Optimal Binary Power Control for Underlay CR With Different Interference Constraints and Impact of Channel Estimation Errors

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***Abstract*—Adapting the power of secondary users (SUs) while adhering to constraints on the interference caused to primary receivers (PRxs) is a critical issue in underlay cognitive radio (CR). This adaptation is driven by the interference and transmit power constraints imposed on the secondary transmitter (STx). Its performance also depends on the quality of channel state information (CSI) available at the STx of the links from the STx to the secondary receiver and to the PRxs. For a system in which an STx is subject to an average interference constraint or an interference outage probability constraint at each of the PRxs, we derive novel symbol error probability (SEP)-optimal, practically motivated binary transmit power control policies. As a reference, we also present the corresponding SEP-optimal continuous transmit power control policies for one PRx. We then analyze the robustness of the optimal policies when the STx knows noisy channel estimates of the links between the SU and the PRxs. Altogether, our work develops a holistic understanding of the critical role played by different transmit and interference constraints in driving power control in underlay CR and the impact of CSI on its performance.**

***Index Terms*—Cognitive radio, underlay, fading, interference, estimation errors, symbol error probability, power control.**

I. Introduction

COGNITIVE RADIO (CR) promises efficient usage of scarce spectral resources [1]. One mode of access in CR that has attracted considerable interest is underlay CR, in which secondary users (SUs) share the same spectrum with primary users (PUs) and transmit opportunistically under tight interference constraints [2], [3]. These constraints limit the data rate and reliability of communication between the SUs. Therefore, interference-aware transmit power control by the secondary transmitter (STx) is an important problem in underlay CRs. The interference constraint and the transmit power constraint imposed on the STx, together drive this power control.

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Different interference constraints have been studied in the literature, and lead to different transmit power policies. Broadly, they can be classified into the following categories:

1) Interference Power Constraint: In which the interference that the STx causes to the primary receiver (PRx) is constrained. In the peak interference constraint, the instantaneous interference power is constrained to lie below a threshold [3]–[7], whereas in the average interference constraint, the fading-averaged interference power is constrained to lie below a threshold [4]–[6], [8]. The average interference constraint is less restrictive than the peak power constraint, and is well-suited for situations in which transmissions between the PUs span multiple coherence intervals.

2) Interference Outage Probability Constraint: In which the instantaneous interference power at the PRx cannot exceed a threshold τ more than P_{out} fraction of time [2], [9], [10]. This constraint is suited for situations in which the PU transmissions do not last long enough to average over multiple coherence intervals [5]. It has been used to design the primary exclusive zone to protect the PUs in CR networks [1], [11]. Furthermore, it includes the peak interference constraint [3]–[7], [12], as a special case ($P_{\text{out}} = 0$).

3) Signal-to-Interference-Plus-Noise-Ratio (SINR)-Based Outage Constraint: In which the instantaneous SINR of the primary signal at the PRx cannot fall below a threshold [13]–[16]. While this can guarantee a quality of service at the PU, it requires channel state information (CSI) about the primary transmitter (PTx)–PRx link at the STx. This is practically infeasible when the PUs operate oblivious to the presence of the SUs, as is often assumed in CR.

We now discuss the relevant literature on underlay CR.

A. Related Literature on Optimal Transmission Policies for Underlay CR

Several papers on CR have developed different optimal transmit power control policies for some of the above interference constraints [4]–[8], [13]–[15]. The papers also differ in the transmit power constraint imposed on the STx. We refer the reader to [9] and the references therein for a survey of the optimal policies that have been developed assuming perfect CSI. In the following, we discuss the papers that investigate underlay CR with imperfect CSI.

The effect of various channel knowledge scenarios, peak transmit power, and SINR-based outage constraints on the capacity of a CR system is investigated in [16]. In [17], the outage probability of the primary link and the channel capacity of a peak interference constrained SU is evaluated. The mean capacity of an SU under peak transmit power and peak interference constraints is studied in [3]. In [2], the ergodic capacity of CR is evaluated under an interference outage probability constraint and an average or peak transmit power constraint. In [9], a symbol error probability (SEP)-optimal transmit power control policy is developed for an SU that is subject to the interference outage probability and peak transmit power constraints.

While [13] develops a transmit power control policy that maximizes the rate of the SU under peak transmit power and SINR-based outage probability constraints, [14] derives policies that maximize the ergodic capacity and outage rate of the SU subject to average transmit power and SINR-based outage probability constraints. The problem of maximizing the SU ergodic rate subject to average interference and average sum transmit power constraints and with multiple PRxs that operate on orthogonal bands is studied in [18]. This is done for perfect CSI and then for quantized CSI. In [19], SU ergodic rate is maximized under average transmit power constraint and peak interference constraints at multiple PRxs that operate over the same frequency band for perfect and quantized CSI. In [10], expressions are developed for the optimal power allocation and ergodic capacity of a CR under different levels of knowledge about the STx to secondary receiver (SRx) link and STx–PRx link subject to an average or a peak transmit power constraint and an interference outage probability constraint.

B. Focus and Contributions

From the discussion above, we see that several different combinations of objective functions and constraints have been studied in the literature. However, optimal binary power control policies, in which the STx transmits with a fixed power P_s or with zero power, subject to either average interference or interference outage probability constraints have not been investigated fully in the literature.

Binary power control (BPC) is practically important because it enables the STx to employ more energy-efficient power amplifiers (PAs) than linear PAs. It is also theoretically interesting and different from continuous power control (CPC) because the analytical techniques required for arriving at the optimal policies and the optimal policies themselves are different. For example, for CPC, the optimal power is often obtained by differentiating a Lagrangian type function that incorporates the objective function and constraints [18], [20]. On the other hand, no such differentiation is possible for BPC since the power takes a discrete set of values. It is for this reason that BPC is studied separately in the literature, and has been referred to as bang-bang power control [21], [22].

Our key contributions are as follows. We consider a novel model in which the underlay CR shares the same frequency band with N PUs and is subject to an average interference constraint or an interference outage probability constraint. This is more general than the single PRx model, which has been considered in [2]–[5], [8], [13], [14].

We first develop two novel SEP-optimal BPC policies for the two interference constraints when the STx has perfect CSI about the STx–SRx link and the links between the STx and the PRxs. As our results show, these two policies behave differently. For example, for $N = 1$, unlike the optimal policy for the average interference constraint, the optimal policy for the interference outage probability constraint makes the STx transmit even when the channel power gain of the STx–PRx link is large.

As a reference, we then present the corresponding optimal CPC policies, in which the STx is allowed to transmit with any power between 0 and P_s, for the two interference constraints for $N = 1$. This also enables us to characterize the loss in performance incurred by the restriction to BPC.

We then analyze the performance of the above binary policies with imperfect CSI, both in terms of the fading-averaged SEP of the secondary system and the interference caused to the PRx.[1] This is an essential step in understanding their robustness in practical scenarios, in which the CSI is imperfect. We show that the impact of imperfect CSI depends on the interference constraint.

C. Outline and Notation

We present the system model in Section II. The optimal BPC policies are developed in Section III. The corresponding optimal CPC policies are developed in Section IV. The impact of imperfect CSI is analyzed in Section V. Numerical results and our conclusions follow in Sections VI and VII, respectively.

The notation $X \sim \exp(\lambda_0)$ means that the random variable (RV) X is exponentially distributed with mean λ_0, and $X \sim \mathcal{CN}(0, \delta)$ means that X is a circular symmetric complex Gaussian RV with zero mean and variance δ. The expectation with respect to X is denoted by $\mathbf{E}_X[.]$. The probability of an event A is denoted by $\Pr(A)$.

II. System Model

We consider an underlay CR network in which the SU shares the same frequency band with N PRxs. The STx transmits data to the SRx, and, in the process, interferes with the PRxs. The STx, SRx, and PRxs are single antenna terminals. Let h denote the instantaneous channel power gain of the STx–SRx link, which we shall refer to as the *data link*, and let g_i denote the instantaneous channel power gain of the link between the STx and the i^{th} PRx, which we shall refer to as the i^{th} *interference link*. The channel power gains $h, g_1, \ldots, g_N$ are assumed to be mutually independent of each other, which is

[1] An alternate problem formulation would be to minimize the fading-averaged SEP at the SRx given imperfect CSI itself. Examples of related approaches are [2], [23], though the problem formulations in these papers are different from ours. Such a formulation is beyond the scope of this paper because it involves developing optimal power control policies as a function of the true channel given its estimate, which makes the problem more involved. We also note that other objective functions such as signal-to-noise-ratio (SNR), outage probability, and capacity have been considered in the literature. We focus on the fading-averaged SEP because it is a classical measure of the reliability of communication and has been studied in the underlay CR literature [7], [8], [24].

justified when the nodes are sufficiently spatially separated. We assume that $g_1, \ldots, g_N$ are identically distributed to ensure analytical tractability. All the links undergo Rayleigh fading. Thus, $h \sim \exp(\sigma_h^2)$ and $g_i \sim \exp(\sigma_g^2)$.

A. Data Transmission and Interference Model

The STx transmits a data symbol x, which is chosen with equal probability from an MPSK or MQAM constellation, to the SRx. The signal y_{cr} received at the SRx and the interference I_i seen by the i^{th} PRx are given by

$$y_{\text{cr}} = \sqrt{P(h, \mathbf{g})}\sqrt{h}e^{j\varsigma_h}x + n_{\text{cr}} + n_{\text{P}}, \tag{1}$$

$$I_i = \sqrt{P(h, \mathbf{g})}\sqrt{g_i}e^{j\varsigma_{g_i}}x, \quad 1 \le i \le N, \tag{2}$$

where $P(h, \mathbf{g}) \in \{0, P_s\}$ is the transmit power of the STx, which depends in general on h and $\mathbf{g}$, and $\mathbf{E}[|x|^2] = 1$. Also, ς_h and ς_{g_i} are the phases of the complex baseband channel gains of the data and the i^{th} interference link, respectively. Furthermore, $n_{\text{cr}} \sim \mathcal{CN}(0, \sigma^2)$ is the noise at the SRx and $n_{\text{P}} \sim \mathcal{CN}(0, \sigma_0^2)$ is the net interference at the SRx due to transmissions from the PTxs.

The Gaussian interference assumption, in general, is a worst case model for the interference [1], [5], [25]. Its validity depends on the channel fading statistics of the PTx–SRx link, the statistics of the signal transmitted by the PTx, and the number of PTxs. For example, it is valid when the PTxs are far away from the SRx [2], [5]–[8], [12]. Even with one PTx, the Gaussian model is valid when a constant amplitude signal is sent by the PTx [9]. It ensures analytical tractability and gives valuable insights. Thus, $n_{cr} + n_{\text{P}} \sim \mathcal{CN}(0, \sigma^2 + \sigma_0^2)$. Note that shadowing is not modeled in order to make the SEP analysis tractable [4], [13], [14].

B. BPC Policy and Interference Constraints

A BPC policy θ is a mapping $\theta : (\mathbb{R}^+)^{N+1} \to \{0, P_s\}$ that determines the STx transmit power $P(h, \mathbf{g})$ for every realization of h and $\mathbf{g}$. And, a *feasible BPC policy* is defined as one that satisfies the interference constraint under consideration. When required for clarity of exposition, for a policy θ, we shall denote its fading-averaged SEP by $\overline{\text{SEP}}_\theta$ and its transmit power by $P_\theta(h, \mathbf{g})$.

The STx is subject to one of the following two constraints:

1) *Average interference constraint*: This constraint mandates that the fading-averaged interference power at the i^{th} PRx due to transmissions from the STx is less than or equal to a threshold I_{th}, i.e., $\mathbf{E}_{h,\mathbf{g}}[P(h, \mathbf{g})g_i] \le I_{\text{th}}$.
2) *Interference outage probability constraint*: This constraint mandates that the probability that the instantaneous interference power at the i^{th} PRx exceeds an interference threshold τ is less than or equal to a target outage probability P_{out}, i.e., $\Pr(P(h, \mathbf{g})g_i > \tau) \le P_{\text{out}}$.

C. CSI Assumptions

We initially assume perfect CSI, in which the STx knows the channel power gain h of its local data link, as has been assumed in [4]–[6], [13]–[15], and the interference links' power gains $g_1, \ldots, g_N$. No knowledge of the phase of any channel gain is required at the STx. The STx can acquire knowledge of $\mathbf{g}$ using reciprocity, which implies that the channel gain from the STx to the PRx is the same as the channel gain from the PRx to the STx [26], [27]. Thus, the STx can estimate g_i by overhearing transmissions by the i^{th} PRx when this PRx transmits. Such transmissions by the PRxs occur when the PTxs and PRxs are engaged in a two-way communication. In a similar manner, the STx can also know h. This estimation precedes the data transmission by the STx.

In order to coherently demodulate the transmissions by the STx, the SRx knows the complex baseband channel gain $\sqrt{h}e^{j\varsigma_h}$, which is a classical assumption [28]. However, it knows neither g_i nor ς_{g_i}, for $1 \le i \le N$. Subsequently, in Section V, we also analyze the scenario with imperfect CSI.

D. SEP: Preliminaries

For ease of exposition, we develop the theory for MPSK. Generalizations to other constellations such as MQAM are discussed later in Section V-C.

Let $\text{SEP}(P(h, \mathbf{g}), h)$ denote the resultant instantaneous SEP given the transmit power $P(h, \mathbf{g})$ and h. For MPSK with constellation size M, it is given by [29]

$$\text{SEP}(P(h, \mathbf{g}), h) = \frac{1}{\pi}\int_0^{\Lambda\pi} \exp\left(-\frac{P(h, \mathbf{g})h\sin^2\left(\frac{\pi}{M}\right)}{(\sigma^2 + \sigma_0^2)\sin^2\phi}\right) d\phi, \tag{3}$$

where $\Lambda = 1 - \frac{1}{M}$. Setting $P(h, \mathbf{g}) = 0$ in (3), we get the SEP for zero transmit power as Λ.[2]

III. Optimal BPC Policies With Perfect CSI

We now present an optimal BPC policy θ^* that minimizes the fading-averaged SEP of the SU subject to an average interference constraint at each of the N PRxs. The optimization problem, which optimizes over the space of all policies, can be formulated as follows:

$$\begin{aligned} \min_{\theta} \quad & \mathbf{E}_{h,\mathbf{g}}\left[\text{SEP}\left(P_\theta(h, \mathbf{g}), h\right)\right], \\ \text{s.t.} \quad & \mathbf{E}_{h,\mathbf{g}}\left[P_\theta(h, \mathbf{g})g_i\right] \le I_{\text{th}}, \quad i = 1, 2, \ldots, N, \\ & P_\theta(h, \mathbf{g}) \in \{0, P_{\text{s}}\}. \end{aligned} \tag{4}$$

A. Optimal BPC Policy for Average Interference Constraint

Let $\mathcal{R}(p_1, \ldots, p_N; \mu_1, \ldots, \mu_N)$ denote an N-dimensional hypercube whose left lower corner is at $(p_1, \ldots, p_N)$ and the length along the i^{th} dimension is μ_i. Let $\Omega \triangleq (\mathbb{R}^+)^N$. Let $P^*(h, \mathbf{g})$ denote the optimal transmit power given h and $\mathbf{g}$.

[2] In effect, the system is penalized with the worst case SEP value of Λ every time the STx transmits with zero power [8]. The zero transmit power case needs to be accounted for in the fading-averaged SEP calculations since the SRx is not *a priori* aware of when the STx transmits with zero power. This also ensures that the trivial policy in which the STx never transmits and, thus, never interferes with the PRxs, does not become the optimal solution, as this would be unreasonable.

Result 1: If $P_s\sigma_g^2 \le I_{\rm th}$, then $P^*(h,\mathbf{g}) = P_s, \forall \mathbf{g}, h$. Else, let $\beta > 0$ be the unique solution of the equation

$$P_s \int_0^{\beta} \dots \int_0^{\beta} g_1 \mathrm{p}(g_1)\dots \mathrm{p}(g_N)\, dg_1 \dots dg_N = I_{\rm th}, \quad (5)$$

where $\mathrm{p}(g_i)$ denotes the PDF of g_i. Then,

$$P^*(h,\mathbf{g}) = \begin{cases} P_s, & \mathbf{g} \in \mathcal{R}(0,\dots,0;\beta,\dots,\beta), \forall h, \\ 0, & \mathbf{g} \in \Omega \setminus \mathcal{R}(0,\dots,0;\beta,\dots,\beta), \forall h. \end{cases} \quad (6)$$

For Rayleigh fading, (5) simplifies to

$$P_s\left(1 - e^{-\beta/\sigma_g^2}\right)^{N-1}\left(\sigma_g^2 - \sigma_g^2 e^{-\beta/\sigma_g^2} - \beta e^{-\beta/\sigma_g^2}\right) = I_{\rm th}, \quad (7)$$

and the solution to this equation gives the value of β. For $N = 1$, β is given in closed-form as

$$\beta = -\sigma_g^2\left(1 + W\left(\frac{1}{e}\left[\frac{I_{\rm th}}{P_s\sigma_g^2} - 1\right]\right)\right), \quad (8)$$

where $W(\cdot)$ is the Lambert-W function [30].

Proof: The proof is relegated to Appendix A. ■

B. Optimal BPC Policy for Interference Outage Probability Constraint

Next, we determine an optimal BPC policy that minimizes the fading-averaged SEP of the SU subject to an interference outage probability constraint at each of the N PRxs. The optimization problem can be stated as follows:

$$\begin{aligned} \min_{\theta} \quad & \mathbf{E}_{h,\mathbf{g}}\left[\mathrm{SEP}(P_\theta(h,\mathbf{g}),h)\right], \\ \text{s.t.} \quad & \Pr(P_\theta(h,\mathbf{g})g_i > \tau) \le P_{\rm out}, \quad i = 1,2,\dots,N, \\ & P_\theta(h,\mathbf{g}) \in \{0, P_{\rm s}\}. \end{aligned} \quad (9)$$

Result 2: Let α be the unique solution of the equation

$$\begin{aligned} P_{\rm out} = & \Pr(P_s g_1 > \tau, g_1 \ge \alpha) \\ & + \Pr(P_s g_1 > \tau, 0 \le g_1 < \alpha, g_2 \ge \alpha) + \cdots \\ & + \Pr(P_s g_1 > \tau, 0 \le g_1 < \alpha, 0 \le g_2 < \alpha, \dots, g_N \ge \alpha). \end{aligned} \quad (10)$$

If $\tau/P_s \ge \alpha$, then the optimal transmit power $P^*(h,\mathbf{g})$ is

$$P^*(h,\mathbf{g}) = P_s, \ \forall \mathbf{g}, h. \quad (11)$$

Else, if $\tau/P_s < \alpha$,

$$P^*(h,\mathbf{g}) = \begin{cases} P_s, & \mathbf{g} \in \mathcal{R}(0,\dots,0;\frac{\tau}{P_s},\dots,\frac{\tau}{P_s}), \ \forall h, \\ P_s, & \mathbf{g} \in \Omega \setminus \mathcal{R}(0,\dots,0;\alpha,\dots,\alpha), \forall h, \\ 0, & \text{otherwise}. \end{cases} \quad (12)$$

For Rayleigh fading, (10) simplifies to

$$e^{\frac{-\alpha}{\sigma_g^2}} + \left(e^{\frac{-\tau}{P_s\sigma_g^2}} - e^{\frac{-\alpha}{\sigma_g^2}}\right)\left[1 - \left(1 - e^{\frac{-\alpha}{\sigma_g^2}}\right)^{N-1}\right] = P_{\rm out}, \quad (13)$$

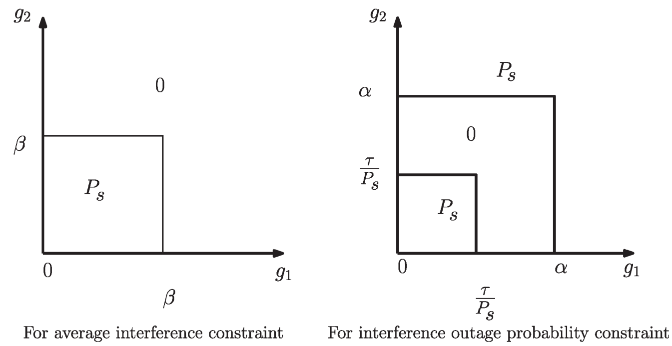

Fig. 1. Structure of optimal BPC policies ($N = 2$).

and the solution to this equation gives the value of α. For $N = 1$, α is given in closed-form as $\alpha = -\sigma_g^2 \ln(P_{\rm out})$.

Proof: The proof is relegated to Appendix B. ■

Comments: Fig. 1 illustrates the optimal BPC policies for both interference constraints for $N = 2$. Both are independent of h, as has also been seen in [4]–[7]. To compare the two policies, consider $N = 1$. While the average interference constraint mandates the STx to transmit at P_s only for low to moderate values of g_1, the interference outage probability constraint mandates the STx to transmit at P_s not only when g_1 is weak but also when it is strong. Intuitively, this is because the latter constraint penalizes the fraction of time the instantaneous interference exceeds the threshold τ, but not the extent by which it exceeds τ.

IV. Optimal CPC Policies With Perfect CSI

As a reference, we now develop optimal CPC policies for the aforementioned interference constraints. Now, the STx can transmit with any real-valued power in the interval $[0, P_s]$. Thus, the STx is subject to a peak-power constraint, which is also well-motivated by PA design constraints. We focus on the case with $N = 1$ PRx in this section. The general case with multiple PRxs is an interesting avenue for future work.

A CPC policy θ is now a mapping $\theta : (\mathbb{R}^+)^2 \to [0, P_s]$ that determines the STx transmit power $P(h,g)$ for every realization of h and g. And, a *feasible CPC policy* is one that satisfies the interference constraint under consideration.

A. Optimal CPC Policy for Interference Outage Probability Constraint

The optimal CPC policy is as follows.

Proposition 1: Let α' be the unique solution of the equation $1 - F_g(\alpha') = P_{\rm out}$, where $F_g(\cdot)$ denotes the cumulative distribution function (CDF) of g. If $\frac{\tau}{P_s} \ge \alpha'$, then the optimal transmit power $P^*(h,g)$ is $P^*(h,g) = P_s, \forall g, h$. Else,

$$P^*(h,g) = \begin{cases} P_s, & 0 \le g \le \frac{\tau}{P_s} \text{ or } g > \alpha', \ \forall h, \\ \frac{\tau}{g}, & \frac{\tau}{P_s} < g \le \alpha', \ \forall h. \end{cases} \quad (14)$$

For Rayleigh fading, $\alpha' = -\sigma_g^2 \ln(P_{\rm out})$.

Proof: The proof is given in [9]. ■

We again see that the optimal policy for the interference outage probability constraint does not depend on h.

B. Optimal CPC Policy for Average Interference Constraint

The exact SEP expression in (3) is in the form of a single integral, and is intractable for the purposes of finding the optimal CPC policy for the average interference constraint. To gain analytical insights, we, therefore, minimize its integral-free Chernoff upper bound:

$$\mathrm{SEP}(P(h,g),h) \leq \Lambda \exp\left(\frac{-P(h,g)h\sin^2\left(\frac{\pi}{M}\right)}{(\sigma^2+\sigma_0^2)}\right). \quad (15)$$

This form is similar to the integral-free SEP approximations used in [28]. One of its advantages is that it directly applies to several other constellations such as MPAM, MQAM, M-DPSK, and MFSK [28]. The optimization problem is then

$$\begin{aligned} \min_{\theta} \quad & \mathbf{E}_{h,g}\left[\Lambda \exp\left(\frac{-P_\theta(h,g)h\sin^2\left(\frac{\pi}{M}\right)}{(\sigma^2+\sigma_0^2)}\right)\right], \\ \text{s.t.} \quad & \mathbf{E}_{h,g}\left[P_\theta(h,g)g\right] \leq I_{\text{th}}, \\ & P_\theta(h,g) \in [0,P_s]. \end{aligned} \quad (16)$$

Proposition 2: The SEP-optimal CPC policy for an average interference and a peak transmit power constrained CR is as follows. If $P_s\sigma_g^2 \leq I_{\text{th}}$, then the optimal transmit power is $P^*(h,g) = P_s, \forall g,h$. Else,

$$P^*(h,g) = \begin{cases} 0, \ g \geq \frac{h\Lambda\sin^2\left(\frac{\pi}{M}\right)}{\lambda(\sigma^2+\sigma_0^2)}, \forall h, \\ P_s, g \leq \frac{h\Lambda\sin^2\left(\frac{\pi}{M}\right)}{\lambda(\sigma^2+\sigma_0^2)} e^{\frac{-P_s h \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2+\sigma_0^2}}, \forall h, \\ \frac{(\sigma^2+\sigma_0^2)}{h\sin^2\left(\frac{\pi}{M}\right)} \ln\left(\frac{h\Lambda\sin^2\left(\frac{\pi}{M}\right)}{g\lambda(\sigma^2+\sigma_0^2)}\right), \ \text{else.} \end{cases} \quad (17)$$

The value of the constant $\lambda > 0$ is set such that $\mathbf{E}_{h,g}[P^*(h,g)g] = I_{\text{th}}$.

Proof: The proof is given in Appendix C. ∎

Comments: A comparison of the optimal BPC and CPC policies reveals several interesting similarities and differences. While the transmit power is not dependent on h for CPC with the interference outage probability constraint and for BPC with the interference outage probability or average interference constraints, it does depend on h for the average interference constraint. For the interference outage probability constraint, the optimal BPC and CPC policies both make the STx transmit at P_s not only when the interference link is weak but also when it is strong. On the other hand, for the average interference constraint, the optimal policies shut the STx down when the interference link is strong.

V. Impact of Imperfect CSI of Interference Links

We now analyze how robust the BPC policies are to imperfect estimates of the N interference links. We develop expressions for the fading-averaged SEP, and the average interference or the interference outage probability with imperfect CSI. Perfect knowledge about h, which is a link internal to the secondary system, is assumed as before.

Channel Estimation Model: Let $\omega_i = \sqrt{g_i}e^{j\varsigma_{g_i}}$ denote the complex baseband channel gain of the link from the STx to the i^{th} PRx. Then, the signal y_{p_i} received at the STx when the i^{th} PRx transmits a pilot p is

$$y_{\text{p}_i} = \sqrt{E_p}\sqrt{g_i}e^{j\varsigma_{g_i}}p + n_i, \quad (18)$$

where E_p is the pilot SNR, $|p|^2 = 1$, and $n_i \sim \mathcal{CN}(0,1)$ is the noise during estimation.

Given the observable y_{p_i} at the STx, the minimum mean square error (MMSE) estimate $\widehat{\omega}_i$ is given by [28]

$$\widehat{\omega}_i = \frac{\sqrt{E_p}\sigma_g^2}{E_p\sigma_g^2+1}p^* y_{p_i}. \quad (19)$$

The estimate $\hat{g}_i$ of the channel power gain is then $\hat{g}_i = |\widehat{\omega}_i|^2$. Let $\hat{\mathbf{g}} = [\hat{g}_1, \ldots, \hat{g}_N]$.

A. Impact of Imperfect CSI on BPC Designed for Average Interference Constraint

For $P_s\sigma_g^2 \leq I_{\text{th}}$, the average interference of the BPC policy derived in Result 1 at any PRx is $P_s\sigma_g^2$, since the STx transmits with power P_s always irrespective of $\hat{\mathbf{g}}$. For $P_s\sigma_g^2 > I_{\text{th}}$, the average interference at the i^{th} PRx is given as follows.

Proposition 3: For $P_s\sigma_g^2 > I_{\text{th}}$, the average interference $\widehat{I}_i$ at the i^{th} PRx with imperfect CSI is given by

$$\widehat{I}_i = P_s\sigma_g^2\left(1-e^{-\frac{\beta}{\eta_2}}\right)^{N-1}\left(1-e^{-\frac{\beta}{\eta_2}} - \frac{\beta}{\sigma_g^2}e^{-\frac{\beta}{\eta_2}}\right), \quad (20)$$

where $\eta_2 = E_p\sigma_g^4/(E_p\sigma_g^2+1)$.

Proof: The proof is relegated to Appendix D. ∎

We now derive the fading-averaged SEP of the optimal transmit power policy for MPSK. For $P_s\sigma_g^2 \leq I_{\text{th}}$, the STx transmits at power P_s regardless of $\hat{\mathbf{g}}$. In this regime, the fading-averaged SEP can be shown to be equal to

$$\overline{\mathrm{SEP}} = \Lambda - \frac{\Upsilon}{2} - \frac{\Upsilon}{\pi}\tan^{-1}\left(\Upsilon\cot\left(\frac{\pi}{M}\right)\right), \quad (21)$$

where $\Upsilon = \sqrt{\frac{P_s\sin^2(\frac{\pi}{M})\sigma_h^2}{P_s\sin^2(\frac{\pi}{M})\sigma_h^2+\sigma^2+\sigma_0^2}}$. Otherwise, it is given in closed-form as follows.

Proposition 4: For $P_s\sigma_g^2 > I_{\text{th}}$ and MPSK, the fading-averaged SEP of the SU with imperfect CSI is given by

$$\overline{\mathrm{SEP}} = \Lambda - \left(\frac{\Upsilon}{2} + \frac{\Upsilon}{\pi}\tan^{-1}\left[\Upsilon\cot\left(\frac{\pi}{M}\right)\right]\right)\left(1-e^{-\frac{\beta}{\eta_2}}\right)^N. \quad (22)$$

Proof: The proof is relegated to Appendix E. ∎

B. Impact of Imperfect CSI on BPC Designed for Interference Outage Probability Constraint

For $\tau/P_s > \alpha$, the interference outage probability of the policy in Result 2 can be shown to be equal to $e^{-\tau/(P_s\sigma_g^2)}$, since the STx transmits at P_s regardless of $\hat{g}$. For $\tau/P_s \leq \alpha$, it is given in closed-form as follows.

Proposition 5: For $\tau/P_s \leq \alpha$, the interference outage probability at the i^{th} PRx with imperfect CSI is given by

$$\begin{aligned}\widehat{O}_i =& \left[1-\left(1-e^{-\frac{\alpha}{\eta_2}}\right)^{N-1}\right] e^{\frac{-\tau}{P_s\sigma_g^2}} + \left(1-e^{-\frac{\alpha}{\eta_2}}\right)^{N-1} \\ &\times \left[e^{\frac{-\tau}{P_s\sigma_g^2}} Q_1\left(\sqrt{a'\rho}, \sqrt{\frac{2\alpha}{\eta_2(1-\rho)}}\right)\right. \\ &\left. + e^{\frac{-\alpha}{P_s\sigma_g^2}}\left(1-Q_1\left(\sqrt{a'}, \sqrt{\frac{2\rho\alpha}{\eta_2(1-\rho)}}\right)\right)\right] \\ &+ \left(1-e^{\frac{-\tau}{\eta_2 P_s}}\right)^{N-1} \\ &\times \left[e^{\frac{-\tau}{P_s\sigma_g^2}}\left(1-Q_1\left(\sqrt{a'\rho}, \sqrt{a'\sigma_g^2/\eta_2}\right)\right)\right. \\ &\left. - e^{\frac{-\tau}{\eta_2 P_s}}\left(1-Q_1\left(\sqrt{a'}, \sqrt{a'\rho\sigma_g^2/\eta_2}\right)\right)\right], \quad (23)\end{aligned}$$

where $a' = 2\tau/(P_s\sigma_g^2(1-\rho))$ and $Q_1(\cdot,\cdot)$ is the Marcum-Q function [29].

Proof: The proof is relegated to Appendix F. ■

We now derive the fading-averaged SEP with imperfect CSI for MPSK. For $\tau/P_s > \alpha$, the STx transmits at power P_s regardless of $\hat{\mathbf{g}}$. In this regime, it is given by (21). Otherwise, it is given in closed-form as follows.

Proposition 6: For $\tau/P_s \leq \alpha$ and MPSK, the fading-averaged SEP of the SU with imperfect CSI is given by

$$\begin{aligned}\overline{\text{SEP}} =& \Lambda - \left(\frac{\Upsilon}{2} + \frac{\Upsilon}{\pi}\tan^{-1}\left(\Upsilon\cot\left(\frac{\pi}{M}\right)\right)\right) \\ &\times \left[\left(1-e^{\frac{-\tau}{P_s\eta_2}}\right)^N + 1 - \left(1-e^{-\frac{\alpha}{\eta_2}}\right)^N\right]. \quad (24)\end{aligned}$$

Proof: The proof is relegated to Appendix G. ■

C. Generalizations to Square-MQAM

For square-MQAM, the instantaneous SEP is given by [29]

$$\begin{aligned}&\text{SEP}(P(h,\mathbf{g}),h) \\ &= \frac{4m}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(\frac{-1.5P(h,\mathbf{g})h}{(M-1)(\sigma^2+\sigma_0^2)\sin^2\phi}\right) d\phi \\ &- \frac{4m^2}{\pi}\int_0^{\frac{\pi}{4}} \exp\left(\frac{-1.5P(h,\mathbf{g})h}{(M-1)(\sigma^2+\sigma_0^2)\sin^2\phi}\right) d\phi, \end{aligned} \quad (25)$$

where $m = 1-(1/\sqrt{M})$. From (3) and (25), we see that the instantaneous SEP of square-MQAM has a form similar to that for MPSK. Specifically, for both MPSK and MQAM, the SEP is an exponentially decaying function of power. Therefore, the development and analysis of the optimal binary and continuous power control policies for MQAM turns out to be similar to that for MPSK, and is not repeated here to conserve space.

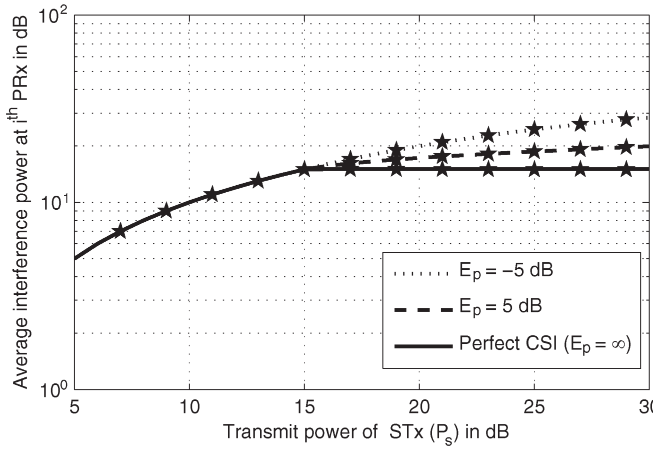

Fig. 2. Impact of imperfect CSI and transmit power of STx on the average interference at the i^{th} PRx ($I_{\text{th}} = 15$ dB and $N = 2$). Simulation results are shown using $\star$ and analytical results using lines.

VI. Numerical Results and Discussion

We now present Monte Carlo simulation results that use 10^5 samples to verify the analytical results and quantify the combined effect of transmit power and interference constraints, and imperfect CSI. Unless mentioned otherwise, $\sigma_g^2 = 1$, $\sigma_h^2 = 5$, and $\sigma^2 + \sigma_0^2 = 1$. We first present results for the average interference constraint and then for the interference outage probability constraint. We then compare the performance of the optimal binary and CPC policies.

A. BPC for Average Interference Constraint

Fig. 2 plots the average interference at the i^{th} PRx as a function of the transmit power P_s for different pilot SNRs E_p for $I_{\text{th}} = 15$ dB and $N = 2$. The analysis and the simulation results match very well. For $P_s\sigma_g^2 \leq I_{\text{th}}$, the STx transmits at P_s always. Therefore, the average interference is $P_s\sigma_g^2$ and it increases with P_s for both perfect ($E_p = \infty$) and imperfect CSI. The trends are different, however, in the interference-constrained regime ($P_s\sigma_g^2 > I_{\text{th}}$). Here, with perfect CSI, the average interference remains constant at I_{th}. However, with imperfect CSI, the average interference increases as P_s increases and exceeds that with perfect CSI. As expected, the larger the pilot SNR, the closer is the average interference curve to that for the perfect CSI case.

Fig. 3 plots the fading-averaged SEP as a function of P_s for different E_p for $I_{\text{th}} = 15$ dB, $N = 2$, and QPSK. Again, the analysis and the simulation results match well. For $P_s\sigma_g^2 \leq I_{\text{th}}$, the STx transmits at P_s always, irrespective of the channel states of the STx–PRx links for both perfect and imperfect CSI. Therefore, the fading-averaged SEP decreases as P_s increases for all E_p. The behavior changes in the interference-constrained regime ($P_s\sigma_g^2 > I_{\text{th}}$). With perfect CSI, as P_s increases, the fading-averaged SEP increases. This is because as P_s increases, the region over which the STx transmits with non-zero power P_s shrinks (cf. Result 1) so as to satisfy the average interference constraint. Thus, if P_s is allowed to be optimized, then it is optimal for the STx to set it to 15 dB even if the STx is capable of transmitting with a higher power. With imperfect CSI, the

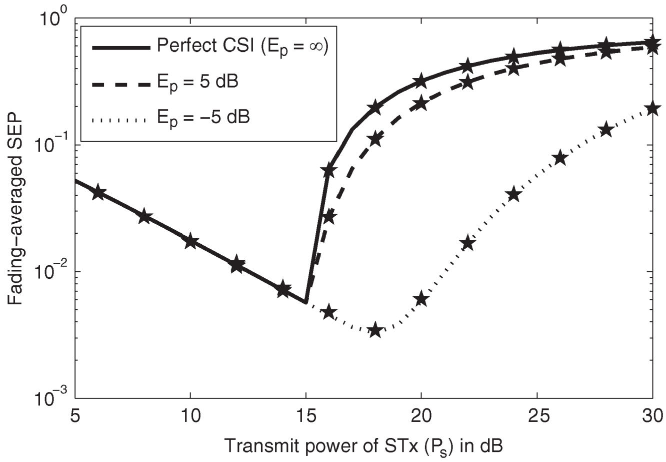

Fig. 3. Impact of imperfect CSI and transmit power of STx on the SEP of an average interference constrained SU ($I_{\rm th} = 15$ dB, QPSK, and $N = 2$). Simulation results are shown using $\star$ and analytical results using lines.

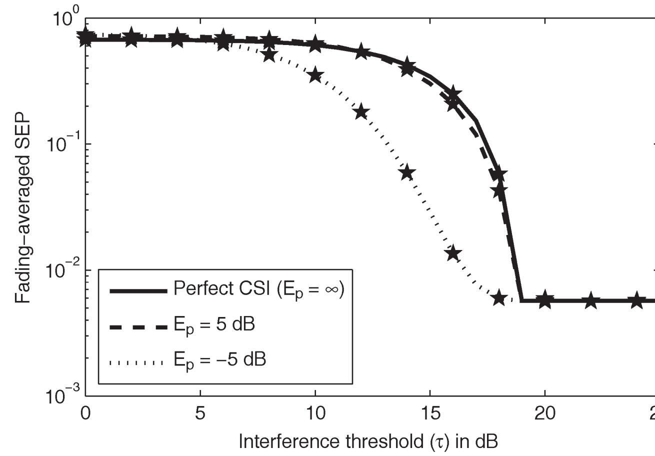

Fig. 5. Impact of imperfect CSI and interference threshold on the SEP of an interference outage probability constrained SU ($P_{\rm out} = 0.1$, $P_s = 15$ dB, QPSK, and $N = 2$). Simulation results are shown using $\star$ and analytical results using lines.

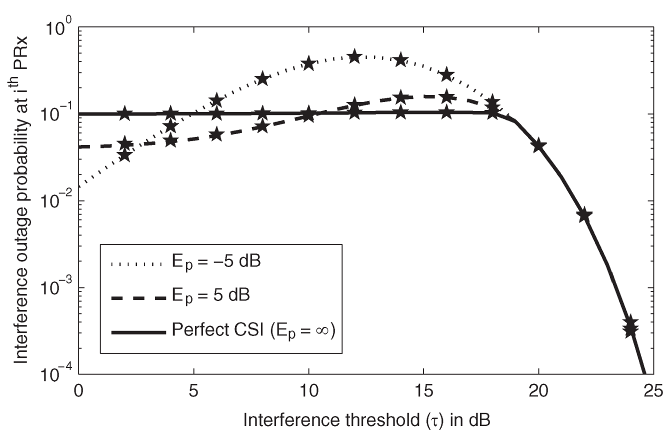

Fig. 4. Impact of imperfect CSI and interference threshold on the interference outage probability at each PRx ($P_{\rm out} = 0.1$, $P_s = 15$ dB, and $N = 2$). Simulation results are shown using $\star$ and analytical results using lines.

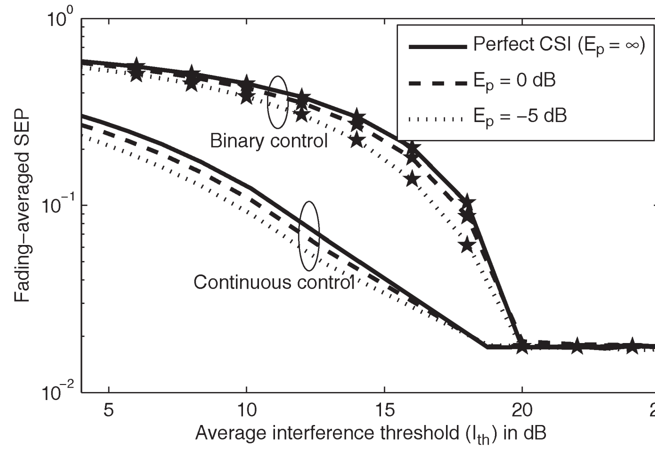

Fig. 6. Impact of binary and continuous transmit power constraints on the SEP of an average interference constrained CR ($\sigma_g^2 = 10$, $\sigma_h^2 = 5$, $P_s = 10$ dB, QPSK, and $N = 1$). Simulation results are shown using lines and analytical results using $\star$.

fading-averaged SEP is lower than for the perfect CSI case. This behavior is related to the trends observed in Fig. 2 for the interference-constrained regime in which the STx causes excessive interference. As expected, the larger the pilot SNR, the closer is the fading-averaged SEP curve to that for the perfect CSI case.

B. BPC for Interference Outage Probability Constraint

Fig. 4 plots the interference outage probability at the $i^{\rm th}$ PRx as a function of the interference threshold τ for different pilot SNRs for $P_{\rm out} = 0.1$, $P_s = 15$ dB, and $N = 2$. The simulation and analytical results match well. With perfect CSI, as τ increases, the interference outage probability initially remains constant at $P_{\rm out} = 0.1$ until $\tau = 19$ dB, i.e., as long as $\tau/P_s < \alpha$. Thereafter, it decreases as τ increases. However, with imperfect CSI, unlike Fig. 2, the interference outage probability can be higher or lower relative to $P_{\rm out}$, with the deviation being more for lower pilot SNRs. However, once $\tau/P_s \geq \alpha$, the outage probability is the same for all E_p because the STx transmits with power P_s for all $\hat{\mathbf{g}}$. As the pilot SNR increases, the interference outage probability curve approaches that for perfect CSI.

Fig. 5 plots the fading-averaged SEP as a function of τ for different pilot SNRs for $P_{\rm out} = 0.1$, $P_s = 15$ dB, QPSK, and $N = 2$. We observe a good match between the analysis and the simulation results. With perfect CSI and as τ increases, the fading-averaged SEP decreases and eventually reaches an error floor because the STx always transmits with power P_s. This is unlike Fig. 3. With imperfect CSI, we see that the fading-averaged SEP can be higher or lower than that for perfect CSI depending on the values of τ and E_p.

C. Comparison of BPC and CPC Policies for $N = 1$

Fig. 6 plots the fading-averaged SEP of an average interference constrained STx as a function of the average interference threshold $I_{\rm th}$ for BPC and CPC policies and for different pilot SNRs. As $I_{\rm th}$ increases, the SEP decreases and eventually reaches an error floor for both transmit power constraints. CPC outperforms BPC because the latter imposes a tighter

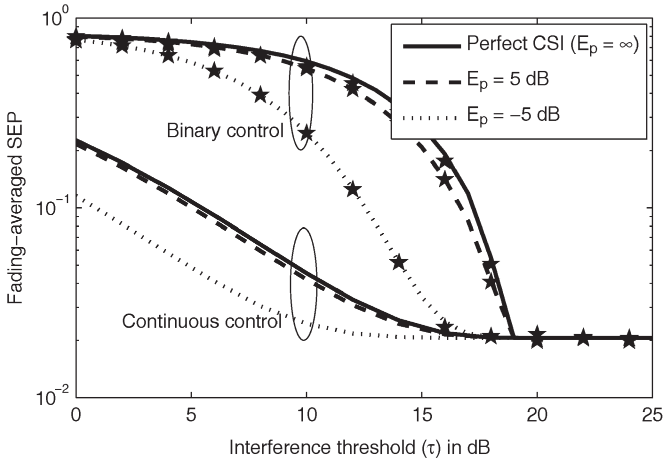

Fig. 7. Impact of binary and continuous transmit power constraints on the SEP of an interference outage probability constrained CR ($\sigma_g^2 = 1$, $\sigma_h^2 = 5$, $P_{\text{out}} = 0.1$, $P_s = 15$ dB, 8PSK, and $N = 1$). Simulation results are shown using lines and analytical results using $\star$.

constraint on the STx transmit power. Also, as the pilot SNR decreases, the SEP improves marginally for both transmit power constraints.

Fig. 7 plots the fading-averaged SEP of an interference outage probability constrained STx as a function of the interference threshold τ for BPC and CPC policies and for different pilot SNRs. As τ increases, the SEP decreases and eventually reaches an error floor for both policies. Again, as expected, CPC outperforms BPC. And, as the pilot SNR decreases, the fading-averaged SEP improves for both policies.

VII. Conclusion

We developed novel, practically-motivated, SEP-optimal BPC policies for the average interference and interference outage probability constraints when perfect CSI about the links between the STx and multiple PRxs is available at the STx. As a benchmark, we also presented the optimal CPC policies for these interference constraints for the case with one PRx. Finally, we developed closed-form expressions to characterize the impact of imperfect CSI on the binary policies. We observed that the impact of imperfect CSI depends on the interference constraints as well as the system parameter values.

The sum total of our results help understand how the choice of the interference and transmit power constraints affects the optimal transmit power policy and how imperfect CSI impacts its performance. These results are of interest to regulators, who, in principle, specify the interference constraints that protect the primary, and the system designers, who strive to maximize secondary system performance subject to these constraints and grapple with issues such as noisy channel estimates.

Appendix

A. Proof of Result 1

If $P_s\sigma_g^2 \le I_{\text{th}}$, it is clear that the optimal policy is to transmit with power P_s always irrespective of $\mathbf{g}$, since it is feasible and clearly yields the lowest possible SEP among all feasible policies. Henceforth, we focus on $P_s\sigma_g^2 > I_{\text{th}}$. We prove the result using the following series of claims.

Claim 1: Any feasible policy that does not meet the average interference constraint with equality at each of the N PRxs is sub-optimal.

Proof: If all the N interference constraints are inactive, then the policy is clearly sub-optimal because the power of the STx can be scaled up by a factor greater than unity for all $\mathbf{g}$, as this lowers the fading-averaged SEP without violating any of the N interference constraints.

Let us consider a policy θ, in which the average interference constraint for PRx 2 is active, i.e., $\overline{I}_2 = I_{\text{th}}$, but is inactive for PRx 1, i.e., $\overline{I}_1 \le I_{\text{th}}$. Without loss of generality, let $l_1 < \beta < l_2$ such that $P(h,\mathbf{g}) = P_s$, for $\mathbf{g} \in \mathcal{R}(0,\ldots,0; l_1,\ldots,l_N)$. For this policy, the average interference $\overline{I}_i$ at the i^{th} PRx equals

$$\overline{I}_i = \mathbf{E}_{h,\mathbf{g}}\left[P_\theta(h,\mathbf{g})g_i\right] = P_s\left(\sigma_g^2 - \sigma_g^2 e^{-l_i/\sigma_g^2} - l_i e^{-l_i/\sigma_g^2}\right) \times \prod_{k=1,k\neq i}^{N}\left(1 - e^{-l_k/\sigma_g^2}\right), \quad i = 1,\ldots,N. \tag{26}$$

We now show that θ is a sub-optimal policy. Consider an alternate policy θ' in which l_2 is reduced by Δl_2 and l_1 is increased by Δl_1, and l_i, for $i = 3,\ldots,N$, are left unperturbed. To ensure that θ' is feasible we set

$$-\frac{\partial \overline{I}_2}{\partial l_2}\Delta l_2 + \frac{\partial \overline{I}_2}{\partial l_1}\Delta l_1 = 0. \tag{27}$$

This ensures that $\overline{I}_2 = I_{\text{th}}$. Simplifying (27) above yields

$$\frac{\Delta l_2}{\Delta l_1} = \frac{e^{-l_1/\sigma_g^2}\left(\sigma_g^2 - \sigma_g^2 e^{-l_2/\sigma_g^2} - l_2 e^{\frac{-l_2}{\sigma_g^2}}\right)}{l_2 e^{-l_2/\sigma_g^2}\left(1 - e^{-l_1/\sigma_g^2}\right)}. \tag{28}$$

It can be shown that, with this perturbation, the average interference caused to the i^{th} PRx, for $3 \le i \le N$, does not exceed I_{th}. Therefore, θ' is feasible. It can also be shown that

$$\overline{\text{SEP}}_{\theta'} = \overline{\text{SEP}}_\theta + \left[\prod_{k=3}^{N}\left(1 - e^{-l_k/\sigma_g^2}\right)\right]\left(\Lambda - \frac{1}{\pi} \times \int_0^{\Lambda\pi}\left(1 + \frac{P_s \sin^2\left(\frac{\pi}{M}\right)\sigma_h^2}{(\sigma^2+\sigma_0^2)\sin^2\phi}\right)^{-1} d\phi\right) \times \left(\mathrm{p}(l_2)\Delta l_2\left(1 - e^{-l_1/\sigma_g^2}\right) - \mathrm{p}(l_1)\Delta l_1\left(1 - e^{-l_2/\sigma_g^2} - \frac{\Delta l_2}{\sigma_g^2}e^{-l_2/\sigma_g^2}\right)\right). \tag{29}$$

Substituting (28) in (29) and using the inequalities $\Lambda > \frac{1}{\pi}\int_0^{\Lambda\pi}\left(1 + \frac{P_s\sin^2(\frac{\pi}{M})\sigma_h^2}{(\sigma^2+\sigma_0^2)\sin^2\phi}\right)^{-1} d\phi$ and $e^{-x} > 1 - x$, for $x > 0$, it can be shown that $\mathrm{p}(l_2)\Delta l_2(1 - e^{-l_1/\sigma_g^2}) < \mathrm{p}(l_1)\Delta l_1(1 - e^{-l_2/\sigma_g^2} - \Delta l_2 e^{-l_2/\sigma_g^2}/\sigma_g^2)$. Therefore, $\overline{\text{SEP}}_{\theta'} < \overline{\text{SEP}}_\theta$, which proves that θ is sub-optimal. ■

Claim 2: For a policy θ to be optimal, if there is a region $\mathcal{R}(a_1,\ldots,a_N;\delta_1,\ldots,\delta_N) \subset \mathcal{R}(0,\ldots,0;\beta,\ldots,\beta)$ for $\mathbf{g}$ in which the STx transmits with zero power, then there must exist another region $\mathcal{R}(b_1,\ldots,b_N;\epsilon_1,\ldots,\epsilon_N) \subset \Omega \setminus \mathcal{R}(0,\ldots,0;\beta,\ldots,\beta)$ in which $P(h,\mathbf{g}) = P_s$ such that $a_i \prod_{k=1}^{N} \mathrm{p}(a_k)\delta_k = b_i \prod_{k=1}^{N} \mathrm{p}(b_k)\epsilon_k$.

Proof: Let us say that there is a region $\mathcal{R}(a_1,\ldots,a_N, \delta_1,\ldots,\delta_N) \subset \mathcal{R}(0,\ldots,0,\beta,\ldots,\beta)$ for $\mathbf{g}$ in which $P(h,\mathbf{g}) = 0$, but there is no region $\mathcal{R}(b_1,\ldots,b_N,\epsilon_1,\ldots,\epsilon_N) \subset \Omega \setminus \mathcal{R}(0,\ldots,0,\beta,\ldots,\beta)$ in which $P(h,\mathbf{g}) = P_s$. In that case, the average interference at the i^{th} PRx is given by $I_i' = I_{\text{th}} - P_s a_i \prod_{k=1}^{N} \mathrm{p}(a_k)\delta_k < I_{\text{th}}$. Thus, from Claim 1, θ cannot be optimal.

Hence, there must exist another region $\mathcal{R}(b_1,\ldots,b_N,\epsilon_1,\ldots,\epsilon_N) \subset \Omega \setminus \mathcal{R}(0,\ldots,0,\beta,\ldots,\beta)$ for $\mathbf{g}$ in which $P(h,\mathbf{g}) = P_s$. In that case, we have $I_i' = I_{\text{th}} - P_s a_i \prod_{k=1}^{N} \mathrm{p}(a_k)\delta_k + P_s b_i \prod_{k=1}^{N} \mathrm{p}(b_k)\epsilon_k = I_{\text{th}}$. Therefore, $a_i \prod_{k=1}^{N} \mathrm{p}(a_k)\delta_k = b_i \prod_{k=1}^{N} \mathrm{p}(b_k)\epsilon_k$. ■

We next show that shrinking δ_i and ϵ_i, for $i = 1,\ldots,N$, to zero reduces the fading-averaged SEP. Let θ^* denote the resulting policy. $\overline{\text{SEP}}_\theta$ can be written in terms of $\overline{\text{SEP}}_{\theta^*}$ as

$$\overline{\text{SEP}}_\theta = \overline{\text{SEP}}_{\theta^*} + \left[\Lambda - \frac{1}{\pi}\int_0^{\Lambda\pi}\left(1 + \frac{P_s \sin^2\left(\frac{\pi}{M}\right)\sigma_h^2}{(\sigma^2+\sigma_0^2)\sin^2\phi}\right)^{-1} d\phi\right] \times \left(\frac{b_i}{a_i} - 1\right)\prod_{k=1}^{N}\mathrm{p}(b_k)\epsilon_k. \quad (30)$$

Since $b_i > a_i$ and $\Lambda > \frac{1}{\pi}\int_0^{\Lambda\pi}\left(1 + \frac{P_s \sin^2\left(\frac{\pi}{M}\right)\sigma_h^2}{(\sigma^2+\sigma_0^2)\sin^2\phi}\right)^{-1} d\phi$, we get $\overline{\text{SEP}}_\theta > \overline{\text{SEP}}_{\theta^*}$. Thus, θ is sub-optimal.

B. Proof of Result 2

The proof is through the following series of claims.

Claim 3: When $\tau/P_s \geq \alpha$, the always-on policy, in which $P(h,\mathbf{g}) = P_s, \forall \mathbf{g}, h$, is optimal.

Proof: For the always-on policy, the outage probability O_i at the i^{th} PRx is $O_i = \Pr(g_i > \tau/P_s) = e^{-\frac{\tau}{P_s\sigma_g^2}}$. When $\tau/P_s \geq \alpha$, we have $e^{-\frac{\tau}{P_s\sigma_g^2}} < e^{-\frac{\alpha}{\sigma_g^2}} + \left(e^{-\frac{\tau}{P_s\sigma_g^2}} - e^{-\frac{\alpha}{\sigma_g^2}}\right)\left[1 - \left(1 - e^{-\frac{\alpha}{\sigma_g^2}}\right)^{N-1}\right] = P_{\text{out}}$. This is, therefore, a feasible policy. Hence, it is optimal as it yields the lowest fading-averaged SEP. ■

Henceforth, we focus on the regime $\tau/P_s < \alpha$, in which the always-on policy is not feasible. Next, we characterize the optimal transmit power when the interference links are weak.

Claim 4: For $g_i \in [0,\tau/P_s)$, $i = 1,2,\ldots,N$, we have $P^*(h,\mathbf{g}) = P_s$.

Proof: When $g_i \in [0,\tau/P_s)$, $i = 1,2,\ldots,N$, even when the STx transmits with power P_s, the instantaneous interference that it causes at each of the N PRxs is less than or equal to the threshold τ. It, therefore, does not contribute to the interference outage probability at any of the PRxs. Therefore, it is sub-optimal in terms of the fading-averaged SEP to transmit with zero power in this region. ■

Claim 5: Any feasible policy that satisfies Claim 4 and does not meet the interference outage probability constraint with equality at each of the N PRxs is sub-optimal.

Proof: The proof is along lines similar to that for Claim 1. We skip it to conserve space. ■

Let Ψ denote the subset of feasible policies that satisfy Claims 4 and 5. Let $\mathcal{A}$ denote the "annular" region $\mathcal{R}(0,\ldots,0;\alpha,\ldots,\alpha) \setminus \mathcal{R}(0,\ldots,0;\tau/P_s,\ldots,\tau/P_s)$.

Claim 6: For a policy $\theta \in \Psi$ to be optimal, if there exists a region $\mathcal{R}(a_1,\ldots,a_N;\delta_1,\ldots,\delta_N) \subset \Omega \setminus \mathcal{R}(0,\ldots,0;\alpha,\ldots,\alpha)$ such that $P(h,\mathbf{g}) = 0$, for $\mathbf{g} \in \mathcal{R}(a_1,\ldots,a_N;\delta_1,\ldots,\delta_N)$, then there must also exist another region $\mathcal{R}(b_1,\ldots,b_N;\epsilon_1,\ldots,\epsilon_N) \subset \mathcal{A}$ such that $P(h,\mathbf{g}) = P_s$, for $\mathbf{g} \in \mathcal{R}(b_1,\ldots,b_N;\epsilon_1,\ldots,\epsilon_N)$, where $\prod_{k=1}^{N}\mathrm{p}(a_k)\delta_k = \prod_{k=1}^{N}\mathrm{p}(b_k)\epsilon_k$.

Proof: Let us assume that there is no region $\mathcal{R}(b_1,\ldots,b_N;\epsilon_1,\ldots,\epsilon_N) \subset \mathcal{A}$ in which $P(h,\mathbf{g}) = P_s$. Since this region does not contribute to the interference outage probability at any PRx, the outage probability at the i^{th} PRx of the policy θ is upper bounded by $P_{\text{out}} - \prod_{k=1}^{N}\mathrm{p}(a_k)\delta_k < P_{\text{out}}$. From Claim 5, it follows that θ is sub-optimal.

Hence, for $\theta \in \Psi$ to be optimal, there must exist another region $\mathcal{R}(b_1,\ldots,b_N;\epsilon_1,\ldots,\epsilon_N) \subset \mathcal{A}$ such that $P(h,\mathbf{g}) = P_s$, for $\mathbf{g} \in \mathcal{R}(b_1,\ldots,b_N,\epsilon_1,\ldots,\epsilon_N)$. In that case, the interference outage probability at the i^{th} PRx is equal to $P_{\text{out}} - \prod_{k=1}^{N}\mathrm{p}(a_k)\delta_k + \prod_{k=1}^{N}\mathrm{p}(b_k)\epsilon_k$. Since this equals P_{out}, we get $\prod_{k=1}^{N}\mathrm{p}(a_k)\delta_k = \prod_{k=1}^{N}\mathrm{p}(b_k)\epsilon_k$. ■

Claim 7: For an optimal transmit power policy in Ψ, the regions $\mathcal{R}(b_1,\ldots,b_N;\epsilon_1,\ldots,\epsilon_N) \subset \mathcal{A}$ for $\mathbf{g}$ in which $P(h,\mathbf{g}) = P_s$, and $\mathcal{R}(a_1,\ldots,a_N;\delta_1,\ldots,\delta_N) \subset \Omega \setminus \mathcal{R}(0,\ldots,0;\alpha,\ldots,\alpha)$ in which $P(h,\mathbf{g}) = 0$ can both be shrunk to zero without affecting the fading-averaged SEP.

Proof: Let $\theta \in \Psi$ be a policy in which $P(h,\mathbf{g}) = 0$, for all $\mathbf{g} \in \mathcal{A} \setminus \mathcal{R}(b_1,\ldots,b_N;\epsilon_1,\ldots,\epsilon_N)$ and for all $\mathbf{g} \in \mathcal{R}(a_1,\ldots,a_N;\delta_1,\ldots,\delta_N)$; and $P(h,\mathbf{g}) = P_s$, otherwise.

Then, the fading-averaged SEP of θ can be written in terms of that of θ^* as follows:

$$\overline{\text{SEP}}_\theta = \overline{\text{SEP}}_{\theta^*} + \left[\Lambda - \frac{1}{\pi}\int_0^{\Lambda\pi}\left(1 + \frac{P_s \sin^2\left(\frac{\pi}{M}\right)\sigma_h^2}{(\sigma^2+\sigma_0^2)\sin^2\phi}\right)^{-1} d\phi\right] \times \left(\prod_{k=1}^{N}\mathrm{p}(a_k)\delta_k - \prod_{k=1}^{N}\mathrm{p}(b_k)\epsilon_k\right). \quad (31)$$

Since $\prod_{k=1}^{N}\mathrm{p}(a_k)\delta_k = \prod_{k=1}^{N}\mathrm{p}(b_k)\epsilon_k$, we get $\overline{\text{SEP}}_\theta = \overline{\text{SEP}}_{\theta^*}$. Thus, θ^* is also optimal. ■

This result also implies that the optimal policy is not unique.

C. Proof of Proposition 2

If $P_s\sigma_g^2 \leq I_{\text{th}}$, then the always-on policy in which $P^*(h,g) = P_s$, $\forall g, h$, satisfies the average interference and transmit power constraints. It clearly yields the lowest fading-averaged SEP among all feasible transmission policies.

Let us now consider the case when $P_s\sigma_g^2 > I_{\text{th}}$, in which the always-on policy is not feasible. The set of all feasible CPC policies, $\mathcal{S}$, is a non-empty set because the policy in which the STx always transmits with zero power is clearly feasible.

Let $\theta \in \mathcal{S}$ be a feasible CPC policy. Given a $\lambda > 0$, define an auxiliary function $L_\theta(P(h,g),\lambda)$ as

$$L_\theta\left(P(h,g),\lambda\right) \triangleq \mathbf{E}_{h,g}\left[\chi\left(P(h,g),h\right) + \lambda P(h,g)g\right], \quad (32)$$

where $\chi(P(h,g),h) = \Lambda \exp\left(\frac{-P(h,g)h\sin^2\left(\frac{\pi}{M}\right)}{\sigma^2+\sigma_0^2}\right)$ is the SEP upper bound given in (15). Let θ^* be a policy whose transmit power $P^*(h,g)$ is defined as

$$P^*(h,g) = \arg\min_{x\in[0,P_s]}\left\{\chi(x,h) + \lambda xg\right\}. \quad (33)$$

From the definition of θ^*, it follows that $L_{\theta^*}(P^*(h,g),\lambda) \leq L_\theta(P(h,g),\lambda)$. Therefore,

$$\mathbf{E}_{h,g}\left[\chi(P^*(h,g),h)\right] + \lambda\mathbf{E}_{h,g}\left[P^*(h,g)g\right] \leq \mathbf{E}_{h,g}\left[\chi\left(P(h,g),h\right)\right] + \lambda\mathbf{E}_{h,g}\left[P(h,g)g\right]. \quad (34)$$

Choose λ such that the average interference generated by θ^* is $I^* = \mathbf{E}_{h,g}[P^*(h,g)g] = I_{\text{th}}$.[3] Thus, θ^* is a feasible CPC policy. Rearranging the terms in (34), we get

$$\mathbf{E}_{h,g}\left[\chi\left(P^*(h,g),h\right)\right] \leq \mathbf{E}_{h,g}\left[\chi\left(P(h,g),h\right)\right] + \lambda\left(\mathbf{E}_{h,g}\left[P(h,g)g\right] - I_{\text{th}}\right). \quad (35)$$

Since θ is feasible, we know that $\mathbf{E}_{h,g}[P(h,g)g] \leq I_{\text{th}}$. Hence, (35) implies that $\mathbf{E}_{h,g}[\chi(P^*(h,g),h)] \leq \mathbf{E}_{h,g}[\chi(P(h,g),h)]$. Thus, the fading-averaged SEP of θ^* is less than that of any other feasible policy. Hence, θ^* is optimal.

Structure of Optimal Policy: Define

$$\Xi(x,h) = \chi(x,h) + \lambda(xg - I_{\text{th}}). \quad (36)$$

Equating $\frac{\partial \Xi(x,h)}{\partial x}$ to zero, we get

$$\exp\left(\frac{-xh\sin^2\left(\frac{\pi}{M}\right)}{\sigma^2+\sigma_0^2}\right) = \frac{\lambda g(\sigma^2+\sigma_0^2)}{\Lambda h\sin^2\left(\frac{\pi}{M}\right)}. \quad (37)$$

Furthermore, it can be verified that $\frac{\partial \Xi(x,h)}{\partial x}$ is non-negative at $x = 0$ and is non-positive at $x = P_s$. Rearranging (37) to write the solution x in terms of the other terms, and applying the Karush–Kuhn–Tucker (KKT) conditions yields (17).

D. Proof of Proposition 3

The average interference at the i^{th} PRx with imperfect CSI is given by

$$\widehat{I}_i = \mathbf{E}_{h,\hat{\mathbf{g}},g_i}\left[P(h,\hat{\mathbf{g}})g_i\right]. \quad (38)$$

[3]The existence of such a unique λ can be proved by showing that I^* is a continuous function of λ, and then applying the intermediate value theorem [31].

Using (6), the average interference can be written as

$$\widehat{I}_i = \int_0^\infty \int_0^\beta \cdots \int_0^\beta P_s g_i \times \mathrm{p}(g_i,\hat{g}_1,\ldots,\hat{g}_i,\ldots,\hat{g_N})\,d\hat{g}_1\ldots d\hat{g_N}\,dg_i.$$

Using the fact that $\hat{g}_i$, $i = 1,\ldots,N$, are mutually independent, the equation above can be simplified to obtain

$$\widehat{I}_i = \left(1 - \exp\left(\frac{-\beta(E_p\sigma_g^2+1)}{E_p\sigma_g^4}\right)\right)^{N-1} \times \int_0^\infty \int_0^\beta P_s g_i \mathrm{p}(g_i,\hat{g}_i)\,d\hat{g}_i dg_i. \quad (39)$$

Further, g_i and $\hat{g}_i$ are correlated exponentials. Using [29, (6.2)], it can be shown that their joint PDF $\mathrm{p}(g_i,\hat{g}_i)$ is

$$\mathrm{p}(g_i,\hat{g}_i) = \frac{1}{\eta_1\eta_2(1-\rho)}\exp\left(\frac{-1}{1-\rho}\left(\frac{g_i}{\eta_1}+\frac{\hat{g}_i}{\eta_2}\right)\right) \times I_0\left(\frac{2\sqrt{\rho g_i\hat{g}_i}}{(1-\rho)\sqrt{\eta_1\eta_2}}\right),\ g_i \geq 0, \hat{g}_i > 0, \quad (40)$$

where $\eta_1 = \mathbf{E}[g_i] = \sigma_g^2$, $\eta_2 = \mathbf{E}[\hat{g}_i] = E_p\sigma_g^4/(E_p\sigma_g^2+1)$, $\rho = E_p\sigma_g^2/(E_p\sigma_g^2+1)$, and $I_0(\cdot)$ is the modified Bessel function of the zeroth order. Substituting (40) in (39) and using the identity in [32, (6.631.1)], (39) can be simplified to obtain (20).

E. Brief Proof of Proposition 4

The fading-averaged SEP of MPSK is given by

$$\overline{\text{SEP}} = \mathbf{E}_{h,\hat{\mathbf{g}}}\left[\frac{1}{\pi}\int_0^{\Lambda\pi}\exp\left(-\frac{P^*(h,\hat{\mathbf{g}})h\sin^2\left(\frac{\pi}{M}\right)}{(\sigma^2+\sigma_0^2)\sin^2\phi}\right)d\phi\right]. \quad (41)$$

Using (6), the equation above can be written as

$$\overline{\text{SEP}} = \frac{1}{\pi}\int_0^\beta \cdots \int_0^\beta \int_0^\infty \int_0^{\Lambda\pi} \exp\left(-\frac{P^*(h,\hat{\mathbf{g}})h\sin^2\left(\frac{\pi}{M}\right)}{(\sigma^2+\sigma_0^2)\sin^2\phi}\right) \times \mathrm{p}(h)\mathrm{p}(\hat{g_1})\ldots\mathrm{p}(\hat{g_N})\,d\phi\,dh\,d\hat{g}_1\ldots d\hat{g_N}$$
$$+ \frac{1}{\pi}\int_\beta^\infty \int_0^\infty \int_0^{\Lambda\pi} \mathrm{p}(h)\mathrm{p}(\hat{g_1})\,d\phi\,dh\,d\hat{g}_1 + \ldots$$
$$+ \frac{1}{\pi}\int_\beta^\infty \cdots \int_0^\beta \int_0^\infty \int_0^{\Lambda\pi} \mathrm{p}(h) \times \mathrm{p}(\hat{g_1})\ldots\mathrm{p}(\hat{g_N})\,d\phi\,dh\,d\hat{g}_1\ldots d\hat{g_N}.$$

Using the identity in [29, (5A.15)] and carefully simplifying the integrals over $\mathbf{g}$ and h yields (22).

F. Brief Proof of Proposition 5

The interference outage probability $\widehat{O}_i$ at the i^{th} PRx is given by

$$\begin{aligned}\widehat{O}_i &= \Pr\left(P_s g_i > \tau, 0 \le \hat{g}_1 < \frac{\tau}{P_s}, \ldots, 0 \le \hat{g_N} < \frac{\tau}{P_s}\right)\\ &\quad + \Pr(P_s g_i > \tau, \hat{g}_i \ge \alpha)\\ &\quad + \Pr(P_s g_i > \tau, 0 \le \hat{g}_i < \alpha, \hat{g_1} \ge \alpha) + \ldots\\ &\quad + \Pr(P_s g_i > \tau, 0 \le \hat{g}_i < \alpha, 0 \le \hat{g_1} < \alpha, \ldots, \hat{g_N} \ge \alpha).\end{aligned}$$

Simplifying the equation above, we get

$$\begin{aligned}\widehat{O}_i &= \left(1 - e^{\frac{-\tau}{\eta_2 P_s}}\right)^{N-1} \int_{\frac{\tau}{P_s}}^{\infty} \int_0^{\frac{\tau}{P_s}} \mathrm{p}(g_i, \hat{g}_i)\, d\hat{g}_i\, dg_i\\ &\quad + \int_{\frac{\tau}{P_s}}^{\infty} \int_0^{\alpha} \mathrm{p}(g_i, \hat{g}_i)\, d\hat{g}_i\, dg_i\\ &\quad + e^{-\frac{\alpha}{\eta_2}} \int_{\frac{\tau}{P_s}}^{\infty} \int_0^{\alpha} \mathrm{p}(g_i, \hat{g}_i)\, d\hat{g}_i\, dg_i\\ &\quad \times \left(1 + \left[1 - e^{-\frac{\alpha}{\eta_2}}\right] + \ldots + \left[1 - e^{-\frac{\alpha}{\eta_2}}\right]^{N-2}\right), \qquad (42)\end{aligned}$$

where η_2 is defined in Appendix D. Substituting the joint distribution of g_i and $\hat{g}_i$ from (40) and using the identity in [33, (B.18)], (42) simplifies to (23).

G. Brief Proof of Proposition 6

Starting from (41) and using (12), the fading-averaged SEP for an interference outage probability constrained SU can be written as a sum of three terms:

$$\overline{\mathrm{SEP}} = T_1 + T_2 + T_3, \qquad (43)$$

where

$$\begin{aligned}T_1 &= \frac{1}{\pi} \int_0^{\frac{\tau}{P_s}} \cdots \int_0^{\frac{\tau}{P_s}} \int_0^{\infty} \int_0^{\Lambda\pi} \zeta \mathrm{p}(h)\\ &\qquad \times \mathrm{p}(\hat{g_1}) \ldots \mathrm{p}(\hat{g_N})\, d\phi\, dh\, d\hat{g_1} \ldots d\hat{g_N},\\ T_2 &= \frac{1}{\pi} \int_{\alpha}^{\infty} \int_0^{\infty} \int_0^{\Lambda\pi} \zeta \mathrm{p}(h) \mathrm{p}(\hat{g_1})\, d\phi\, dh\, d\hat{g_1} + \ldots\\ &\quad + \frac{1}{\pi} \int_{\alpha}^{\infty} \cdots \int_0^{\alpha} \int_0^{\infty} \int_0^{\Lambda\pi} \zeta \mathrm{p}(h)\\ &\qquad \times \mathrm{p}(\hat{g_1}) \ldots \mathrm{p}(\hat{g_N})\, d\phi\, dh\, d\hat{g_1} \ldots d\hat{g_N},\\ T_3 &= \frac{1}{\pi} \int_0^{\alpha} \cdots \int_0^{\alpha} \int_0^{\infty} \int_0^{\Lambda\pi} \mathrm{p}(h)\\ &\qquad \times \mathrm{p}(\hat{g_1}) \ldots \mathrm{p}(\hat{g_N})\, d\phi\, dh\, d\hat{g_1} \ldots d\hat{g_N}\\ &\quad - \frac{1}{\pi} \int_0^{\frac{\tau}{P_s}} \cdots \int_0^{\frac{\tau}{P_s}} \int_0^{\infty} \int_0^{\Lambda\pi} \mathrm{p}(h)\\ &\qquad \times \mathrm{p}(\hat{g_1}) \ldots \mathrm{p}(\hat{g_N})\, d\phi\, dh\, d\hat{g_1} \ldots d\hat{g_N},\end{aligned}$$

and $\zeta = \exp\left(-\frac{P^*(h, \hat{\mathbf{g}}) h \sin^2(\frac{\pi}{M})}{(\sigma^2 + \sigma_0^2) \sin^2 \phi}\right)$. Using the identity in [29, (5A.15)] to simplify the three integrals above involving ϕ, then integrating over h, $\hat{g}_1, \ldots, \hat{g}_N$, and simplifying further yields (24).

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