Capture-Induced, Fast, Distributed, Splitting Based Selection with Imperfect Power Control

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***Abstract*—Opportunistic selection selects the node that improves the overall system performance the most. Selecting the best node is challenging as the nodes are geographically distributed and have only local knowledge. Yet, selection must be fast to allow more time to be spent on data transmission, which exploits the selected node's services. We analyze the impact of imperfect power control on a fast, distributed, splitting based selection scheme that exploits the capture effect by allowing the transmitting nodes to have different target receive powers and uses information about the total received power to speed up selection. Imperfect power control makes the received power deviate from the target and, hence, affects performance. Our analysis quantifies how it changes the selection probability, reduces the selection speed, and leads to the selection of no node or a wrong node. We show that the effect of imperfect power control is primarily driven by the ratio of target receive powers. Furthermore, we quantify its effect on the net system throughput.**

***Index Terms*—Selection, splitting, imperfect power control, capture, medium access control protocols, throughput.**

I. Introduction

OPPORTUNISTIC selection finds applications in many wireless systems. In it, nodes are ordered according to their ability to improve the system performance, and the best one among them is selected. Formally, this ability is quantified in terms of a real-valued local *metric* that quantifies how useful the node will be if selected. For example, in the downlink of a cellular system, the metric of a user is its downlink signal-to-noise ratio (SNR). Selecting the user with the highest SNR exploits multiuser diversity and improves system throughput [1, Chap. 6]. Instead, in the proportional-fair scheduler, the ratio of the demanded to the average assigned rate is the metric [1, Chap. 6]. In amplify-and-forward relaying, which exploits spatial diversity, the metric is the harmonic mean of the source-to-relay and relay-to-destination channel gains [2]. In sensor networks, where sensor selection reduces energy consumption and increases network lifetime, a node's metric is a function of its residual energy [3].

However, since the nodes are geographically distributed, each node knows only its metric and not that of the others. Hence, distributed selection schemes are required. Broadly, two types of distributed, multiple access selection schemes have been considered in literature. The first category is timer based [2], [4]. In it, a node sets its timer as a monotonically non-increasing function of its metric, and transmits its packet to a coordinating node called sink when its timer expires. This ensures that the best node transmits first. However, the scheme can fail to select the best node if any other node's timer expires within a vulnerability window after the transmission by the best node or its timer does not expire at all [2].

The second category is splitting based [5], [6]. In it, only those nodes whose metrics lie between two thresholds transmit in a slot. At the end of every slot, the sink broadcasts an idle, success, or collision outcome to all the nodes based on whether zero, one, or multiple nodes transmitted in that slot. Accordingly, the thresholds are adjusted for the next slot. The algorithm continues until the best node is selected. It is scalable and, unlike the timer scheme, it guarantees that the best node will get selected. For example, it takes 2.5 slots to select, on average, even when the number of nodes is asymptotically large.

The splitting scheme was speeded up significantly by exploiting capture and power control by the Variable Power Multiple Access Selection Power Based Splitting scheme (VPMAS-PS) [7]. In VPMAS-PS, a node transmits only if its metric lies between two thresholds. Further, a node that transmits also adjusts its transmit power so that its received power is one of Q possible target receive power levels. The different target receive powers are such that the node with a higher target receive power can be selected even in the presence of multiple simultaneous transmissions by nodes with lower target receive powers. These would otherwise have resulted in a collision and a wasted slot.

For example, consider a system with two target receive powers, P_H and P_L, where $P_L = \gamma\sigma^2$, $P_H = \gamma(\eta P_L + \sigma^2)$, γ is the minimum signal-to-interference-plus-noise power ratio (SINR) at which the sink can decode, and σ^2 is the noise power. With this, even when as many as η nodes have target receive power P_L, a node with target receive power P_H will get selected. We shall refer to $\eta \in \mathbb{R}^+$ as the *adversary order*. Further, VPMAS-PS uses the total received power to estimate the best node's target receive power and speed up selection even more. It requires just 1.98, 1.67, and 1.55 slots, on average, to select the best node with two, three and four target receive power levels, respectively. However, in practice, power control errors at the transmitter make its receive power deviate from its target. This affects the performance of VPMAS-PS and, in general, multiple access (MAC) protocols that exploit power control [8]–[12].

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Contributions and Focus: In this paper, we analyze the effect of imperfect power control on splitting based selection. To gain quantitative insights, we focus on VPMAS-PS because it exploits power control to speed up selection. We show that imperfect power control affects it in three ways: (i) it changes the selection probability of the best node in a slot, (ii) it leads to a wrong node getting selected, and (iii) it leads to a selection outage, where no node is selected. To this end, we first characterize the time taken to select the best node. As this is a random variable (RV), which depends on the realizations of the metrics and power control errors, we derive the probability of selecting the best node in each slot. From this, statistical measures such as mean and standard deviation of the selection time can be computed. We also develop a tight lower bound for the probability of selection outage.

Our analysis is novel in two respects. Firstly, it uses an SINR based reception model [13], [14] to determine whether a packet is captured or not. It is more realistic than the abstract MAC layer collision model, in which a collision is assumed to occur anytime two or more nodes transmit simultaneously, or the protocol model for capture, in which the distances between the transmitters and the sink entirely determine when capture occurs [5], [6], [15]. Secondly, it incorporates power control error into the capture model. The effect of the peak power constraint on selection is also evaluated numerically.

While imperfect power control has been studied extensively, the focus has been on MAC algorithms, the goals of which are quite different from a selection algorithm [16]–[19]. Our analysis, while involved, helps quantify the impact of system parameters such as the number of target receive power levels, adversary order, and power control error variance. It also shows that η, which was conceived as a means to speed up selection, can also be used to improve robustness to power control errors. To the best of our knowledge, power control error has not been analyzed for MAC based selection schemes.

Our second contribution is an analysis of the net throughput of a rate-adaptive downlink, which is relevant to a system designer. It accounts for the time spent on selection and the penalties associated with selection outage or selection of a wrong node. It brings out the trade-off between the time allocated for selection and the net throughput. Our third contribution is a new, recursive expression for the average selection duration of VPMAS-PS under perfect power control. This serves as a benchmark for imperfect power control, and was hitherto determined using simulations [7].

The paper is organized as follows. Section II presents the system model. In Section III, we analyze the performance of VPMAS-PS with perfect and imperfect power control. Results in Section IV are followed by our conclusions in Section V.

II. Model

Consider a wireless system with N nodes and a sink. Each node has a real-valued metric that is uniformly distributed in $[0,1)$ [4]. This assumption does not incur a loss in generality because of the following reason. In general, let the metric μ have a cumulative distribution function (CDF) F. Then the variable transformation $\nu = F(\mu)$ yields a new metric ν that is uniformly distributed between $[0,1)$ [20]. Further, since F is a monotonically non-decreasing function, selecting the node with highest ν is the same as selecting the node with the highest μ. Assuming that the CDF F is known is reasonable because it changes at a time scale that is several orders of magnitude slower than the instantaneous metrics, and has also been assumed in [5], [21]. The metrics are independent and identically distributed (i.i.d.) RVs. Following order statistics notation, $[i]$ denotes the node with the i^{th} largest metric $\mu_{[i]}$. Selection is successful if and only if node $[1]$ is selected. Each transmission lasts for one slot duration.

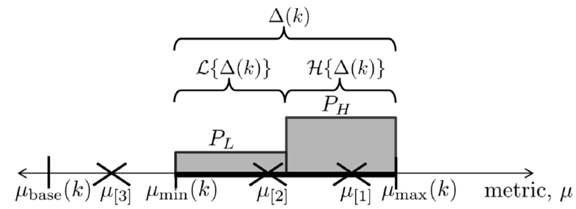

Fig. 1. Target receive powers and transmissions by different nodes based on the location of their metrics in the interval $[\mu_{\min}(k), \mu_{\max}(k))$.

In each slot $k \geq 1$, every node maintains two thresholds, namely, $\mu_{\max}(k)$ and $\mu_{\min}(k)$. The interval $\Delta(k) = [\mu_{\min}(k), \mu_{\max}(k))$ is referred to as the transmission interval in slot k, and $|\Delta(k)|$ represents its length. A node $[i]$ transmits in slot k if and only if $\mu_{[i]} \in \Delta(k)$. If it transmits in slot k, its target receive power $P_{[i]}(k)$ is either P_H or P_L, and is determined as follows:

$$P_{[i]}(k) = \begin{cases} P_H, & \mu_{[i]} \in \mathcal{H}\{\Delta(k)\} \\ P_L, & \mu_{[i]} \in \mathcal{L}\{\Delta(k)\} \end{cases}, \quad (1)$$

where $\mathcal{H}\{\Delta(k)\}$and $\mathcal{L}\{\Delta(k)\}$ represent the upper and lower halves of $\Delta(k)$, respectively.[1]

Thus, a node whose metric lies in $\mathcal{H}\{\Delta(k)\}$ has a higher target receive power, which improves the chances of it getting captured. Further, when the best node transmits, its target receive power is never less than any other transmitting node's target receive power. For example, in Fig. 1, nodes $[1]$ and $[2]$ have target receive powers P_H and P_L in slot k, as $\mu_{[1]}$ and $\mu_{[2]}$ are in $\mathcal{H}\{\Delta(k)\}$ and $\mathcal{L}\{\Delta(k)\}$, respectively. Note that the model can be generalized to cover more than two target receive power levels. However, its analysis is considerably more involved and yields limited additional insights. We, therefore, study it numerically in Section IV.

When M nodes, $[1], [2], \ldots, [M]$, transmit in slot k, the SINR of a node $[i]$ in slot k, $\text{SINR}_{[i]}(k)$, is given by

$$\text{SINR}_{[i]}(k) = \frac{P_{[i]}(k)}{\sum_{j=1, j\neq i}^{M} P_{[j]}(k) + \sigma^2}. \quad (2)$$

Node $[1]$ gets selected if $\text{SINR}_{[1]}(k) \geq \gamma$. The threshold γ depends on the modulation and coding scheme and is of the order of 8-10 dB. As in VPMAS-PS, we assume that in slot k, the sink can measure the total received power $P^{\text{tot}}(k)$ and can sense whether any transmission has occurred or not. This

[1] In general, $\Delta(k)$ can be partitioned into unequal intervals based on η. However, this yields marginal gains [7], and makes the analysis more involved.

capability exists in receivers today in the form of the receive signal strength indicator.

Transmit Power Setting: In slot k, to achieve a target receive power P at sink, a node i sets its transmit power as $\frac{P}{h_i}$, where h_i is the channel power gain of node i from itself to the sink. With this transmit power, the receive power of node i is $h_i \frac{P}{h_i} = P$. The knowledge of h_i can be acquired by periodic feedback from the sink or by exploiting reciprocity. This knowledge can be imperfect, and contributes to the power control error.

Notation: The number of nodes that have target receive power P_H and P_L in slot k are denoted by $n_{\mathrm{H}}(k)$ and $n_{\mathrm{L}}(k)$, respectively. The total number of nodes that transmit in slot k is denoted by $n_{\mathrm{T}}(k) = n_{\mathrm{H}}(k) + n_{\mathrm{L}}(k)$. Further, $X \sim \mathcal{N}(\mu, \sigma^2)$ implies that X is a Gaussian RV with mean μ and variance σ^2. The multinomial $\binom{N}{i_1, i_2, \ldots, i_p}$ denotes $\frac{N!}{(N-i_1-i_2-\cdots-i_p)! i_1! i_2! \cdots i_p!}$. The probability of an event A is denoted by $\Pr\{A\}$, and the probability of an event A conditioned on event B is denoted by $\Pr\{A|B\}$. The mean of an RV X is denoted by $\mathbb{E}[X]$. The indicator function $I_{\{x\}}$ equals 1 when x is true, and is 0 otherwise.

A. VPMAS-PS Overview

We now give a brief summary of VPMAS-PS with perfect power control, which helps set up the notation and conveys the fundamental concepts. In every slot k, each node maintains three thresholds $\mu_{\min}(k)$, $\mu_{\max}(k)$, and $\mu_{\text{base}}(k)$. As we shall see, $\mu_{[1]} \in [\mu_{\text{base}}(k), \mu_{\max}(k))$ always. The scheme is such that in any non-idle slot, the best node always transmits, and is the one that eventually gets selected.

1) Initialization: In slot 1, metrics of all the nodes lie in $[0, 1)$. Therefore, $\mu_{\text{base}}(1) = 0$ and $\mu_{\max}(1) = 1$. The threshold $\mu_{\min}(1)$ is set so that a fraction z of the N nodes, on average, transmit in slot 1. Since the metrics are uniform RVs, this implies that $\mu_{\min}(1) = \mu_{\max}(1) - (\mu_{\max}(1) - \mu_{\text{base}}(1))\, z$. Thus, $|\Delta(1)| = z$. Appendix A shows how z is chosen to maximize the success probability in slot 1 as a function of η and N. As is typical of splitting algorithms, the probability that m nodes transmit in slot 1 turns out to be insensitive to N for $N \geq 10$. This is one of the reasons why splitting based selection schemes such as VPMAS-PS are scalable.

2) Updation of Thresholds: Each node updates its thresholds for slot $k+1$ based on the outcome of slot k, which is broadcasted by the sink, as follows. The broadcast is assumed to be error-free due to its low payload [5], [6], [8].

Success in slot k, $\mathcal{S}(k)$: The algorithm terminates. The decoded node is the best node.

Collision in slot k, $\mathcal{C}(k)$: This implies that at least two nodes have their metrics in $\Delta(k)$. Since $\mu_{\min}(k) \leq \mu_{[1]}$, all nodes set $\mu_{\text{base}}(k+1) = \mu_{\min}(k)$. The sink then uses $P^{\text{tot}}(k)$ to locate where $\mu_{[1]}$ lies as follows: if $P^{\text{tot}}(k) \leq P_H$, then all the transmitting nodes have target receive power P_L and their metrics must lie in $\mathcal{L}\{\Delta(k)\}$, which becomes $\Delta(k+1)$. If $P^{\text{tot}}(k) > P_H$, then it is very likely that $\mu_{[1]}$ lies in $\mathcal{H}\{\Delta(k)\}$, which then becomes $\Delta(k+1)$. We shall refer to this as *correct splitting*.[2] Note that when a collision occurs, the length of the transmission interval is *halved* and the sink broadcasts an *additional* 1-bit information about whether or not $P^{\text{tot}}(k)$ exceeds P_H.

Idle in slot k, $\mathcal{I}(k)$: If no collision has occurred in any of the previous slots, then the N nodes' metrics must be uniformly distributed in $[\mu_{\text{base}}(k), \mu_{\min}(k))$. Hence,[3]

$$\mu_{\max}(k+1) = \mu_{\min}(k), \qquad \mu_{\text{base}}(k+1) = \mu_{\text{base}}(k), \tag{3}$$

$$\mu_{\min}(k+1) = \mu_{\max}(k+1) - [\mu_{\max}(k+1) - \mu_{\text{base}}(k+1)]z. \tag{4}$$

The above thresholds ensure that the metrics are i.i.d. and uniformly distributed in the interval $[\mu_{\text{base}}(k+1), \mu_{\max}(k+1))$. Hence, VPMAS-PS statistically behaves as it did in slot 1.

B. Imperfect Power Control

Power control errors arise due to a combination of many factors such as feedback delay, quantization, imperfect estimation, power control algorithm used, rate of adaptation of power, dynamic range of the transmitter, feedback errors, and even filtering effects at the receiver. Each of these factors affects the receive power in dB [18]. Though these factors are system-dependent and are difficult to quantify separately, by virtue of the central limit theorem, the power control error in dB is well modeled as a Gaussian RV. This implies that the power control error in linear scale is a lognormal RV. This model has been validated in systems that employ open-loop or closed-loop power control [22], [23]. It has also been verified by analysis and extensive field measurements [24]–[26].

Therefore, the power received from transmitting node $[i]$ in slot k, becomes $P_{[i]}(k)e^{l_{[i]}}$, where $e^{l_{[i]}}$ models its power control error and $l_{[i]} \sim \mathcal{N}(0, \sigma_l^2)$. When nodes $[1], [2], \ldots, [M]$ transmit, (2) changes to

$$\text{SINR}_{[i]}(k) = \frac{P_{[i]}(k)e^{l_{[i]}}}{\sum_{j=1, j\neq i}^{M} P_{[j]}(k)e^{l_{[j]}} + \sigma^2}. \tag{5}$$

Now a suboptimal node $[i] \neq [1]$ may get selected in slot k if $\text{SINR}_{[i]}(k) \geq \gamma$. Also note that even if only one node transmits in a slot k, it may not get selected due to its power control error. Henceforth, with a little abuse of terminology, this case is also referred to as a collision. Further, power control error can lead to $P^{\text{tot}}(k) \leq P_H$ despite some or all of the nodes transmitting with target receive power P_H in slot k. As a result, $\Delta(k+1) = \mathcal{L}\{\Delta(k)\}$, which is an example of an *incorrect splitting*. It can lead to a wrong node getting selected.[4]

[2]A rare event that can lead to $P^{\text{tot}}(k) > P_H$ is when all the transmitting nodes have a target receive power P_L. However, this requires that more than $\frac{P_H - \sigma^2}{P_L} = \eta\gamma + 1 - \frac{1}{\gamma} \approx \eta\gamma$ nodes have their metrics in $\mathcal{L}\{\Delta(k)\}$, which is highly unlikely. For example, for $\eta = 2$ and $\gamma = 10$, we have $\eta\gamma = 20 \gg 1$.

[3]The following rare event can also lead to an idle in slot k. A collision occurs in slot $k-1$ with $P^{\text{tot}}(k-1) > P_H$ but all the transmitting nodes' metrics lie in $\mathcal{L}\{\Delta(k-1)\}$. Hence, $\mathcal{H}\{\Delta(k-1)\}$ becomes $\Delta(k)$ and results in an idle in slot k, $\mathcal{I}(k)$. In such a case, $\mathcal{L}\{\Delta(k-1)\}$ is made $\Delta(k+1)$ and $\mu_{\text{base}}(k+1) = \mu_{\text{base}}(k)$.

[4]This issue also triggers a change in VPMAS-PS to address the case when $n_{\mathrm{T}}(k) = n_{\mathrm{H}}(k)$. In this case, an idle occurs in slot $k+1$, which would otherwise have been impossible with perfect power control. In this case, $\Delta(k+2) = \mathcal{H}\{\Delta(k)\}$.

III. Analysis of VPMAS-PS

A. With Perfect Power Control

We first develop a new, simple expression for the average number of slots required by VPMAS-PS to select the best node with perfect power control. This serves as a useful benchmark for understanding the impact of imperfect power control.

Let the RV X be the number of slots required to select the best node for a given realization of metrics. Given that m nodes transmit in a slot, let $Y(m)$ denote the number of slots required for selection including the slot in which the transmission occurred. Clearly, $Y(1) = 1$.

Result *1:* The average number of slots $\mathbb{E}[X]$ required to select the best node is given by

$$\mathbb{E}[X] = \Bigg[1 + \sum_{k=\eta+1}^{N-1} \binom{N}{1,k} \left(\frac{z}{2}\right)^{1+k} (1-z)^{N-1-k} + \sum_{i=2}^{N} \binom{N}{i} \left(\frac{z}{2}\right)^{i} \left(\mathbb{E}[Y(i)] \left(1-\frac{z}{2}\right)^{N-i} + \left(I_{\{i>\eta\gamma\}} + \mathbb{E}[Y(i)]\right)(1-z)^{N-i}\right)\Bigg] \frac{1}{1-(1-z)^N}, \quad (6)$$

where $\mathbb{E}[Y(m)] = \frac{2^m + I_{\{m>\eta\gamma\}} + m I_{\{m>\eta+1\}} + \sum_{i=2}^{m-1}\binom{m}{i}\mathbb{E}[Y(i)]}{2^m-2}$, for $m \geq 2$, and $\mathbb{E}[Y(1)] = 1$.

Proof: The proof is given in Appendix B. ■

The recursive nature of the above result is typical of splitting algorithms [5], [6].

B. With Imperfect Power Control ($\sigma_l > 0$)

We now evaluate the probability of success as a function of the slot number with imperfect power control. We then derive the selection outage probability.

1) Assumptions: To ensure analytical tractability, we make the following assumptions. While these are intuitive, in order to provide quantitative insights for the justifications, we set $\gamma = 10$ dB, $N = 100$, $\sigma_l = 3$, and $\eta = 5$ below. (i) The effect of noise when two or more nodes transmit is negligible. This is reasonable because even when two nodes have target receive power P_L, the ratio of noise power to the total received power with perfect power control is $\frac{1}{2\gamma+1} \ll 1$. (ii) The probability of the event in which nodes [1], [2], and [3] have the same target receive power and one of them gets selected is negligible. In slot 1, it is less than 0.005, and is zero with perfect power control. (iii) The probability of a collision and incorrect splitting in a slot when at least three nodes have target receive power P_H and one has target receive power P_L is negligible. In slot 1, when three nodes have target receive power P_H and one node has target receive power P_L, it is less than 0.001, and it is zero with perfect power control. (iv) The probability that at least five nodes transmit simultaneously is negligible. In slot 1, it is less than 0.015.

2) Probability of Success in Slot $k \geq 1$: We first evaluate $\Pr\{\mathcal{S}(k)\}$ for $k = 1$ and 2 exactly. Then, for $k \geq 3$, we calculate the probabilities of some sequences that contribute the most to $\Pr\{\mathcal{S}(k)\}$. This tackles the exponential increase in the number of sequences that lead to $\mathcal{S}(k)$ as k increases.

(a) Probability of Success in Slot 1: We know that

$$\Pr\{\mathcal{S}(1)\} = \Pr\{\mathcal{S}(1), n_{\mathrm{H}}(1) > 0\} + \Pr\{\mathcal{S}(1), n_{\mathrm{H}}(1) = 0\}. \quad (7)$$

The term $\Pr\{\mathcal{S}(1), n_{\mathrm{H}}(1) \geq 1\}$ is the probability of success in slot 1 when at least one node, including the best node, has target receive power P_H. The total number of ways in which $n_{\mathrm{H}}(1)$ and $n_{\mathrm{L}}(1)$ nodes can be chosen from the N nodes is $\binom{N}{n_{\mathrm{L}}(1), n_{\mathrm{H}}(1)}$. The probability that $N - n_{\mathrm{H}}(1) - n_{\mathrm{L}}(1)$ nodes are silent in slot 1 is $(1-z)^{N-n_T(1)}$. The probability that $n_{\mathrm{H}}(1)$ nodes have target receive power P_H and $n_{\mathrm{L}}(1)$ nodes have target receive power P_L in slot 1 is $\left(\frac{z}{2}\right)^{n_{\mathrm{H}}(1)}$ and $\left(\frac{z}{2}\right)^{n_{\mathrm{L}}(1)}$, respectively. A success occurs if $\mathrm{SINR}_{[1]}(1) \geq \gamma$. Hence,

$$\Pr\{\mathcal{S}(1), n_{\mathrm{H}}(1) \geq 1\} = \sum_{n_{\mathrm{L}}(1)=0}^{N-n_{\mathrm{H}}(1)} \sum_{n_{\mathrm{H}}(1)=1}^{N} \binom{N}{n_{\mathrm{L}}(1), n_{\mathrm{H}}(1)} \times (1-z)^{N-n_T(1)} \left(\frac{z}{2}\right)^{n_T(1)} \Pr\{\mathrm{SINR}_{[1]}(1) \geq \gamma\}. \quad (8)$$

The following two cases lead to the event $\{\mathcal{S}(1), n_{\mathrm{H}}(1) \geq 1\}$:

(i) $n_T(1) = n_H(1) = 1$: In this case, $\Pr\{\mathrm{SINR}_{[1]}(1) \geq \gamma\} = \Pr\left\{\frac{P_H e^{l_{[1]}}}{\sigma^2} \geq \gamma\right\} = Q\left(\frac{\ln\left(\frac{\gamma\sigma^2}{P_H}\right)}{\sigma_l}\right)$, where $Q(.)$ is the Gaussian Q-function.

(ii) $n_T(k) \geq 2$ and $n_H(1) \geq 1$: Neglecting σ^2 yields

$$\Pr\{\mathrm{SINR}_{[1]}(1) \geq \gamma\} \approx \Pr\left\{\frac{e^{l_{[1]}}}{\sum_{i=2}^{n_{\mathrm{H}}(1)} e^{l_{[i]}} + \frac{P_L}{P_H}\sum_{j=n_{\mathrm{H}}(1)+1}^{n_{\mathrm{H}}(1)+n_{\mathrm{L}}(1)} e^{l_{[j]}}} \geq \gamma\right\}. \quad (9)$$

The term $\sum_{i=2}^{n_{\mathrm{H}}(1)} e^{l_{[i]}} + \frac{P_L}{P_H}\sum_{j=n_{\mathrm{H}}(1)+1}^{n_{\mathrm{H}}(1)+n_{\mathrm{L}}(1)} e^{l_{[j]}}$ in (9) can be approximated by a lognormal RV e^{l_α}, where $l_\alpha \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$. Here, μ_α and σ_α^2 are given in closed-form by the Fenton-Wilkinson (F-W) method [27, Chap. 3]. Hence, $\Pr\{\mathrm{SINR}_{[1]}(1) \geq \gamma\} \approx \Pr\left\{\frac{e^{l_{[1]}}}{e^{l_\alpha}} \geq \gamma\right\} = Q\left(\frac{\ln(\gamma)+\mu_\alpha}{\sqrt{\sigma_l^2+\sigma_\alpha^2}}\right)$.

Similarly, the second term in (7) is given by

$$\Pr\{\mathcal{S}(1), n_{\mathrm{H}}(1) = 0\} = \sum_{n_{\mathrm{L}}(1)=1}^{N} \binom{N}{n_{\mathrm{L}}(1)} \left(\frac{z}{2}\right)^{n_{\mathrm{L}}(1)} \times (1-z)^{N-n_{\mathrm{L}}(1)} \Pr\{\mathrm{SINR}_{[1]}(1) \geq \gamma\}. \quad (10)$$

Here, for $n_{\mathrm{L}}(1) = 1$, $\Pr\{\mathrm{SINR}_{[1]}(1) \geq \gamma\} = Q\left(\frac{\ln\left(\frac{\gamma\sigma^2}{P_L}\right)}{\sigma_l}\right)$.

For $n_{\mathrm{L}}(1) \geq 2$, using the F-W method again, we get $\Pr\{\mathrm{SINR}_{[1]}(1) \geq \gamma\} \approx Q\left(\frac{\ln(\gamma)+\mu_\beta}{\sqrt{\sigma_l^2+\sigma_\beta^2}}\right)$, where $e^{l_\beta} \approx \sum_{j=2}^{n_{\mathrm{L}}(1)} e^{l_{[j]}}$ and $l_\beta \sim \mathcal{N}(\mu_\beta, \sigma_\beta^2)$.

(b) Probability of Success in Slot 2: A success in slot 2 is preceded by an idle or a collision in slot 1. Hence,

$$\Pr\{\mathcal{S}(2)\} = \Pr\{\mathcal{I}(1)\}\Pr\{\mathcal{S}(2)|\mathcal{I}(1)\} + \Pr\{\mathcal{C}(1), \mathcal{S}(2)\}. \quad (11)$$

Since the metrics are i.i.d. and uniformly distributed, VPMAS-PS effectively restarts if an idle occurs in slot 1. Hence, $\Pr\{\mathcal{S}(2)|\mathcal{I}(1)\} = \Pr\{\mathcal{S}(1)\}$. An idle in slot 1 occurs when all the N nodes do not transmit in slot 1. Hence, $\Pr\{\mathcal{I}(1)\} = (1-z)^N$.

The event $\{\mathcal{C}(1), \mathcal{S}(2)\}$ above can occur only if all the following three conditions are satisfied:

E1) In slot 1, no node gets selected, i.e., $\text{SINR}_{[i]}(1) < \gamma$, for $i = 1, \ldots, n_{\text{T}}(1)$.

E2) For a success to occur in slot 2, $\mu_{[1]}$ must lie in $\Delta(2)$. This happens if $P^{\text{tot}}(1) > P_H$ when $P_{[1]}(1) = P_H$, and if $P^{\text{tot}}(1) \leq P_H$ when $P_{[1]}(1) = P_L$.

E3) In slot 2, the best node is selected, i.e., $\text{SINR}_{[1]}(2) \geq \gamma$.

From the law of total probability, $\Pr\{\mathcal{C}(1), \mathcal{S}(2)\}$ in (11) is given by

$$\Pr\{\mathcal{C}(1), \mathcal{S}(2)\} = \Pr\{n_{\text{H}}(1) = 0, \mathcal{C}(1), \mathcal{S}(2)\} + \Pr\{n_{\text{H}}(1) \geq 1, \mathcal{C}(1), \mathcal{S}(2)\}. \quad (12)$$

In (12), the first event $\{n_{\text{H}}(1) = 0, \mathcal{C}(1), \mathcal{S}(2)\}$ occurs if all the $n_{\text{T}}(1)$ nodes have target receive power P_L and collide in slot 1, which is followed by a success in slot 2. As all the $n_{\text{T}}(1)$ nodes' metrics are in $\mathcal{L}\{\Delta(1)\}$, E2 requires $\mathcal{L}\{\Delta(1)\}$ to be $\Delta(2)$. All of these $n_{\text{T}}(1)$ nodes then transmit again in slot 2. Thus, $n_{\text{T}}(1) = n_{\text{L}}(1) = n_{\text{T}}(2) = n_{\text{H}}(2) + n_{\text{L}}(2)$. Among these $n_{\text{T}}(2)$ nodes, at least the best node should have target receive power P_H in slot 2, as otherwise a collision will occur again in slot 2. This implies that $P_{[1]}(2) = P_H$. As the length of the transmission interval is halved with each collision, the probability that a node has target receive power P_H in slot 2 is $\frac{z}{4}$, which also is the probability that a node has target receive power P_L in slot 2. Therefore,

$$\Pr\{n_{\text{H}}(1){=}0, \mathcal{C}(1), \mathcal{S}(2)\} = \sum_{n_{\text{L}}(1)=1}^{N} \sum_{n_{\text{H}}(2)=1}^{n_{\text{L}}(1)} \binom{N}{n_{\text{L}}(2), n_{\text{H}}(2)} \times \left(\frac{z}{4}\right)^{n_{\text{L}}(1)} (1-z)^{N-n_{\text{L}}(1)} \Pr\left\{\text{SINR}_{[1]}(1) < \gamma, \ldots, \text{SINR}_{[n_{\text{L}}(1)]}(1) < \gamma, P^{\text{tot}}(1) \leq P_H, \text{SINR}_{[1]}(2) \geq \gamma\right\}. \quad (13)$$

To calculate the probability term in (13), we consider the following three cases separately:

(i) $n_L(1) = 1$*:* Here, E1, E2, and E3 require $\frac{P_L e^{l_{[1]}}}{\sigma^2} < \gamma$, $P_L e^{l_{[1]}} + \sigma^2 \leq P_H$, and $\frac{P_H e^{l_{[1]}}}{\sigma^2} \geq \gamma$, respectively. These together can be satisfied if $\kappa_1 \leq l_{[1]} < \kappa_2$, where $\kappa_1 = \ln\left(\frac{\sigma^2\gamma}{P_H}\right)$ and $\kappa_2 = \ln\left(\min\left\{\frac{\sigma^2\gamma}{P_L}, \frac{P_H - \sigma^2}{P_L}\right\}\right)$. The probability α_1 that $l_{[1]}$ satisfies these constraints is

$$\alpha_1 = Q\left(\frac{\kappa_1}{\sigma_l}\right) - Q\left(\frac{\kappa_2}{\sigma_l}\right). \quad (14)$$

(ii) $n_L(1) = 2$*:* In this case, E1 requires $\frac{e^{l_{[1]}}}{e^{l_{[2]}}} < \gamma$ and $\frac{e^{l_{[2]}}}{e^{l_{[1]}}} < \gamma$, E2 requires $P_L\left(e^{l_{[1]}} + e^{l_{[2]}}\right) \leq P_H$, and E3 requires $\frac{P_H e^{l_{[1]}}}{P_L e^{l_{[2]}}} \geq \gamma$. Putting these together, we get, $l_{[2]} < \ln\left(\frac{P_H}{P_L}\right)$ and $\vartheta_1(l_{[2]}) < l_{[1]} < \vartheta_2(l_{[2]})$, where $\vartheta_1(l_{[2]}) = \ln\left(\max\left\{\frac{e^{l_{[2]}}}{\gamma}, \frac{\gamma P_L e^{l_{[2]}}}{P_H}\right\}\right)$ and $\vartheta_2(l_{[2]}) = \ln\left(\min\left\{\gamma e^{l_{[2]}}, \frac{P_H}{P_L} - e^{l_{[2]}}\right\}\right)$. Therefore, the probability α_2 that $l_{[1]}$ and $l_{[2]}$ satisfy these constraints is

$$\alpha_2 = \frac{1}{\sqrt{2\pi\sigma_l^2}} \int_{-\infty}^{\ln\left(\frac{P_H}{P_L}\right)} \left[Q\left(\frac{\vartheta_1(l_{[2]})}{\sigma_l}\right) - Q\left(\frac{\vartheta_2(l_{[2]})}{\sigma_l}\right)\right] \times e^{-\frac{l_{[2]}^2}{2\sigma_l^2}} \mathrm{d}l_{[2]}. \quad (15)$$

(iii) $n_L(1) \geq 3$*:* Here, E1 is assumed to be true (cf. Section III-B1). E2 requires $P^{\text{tot}}(1) \leq P_H$, where $P^{\text{tot}}(1) = P_L \sum_{i=1}^{n_{\text{L}}(1)} e^{l_{[i]}} = P_L\left(e^{l_{[1]}} + \sum_{i=2}^{n_{\text{H}}(2)} e^{l_{[i]}} + \sum_{j=1}^{n_{\text{L}}(2)} e^{l_{[1]}}\right) \approx P_L\left(e^{l_{[1]}} + e^{l_\varepsilon} + e^{l_\lambda}\right)$, where the lognormal RVs e^{l_ε} and e^{l_λ} approximate the sums $\sum_{i=2}^{n_{\text{H}}(2)} e^{l_{[i]}}$ and $\sum_{j=1}^{n_{\text{L}}(2)} e^{l_{[j]}}$, respectively, with $l_\varepsilon \sim \mathcal{N}(\mu_\varepsilon, \sigma_\varepsilon^2)$ and $l_\lambda \sim \mathcal{N}(\mu_\lambda, \sigma_\lambda^2)$. E3 requires $\text{SINR}_{[1]}(2) \approx \frac{P_H e^{l_{[1]}}}{P_H e^{l_\varepsilon} + P_L e^{l_\lambda}} \geq \gamma$. Altogether, it is required that $e^{l_\lambda} < \frac{P_H}{P_L}, e^{l_\varepsilon} < \frac{P_H}{P_L} - e^{l_\lambda}$ and $\vartheta_3(l_\varepsilon, l_\lambda) \leq l_{[1]} \leq \vartheta_4(l_\varepsilon, l_\lambda)$, where $\vartheta_3(l_\varepsilon, l_\lambda) = \ln\left(\gamma e^{l_\varepsilon} + \frac{\gamma P_L}{P_H} e^{l_\lambda}\right)$ and $\vartheta_4(l_\varepsilon, l_\lambda) = \ln\left(\frac{P_H}{P_L} - e^{l_\varepsilon} - e^{l_\lambda}\right)$. The probability α_3 that e^{l_λ}, e^{l_ε}, and $e^{l_{[1]}}$ satisfy the above constraints is

$$\alpha_3 \approx \frac{1}{2\pi\sigma_\varepsilon\sigma_\lambda} \int_{-\infty}^{\ln\left(\frac{P_H}{P_L}\right)} \int_{-\infty}^{\ln\left(\frac{P_H}{P_L} - e^{l_\lambda}\right)} \left[Q\left(\frac{\vartheta_3(l_\varepsilon, l_\lambda)}{\sigma_l}\right) - Q\left(\frac{\vartheta_4(l_\varepsilon, l_\lambda)}{\sigma_l}\right)\right] e^{-\frac{(l_\eta - \mu_\varepsilon)^2}{2\sigma_\varepsilon^2}} e^{-\frac{(l_\lambda - \mu_\lambda)^2}{2\sigma_\lambda^2}} \mathrm{d}l_\varepsilon \mathrm{d}l_\lambda, \quad (16)$$

where the two integrals above are evaluated numerically. Summing over the different possible values of $n_{\text{L}}(1)$ in (13) yields $\Pr\{n_{\text{H}}(1) = 0, \mathcal{C}(1), \mathcal{S}(2)\}$. Similarly $\Pr\{n_{\text{H}}(1) \geq 1, \mathcal{C}(1), \mathcal{S}(2)\}$ in (12) is derived. Summing over the probabilities of both these cases yields the expression for $\Pr\{\mathcal{C}(1), \mathcal{S}(2)\}$ in (12).

(c) Probability of Success in Slot k, $k \geq 3$: Here,

$$\Pr\{\mathcal{S}(k)\} = \Pr\{\mathcal{I}(1), \mathcal{S}(k)\} + \Pr\{\mathcal{C}(1), \mathcal{S}(k)\}. \quad (17)$$

In (17), $\Pr\{\mathcal{I}(1), \mathcal{S}(k)\} = \Pr\{\mathcal{I}(1)\}\Pr\{\mathcal{S}(k-1)\}$ because VPMAS-PS effectively starts afresh after an idle event in slot 1. The term $\Pr\{\mathcal{C}(1), \mathcal{S}(k)\}$ in (17) is the probability that a collision occurs in slot 1 and a success occurs in slot k. To evaluate it, we now analyze the following four likely sequences, namely, $A_{1,k}$, $A_{2,k}$, $A_{3,k}$, and $A_{4,k}$, that result in the event $\{\mathcal{C}(1), \mathcal{S}(k)\}$.

Sequence $A_{1,k}$: It consists of the following three events:

(i) Only one node transmits in the first $k-1$ slots and it does so with target receive power P_L. Its SINR is below γ in each of these slots.

(ii) VPMAS-PS correctly splits the transmission interval in each of these $k-1$ slots, and

(iii) In slot k, the above node transmits with target receive power P_H and gets selected.

The first, second, and third events require $\frac{P_L e^{l_{[1]}}}{\sigma^2} < \gamma$, $P_L e^{l_{[1]}} + \sigma^2 \leq P_H$, and $\frac{P_H e^{l_{[1]}}}{\sigma^2} \geq \gamma$, respectively. The probability α_1 that $l_{[1]}$ satisfies these conditions is given in (14). As the transmission interval is halved in length with each split, $|\Delta(k)| = \frac{|\Delta(1)|}{2^{k-1}} = \frac{z}{2^{k-1}}$. The probability that a node's metric lies in $\mathcal{H}\{\Delta(k)\}$ is $\frac{|\Delta(k)|}{2} = \frac{z}{2^k}$. Hence,

$$\Pr\{A_{1,k}\} = \binom{N}{1}(1-z)^{N-1}\frac{z}{2^k}\alpha_1.$$

Sequence $A_{2,k}$: It consists of the following three events:

(i) Two nodes have same target receive power and collide for the first $k-1$ slots,

(ii) The transmission interval is split correctly in each of these $k-1$ slots, and

(iii) In slot k, the above two nodes have different target receive powers and a success occurs.

As shown in Appendix C, the probability of $A_{2,k}$ is

$$\Pr\{A_{2,k}\} = \binom{N}{1,1}(1-z)^{N-2}\left(\frac{z}{2^k}\right)^2 2^{k-1}\alpha_4, \quad (18)$$

where

$$\alpha_4 = \frac{1}{\sqrt{2\pi\sigma_l{}^2}}\int_{-\infty}^{\ln\left(\frac{P_H}{P_L}\right)}\left[Q\left(\frac{\omega_1(l_{[2]})}{\sigma_l}\right) - Q\left(\frac{\omega_2(l_{[2]})}{\sigma_l}\right)\right] \times e^{-\frac{l_{[2]}^2}{2\sigma_l{}^2}}\,\mathrm{d}l_{[2]}, \quad (19)$$

$\omega_1(l_{[2]}) = \ln\left(\max\left\{\frac{e^{l_{[2]}}}{\gamma}, 1-e^{l_{[2]}}, \frac{P_L\gamma e^{l_{[2]}}}{P_H}\right\}\right)$, and $\omega_2(l_{[2]}) = \ln\left(\min\left\{\gamma e^{l_{[2]}}, \frac{P_H}{P_L} - e^{l_{[2]}}\right\}\right)$. The single integral in (19) is evaluated numerically.

Sequence $A_{3,k}$: This consists of the following three events:

(i) Three nodes have the same target receive power and collide for the first $k-1$ slots,

(ii) The transmission interval is split correctly in each of these $k-1$ slots, and

(iii) In slot k, one node has target receive power P_H and the other two nodes have target receive power P_L, which results in a success.

Collision in event (i) is assumed to be true (cf. Section III-B1). Also note that $P_{[2]}(n) = P_{[3]}(n)$, for $n = 1,\ldots,k$. Hence, nodes $[2]$ and $[3]$ can be treated as a single node with power control error $e^{l_\iota} \approx e^{l_{[2]}} + e^{l_{[3]}}$, where $l_\iota \sim \mathcal{N}(\mu_\iota, \sigma_\iota{}^2)$. As done for $A_{2,k}$, it can be shown that

$$\Pr\{A_{3,k}\} \approx \binom{N}{2,1}(1-z)^{N-3}\left(\frac{z}{2^k}\right)^3 2^{k-1}\alpha_5. \quad (20)$$

Here,

$$\alpha_5 = \frac{1}{\sqrt{2\pi\sigma_\iota{}^2}}\int_{-\infty}^{\ln\left(\frac{P_H}{P_L}\right)}\left[Q\left(\frac{\omega_3(l_\iota)}{\sigma_l}\right) - Q\left(\frac{\omega_4(l_\iota)}{\sigma_l}\right)\right] \times e^{-\frac{(l_\iota-\mu_\iota)^2}{2\sigma_\iota{}^2}}\,\mathrm{d}l_\iota, \quad (21)$$

where $\omega_3(l_\iota) = \ln\left(\max\left\{1-e^{l_\iota}, \frac{\gamma P_L e^{l_\iota}}{P_H}\right\}\right)$, and $\omega_4(l_\iota) = \ln\left(\frac{P_H}{P_L} - e^{l_\iota}\right)$. Here, the integral is evaluated numerically.

Sequence $A_{4,k}$: It consists of the following four events:

(i) Three nodes transmit and collide in slot 1,

(ii) Best node and at least one other node have the same target receive power, causing collisions in slots $2,\ldots,k-1$.

(iii) VPMAS-PS correctly splits the transmission interval in first $k-1$ slots, and

(iv) In slot k, only two nodes transmit, one with target receive power P_H and another with target receive power P_L, and a success occurs.

As shown in Appendix D, the probability of $A_{4,k}$ is

$$\Pr\{A_{4,k}\} \approx \binom{N}{1,1,1}(1-z)^{N-3}\frac{\alpha_6 z^3}{2^{2k}}\left(\frac{2^{k-1}-1}{2}\right), \quad (22)$$

where

$$\alpha_6 = \frac{1}{2\pi\sigma_l{}^2}\int_{-\infty}^{\ln\left(\frac{P_H}{P_L}\right)}\int_{-\infty}^{\ln\left(\frac{P_H}{P_L}-e^{l_{[2]}}\right)}\left[Q\left(\frac{\omega_5(l_{[2]},l_{[3]})}{\sigma_l}\right) - Q\left(\frac{\omega_6(l_{[2]},l_{[3]})}{\sigma_l}\right)\right] e^{-\frac{l_{[2]}^2+l_{[3]}^2}{2\sigma_l{}^2}}\,\mathrm{d}l_{[3]}\mathrm{d}l_{[2]}, \quad (23)$$

$\omega_5(l_{[2]},l_{[3]}) = \ln\left(\max\left\{\frac{e^{l_{[2]}}}{\gamma}, 1-e^{l_{[2]}}, \frac{\gamma P_L e^{l_{[2]}}}{P_H}\right\}\right)$, and $\omega_6(l_{[2]},l_{[3]}) = \ln\left(\min\left\{\gamma e^{l_{[2]}}, \frac{P_H}{P_L} - e^{l_{[2]}} - e^{l_{[3]}}\right\}\right)$.

3) Selection Outage Probability and Incorrect Node Selection Probability: We now derive a lower bound for the selection outage probability $\Pr\{\text{Outage}\}$, which is zero for perfect power control.

Result *2:* $\Pr\{\text{Outage}\}$ is lower bounded by

$$\Pr\{\text{Outage}\} > \frac{1}{1-(1-z)^N}\left(\sum_{\substack{i=1\\ i\neq 2}}^{N}\binom{N}{i}(1-z)^{N-i}z^i\beta_0^i + \binom{N}{2}(1-z)^{N-2}z^2(\beta_1+\beta_2)\right), \quad (24)$$

where $\beta_0 = 1 - Q\left(\frac{\ln\left(\frac{\sigma^2\gamma}{P_H}\right)}{\sigma_l}\right)$,

$$\beta_1 = \frac{1}{\sqrt{2\pi\sigma_l^2}}\int_{-\infty}^{\infty}\left[Q\left(\frac{f_L(l_{[2]})}{\sigma_l}\right) - Q\left(\frac{f_H(l_{[2]})}{\sigma_l}\right)\right] \times e^{-\frac{(l_{[2]})^2}{2\sigma_l^2}}\,dl_{[2]},$$

$$\beta_2 = \frac{1}{\sqrt{2\pi\sigma_l^2}}\int_{-\infty}^{0}\left[Q\left(\frac{g_L(l_{[1]})}{\sigma_l}\right) - Q\left(\frac{g_H(l_{[1]})}{\sigma_l}\right)\right] \times e^{-\frac{(l_{[1]})^2}{2\sigma_l^2}}\,dl_{[1]}.$$

Further, $f_L(l_{[2]}) = \ln\left(\max\left\{\frac{e^{l_{[2]}}}{\gamma}, 1 - \frac{P_L e^{l_{[2]}}}{P_H}\right\}\right)$, $f_H(l_{[2]}) = \ln\left(\min\left\{\frac{\gamma P_L e^{l_{[2]}}}{P_H}, \frac{\gamma\sigma^2}{P_H}\right\}\right)$, $g_L(l_{[1]}) = \ln\left(\frac{P_H e^{l_{[1]}}}{P_L\gamma}\right)$, and $g_H(l_{[1]}) = \ln\left(\min\left\{\gamma e^{l_{[1]}}, \frac{P_H(1-e^{l_{[1]}})}{P_L}, \frac{\gamma\sigma^2}{P_H}\right\}\right)$,

Proof: The proof is relegated to Appendix E. ■

The probability that a wrong node is selected is then given by $1 - \sum_{k=1}^{\infty}\Pr\{\mathcal{S}(k)\} - \Pr\{\text{Outage}\}$.

C. Net Throughput Analysis

We now analyze the net throughput when the metric of a node i is its downlink channel power gain g_i. The base station (BS) uses VPMAS-PS to select the node with the highest downlink channel power gain and transmits data to it. This quantifies the collective impact of delayed or incorrect node selection and selection outage on the system. We consider Rayleigh fading. Thus, g_i is modeled as an exponential RV with unit mean. The coherence time of the channel is C slots and T out of C slots are allocated for the selection process. The net throughput S is equal to

$$S = \frac{C-T}{C}\sum_{i=1}^{N}\Pr\{\text{node } [i] \text{ is selected within } T \text{ slots}\}\,\tilde{s}_{[i]},$$

where $\tilde{s}_{[i]}$ is the average rate of the system given node $[i]$ has been selected. It is given in bits/symbol by

$$\tilde{s}_{[i]} = \int_0^\infty \log_2(1 + g_{[i]}\mathrm{SNR}_{\mathrm{dl}}) f_{[i]}(g)\, dg, \tag{25}$$

where $\mathrm{SNR}_{\mathrm{dl}}$ is the fading-averaged downlink SNR and $f_{[i]}(g)$ is the probability density function of $g_{[i]}$. Using order statistics, $f_{[i]}(g) = \frac{N!}{(i-1)!(N-i)!}\sum_{m=0}^{N-i}\binom{N-i}{m}(-1)^m e^{-g(m+i)}$, for $g \geq 0$. Substituting this in (25), we get

$$\tilde{s}_{[i]} = \frac{1}{\log(2)}\frac{N!}{(i-1)!(N-i)!} \times \sum_{m=0}^{N-i}\binom{N-i}{m}(-1)^{m+1}\frac{e^{\frac{(m+i)}{\mathrm{SNR}_{\mathrm{dl}}}}}{(m+i)}\mathrm{Ei}\left(-\frac{(m+i)}{\mathrm{SNR}_{\mathrm{dl}}}\right), \tag{26}$$

where $\mathrm{Ei}\,(x) = -\int_{-x}^{\infty}\frac{e^{-t}}{t}\,dt$ is the standard exponential integral function [28].

We now evaluate $\Pr\{\text{node } [i] \text{ is selected within } T \text{ slots}\}$. For $i = 1$, the results presented in Section III-B2 for $\Pr\{\mathcal{S}(n)\}$ directly yield $\Pr\{\text{node } [1] \text{ is selected within } T \text{ slots}\}$ because it is equal to $\sum_{n=1}^{T}\Pr\{\mathcal{S}(n)\}$. The analysis of the corresponding probability terms for nodes [2] and [3] is similar except for the following two differences. Firstly, if node $[i]$ is to be selected, then, from the design of VPMAS-PS, the nodes $[1], \ldots, [i-1]$ will transmit with it so long as correct splitting occurs. Secondly, unlike the analysis for best node selection, incorrect splitting does contribute to the probability that other nodes get selected. The detailed derivations of these terms are not shown here to avoid repetition and to conserve space. The probability that node $[i]$, for $i \geq 4$, gets selected is negligible, and does not affect the net throughput.

IV. Numerical Results

We now verify the analytical results using Monte Carlo simulations that use $25,000$ runs. Unless mentioned otherwise, we use $N = 50$, $\gamma = 10$ dB, and $\sigma^2 = -110$ dBm. In each slot, the SINR, given in (5), is compared with the threshold γ to determine its outcome. All results are presented in terms of the slot duration. Typically, a slot is much smaller than the coherence interval. For example, in the IEEE 802.11 standard [29, Tbl. 17-15], it is of the order of 200 μs, after accounting for physical and MAC layer overheads.

A. Impact on Selection and Outage Probabilities

The probability of selecting the best node in a slot as a function of the slot number is plotted in Fig. 2 for $\eta = 5$ and in Fig. 3 for $\eta = 2$. This is done for different values of the power control error standard deviation σ_l. We see that the analytical and simulation results match each other well. The marginal difference between the two arises due to the lognormal approximation used in the analysis and because some unlikely sequences are not accounted for. As σ_l increases, the probability of selecting the best node in any slot decreases. Furthermore, with smaller η, VPMAS-PS is more susceptible to power control error because $\frac{P_H}{P_L}$ decreases.

With power control errors, there is a non-zero probability of selection outage and, hence, the average number of slots required to select the best node becomes unbounded unless

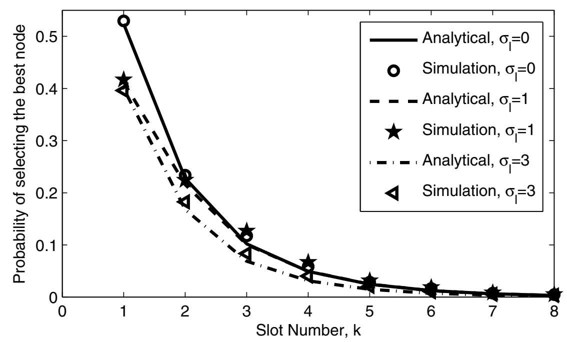

Fig. 2. Probability of selecting the best node in different slots for $\eta = 5$.

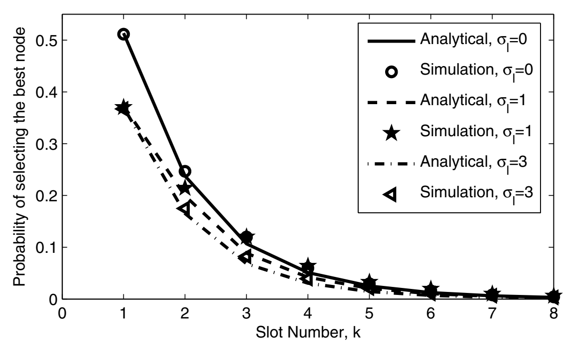

Fig. 3. Probability of selecting the best node in different slots for $\eta = 2$.

the algorithm is terminated after a maximum duration. We, therefore, compare the average number of slots required to select the best node given that the best node is selected. This is plotted in Fig. 4 for different σ_l and Q for $\eta = 2$. For two power levels, the average number increases by 11.1% from 1.98 to 2.2 slots when σ_l increases from 0 to 1. For three and four target receive power levels, the corresponding increase is 7.2% and 5.1%, respectively. As a benchmark, we also plot the average number of slots required for success for perfect power control (cf. (6)). It matches the corresponding simulation results well. We also see that VPMAS-PS is scalable, i.e., the average number of slots required for success does not decrease as N increases. As mentioned, this is because of the manner in which z depends on N (cf. Appendix A).

The selection outage probability as a function of σ_l for different η is shown in Fig. 5. Also plotted is its lower bound, which is given in (24). As before, the outage probability decreases as η increases. Consider, for example, $Q = 2$. For $\eta = 2$ and $\sigma_l = 2$, it is 2.9%, and decreases to 1% for $\eta = 5$ for the same σ_l. Similarly, the probability of selecting the wrong node is 16% for $\eta = 2$ and $\sigma_l = 1$. When η increases to 5, it decreases to 10%. Furthermore, it can also be seen that VPMAS-PS with more number of power levels is less vulnerable to power control errors. For example, for $\sigma_l = 3$ and $\eta = 2$, the selection outage probabilities for $Q = 2$, 3, and 4 are 0.08, 0.01, and 0.00, respectively.

Note: Even $\sigma_l = 2$ corresponds to a large variation in power error. This can be understood by evaluating the amount of fading (AF) [30, Chap. 1]. Intuitively, the larger the AF, the more the RV fluctuates. The AF of a lognormal RV with $\sigma_l =$

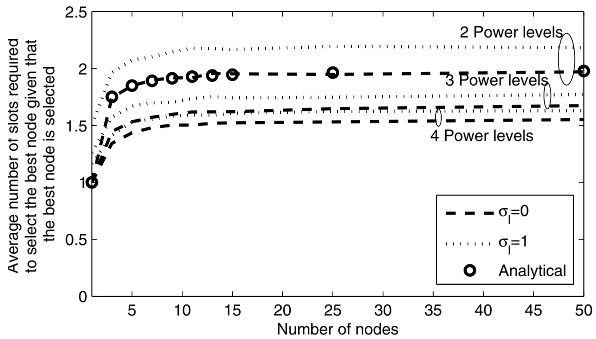

Fig. 4. Average number of slots required for selection given the best node is selected for different number of nodes N and different number of power levels Q. Analytical results for $Q = 2$ are shown using the marker ∘.

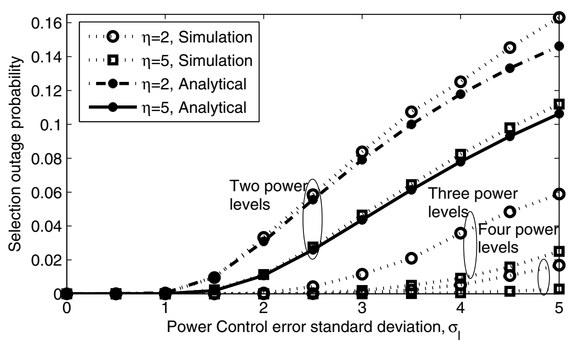

Fig. 5. Outage probability vs. power control error standard deviation, σ_l, for different adversary orders.

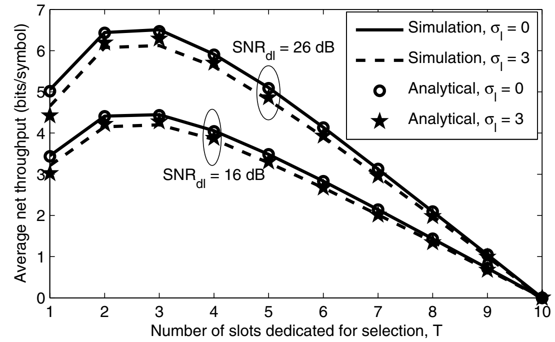

Fig. 6. Average throughput as a function of the selection duration T for different values of $\mathrm{SNR}_{\mathrm{dl}}$ ($\eta = 5$ and $N = 25$).

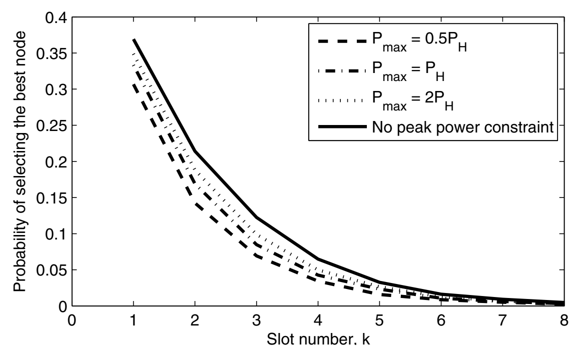

Fig. 7. Effect of peak power constraint when metric μ_i is not coupled to h_i: Probability of selecting the best node for different values of $P_{\max}$ ($\eta = 2$ and $\sigma_l = 1$).

2 is $e^{\sigma_l^2} - 1 = 53.6$, while it is just unity for a Rayleigh RV.

B. Impact on Net System Throughput

Figure 6 plots the net throughput as a function of T for different σ_l and $\mathrm{SNR}_{\mathrm{dl}}$ for $C = 10$. We see that a trade-off exists between T and the net throughput. Too small a T often leads to no node being selected, while too large a T reduces the fraction of time available for data transmission. We observe that power control error does reduce the net throughput. The difference between the analytical and simulation results is less than 7%. The minor difference arises because of the F-W approximation and because some low probability events have been neglected to make the analysis tractable.

C. Impact of Peak Power Constraint

In practice, a node's transmit power cannot exceed a peak value of $P_{\max}$. The peak power constraint affects VPMAS-PAS because a node may not be able to achieve its target receive power if its channel power gain to the sink is very low. Now, if the target receive power is P_H, then the transmit power of the node gets set as $\min\left\{\frac{P_H}{h_i}, P_{\max}\right\}$. The receive power with imperfect power control is then given by $\min\left\{P_H e^{l_i}, P_{\max} h_i\right\}$. If a node cannot even meet the lowest target receive power level of P_L, then it does not transmit.

The effect of $P_{\max}$ is negligible when the metric μ_i of a node i is tightly coupled with its channel power gain h_i to the sink. This is because the best node, which has the highest metric, will also have the highest channel gain. Thus, its transmit power is unlikely to be limited by $P_{\max}$. The other extreme, when μ_i and h_i are independent or weakly correlated, is investigated in Fig. 7 for $\sigma_l = 1$. Here, h_i is modeled as a unit mean exponential RV. Now, the lower the value of $P_{\max}$, the more is its impact on VPMAS-PS. We observe that when $P_{\max}/P_H$ is 0.5, the probability of selecting the best node in slot 1 decreases by 17.1% when compared to $P_{\max} = \infty$.

V. Conclusions

The multiple access based distributed selection algorithm VPMAS-PS facilitates capture by making each node control its transmit power such that the receive power level takes one out of $Q \geq 2$ values. By doing so, it quickly selects the best node. However, imperfect power control can cause the receive power to deviate from the target receive power. This reduces the selection probability of the best node in a slot and increases the time required to select the best node. In some cases, a suboptimal node or even no node gets selected.

We derived the success probability in each slot for the lognormal power control error model and investigated the collective impact of this error on the downlink throughput. We saw that the effect of power control error can be ameliorated by increasing the adversary order.

Appendix

A. Choosing z

Let S_r denote the probability of success given that r nodes transmitted. When $r = 1$, success always occurs. Hence, $S_r = 1$. When $2 \le r \le \eta + 1$, a success occurs when one node has target receive power P_H and the remaining $r - 1$ nodes have target receive power P_L. Given that a node transmits, the probability that the node has a target receive power P_H is $\frac{1}{2}$, which is also the probability that the node has a target receive power P_L. Hence, S_r in this case is $\frac{r}{2^r}$. For $r \ge \eta + 2$, a collision is inevitable; hence, $S_r = 0$.

If ϵ is the transmission probability of a node in slot 1, then the probability that r nodes transmit is $\binom{N}{r}\epsilon^r(1-\epsilon)^{N-r}$. Hence, the probability of success is $\sum_{r=1}^{N} S_r\binom{N}{r}\epsilon^r(1-\epsilon)^{N-r}$, and z is the value of ϵ that maximizes it.

B. Proof of Result 1

The expected number of slots required for selection, $\mathbb{E}[X]$, depends on the outcome of slot 1 as follows: (i) $\mathcal{S}(1)$: The algorithm terminates. (ii) $\mathcal{I}(1)$: Statistically, the algorithm starts afresh from slot 2. Hence, the expected number of slots required for selection is $1 + \mathbb{E}[X]$. (iii) $\mathcal{C}(1)$: For this, one of the following three mutually exclusive events occur in slot 1:

Case (i): Only one node transmits with target receive power P_H and more than η nodes have a target receive power P_L in slot 1, i.e., $n_{\mathrm{H}}(1) = 1$ and $n_{\mathrm{L}}(1) > \eta$.

Case (ii): More than one node transmits with target receive power P_H in slot 1, i.e., $n_{\mathrm{H}}(1) \ge 2$.

Case (iii): No node transmits with target receive power P_H but there are at least two nodes have a target receive power P_L in slot 1, i.e., $n_{\mathrm{H}}(1) = 0$ and $n_{\mathrm{L}}(1) \ge 2$.

In case (i), the algorithm splits $\Delta(1)$ correctly and $\Delta(2) = \mathcal{H}\{\Delta(1)\}$. Here, the node with target receive power P_H in slot 1 is the only node that transmits again in slot 2. Hence, a success occurs in slot 2. Thus, $\mathbb{E}[X] = 2$. In case (ii), $\Delta(1)$ is split correctly. Thus, $\Delta(2) = \mathcal{H}\{\Delta(1)\}$. All the $n_{\mathrm{H}}(1)$ nodes transmit again in slot 2, and the average number of slots required for selection here is $1 + \mathbb{E}[Y(n_{\mathrm{H}}(1))]$. Case (iii) can occur in two different ways, which are handled differently by VPMAS-PS: (a) $n_{\mathrm{L}}(1) \ge \eta\gamma + 1$: In this case, $P^{\mathrm{tot}}(1) = n_{\mathrm{L}}(1)P_L + \sigma^2 \ge (\eta\gamma + 1)P_L + \sigma^2 > P_H$. Hence, incorrect splitting occurs and $\Delta(2) = \Delta(1)$. In this case, an idle occurs in slot 2. VPMAS-PS then sets $\Delta(3) = \mathcal{L}\{\Delta(1)\}$, and all the $n_{\mathrm{L}}(1)$ nodes transmit in slot 3. Hence, $\mathbb{E}[X] = 2 + \mathbb{E}[Y(n_{\mathrm{L}}(1))]$. (b) $n_{\mathrm{L}}(1) \le \eta\gamma$: In this case, VPMAS-PS correctly infers that the metric of the best node lies in the lower half of $\Delta(1)$, chooses $\Delta(2)$ accordingly, and all the $n_{\mathrm{L}}(1)$ nodes transmit again in slot 2. Hence, $\mathbb{E}[X] = 1 + \mathbb{E}[Y(n_{\mathrm{L}}(1))]$. Therefore,

$$\begin{aligned}\mathbb{E}[X] &= \Pr\{\mathcal{S}(1)\} + 2\sum_{k=\eta+1}^{N-1}\Pr\{n_{\mathrm{H}}(1) = 1, n_{\mathrm{L}}(1) = k\} \\ &+ (1 + \mathbb{E}[X])\Pr\{\mathcal{I}(1)\} + \sum_{i=2}^{N}\left[(1 + \mathbb{E}[Y(i)])\Pr\{n_{\mathrm{H}}(1) = i\}\right. \\ &\left.+ \left(1 + I_{\{i>\eta\gamma\}} + \mathbb{E}[Y(i)]\right)\Pr\{n_{\mathrm{H}}(1) = 0, n_{\mathrm{L}}(1) = i\}\right]. \quad (27)\end{aligned}$$

As $\Pr\{\mathcal{C}(1)\} = \sum_{k=\eta+1}^{N-1}\Pr\{n_{\mathrm{H}}(1) = 1, n_{\mathrm{L}}(1) = k\} + \sum_{i=2}^{N}(\Pr\{n_{\mathrm{H}}(1) = i\} + \Pr\{n_{\mathrm{H}}(1) = 0, n_{\mathrm{L}}(1) = i\})$ and $\Pr\{\mathcal{S}(1)\} + \Pr\{\mathcal{I}(1)\} + \Pr\{\mathcal{C}(1)\} = 1$, (27) becomes

$$\begin{aligned}\mathbb{E}[X] &= 1 + \mathbb{E}[X]\Pr\{\mathcal{I}(1)\} + \sum_{k=\eta+1}^{N-1}\Pr\{n_{\mathrm{H}}(1) = 1, n_{\mathrm{L}}(1) = k\} \\ &+ \sum_{i=2}^{N}\left[\mathbb{E}[Y(i)]\Pr\{n_{\mathrm{H}}(1) = i\}\right. \\ &\left.+ \left(I_{\{i>\eta\gamma\}} + \mathbb{E}[Y(i)]\right)\Pr\{n_{\mathrm{H}}(1) = 0, n_{\mathrm{L}}(1) = i\}\right]. \quad (28)\end{aligned}$$

In (28), substituting $\Pr\{\mathcal{I}(1)\} = (1 - z)^N$, $\Pr\{n_{\mathrm{H}}(1) = 1, n_{\mathrm{L}}(1) = k\} = \binom{N}{1,k}\left(\frac{z}{2}\right)^{1+k}(1 - z)^{N-1-k}$, $\Pr\{n_{\mathrm{H}}(1) = i\} = \binom{N}{i}\left(\frac{z}{2}\right)^i\left(1 - \frac{z}{2}\right)^{N-i}$, and $\Pr\{n_{\mathrm{H}}(1) = 0, n_{\mathrm{L}}(1) = i\} = \binom{N}{i}\left(\frac{z}{2}\right)^i(1 - z)^{N-i}$, and then rearranging its terms yields (6).

Derivation of $\mathbb{E}[Y(m)]$: Given $m \ge 2$ nodes transmitted in slot k, i.e., $n_{\mathrm{T}}(k) = n_{\mathrm{H}}(k) + n_{\mathrm{L}}(k) = m$, only one of the following three cases could have occurred:

(a) $n_H(k) = 0$: All the m nodes have target receive power P_L in slot k. If $n_{\mathrm{L}}(k) \le \eta\gamma$, VPMAS-PS splits $\Delta(k)$ correctly, and all of them transmit again in slot $k+1$. Otherwise, an idle occurs in slot $k + 1$, and all the m nodes transmit again in slot $k + 2$. Hence, the average number of slots required for selection is $1 + I_{\{m>\eta\gamma\}} + \mathbb{E}[Y(m)]$.

(b) $n_H(k) = 1$: One node has target receive power P_H, while the other $m - 1$ nodes have target receive power P_L. The node with target receive power P_H gets decoded in slot k only if $m - 1 \le \eta$. Otherwise, its SINR is less than γ and it gets selected only in slot $k+1$. Hence, in this case, the average number of slots required for selection is $1 + I_{\{m-1>\eta\}}$.

(c) $n_H(k) = i$, for $i \ge 2$: In this case, i nodes transmit in slot $k + 1$. Hence, $1 + \mathbb{E}[Y(i)]$ slots on average are required to select the best node.

As the probability that i out of m nodes have target receive power P_H is $\binom{m}{i}\frac{1}{2^m}$, we get

$$\begin{aligned}\mathbb{E}[Y(m)] &= \frac{1}{2^m}\left[1 + I_{\{m>\eta\gamma\}} + \mathbb{E}[Y(m)]\right. \\ &\left.+ m\left(1 + I_{\{m-1>\eta\}}\right) + \sum_{i=2}^{m}\binom{m}{i}(1 + \mathbb{E}[Y(i)])\right]. \quad (29)\end{aligned}$$

Rearranging terms and simplifying yields the desired recursion for $\mathbb{E}[Y(m)]$.

C. Probability of Sequence $A_{2,k}$

The first event of $A_{2,k}$ requires $\frac{e^{l_{[1]}}}{e^{l_{[2]}}} < \gamma$ and $\frac{e^{l_{[2]}}}{e^{l_{[1]}}} < \gamma$. The second event requires $P^{\mathrm{tot}} = P_H(e^{l_{[1]}} + e^{l_{[2]}}) > P_H$, when both the nodes that transmit have target receive power P_H, and $P_L(e^{l_{[1]}} + e^{l_{[2]}}) \le P_H$ when both the nodes have target receive power P_L. The third event requires $\frac{P_H e^{l_{[1]}}}{P_L e^{l_{[2]}}} \ge \gamma$. Altogether, if $l_{[1]}$ and $l_{[2]}$ are constrained as $\omega_1(l_{[2]}) < l_{[1]} < \omega_2(l_{[2]})$, $l_{[2]} < \ln\left(\frac{P_H}{P_L}\right)$, where $\omega_1(l_{[2]})$ and $\omega_2(l_{[2]})$ are given just below (19), then the three events that define $A_{2,k}$ will occur. The probability α_4 that $l_{[1]}$ and $l_{[2]}$ satisfy the above mentioned constraints is given in (19).

For the nodes to have the same target receive power in the first $k-1$ slots, their metrics must lie in one of the 2^{k-1} possible transmission intervals, namely, $\mathcal{R}_0^{(k)}, \mathcal{R}_1^{(k)}, \ldots, \mathcal{R}_{2^{k-1}-1}^{(k)}$, each of length $\frac{z}{2^{k-1}}$, in slot k, where $\mathcal{R}_i^{(k)} = \left[\mu_{\min}(1) + i\frac{z}{2^{k-1}}, \mu_{\min}(1) + (i+1)\frac{z}{2^{k-1}}\right)$.[5] Each of these intervals defines a unique sequence of target receive powers for the two nodes in the first $k-1$ slots. This is illustrated in Fig. 8 for $k=4$. For example, if metrics of both the nodes lie in $\mathcal{R}_1^{(4)}$, then both will have target receive powers P_L, P_L, and P_H in slots 1, 2, and 3, respectively.

The third event requires that, in slot k, the metrics of the two nodes lie in two different halves of $\Delta(k)$, each of length $\frac{z}{2^k}$. This occurs with probability $\frac{z^2}{2^{2k}}$. The probability that $\mu_{[1]}$ and $\mu_{[2]}$ lie in $\mathcal{R}_i^{(k)}$, $0 \le i \le 2^{k-1}-1$, and $A_{2,k}$ occurs is then $\binom{N}{1,1}(1-z)^{N-2}\frac{z^2}{2^{2k}}\alpha_4$. Hence, adding up the probabilities over all the transmission intervals yields (18).

D. Probability of Sequence $A_{4,k}$

In $A_{4,k}$, the first event $\mathcal{C}(1)$ occurs if:

Case (a): All the three nodes that transmit have the same target receive power, in which case a collision is certain, or

Case (b): Two nodes have target receive power P_H while the third node has target receive power P_L, and $\frac{P_H e^{l_{[1]}}}{P_H e^{l_{[2]}} + P_L e^{l_{[3]}}} < \gamma$, $\frac{P_H e^{l_{[2]}}}{P_H e^{l_{[1]}} + P_L e^{l_{[3]}}} < \gamma$, and $\frac{P_L e^{l_{[3]}}}{P_H e^{l_{[1]}} + P_H e^{l_{[2]}}} < \gamma$. The last condition is very likely and can, thus, be ignored (cf. Section III-B1). Combining the first two conditions, we get $\frac{e^{l_{[2]}}}{\gamma} - \frac{P_L}{P_H}e^{l_{[3]}} < e^{l_{[1]}} < \gamma\left(e^{l_{[2]}} + \frac{P_L}{P_H}e^{l_{[3]}}\right)$.

The second event involves collisions in slots $2, \ldots, k-1$, in which the best node and at least one other node have the same target receive power. In case three nodes transmit, then the conditions given in cases (a) and (b) of the first event of sequence $A_{4,k}$ ensure that a collision occurs. In case two nodes transmit, then a collision occurs if $\frac{e^{l_{[1]}}}{e^{l_{[2]}}} < \gamma$ and $\frac{e^{l_{[2]}}}{e^{l_{[1]}}} < \gamma$, i.e., $\frac{e^{l_{[2]}}}{\gamma} < e^{l_{[1]}} < \gamma e^{l_{[2]}}$.

The third event requires correct splitting of $\Delta(m)$, for $1 \le m \le k-1$. This happens if $P^{\text{tot}}(m) > P_H$ when $P_{[1]}(m) = P_H$, and $P^{\text{tot}}(m) \le P_H$ when $P_{[1]}(m) = P_L$. If three nodes have a target receive power P_H, correct splitting happens if $P_H(e^{l_{[1]}} + e^{l_{[2]}} + e^{l_{[3]}}) > P_H$, which is equivalent to $1 - e^{l_{[2]}} - e^{l_{[3]}} < e^{l_{[1]}}$. Instead, if the three nodes have a target receive power P_L, then correct splitting happens if $e^{l_{[2]}} < \frac{P_H}{P_L}$, $e^{l_{[3]}} < \frac{P_H}{P_L} - e^{l_{[2]}}$, and $e^{l_{[1]}} < \frac{P_H}{P_L} - e^{l_{[2]}} - e^{l_{[3]}}$. Similarly, in case the collision is between two nodes with target receive power P_H and one node with target receive power P_L, then correct splitting requires $1 - e^{l_{[2]}} - \frac{P_L}{P_H}e^{l_{[3]}} < e^{l_{[1]}}$. Similarly, when the collision is due to two nodes transmitting at target receive power P_H, then correct splitting requires $1 - e^{l_{[2]}} < e^{l_{[1]}}$, and when it is between two nodes with target receive power P_L, correct splitting requires $e^{l_{[2]}} < \frac{P_H}{P_L}$ and $e^{l_{[1]}} < \frac{P_H}{P_L} - e^{l_{[2]}}$.

[5]If both the nodes have target receive power P_L for the first $k-1$ slots, then only $P_L(e^{l_{[1]}} + e^{l_{[2]}}) \le P_H$ needs to be satisfied for correct splitting in second event. Similarly, if both nodes have target receive power P_H for first $k-1$ slots, then only $P_L(e^{l_{[1]}} + e^{l_{[2]}}) \ge P_H$ needs to be satisfied. However, we include both these conditions in the constraints mentioned above for $l_{[1]}$ and $l_{[2]}$ to simplify the calculations. As a result, the calculated probability of sequence $A_{2,k}$ is marginally lower than its actual value.

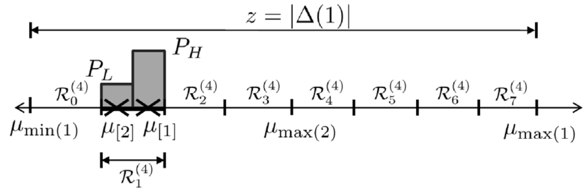

Fig. 8. Illustration of metric realizations that lead to the sequence $A_{2,4}$.

The fourth event requires $\frac{P_H e^{l_{[1]}}}{P_L e^{l_{[2]}}} \ge \gamma$.

As in Appendix C, we require that all the conditions on $l_{[1]}$, $l_{[2]}$, and $l_{[3]}$ that come out of the four events hold simultaneously. We then get $\omega_5(l_{[2]}, l_{[3]}) < l_{[1]} < \omega_6(l_{[2]}, l_{[3]})$, $l_{[2]} < \ln\left(\frac{P_H}{P_L}\right)$, and $l_{[3]} < \ln\left(\frac{P_H}{P_L} - l_{[2]}\right)$, where $\omega_5(l_{[2]}, l_{[3]})$ and $\omega_6(l_{[2]}, l_{[3]})$ are given just below (23). The probability α_6 that the above constraints are satisfied is given by (23). As in Appendix C, the intervals in which the metrics of the three nodes should lie for all the above events to occur can be enumerated. Summing the probabilities over all these intervals yields (22).

E. Proof of Result 2

We arrive at a lower bound for the outage probability by considering the events when exactly i, for $1 \le i \le N$, out of N nodes transmit in the first non-idle slot. Let slot $k \ge 1$ be the first non-idle slot. Thus, $n_{\mathrm{T}}(k) = i$. We treat the cases of $n_{\mathrm{T}}(k) = 1, 2$, and ≥ 3 separately below.

1) $n_T(k) = 1$: Here, an outage occurs only if the SNR of the transmitting node does not exceed γ even when it has target receive power P_H, i.e., $\frac{P_H e^{l_{[1]}}}{\sigma^2} < \gamma$. Hence, it can never be decoded. The probability that slot k is the first non-idle slot and only one node transmits in it is $(1-z)^{N(k-1)}\binom{N}{1}(1-z)^{N-1}z$. Therefore, $\Pr\{\text{Outage, slot } k \text{ is first non-idle slot}, n_{\mathrm{T}}(k) = 1\} = (1-z)^{N(k-1)}\binom{N}{1}(1-z)^{N-1}z\beta_0$, where β_0 is defined in Result 2.

2) $n_T(k) = 2$: If both nodes have the same target receive power, then $\frac{e^{l_{[1]}}}{e^{l_{[2]}}} < \gamma$ and $\frac{e^{l_{[2]}}}{e^{l_{[1]}}} < \gamma$ ensures that a collision occurs and no node gets selected. As each collision is followed by a splitting of $\Delta(k)$, eventually these two nodes will have different target receive powers in some slot $m > k$. For selection outage, none of the nodes should get selected even when they have different target receive powers. This happens when $\frac{P_H e^{l_{[1]}}}{P_L e^{l_{[2]}}} < \gamma$ and $\frac{P_L e^{l_{[2]}}}{P_H e^{l_{[1]}}} < \gamma$.[6] This is followed by further splitting, which can cause one of the following two events that lead to an outage:

Event A: If $P^{\text{tot}}(m) = P_H e^{l_{[1]}} + P_L e^{l_{[2]}} > P_H$, then $\Delta(m+1) = \mathcal{H}\{\Delta(m)\}$. Hence, only node $[1]$ can transmit in the subsequent slots. Outage will occur only if the power control error of node $[1]$ is such that even if it transmits with the higher target receive power P_H, its received SNR, $\frac{P_H e^{l_{[1]}}}{\sigma^2}$, is less than γ. All of these conditions can be written as $f_L(l_{[2]}) < l_{[1]} <$

[6]It may happen that the two nodes have different target receive powers in the first non-idle slot. In this case, they need not satisfy the conditions $\frac{e^{l_{[1]}}}{e^{l_{[2]}}} < \gamma$ and $\frac{e^{l_{[2]}}}{e^{l_{[1]}}} < \gamma$ together. Including these two conditions together leads to a tractable lower bound on the outage probability, which is the aim of this derivation.

$f_H(l_{[2]})$, where $f_L(l_{[2]})$, $f_H(l_{[2]})$, and the probability β_1 that these conditions are satisfied are given in the result statement. The probability that slot k is the first non-idle slot and only two nodes transmit in it is $(1-z)^{N(k-1)}\binom{N}{2}(1-z)^{N-2}z^2$. Hence, $\Pr\{\text{Slot } k \text{ is } 1^{\text{st}} \text{ non-idle slot}, n_{\mathrm{T}}(k)=2, \text{Event A}\} = (1-z)^{N(k-1)}\binom{N}{2}(1-z)^{N-2}z^2\beta_1$.

Event B: The differences with respect to *A* are $P^{\text{tot}}(m) \leq P_H$ and $\mathcal{L}\{\Delta(m)\}$ becomes $\Delta(m+1)$. It can be shown that $\Pr\{\text{Slot } k \text{ is } 1^{\text{st}} \text{ non-idle slot}, n_{\mathrm{T}}(k)=2, \text{Event B}\} = (1-z)^{N(k-1)}\binom{N}{2}(1-z)^{N-2}z^2\beta_2$, where β_2 is defined in Result 2.

3) $n_T(k) \geq 3$: If the power control error of each transmitting node is such that even if the node transmits alone with target receive power P_H, its SNR does not exceed γ, then an outage is inevitable. This occurs with probability $(1-z)^{N(k-1)}\binom{N}{n_{\mathrm{T}}(k)}(1-z)^{N-n_{\mathrm{T}}(k)}(z\beta_0)^{n_{\mathrm{T}}(k)}$, where β_0 is given in Result 2.

Summing up the probabilities of outage for different $n_{\mathrm{T}}(k)$ and for different k yields (24).

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