2}{\varepsilon E_s \alpha_q^2 |L_{g_q}|^2}} \exp\left(-\frac{1}{\varpi_{sqd}^2}\left[\left|\frac{\hat{g}_k}{L_{g_q}}\right|^2 - \varepsilon E_s \alpha_q^2 z\right]\right) p_{|g_q|^2}(z)\, dz\right)\right], \quad (44)$$

where $\nu = \frac{2}{\sqrt{\varepsilon E_s}\alpha_q \sigma_{sq}\sigma_{qd}|L_{g_q}|}$.

Substituting the PDF of $|g_q|$ from (25), we get

$$P(\text{Err}) = \frac{1}{\pi}\sum_{k=1}^N \int_0^\infty \int_0^{\frac{M-1}{M}\pi}\left[\left(1 + \frac{\varepsilon m^2 \bar{\gamma}_{sd}^2}{(1+(1+\varepsilon)\bar{\gamma}_{sd})\sin^2\theta}\right)^{-1} \times \exp\left(-\frac{\eta_k y^2}{\sin^2\theta}\right) d\theta\right]\left[\prod_{q=1,2,\ldots,N, q\neq k}\left(1 - \nu y K_1(\nu y) - \int_0^{\frac{y^2}{\varepsilon E_s \alpha_q^2 |L_{g_q}|^2}} \exp\left(-\frac{1}{\varpi_{sqd}^2}\left[\frac{y^2}{|L_{g_q}|^2} - \varepsilon E_s \alpha_q^2 z\right]\right) p_{|g_q|^2}(z)\, dz\right)\right] \frac{2y}{\varpi_{skd}^2 |L_{g_k}|^2} \times \left[\int_0^{\frac{y^2}{\varepsilon E_s \alpha_k^2 |L_{g_k}|^2}} \exp\left(-\frac{1}{\varpi_{skd}^2}\left[\left|\frac{y}{L_{g_k}}\right|^2 - \varepsilon E_s \alpha_k^2 z\right]\right) p_{|g_k|^2}(z)\, dz\right] dy. \quad (45)$$

Substituting $y^2 = u$ in (45) and applying Gauss-Laguerre quadrature [33] on the outermost integral over the variable y yields the desired result in (26).

G. Brief Derivation of (27)

Substituting (4) in $I(a_n, q)$ and further simplifying by substituting $z = \frac{y^2 \sigma_{sq}^2 \sigma_{qd}^2}{4}$ in it yields

$$I(a_n, q) = \exp\left(-\frac{a_n}{\varpi_{sqd}^2 |L_{g_q}|^2}\right) \times \int_0^{\frac{2}{\alpha_q \sigma_{sq}\sigma_{qd}}\sqrt{\frac{a_n}{\varepsilon E_s}}} \exp\left(\frac{\varepsilon E_s \alpha_q^2 \sigma_{sq}^2 \sigma_{qd}^2 y^2}{4\varpi_{sqd}^2}\right) y K_0(y)\, dy. \quad (46)$$

In order to evaluate (46), we use the approximation $yK_0(y) \approx \exp\left(-\kappa_1 y^2 - \kappa_2\right)$, where $0 \le \kappa_1, \kappa_2 \le 1$. The parameters κ_1 and κ_2 are numerically determined by minimizing the mean squared error between $yK_0(y)$ and $\exp\left(-\kappa_1 y^2 - \kappa_2\right)$. Using the above approximation in (46) and simplifying further yields the desired result.

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