Novel Relay Selection Rules for Average Interference-Constrained Cognitive AF Relay Networks

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***Abstract*—Cooperative relaying combined with selection exploits spatial diversity to significantly improve the performance of interference-constrained secondary users in an underlay cognitive radio (CR) network. However, unlike conventional relaying, the state of the links between the relay and the primary receiver affects the choice of the relay. Further, while the optimal amplify-and-forward (AF) relay selection rule for underlay CR is well understood for the peak interference-constraint, this is not so for the less conservative average interference constraint. For the latter, we present three novel AF relay selection (RS) rules, namely, symbol error probability (SEP)-optimal, inverse-of-affine (IOA), and linear rules. We analyze the SEPs of the IOA and linear rules and also develop a novel, accurate approximation technique for analyzing the performance of AF relays. Extensive numerical results show that all the three rules outperform several RS rules proposed in the literature and generalize the conventional AF RS rule.**

***Index Terms*—Underlay cognitive radio, interference constraint, cooperative communications, relays, amplify-and-forward, selection, symbol error probability.**

I. Introduction

COGNITIVE RADIO (CR) promises to significantly improve the utilization of scarce wireless spectrum, and has attracted significant interest in academia and industry [1]. In the underlay mode of CR, which is the focus of our paper, a secondary user (SU) can simultaneously transmit on the same band as a higher priority primary user (PU) so long as the interference it causes to the primary receiver is tightly constrained [1]. This interference constraint limits the SU's data rate and reliability.

Cooperative relaying, which is being standardized in next generation wireless local area networks and cellular systems [2], is a promising technique that enhances the performance of the SUs by exploiting spatial diversity. When multiple relays are available, one of them is selected to forward a message from a secondary source S to a destination D based on the instantaneous channel conditions. Relay selection (RS) is practically appealing because it avoids the need for tight synchronization between simultaneously transmitting relays or the spectrally inefficient use of bandwidth when all the relays transmit on orthogonal resources [3]. Even though the relays have only local channel knowledge, RS can be implemented using distributed selection algorithms [4], and achieves full diversity order in conventional relay networks [5]. RS for both amplify-and-forward (AF) and decode-and-forward (DF) relays has been studied in the literature [6]–[10].

Conventionally, the best relay is the one that maximizes the end-to-end signal-to-noise-ratio (SNR) at D. However, in underlay CR, it may not be preferable to select a relay with the largest SNR, if it causes excessive interference to a primary receiver X. Therefore, the RS rule is now also a function of the links between the relays and X. Further, it also depends on the interference constraint, which sets underlay CR apart from conventional wireless communications. In the peak interference constraint, the instantaneous interference from the source to X and from the selected relay to X must not exceed a threshold [11]–[14]. Instead, in the average interference constraint, the *fading-averaged* interference caused to X must not exceed a threshold [8], [15].[1]

A. Literature on AF RS in Underlay CR

In [11], among the fixed-gain relays that satisfy the peak interference constraint, the relay that maximizes the SNR of the relay (R)-to-D (RD) link is selected. In [12], the relay with the maximum end-to-end SNR at D is selected. Further, in [12], the relay transmit power is inversely proportional to the R-to-X (RX) channel power gain to satisfy the peak interference constraint. Selection with fixed-power relays that are subject to a peak interference constraint has also been studied in [13], [14]. In [13], the *max-min rule* is proposed, in which the relays that satisfy the peak interference constraint are short-listed. Among these relays, the one that maximizes the minimum of the S-to-R (SR) and RD link SNRs is selected. Instead,

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[1] Another possible constraint limits how often the signal-to-interference-plus-noise-ratio (SINR) of the signal transmitted by the primary transmitter (T) at X drops below a threshold [9]. However, it requires channel state information of T-to-X link at S and the relays, which can be impractical. We, therefore, do not study this constraint.

in [14], the relays that do not satisfy the peak interference constraint or for whom the minimum of the SR and RD link SNRs is below a threshold are first excluded. Among the remaining relays, the one that maximizes the ratio of the minimum of the SR and RD link SNRs and the instantaneous interference power caused by it to X is selected. We shall refer to this rule as the *quotient rule*.

B. Contributions

We study relay selection for an underlay CR network that uses *fixed-power* AF relays subject to an average interference constraint. We focus on the classical AF relaying protocol [7] because it has attracted considerable interest both in the conventional cooperative networks [6], [16] and in underlay CR [12]–[14]. We note that other cooperative protocols exist such as non-orthogonal relaying [17] and incremental relaying [18], which achieve higher spectral efficiencies at the expense of a more involved receiver. However, they are beyond the scope of this paper.

We study the average interference constraint because it is less restrictive than the conservative peak interference constraint, and, thus, enables the secondary system to perform better by changing both the choice of the relay and its transmit power depending on the channel fades. It is well motivated when T_c/T_p is of the order of 1 or less, where T_c denotes the coherence time of the RX link and T_p denotes the packet transmission duration. When T_c/T_p exceeds 1, its practicality depends on the quality-of-service requirements of the primary traffic between T and X, which anyways needs to be resilient to a prolonged deep fade in the primary link. Given its appeal, it has been studied for non-cooperative CR [15] and DF relay-based CR [8]. However, to the best of our knowledge, the combination of AF RS and the average interference constraint has not been fully addressed in the literature.[2] RS with the peak interference constraint is technically simpler because the transmit power of the relay is simply proportional to the reciprocal of the RX channel power gain [12].

We make the following contributions in this paper:

- We systematically develop a novel, optimal RS rule that minimizes the symbol error probability (SEP) of a secondary system that is subject to an average interference constraint. We note that alternate problem formulations that optimize other performance measures such as capacity [8], [12], outage probability [9], and diversity-multiplexing tradeoff [20] are possible. We focus on the SEP because it is a widely studied classical measure of reliability of communications [13], [14], [21], [22]. Further, it leads to a theoretically rich and insightful problem.
- While the optimal rule's characterization is general and serves as a fundamental benchmark, it is in the form of a single integral involving the end-to-end SINR at D, which makes it difficult to implement. We, therefore, present two novel, integral-free RS rules, namely, the *inverse-of-affine* (IOA) and *linear* rules, that are derived from the optimal rule and have lower implementation complexity. All the three rules reduce to the optimal RS rule for interference-unconstrained conventional cooperative systems, and are, thus, generalizations of this rule.
- We derive asymptotically tight upper and lower bounds for the SEPs of the IOA and linear rules. We find that the IOA rule, despite its simpler closed-form, performs as well as the optimal rule for all the parameters of interest. The linear rule incurs a performance loss compared to the optimal rule. To gain more insights, we analyze the asymptotic SEP of these two rules. We also develop a novel, asymptotically tight, and closed-form SEP approximations for these two rules, which our numerical results show are accurate to within 0.6 dB even at low SINRs. In general, such an analysis is challenging due to the functional form of the SINR obtained from using an AF relay [7], [16]. The additional dependence of the RS rules on the RX links further complicates our analysis.
- We present extensive numerical results to study the effect of parameters such as the number of relays, constellation size, and channel statistics on the SEP. All the three proposed rules outperform the rules proposed in the literature.

We note that while the optimal antenna selection (AS) rule of [15] and our RS rule appear to have functionally similar forms, there are significant differences between the two. In [15], a non-cooperative CR network is considered, in which S selects one among multiple antennas to transmit its data to D. However, we consider a cooperative CR network that uses AF relays, in which one among multiple relays is selected to forward S's data to D in addition to the direct transmission between S and D. The different ad hoc rules considered for AS [23] and RS [13], [14] reconfirm that these two models have been treated as being different in the literature. The SEP analysis in our paper is more involved than that in [15]. This is because the SINR at D in [15] is directly proportional to the channel power gain of the link between the selected antenna and D, while, in our problem, the SINR at D is more involved and so is its probability density function (PDF), which drives the SEP analysis. The proposed IOA and linear RS rules are novel and do not follow from [15]. Further, our novel SEP approximation method, which simplifies the SEP analysis, is not proposed in [15].

Outline: Section II develops the system model and the problem statement. The optimal RS rule and its two variants are given in Section III. The SEP of the IOA and linear rules is analyzed in Section IV. Numerical results are presented in Section V. Our conclusions follow in Section VI.

Notation: The absolute value of a complex number y is denoted by $|y|$. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. For a random variable (RV) $Y, f_Y(y)$ denotes its PDF, $F_Y(y)$ denotes its cumulative distribution function (CDF), and $\mathbb{E}_Y[.]$ denotes expectation with respect to Y. Scalar and vector variables are written in normal and bold fonts, respectively. $Y \sim CN(0, \sigma^2)$ implies that Y is a circular

[2] While [19] also studies AF relaying and the average interference constraint, the RS rule selects the relay with the highest end-to-end SINR at D. It focuses on optimizing the transmit powers of the source and relays.

symmetric zero-mean complex Gaussian RV with variance σ^2, and $1_{\{a\}}$ denotes the indicator function; it is 1 if a is true and is 0 otherwise. $\mathrm{E}_k(y) \triangleq \int_1^\infty e^{-yt}/t^k\, dt$ denotes the exponential integral function [24, (5.1.4)] and ${}_2F_1(a, b; c; z)$ denotes the Gauss hypergeometric function [24, (15.1)].

II. System Model and Problem Statement

Our system comprises of a primary network, in which a primary transmitter T sends data to a primary receiver X, and an underlay secondary network, in which S transmits data to D using L relays $1, 2, \ldots, L$. Each node is equipped with a single antenna. The complex baseband channel gain from S to X is h_{SX}, from S to D is h_{SD}, from S to relay i is h_{Si}, from relay i to D is h_{iD}, and from relay i to X is h_{iX}. Let $\mathbf{h}_S \triangleq [h_{S1}, h_{S2}, \ldots, h_{SL}]$, $\mathbf{h}_D \triangleq [h_{1D}, h_{2D}, \ldots, h_{LD}]$, $\mathbf{h}_X \triangleq [h_{1X}, h_{2X}, \ldots, h_{LX}]$, and $\mathbf{h} \triangleq [\mathbf{h}_S, \mathbf{h}_D, \mathbf{h}_P]$. Further, the complex baseband channel gain from T to relay i is h_{Ti} and from T to D is h_{TD}. All channels are frequency-flat, block fading channels that remain constant over the duration of at least two transmissions. The direct S-to-D (SD) link is independent of all other links.

A. Data Transmissions

The transmission occurs over two slots. In the first time slot, S transmits a data symbol x_s that is drawn with equal probability from MPSK constellation of size M. Let x_p denote the symbol transmitted by T drawn from MPSK or MQAM constellation. After accounting for the interferences caused by the transmissions by T at the relay and destination, the received signals y_{Si} at relay i and y_{SD} at D are given by

$$y_{Si} = \sqrt{E_s} h_{Si} x_s + n_i + \sqrt{E_p} h_{Ti} x_p, \quad 1 \le i \le L, \tag{1}$$

$$y_{SD} = \sqrt{E_s} h_{SD} x_s + n_D + \sqrt{E_p} h_{TD} x_p. \tag{2}$$

Here, E_s and E_p are the transmit powers of S and T, respectively, $\mathbb{E}\left[|x_s|^2\right] = \mathbb{E}\left[|x_p|^2\right] = 1$. The noises at relay i and D are $n_i \sim CN(0, \sigma_0^2)$ and $n_D \sim CN(0, \sigma_0^2)$, respectively.

In the second time slot, the selected relay $\beta \in \{1, \ldots, L\}$ amplifies the signal $y_{S\beta}$ by a factor $\alpha_\beta = \sqrt{\frac{E_r}{E_s|h_{S\beta}|^2 + \sigma_0^2 + E_p|h_{T\beta}|^2}}$ [25] to ensure that its transmission power is fixed at E_r, and forwards it to D. The source does not transmit in the second time slot. The received signal $y_{\beta D}$ at D in the second time slot is given by

$$y_{\beta D} = y_{S\beta} \alpha_\beta h_{\beta D} + n'_D + \sqrt{E_p} h_{TD} x'_p, \tag{3}$$

where $n'_D \sim CN(0, \sigma_0^2)$ is the noise at D, x'_p is the transmitted symbol by T in the second time slot, and $\mathbb{E}\left[|x'_p|^2\right] = 1$. We use the fixed-power relaying model, in which the relay transmit power does not depend on the instantaneous channel gains of any of the links. This enables the use of energy-efficient power-amplifiers, which facilitates the design of cheap, low complexity relays. To make the problem analytically tractable, we make the following two assumptions and explain their specific roles.

A1. *Conditioned on h_{Ti}, the interference from T to relay i, $\sqrt{E_p} h_{Ti} x_p$, is approximated to be Gaussian; and conditioned on h_{TD}, the interference from T to D, $\sqrt{E_p} h_{TD} x_p$, is approximated to be Gaussian.* Without this, even maximal ratio combining (MRC) at D need no longer be maximum-likelihood (ML)-optimal. Even the utility of SINR diminishes since the classical SEP expression for MPSK [22, (8.23)] is no longer applicable [21, Chapter 6.1.1].[3] The output y_{MRC} at D after MRC is given by

$$y_{\mathrm{MRC}} = w_1 y_{SD} + w_2 y_{\beta D}, \tag{4}$$

where the weights can be shown to be $w_1 = \frac{\sqrt{E_s} h_{SD}^*}{\sigma_0^2 + E_p|h_{TD}|^2}$ and $w_2 = \frac{\sqrt{\frac{E_r E_s}{E_s|h_{S\beta}|^2 + \sigma_0^2 + E_p|h_{T\beta}|^2}} h_{S\beta}^* h_{\beta D}^*}{\sigma_0^2 + E_p|h_{TD}|^2 + \frac{E_r|h_{\beta D}|^2 \left(\sigma_0^2 + E_p|h_{T\beta}|^2\right)}{E_s|h_{S\beta}|^2 + \sigma_0^2 + E_p|h_{T\beta}|^2}}$.

A2. *The selected relay β is assumed to know the channel power gains $|h_{S\beta}|^2$ and $|h_{T\beta}|^2$ to compute its gain α_β.* In practice, the relay can estimate these using a training protocol [29]. *The destination is assumed to know the baseband channel gains $h_{SD}, h_{S\beta}$, and $h_{\beta D}$, and the channel power gains $|h_{TD}|^2$ and $|h_{T\beta}|^2$ to compute the weights w_1 and w_2 for coherent demodulation.* Note that the selected relay β does not need to know the channel power gains from S and T to any of the other relays. Further, the relays are assumed to know $\mathbb{E}\left[|h_{TD}|^2\right]$, but they need not know the instantaneous value $|h_{TD}|^2$. This can be communicated to them by the destination over a much longer time scale [9]. Phases of the baseband channel gains are not required at the relays.

Therefore, from A1, A2, and (4), the instantaneous SINR at D after MRC is given by

$$\gamma_{\mathrm{MRC}} = \gamma_{SD} + \gamma_\beta, \tag{5}$$

where $\gamma_{SD} = \frac{E_s|h_{SD}|^2}{\sigma_0^2 + E_p|h_{TD}|^2}$ is the SINR of the SD link and $\gamma_\beta = \frac{\gamma_{S\beta}\gamma_{\beta D}}{\gamma_{S\beta} + \gamma_{\beta D} + 1}$ is the end-to-end SINR of the selected relay link at the destination, with $\gamma_{S\beta} = \frac{E_s|h_{S\beta}|^2}{\sigma_0^2 + E_p|h_{T\beta}|^2}$ and $\gamma_{\beta D} = \frac{E_r|h_{\beta D}|^2}{\sigma_0^2 + E_p|h_{TD}|^2}$ being the SINRs of the first and second hops, respectively, for $\beta \in \{1, \ldots, L\}$.

To evaluate the impact of these assumptions, Fig. 1 plots the SEPs from Monte Carlo simulations as a function of the average SINR of an interference-unconstrained cooperative system consisting of a source S, a fixed-power AF relay β, and a destination D. The relay β and D are affected by the interference from T, which transmits symbols drawn from 8PSK or 64QAM constellations. S transmits symbols drawn from 8PSK constellation. All channels, including the interference

[3] We note that the Gaussian interference approximation has been widely made in the analysis of SEP, capacity, or outage probability. For example, the SEP analysis in [25, (6)] is valid only with this approximation. In [9, (9)], [26, (1)] the interference power from the primary transmitter to relay i is added to the noise power in the expression for the instantaneous mutual information. This holds only for Gaussian interference. In [27], [28], the approximation is used because it provides a worst case, but tractable, model for the interference.

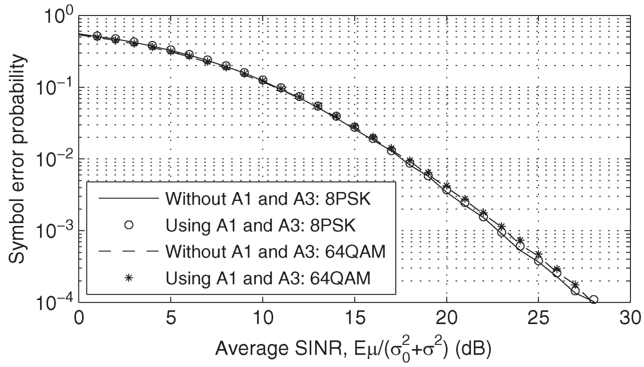

Fig. 1. SEP as a function of average SINR when the primary interference is approximated as Gaussian and T transmits 8PSK and 64QAM symbols ($E_s = E_r = E = 10$ dB, $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 1.98$ dB, $\sigma_0^2 = 2$ dB, and mean channel power gains of SD, SR, RD, and RX links are μ).

channels, undergo Rayleigh fading. These simulations do not make the Gaussian interference approximation. Also plotted is the SEP based on the above assumptions and the following assumption A3. *In the above expressions for the instantaneous SINRs of the first hop, second hop, and the SD link, the terms $E_p|h_{T\beta}|^2$ and $E_p|h_{TD}|^2$ are replaced with their average values σ_1^2 and σ_2^2, respectively, to simplify the optimization and analysis*. Therefore, $\gamma_{S\beta} = \frac{E_s|h_{S\beta}|^2}{\sigma_0^2+\sigma_1^2}, \gamma_{\beta D} = \frac{E_r|h_{\beta D}|^2}{\sigma_0^2+\sigma_2^2}$, and $\gamma_{SD} = \frac{E_s|h_{SD}|^2}{\sigma_0^2+\sigma_2^2}$. We see that the SEP curves, with and without the assumptions A1 and A3, are close to each other over a wide range of SINRs when T transmits 8PSK or 64QAM symbols. Thus, the above assumptions are justified for our problem even with one primary transmitter. With a large number of primary transmitters, the central limit theorem justifies the Gaussian interference approximation [30].

B. Relay Selection

The RS rule selects one among the L available relays. Further, no relay may be selected to avoid interference with X, which is denoted by a virtual relay 0, with $h_{S0} = h_{0D} = h_{0X} \triangleq 0$. Therefore, a RS rule ϕ is a mapping

$$\phi : (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \to \{0, 1, \ldots, L\}, \quad (6)$$

that selects one out of the $L + 1$ relays for every realization of $\{|h_{Si}|^2\}_{i=1}^L, \{|h_{iD}|^2\}_{i=1}^L$, and $\{|h_{iX}|^2\}_{i=1}^L$. Since the relays do not know the channel power gain of the SD link, the RS rule does not take $|h_{SD}|^2$ into account.

C. Optimal RS Rule Problem Statement

Using A1, the instantaneous SEP for MPSK when relay β is selected is given by [22, (8.23)]

$$\mathrm{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) = \frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q\left(\gamma_{SD}+\gamma_\beta\right)}{\sin^2\theta}}\, d\theta, \quad (7)$$

where $q = \sin^2(\pi/M), m = (M-1)/M$, and using A3, $\gamma_{SD} = \frac{E_s|h_{SD}|^2}{\sigma_0^2+\sigma_2^2}$ and $\gamma_\beta = \frac{\gamma_{S\beta}\gamma_{\beta D}}{\gamma_{S\beta}+\gamma_{\beta D}+1}$, with $\gamma_{S\beta} = \frac{E_s|h_{S\beta}|^2}{\sigma_0^2+\sigma_1^2}$ and $\gamma_{\beta D} = \frac{E_r|h_{\beta D}|^2}{\sigma_0^2+\sigma_2^2}$. Averaging over the RV γ_{SD}, which is independent of γ_β, we get the following expression for the SD link-averaged SEP, which is denoted by $\mathrm{SEP}(|h_{S\beta}|^2, |h_{\beta D}|^2)$:

$$\mathrm{SEP}\left(|h_{S\beta}|^2, |h_{\beta D}|^2\right) = \frac{1}{\pi}\int_0^{m\pi} M_{\gamma_{SD}}\left(\frac{q}{\sin^2\theta}\right) e^{-\frac{q\gamma_\beta}{\sin^2\theta}}\, d\theta. \quad (8)$$

Here, $M_{\gamma_{SD}}(.)$ denotes the moment generating function (MGF) of γ_{SD}. For example, for Rayleigh fading, $M_{\gamma_{SD}}(x) = (1 + x\overline{\gamma}_{SD})^{-1}$, where $\overline{\gamma}_{SD} = \mathbb{E}[\gamma_{SD}]$.

We define a *feasible RS rule* ϕ to be a rule whose average interference to X is less than or equal to a threshold I_{avg}. Our goal is to find an optimal RS rule ϕ^* that minimizes the SEP of the secondary system while ensuring that the average interference caused to X is below a threshold I_{avg}.[4] Therefore, our problem can be mathematically stated as the following mixed-integer, stochastic, constrained optimization problem:

$$\begin{aligned} \min_{\phi} \quad & \mathbb{E}_{\mathbf{h}}\left[\mathrm{SEP}\left(|h_{S\beta}|^2, |h_{\beta D}|^2\right)\right], \\ \text{s.t.} \quad & \mathbb{E}_{\mathbf{h}}\left[E_\beta|h_{\beta X}|^2\right] \le I_{\text{avg}}, \\ & \beta = \phi(\mathbf{h}). \end{aligned} \quad (9)$$

We note that the above problem formulation can be easily generalized to other constellations such as MPAM, MQAM, MDPSK, and MFSK, whose SEP upper bound is an exponentially decaying function of the SINR [21, (6.1)], [22, (8.1)].

III. OPTIMAL RS RULE AND SIMPLER VARIANTS

Let us first consider the conventional RS rule that minimizes the SEP at D when the average interference constraint in (9) is not active. From the expression for the instantaneous SD link-averaged SEP in (8), the optimal rule selects the relay with the highest end-to-end SINR [7]. Thus,

$$\beta = \underset{i\in\{1,\ldots,L\}}{\arg\max}\{\gamma_i\}. \quad (10)$$

We shall refer to this as the *unconstrained rule*. In this case, the average interference I_{un} caused to X is $I_{\text{un}} = E_r\mathbb{E}\left[|h_{\beta X}|^2\right]$. However, when $I_{\text{un}} > I_{\text{avg}}$, the unconstrained rule is not feasible, and, thus, cannot be optimal.

[4]The interference caused to X due to transmissions by S can also be accounted for in our model as follows. In the *per slot constraint* [8], the average interference in slot 1 due to transmissions by S and that in slot 2 due to transmissions by the selected relay β are considered separately. Here, $E_s\mathbb{E}_{h_{SX}}[|h_{SX}|^2] \le I_{\text{avg}}$ and $\mathbb{E}_{\mathbf{h}}[E_\beta|h_{\beta X}|^2] \le I_{\text{avg}}$. The above formulation then requires $E_s \le \frac{I_{\text{avg}}}{\mathbb{E}_{h_{SX}}[|h_{SX}|^2]}$. In the *slot-averaged constraint*, the average interference resulting from both slots is constrained, i.e., $(E_s\mathbb{E}_{h_{SX}}[|h_{SX}|^2] + \mathbb{E}_{\mathbf{h}}[E_\beta|h_{\beta X}|^2])/2 \le I'_{\text{avg}}$. This is equivalent to $\mathbb{E}_{\mathbf{h}}[E_\beta|h_{\beta X}|^2] \le I_{\text{avg}}$, where $I_{\text{avg}} = 2I'_{\text{avg}} - E_s\mathbb{E}_{h_{SX}}[|h_{SX}|^2]$.

The optimal RS rule for our model is as follows.

Result 1: The selected relay $\beta^* = \phi^*(\mathbf{h})$, where ϕ^* is an optimal rule, is given as follows:

$$\beta^* = \begin{cases} \arg\max_{i\in\{1,\ldots,L\}}\{\gamma_i\}, & I_{\rm un} \le I_{\rm avg}, \\ \arg\min_{i\in\{0,\ldots,L\}} \left\{ \frac{1}{\pi}\int_0^{m\pi} M_{\gamma_{SD}}\left(\frac{q}{\sin^2\theta}\right) e^{-\frac{q\gamma_i}{\sin^2\theta}} d\theta \right. & \\ \qquad\qquad \left. + \lambda E_i |h_{iX}|^2 \right\}, I_{\rm un} > I_{\rm avg}, & \end{cases} \tag{11}$$

where $E_i = 0$, for $i = 0$, and $E_i = E_r$, for $1 \le i \le L$. Here, $\lambda = 0$ when $I_{\rm un} \le I_{\rm avg}$, and is a strictly positive constant when $I_{\rm un} > I_{\rm avg}$. It is chosen such that the average interference constraint is satisfied with equality, and such a choice exists.

Proof: The proof is relegated to Appendix A. ■

The constant λ is computed numerically, as is typical in several constrained optimization problems in wireless communications [21]. It is a function of the mean channel power gains and the RS rule, and it needs to be computed only once. In general, the smaller the value of $I_{\rm avg}$, the larger the value of λ. We, therefore, treat λ as a system parameter henceforth.

A. Two Simpler Insightful Variants

While the above RS rule is optimal, its single integral form can be difficult to implement as it entails numerical integration. We now derive two simpler and insightful variants, called the *IOA* and *linear* rules, by simplifying (11). As we shall see in Section V, the performance of the IOA rule closely matches that of the optimal rule and is better than the linear rule. Both outperform several existing RS rules.

1) IOA Rule: Applying the Chernoff upper bound to SEP $\left(|h_{S\beta}|^2, |h_{\beta D}|^2\right)$ in (8) and then upper bounding further using the inequality $e^{-x} \le 1/(1+x)$, for $x \ge 0$, we get

$$\text{SEP}\left(|h_{S\beta}|^2, |h_{\beta D}|^2\right) \le m M_{\gamma_{SD}}(q) e^{-q\gamma_\beta} \le \frac{m M_{\gamma_{SD}}(q)}{1+q\gamma_\beta}. \tag{12}$$

Using this bound in (11), we get the equivalent rule

$$\beta = \begin{cases} \arg\max_{i\in\{1,\ldots,L\}}\{\gamma_i\}, & I_{\rm un} \le I_{\rm avg}, \\ \arg\min_{i\in\{0,\ldots,L\}} \left\{ \frac{1}{1+q\gamma_i} + \lambda E_i |h_{iX}|^2 \right\}, & I_{\rm un} > I_{\rm avg}. \end{cases} \tag{13}$$

We call it the *IOA* rule because its first term $1/(1+q\gamma_i)$ is the inverse of an affine function of γ_i.

2) Linear Rule: Another variant of the optimal RS rule can be obtained from (12) by using the approximation $e^{-q\gamma_i} \approx 1 - q\gamma_i$, which is accurate for small values of $q\gamma_i$. We get

$$\beta = \begin{cases} \arg\max_{i\in\{1,\ldots,L\}}\{\gamma_i\}, & I_{\rm un} \le I_{\rm avg}, \\ \arg\max_{i\in\{0,\ldots,L\}} \left\{ \gamma_i - \lambda E_i |h_{iX}|^2 \right\}, & I_{\rm un} > I_{\rm avg}. \end{cases} \tag{14}$$

In both these rules, the constant λ, as before, is strictly positive when $I_{\rm un} > I_{\rm avg}$, and is chosen such that the average interference constraint is satisfied with equality. When $I_{\rm un} \le I_{\rm avg}$, $\lambda = 0$. Therefore, all the three rules reduce to the conventional interference-unconstrained RS rule.[5]

B. Optimal RS Rule for Fixed-Gain AF Relaying

While we focus on fixed-power relaying in this paper, we note that our approach can be extended to fixed-gain AF relaying as well. For it, using assumptions A1 and A3, the SINR at D when the relay β is selected can be shown to be $\gamma_{SD} + \gamma_\beta$, where $\gamma_{SD} = \frac{E_s|h_{SD}|^2}{\sigma_0^2+\sigma_2^2}$, $\gamma_\beta = \frac{\frac{E_s|h_{S\beta}|^2}{\sigma_0^2+\sigma_1^2}\frac{E_s|h_{\beta D}|^2}{\sigma_0^2+\sigma_2^2}}{\frac{E_s|h_{\beta D}|^2}{\sigma_0^2+\sigma_2^2}+\frac{E_s}{g^2(\sigma_0^2+\sigma_1^2)}}$, and g is the constant relay gain. When the interference constraint is inactive, the average interference power at X due to the selected relay's transmission is $I_{\rm un} = E_s g^2 \mathbb{E}\left[|h_{S\beta}|^2|h_{\beta X}|^2\right] + E_p g^2 \mathbb{E}\left[|h_{T\beta}|^2|h_{\beta X}|^2\right] + g^2\sigma_0^2 \mathbb{E}\left[|h_{\beta X}|^2\right]$. The optimal selected fixed-gain relay β^* can be shown to be

$$\beta^* = \begin{cases} \arg\max_{i\in\{1,\ldots,L\}}\{\gamma_i\}, & I_{\rm un} \le I_{\rm avg}, \\ \arg\min_{i\in\{0,\ldots,L\}} \left\{ \frac{1}{\pi}\int_0^{m\pi} M_{\gamma_{SD}}\left(\frac{q}{\sin^2\theta}\right) e^{-\frac{q\gamma_i}{\sin^2\theta}} d\theta \right. & \\ \qquad \left. + \lambda g^2 |h_{iX}|^2 \left(E_s|h_{Si}|^2 + \sigma_0^2 + E_p|h_{Ti}|^2\right) \right\}, & \\ \qquad\qquad I_{\rm un} > I_{\rm avg}. & \end{cases} \tag{15}$$

We do not delve into this further due to space constraints.

IV. SEP Analysis of IOA and Linear Rules

We now analyze the performance of the IOA and linear rules. When $\lambda = 0$, the problem reduces to the SEP analysis of the conventional unconstrained RS rule, which has been extensively studied in the literature, e.g., [7], [16]. We, therefore, focus on $\lambda > 0$ henceforth. In the analysis that follows, we assume that the various links are mutually independent and undergo Rayleigh fading. Thus $h_{Si} \sim CN(0, \mu_{SR})$, $h_{iD} \sim CN(0, \mu_{RD})$, and $h_{iX} \sim CN(0, \mu_{RX})$, for $i = 1, 2, \ldots, L$. Similarly, $h_{SD} \sim CN(0, \mu_{SD})$ and $h_{SX} \sim CN(0, \mu_{SX})$.

A. IOA Rule

From (8), the fading-averaged SEP is given by $\frac{1}{\pi}\int_0^{m\pi} \frac{1}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}} \mathbb{E}_{\mathbf{h}}\left[e^{-\frac{q\gamma_\beta}{\sin^2\theta}}\right] d\theta$, where $\overline{\gamma}_{SD} = \frac{E_s\mu_{SD}}{\sigma_0^2+\sigma_2^2}$ is the mean of γ_{SD}. After considerable simplification and using the approximation $\gamma_\beta \approx \frac{\gamma_{S\beta}\gamma_{\beta D}}{\gamma_{S\beta}+\gamma_{\beta D}}$, the SEP can be reduced to a three integral form with an involved integrand that contains modified Bessel functions. To gain insights, we derive simpler SEP bounds using the following inequalities [6]:

$$\frac{1}{2}\min\{\gamma_{Si}, \gamma_{iD}\} \le \gamma_i \le \min\{\gamma_{Si}, \gamma_{iD}\}. \tag{16}$$

Let $\gamma_{u_i} \triangleq \min\{\gamma_{Si}, \gamma_{iD}\}$ and $\gamma_{l_i} \triangleq \frac{1}{2}\min\{\gamma_{Si}, \gamma_{iD}\}$. It can be shown that $\{\gamma_{u_i}\}_{i=1}^L$ are i.i.d. exponential RVs with mean $\overline{\gamma}_u = \frac{\overline{\gamma}_{Si}\overline{\gamma}_{iD}}{\overline{\gamma}_{Si}+\overline{\gamma}_{iD}}$, where $\overline{\gamma}_{Si} = \frac{E_s\mu_{SR}}{\sigma_0^2+\sigma_1^2}$ and $\overline{\gamma}_{iD} = \frac{E_r\mu_{RD}}{\sigma_0^2+\sigma_2^2}$ are the

[5]We note that the linear rule bears a similarity in form to the ad hoc difference antenna selection (DAS) rule, which was proposed in [31]. However, the system models and analyses are different.

means of γ_{Si} and γ_{iD}, respectively. Similarly, $\{\gamma_{l_i}\}_{i=1}^{L}$ are i.i.d. exponential RVs with mean $\overline{\gamma}_l = \overline{\gamma}_u/2$.

Result 2: Given λ, the SEP of the IOA rule, SEP^{IOA}, is lower and upper bounded by

$$\begin{aligned}\text{SEP}^{\text{IOA}} \geq\ & \text{SEP}_0 J(\overline{\gamma}_u) + \frac{2Le^{\frac{2}{q\overline{\gamma}_u}}}{q\overline{\gamma}_u}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^{k+1} \\ & \times\left[\sum_{i=1}^{2}a_i(q\overline{\gamma}_{SD})b_{i-1,k}(2,\overline{\gamma}_u)\right. \\ & \left.+\sum_{i=3}^{N}1_{\{M>2\}}a_i(q\overline{\gamma}_{SD})b_{i,k}(2,\overline{\gamma}_u)\right], \qquad (17)\end{aligned}$$

$$\begin{aligned}\text{SEP}^{\text{IOA}} \leq\ & \text{SEP}_0 J(\overline{\gamma}_l) + \frac{Le^{\frac{1}{2q\overline{\gamma}_l}}}{2q\overline{\gamma}_l}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^{k+1} \\ & \times\left[\sum_{i=1}^{2}a_i(q\overline{\gamma}_{SD})b_{i,k}\left(\frac{1}{2},\overline{\gamma}_l\right)\right. \\ & \left.+\sum_{i=3}^{N}1_{\{M>2\}}a_i(q\overline{\gamma}_{SD})b_{i-1,k}\left(\frac{1}{2},\overline{\gamma}_l\right)\right], \qquad (18)\end{aligned}$$

where $N=3$ for $M=4, N=4$ for $M>4$, and $\theta_0=0$, $\theta_1=\pi/4, \theta_2=\pi/2, \theta_3=3\pi/4, \theta_4=m\pi, \text{SEP}_0 = m-\left(1+\frac{1}{q\overline{\gamma}_{SD}}\right)^{-\frac{1}{2}}\left[\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\left(\sqrt{\frac{1-q}{q+\overline{\gamma}_{SD}^{-1}}}\right)\right]$, $J(\overline{\gamma}) = \left[\frac{e^{\frac{1}{q\overline{\gamma}}-\frac{1}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^k \text{E}_k\left(\frac{1}{q\overline{\gamma}}\right)\right]^L$, $a_i(x) = \frac{\theta_i-\theta_{i-1}}{\pi} + \frac{1}{\pi}\sqrt{\frac{x}{1+x}}\left[\cot^{-1}\left(\sqrt{\frac{1+x}{x}}\tan\theta_i\right) - \cot^{-1}\left(\sqrt{\frac{1+x}{x}}\tan\theta_{i-1}\right)\right]$, and $b_{i,k}(v,\overline{\gamma}) = \int_0^1 x^{k-1}\text{E}_k\left(\frac{\frac{v}{q\overline{\gamma}}+v\csc^2\theta_i}{x}\right)e^{v\csc^2\theta_i-\frac{x}{\lambda E_r\mu_{RX}}} \times \left[1-e^{-\frac{1}{q\overline{\gamma}}\left(\frac{1}{x}-1\right)}+\frac{e^{\frac{1}{q\overline{\gamma}}-\frac{x}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}}\sum_{k=0}^{\infty}\frac{x^{k-1}}{k!(\lambda E_r\mu_{RX})^k}\text{E}_k\left(\frac{1}{q\overline{\gamma}x}\right)\right]^{L-1} dx$.

Proof: The proof is relegated to Appendix B. ■

The first terms $\text{SEP}_0 J(\overline{\gamma}_u)$ in (17) and $\text{SEP}_0 J(\overline{\gamma}_l)$ in (18) are due to the contributions from the direct SD link, and the second terms in (17) and (18) are due to the contributions from the L relay links. In (17) and (18), the series in k can be truncated up to $K+1$ terms, where K depends on $1/(\lambda E_r\mu_{RX}) \triangleq \nu$. We have found that $K=5$ for $\nu \leq 1$, $K=\lfloor \nu+5 \rfloor$ for $1<\nu\leq 5$, and $K=\lfloor \nu+10\rfloor$, for $\nu>5$, suffice for the SEP up to 10^{-4}, where $\lfloor\cdot\rfloor$ is the floor function.

The SEP bounds are in the form of a single integral because of $b_{i,k}(v,\overline{\gamma})$. Using Gauss-Legendre quadrature [24], it can be evaluated accurately as a sum of a few terms as follows:

$$\begin{aligned}b_{i,k}(v,\overline{\gamma}) \approx\ & \frac{1}{2}\sum_{l=1}^{W}w_l z_l^{k-1}\text{E}_k\left(\frac{\frac{v}{q\overline{\gamma}}+v\csc^2\theta_i}{z_l}\right)e^{v\csc^2\theta_i-\frac{z_l}{\lambda E_r\mu_{RX}}} \\ & \times\left[1-e^{-\frac{1}{q\overline{\gamma}}\left(\frac{1}{z_l}-1\right)}+\frac{e^{\frac{1}{q\overline{\gamma}}-\frac{z_l}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}}\right. \\ & \left.\times\sum_{k=0}^{\infty}\frac{1}{k!}\frac{z_l^{k-1}}{(\lambda E_r\mu_{RX})^k}\text{E}_k\left(\frac{1}{q\overline{\gamma}z_l}\right)\right]^{L-1}, \qquad (19)\end{aligned}$$

where $z_l \triangleq (1+x_l)/2$, and x_l and w_l are W Gauss-Legendre abscissas and weights, respectively. We have found that $W=6$ terms are sufficient for the parameters of interest to accurately compute the bounds over four orders of magnitude of the SEP. As we shall see in Section V, the bounds are asymptotically tight for large SINRs. However, they are relatively loose at low SINRs. We, therefore, also develop an SEP approximation that is considerably more accurate and is also asymptotically exact.

1) SEP Approximation: This approximation is based on the observation that both the upper and lower bounds of γ_i in (16) are exponential RVs. This motivates us to *approximate* γ_i as an *exponential* RV γ_i' whose mean $\overline{\gamma}' = \mathbb{E}[\gamma_i']$ is equal to $\mathbb{E}[\gamma_i] \approx \mathbb{E}\left[\frac{\gamma_{Si}\gamma_{iD}}{\gamma_{Si}+\gamma_{iD}}\right]$. Using the PDF of $\frac{\gamma_{Si}\gamma_{iD}}{\gamma_{Si}+\gamma_{iD}}$, which is given in [6], it can be shown that

$$\begin{aligned}\mathbb{E}\left[\frac{\gamma_{Si}\gamma_{iD}}{\gamma_{Si}+\gamma_{iD}}\right] =\ & \frac{2\sqrt{p}}{15}\,{}_2F_1\left(3,3;\frac{7}{2};\frac{1}{2}-\frac{\sigma'}{4\sqrt{p}}\right) \\ & +\frac{\sigma'}{10}\,{}_2F_1\left(4,2;\frac{7}{2};\frac{1}{2}-\frac{\sigma'}{4\sqrt{p}}\right), \qquad (20)\end{aligned}$$

where $p \triangleq \frac{E_sE_r\mu_{SR}\mu_{RD}}{(\sigma_0^2+\sigma_1^2)(\sigma_0^2+\sigma_2^2)}$ and $\sigma' \triangleq \frac{E_s\mu_{SR}}{\sigma_0^2+\sigma_1^2}+\frac{E_r\mu_{RD}}{\sigma_0^2+\sigma_2^2}$.

Using this approximation, we show in Appendix C that

$$\begin{aligned}\text{SEP}^{\text{IOA}} \approx\ & \text{SEP}_0 J(\overline{\gamma}') + \frac{Le^{\frac{1}{q\overline{\gamma}'}}}{q\overline{\gamma}'}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^{k+1} \\ & \times\left[\sum_{i=1}^{2}a_i(q\overline{\gamma}_{SD})b_{i,k}(1,\overline{\gamma}')\right. \\ & \left.+\sum_{i=3}^{N}1_{\{M>2\}}a_i(q\overline{\gamma}_{SD})b_{i-1,k}(1,\overline{\gamma}')\right], \qquad (21)\end{aligned}$$

where $N, \theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \text{SEP}_0, J(\overline{\gamma}')$, and $a_i(q\overline{\gamma}_{SD})$ are defined in Result 2, and $b_{i,k}(1,\overline{\gamma}')$ is given by (19).

2) Computing λ: Using the exponential approximation of γ_i, the average interference I^{IOA} caused to X due to transmissions by the selected relay can be shown to be

$$\begin{aligned}I^{\text{IOA}} \approx\ & \frac{Le^{\frac{1}{q\overline{\gamma}'}}}{2\lambda q\overline{\gamma}'}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^{k+1}\sum_{l=1}^{W}w_l z_l^k e^{-\frac{z_l}{\lambda E_r\mu_{RX}}} \\ & \times\left[\text{E}_k\left(\frac{1}{q\overline{\gamma}'z_l}\right)-\text{E}_{k+1}\left(\frac{1}{q\overline{\gamma}'z_l}\right)\right] \\ & \times\left[1-e^{-\frac{1}{q\overline{\gamma}'}\left(\frac{1}{z_l}-1\right)}+\frac{e^{\frac{1}{q\overline{\gamma}'}-\frac{z_l}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}'}\right. \\ & \left.\times\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^k z_l^{k-1}\text{E}_k\left(\frac{1}{q\overline{\gamma}'z_l}\right)\right]^{L-1}. \qquad (22)\end{aligned}$$

We have found that $W=5$ terms are sufficient for the parameters of interest to accurately compute I^{IOA}. Since λ is the solution of the equation $I^{\text{IOA}} = I_{\text{avg}}$, it can be easily computed, e.g., using `fsolve` in Matlab.

B. Linear Rule

The SEP bounds for the linear rule are as follows.

Result 3: Given λ, the SEP of the linear rule, SEP^{lin}, is lower and upper bounded by

$$\begin{aligned} \text{SEP}^{\text{lin}} \geq\ & \text{SEP}_0 \left(\frac{\lambda E_r \mu_{RX}}{\overline{\gamma}_u + \lambda E_r \mu_{RX}} \right)^L + \frac{2L}{\overline{\gamma}_u + 2\lambda E_r \mu_{RX}} \\ & \times \left[\sum_{i=1}^{2} a'_i(1, \overline{\gamma}_u) b'_{i-1}(1, \overline{\gamma}_u) \right. \\ & \left. + \sum_{i=3}^{N} 1_{\{M>2\}} a'_i(1, \overline{\gamma}_u) b'_i(1, \overline{\gamma}_u) \right], \end{aligned} \tag{23}$$

$$\begin{aligned} \text{SEP}^{\text{lin}} \leq\ & \text{SEP}_0 \left(\frac{\lambda E_r \mu_{RX}}{\overline{\gamma}_l + \lambda E_r \mu_{RX}} \right)^L + \frac{L}{2\overline{\gamma}_l + \lambda E_r \mu_{RX}} \\ & \times \left[\sum_{i=1}^{2} a'_i(4, \overline{\gamma}_l) b'_i(4, \overline{\gamma}_l) \right. \\ & \left. + \sum_{i=3}^{N} 1_{\{M>2\}} a'_i(4, \overline{\gamma}_l) b'_{i-1}(4, \overline{\gamma}_l) \right], \end{aligned} \tag{24}$$

where $N, \theta_0, \theta_1, \ldots, \theta_4, a_i(x)$, and SEP_0 are defined in Result 2, $a'_i(v, \overline{\gamma}) = \frac{a_i(q\overline{\gamma}_{SD})}{1 - \frac{2\lambda E_r \mu_{RX} \overline{\gamma}}{\overline{\gamma}_{SD}(v\overline{\gamma} + 2\lambda E_r \mu_{RX})}} + \frac{a_i\left(\frac{2q\lambda E_r \mu_{RX} \overline{\gamma}}{v\overline{\gamma} + 2\lambda E_r \mu_{RX}}\right)}{1 - \frac{\overline{\gamma}_{SD}(v\overline{\gamma} + 2\lambda E_r \mu_{RX})}{2\lambda E_r \mu_{RX} \overline{\gamma}}}$, and $b'_i(v, \overline{\gamma}) = \sum_{l=0}^{L-1} \binom{L-1}{l} \left(\frac{-\overline{\gamma}}{\overline{\gamma} + \lambda E_r \mu_{RX}} \right)^l \frac{v}{2\left(\frac{q}{\sin^2 \theta_i} + \frac{1 + \frac{vl}{2}}{\overline{\gamma}} \right)}$.

Proof: The proof is relegated to Appendix D. ■

Notice that the above expressions are in closed-form, unlike those for the IOA rule. As in the IOA rule, the first term of the SEP bound for the linear rule is the contribution from the direct SD link and the second term is the contribution from the relay links. As before, we present an SEP approximation that is accurate, yet simpler in form.

1) SEP Approximation: For a given value of λ, the SEP of the linear rule is approximately given by

$$\begin{aligned} \text{SEP}^{\text{lin}} \approx\ & \text{SEP}_0 \left(\frac{\lambda E_r \mu_{RX}}{\overline{\gamma}' + \lambda E_r \mu_{RX}} \right)^L + L\text{SEP}_0 \\ & \times \frac{\sum_{l=0}^{L-1} \binom{L-1}{l} \left(\frac{-\overline{\gamma}'}{\overline{\gamma}' + \lambda E_r \mu_{RX}} \right)^l \left(q + \frac{l+1}{\overline{\gamma}'} \right)^{-1}}{\overline{\gamma}' + \lambda E_r \mu_{RX} + q\lambda E_r \mu_{RX} \overline{\gamma}'}. \end{aligned} \tag{25}$$

The derivation is similar to Appendix C, and is skipped.

2) Computing λ: The average interference I^{lin} caused to X due to the selected relay's transmission can be shown to be

$$\begin{aligned} I^{\text{lin}} =\ & \frac{L}{E_r \mu_{RX} \overline{\gamma}'} \sum_{l=0}^{L-1} \binom{L-1}{l} \left(\frac{-\overline{\gamma}'}{\overline{\gamma}' + \lambda E_r \mu_{RX}} \right)^l \\ & \times \left(\frac{E_r \mu_{RX} \overline{\gamma}'}{\overline{\gamma}' - l\lambda E_r \mu_{RX}} \right)^2 \left[\frac{\overline{\gamma}'}{l+1} - \frac{\lambda E_r \mu_{RX} \overline{\gamma}'}{\overline{\gamma}' + \lambda E_r \mu_{RX}} \right. \\ & \left. \times \left(1 + \frac{\overline{\gamma}' - l\lambda E_r \mu_{RX}}{\overline{\gamma}' + \lambda E_r \mu_{RX}} \right) \right]. \end{aligned} \tag{26}$$

Therefore, λ can be computed numerically as the solution of the equation $I^{\text{lin}} = I_{\text{avg}}$.

C. Asymptotic SEP Analysis and Insights

To gain further insights, we consider the regime in which $\mu_{SD} = \mu_{SR} = \mu_{RD} = \mu_{RX} = \mu$ and $\mu \to \infty$, with E_s, E_r, and λ (> 0) being fixed. The SEP of the IOA rule in this regime simplifies to

$$\begin{aligned} \text{SEP}^{\text{IOA}} =\ & \frac{\left[1 - \frac{1}{M} + \frac{1}{2\pi} \sin\left(\frac{2\pi}{M}\right)\right] \left(\sigma_0^2 + \sigma_2^2\right)}{2qE_s\mu} \\ & \times \left(1 - \frac{L}{\lambda E_r \mu} \right) + o\left(\frac{1}{\mu^3} \right). \end{aligned} \tag{27}$$

The term $o(1/\mu^3)$ is due to the contributions from the relay links. The dominant first term is due to the contribution from the SD link. Since the SEP in this regime falls as $1/\mu$, the diversity order is unity. The expression also brings out how the SEP decreases as λ decreases or L increases or M decreases.

Similarly, in the above regime, the SEP of the linear rule simplifies to $\text{SEP}^{\text{lin}} = \frac{[1 - \frac{1}{M} + \frac{1}{2\pi}\sin(\frac{2\pi}{M})](\sigma_0^2 + \sigma_2^2)}{2qE_s\mu \left(1 + \frac{E_s}{2\lambda[E_s(\sigma_0^2 + \sigma_2^2) + E_r(\sigma_0^2 + \sigma_1^2)]}\right)^L} + o(1/\mu^3)$. The above insights apply here as well.

It can be shown that in an alternate scaling regime, in which $\mu_{SD} = \mu_{SR} = \mu_{RD} = \mu \to \infty, \mu_{RX} = 1/\mu \to 0, E_s, E_r$, and λ (> 0) are fixed, a full diversity order of $L + 1$ is achieved by the linear rule and, thus, the optimal rule.

V. Numerical Results and Benchmarking

We now present simulation results to verify our analysis and gain quantitative insights. For the sake of illustration, we use $E_s = E_r = E = 10$ dB, $\sigma_0^2 = 2$ dB, $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 1.98$ dB. Therefore, $\sigma_0^2 + \sigma^2 = 5$ dB. Unless mentioned otherwise, we assume $\mu_{SD} = \mu_{SR} = \mu_{RD} = \mu_{RX} = \mu$, and vary μ from -5 to 25 dB. Thus, the average SINR of the various links, $E\mu/(\sigma_0^2 + \sigma^2)$, varies from 0 to 30 dB.

A. Comparison and Benchmarking of Proposed Rules

Fig. 2 compares the SEPs of the optimal, IOA, and linear rules as a function of the average SINR $E\mu/(\sigma_0^2 + \sigma^2)$. These are computed from Monte Carlo simulations, with and without the use of the assumptions A1 and A3. The primary transmitter transmits 8PSK symbols. As in Fig. 1, we see that the SEP curves, with and without these assumptions, are close to each other, though the linear rule is more sensitive than the optimal and IOA rules to these assumptions.

As a reference, the SEPs of the conventional non-cognitive relay network (i.e., $I_{\text{avg}} = \infty$) and a non-cooperative network that uses only the direct SD link (i.e., $I_{\text{avg}} = 0$) are shown. When $E\mu/(\sigma_0^2 + \sigma^2) \leq 10$ dB, the network is not interference-constrained. Hence, the SEPs of all the three rules are the same as that of the unconstrained rule. When $E\mu/(\sigma_0^2 + \sigma^2) > 10$ dB, the network is interference-constrained and the performances of the three rules differ. The optimal rule has the lowest

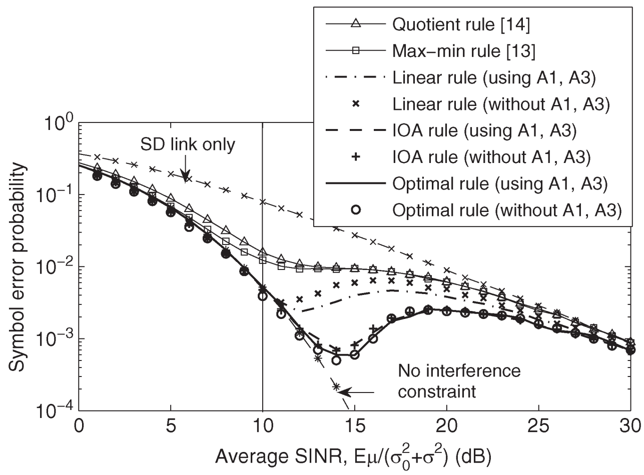

Fig. 2. Comparison of the SEPs of the optimal, IOA, linear, quotient, and max-min rules (QPSK, $L = 4$, and $I_{\text{avg}} = 15$ dB).

SEP. The SEP of the IOA rule is very close to the optimal rule, while that of the linear rule is marginally worse. Notice that the SEPs of all the three rules increase when $E\mu/(\sigma_0^2 + \sigma^2)$ increases from 14 dB to 19 dB. This is because the relay transmit power is fixed. Thus, when a relay transmits, its interference to X increases as μ increases. Consequently, the interference constraint forces all the relays to be shut down more often. As a result, the communication occurs more often through the direct SD link only, which increases the SEP compared to the situation when a relay is selected to aid the direct transmission. This also explains why at high SINRs (> 24 dB), the SEPs of all the three rules approach that using the direct SD link only. The significant reduction in the SEP compared to the direct link at low-to-mid SINRs shows the advantage of using relays in CR. In general, the larger the ratio $I_{\text{avg}}/I_{\text{un}}$, the more the reduction.

The figure also plots the SEPs of the aforementioned max-min rule [13] and quotient rule [14], which have been adapted to our model with the interference threshold set as I_{avg}. To ensure a fair comparison, the SNR threshold used in [14] is determined numerically to minimize the SEP. We see that all the three proposed rules outperform the two benchmark rules for the entire range of average SINRs.

B. Performance Analysis of Proposed Rules

Henceforth, we will focus on the interference-constrained regime and show results using A1 and A3.

1) IOA rule: Fig. 3 plots the SEP from simulations, its bounds in (17) and (18), and its approximation in (21) as a function of λ. When $\lambda = 0$, the system is interference-unconstrained and the SEP is the lowest. As λ increases, the SEP increases due to a tighter interference constraint, which forces all the relays to be shut down more often and communication occurs only over the SD link for the most of the part. Notice that the SEP bounds are relatively loose at lower values of λ but become tighter as λ increases. The SEP approximation is tight for all λ. Further, as L increases, the SEP decreases, which is intuitive.

Fig. 4 plots the SEP from simulations, its upper and lower bounds, and its approximation as a function of the average

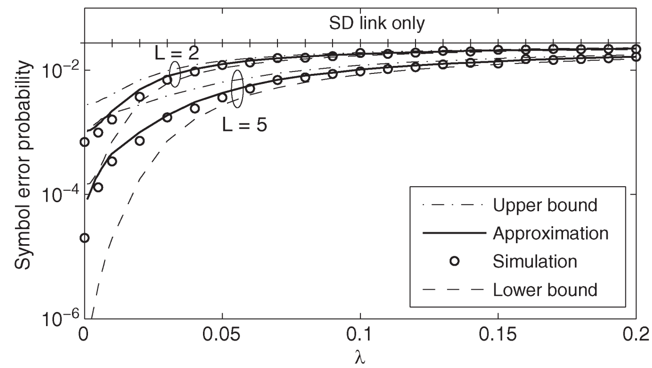

Fig. 3. Effect of λ on SEP of the IOA rule: SEP as a function of λ for two different values of L (QPSK and average SINR of 15 dB).

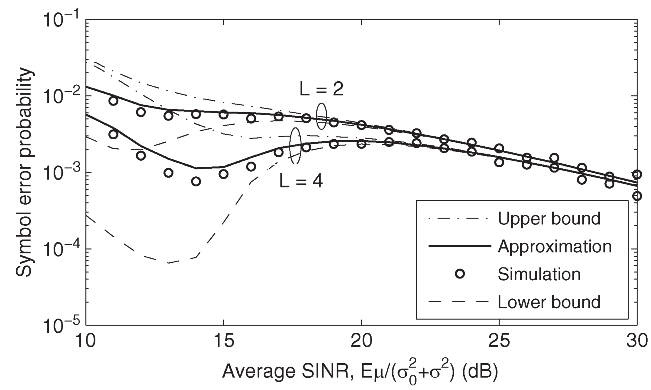

Fig. 4. *IOA rule*: SEP as a function of average SINR for different number of relays (QPSK, $I_{\text{avg}} = 15$ dB, $\mu_{SD} = \mu_{RX} = \mu, \mu_{SR} = 0.75\mu$, and $\mu_{RD} = 1.25\mu$).

SINR for different values of L. As L increases, the SEP decreases. While the SEP bounds are relatively loose at lower SINRs, they are asymptotically tight. Further, the SEP approximation is within 0.6 dB of the SEP from simulations even at lower SINRs, and is asymptotically exact. As $E\mu/(\sigma_0^2 + \sigma^2)$ increases, the SEP initially decreases but then it eventually increases. We refer the reader to the explanation of Fig. 2 for a detailed explanation of this.

We now study the case when the relay transmit power is also optimized. To ensure a meaningful comparison, the relay transmit power E_r is not allowed to exceed a maximum value $E_r^{\max}$. Fig. 5 plots the SEP of the IOA rule from simulations as a function of $E_r^{\max}$ for two values of I_{avg} and $\mu = 10$ dB. Results when E_r is always equal to $E_r^{\max}$ and when it is optimized are shown. Consider, for example, $I_{\text{avg}} = 10$ dB. When $E_r^{\max} < I_{\text{avg}}/\mu$ ($= 0$ dB), the interference constraint is not active and the SEP monotonically decreases as $E_r^{\max}$ increases. When $0 \text{ dB} \leq E_r^{\max} \leq 4$ dB, λ is positive but small. As a result, the SEP decreases as $E_r^{\max}$ increases. The optimal relay power is $E_r^{\max}$ for $E_r^{\max} < 4$ dB. When $E_r^{\max} \geq 4$ dB, the optimal relay power is 4 dB, and this yields a flat SEP curve. Instead, if the relay power is always set to $E_r^{\max}$, then as $E_r^{\max}$ increases, the relay interference to X increases. Consequently, the SEP increases and approaches that of using the non-cooperative network due to the same reason as in Fig. 2. The trends are

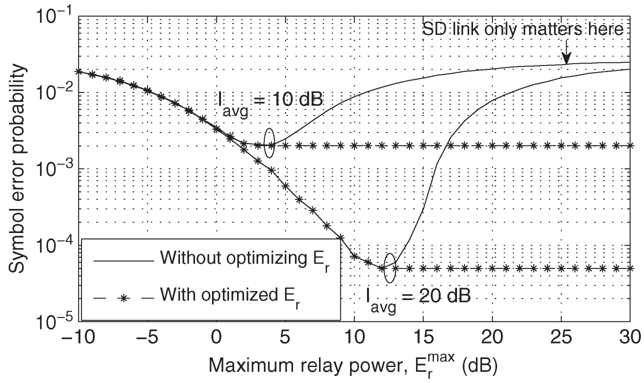

Fig. 5. IOA rule: SEP as a function of maximum relay power and I_{avg}, with and without optimizing E_r (QPSK, $E_s = 10$ dB, $L = 4$, and $\mu = 10$ dB).

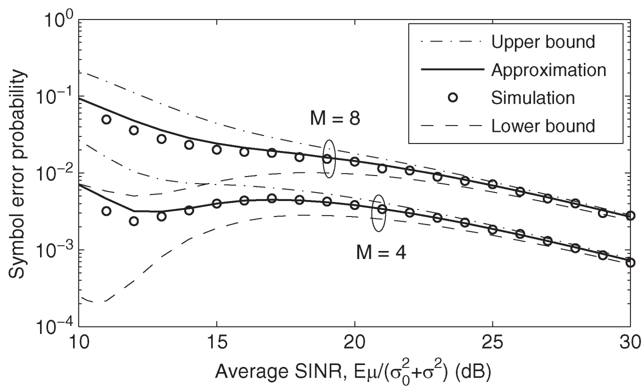

Fig. 6. *Linear rule*: SEP as a function of average SINR for different constellation sizes ($L = 4$, $I_{\text{avg}} = 15$ dB, and $E_s = E_r = E = 10$ dB).

similar for $I_{\text{avg}} = 20$ dB except that the SEP is lower and the interference constraint is active when $E_r^{\max} \geq 10$ dB. Note that the term fixed-power relaying is still appropriate because the relay transmits with the same fixed power for different channel realizations of the SR, RD, and RX links.

2) Linear Rule: Fig. 6 plots the SEP of the linear rule from simulations, its bounds in (23) and (24), and the SEP approximation in (25) as a function of the average SINR for two different constellation sizes M. As expected, the SEP increases as M increases. Notice that the SEP approximation is within 0.5 dB of the SEP from simulations even at low SINRs, where the bounds are loose. At high SINRs, the bounds and the approximation are all tight. The other trends are similar to the IOA rule, and are not repeated here.

VI. Conclusion

We proposed three novel RS rules for an average interference-constrained CR network that uses practically appealing fixed-power AF relays. We first derived the SEP-optimal RS rule, which turned out to be non-linear in form, and was different from the RS rules available in the literature. The IOA and linear rules were simpler, low implementation complexity variants of the optimal rule. The performance of the IOA rule was very close to that of the optimal rule, while the more tractable linear rule had a marginally worse performance. We saw that relay selection was beneficial for underlay CR for low-to-mid SINR values. An interesting avenue for future work is to develop corresponding optimal RS rules for other cooperative relaying protocols.

Appendix

A. Proof of Result 1

When $I_{\text{un}} \leq I_{\text{avg}}$: The unconstrained rule in (10) is feasible. Since it yields the lowest SEP, it is also the optimal rule.

When $I_{\text{un}} > I_{\text{avg}}$: The set of all feasible selection rules, $\mathcal{Z}$, is a non-empty set because a selection rule in which no relay transmits causes zero relay interference to X and is feasible. Let $\phi \in \mathcal{Z}$ be a feasible rule. For a constant $\lambda > 0$, define an auxiliary function $L_\phi(\lambda)$ associated with ϕ as

$$L_\phi(\lambda) \triangleq \mathbb{E}_{\mathbf{h}}\left[\text{SEP}\left(|h_{S\beta}|^2, |h_{\beta D}|^2\right) + \lambda E_\beta |h_{\beta X}|^2\right]. \quad (28)$$

Note that $L_\phi(\lambda)$ is a function of ϕ and λ. Further, define a new rule ϕ^* in terms of the relay β^* it selects as follows:

$$\beta^* = \underset{i\in\{0,\ldots,L\}}{\arg\min}\left\{\text{SEP}\left(|h_{Si}|^2, |h_{iD}|^2\right) + \lambda E_i |h_{iX}|^2\right\}, \quad (29)$$

where λ is chosen such that $\mathbb{E}_{\mathbf{h}}[E_{\beta^*}|h_{\beta^* X}|^2] = I_{\text{avg}}$.[6] Thus, ϕ^* is a feasible rule.

We now prove that ϕ^* is the derived optimal RS rule. From (29), it follows that $L_{\phi^*}(\lambda) \leq L_\phi(\lambda)$. Therefore,

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}&\left[\text{SEP}\left(|h_{S\beta^*}|^2, |h_{\beta^* D}|^2\right)\right] \\ &\leq \mathbb{E}_{\mathbf{h}}\left[\text{SEP}\left(|h_{S\beta}|^2, |h_{\beta D}|^2\right)\right] + \lambda\left(\mathbb{E}_{\mathbf{h}}\left[E_\beta |h_{\beta X}|^2\right] - I_{\text{avg}}\right).\end{aligned} \quad (30)$$

Since ϕ is a feasible rule, $\mathbb{E}_{\mathbf{h}}[E_\beta |h_{\beta X}|^2] \leq I_{\text{avg}}$. Thus,

$$\mathbb{E}_{\mathbf{h}}\left[\text{SEP}\left(|h_{S\beta^*}|^2, |h_{\beta^* D}|^2\right)\right] \leq \mathbb{E}_{\mathbf{h}}\left[\text{SEP}\left(|h_{S\beta}|^2, |h_{\beta D}|^2\right)\right]. \quad (31)$$

Hence, ϕ^* yields the lowest average SEP among all feasible rules. It is, therefore, optimal.

B. Proof of Result 2

First, we derive the SEP lower bound. The SEP, $\Pr(\text{Err}|h_{SD}, \mathbf{h})$, conditioned on $h_{SD}, \mathbf{h}$ can be written as

$$\Pr(\text{Err}|h_{SD}, \mathbf{h}) = \Pr(\beta = 0, \text{Err}|h_{SD}, \mathbf{h}) + \sum_{i=1}^{L} \Pr(\beta = i, \text{Err}|h_{SD}, \mathbf{h}). \quad (32)$$

Averaging over $h_{SD}, \mathbf{h}$ and using the chain rule, we get

$$\text{SEP} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[\Pr(\text{Err}|h_{SD}, \mathbf{h})\right] = T_1 + LT_2, \quad (33)$$

[6] Such a unique choice of λ exists can be proved using the intermediate value theorem by observing that $0 \leq I_{\text{avg}} < I_{\text{un}}$, and proving that the average interference is a continuous and monotonically decreasing function of $\lambda \geq 0$.

where $T_1 = \mathbb{E}_{h_{SD},\mathbf{h}}[\Pr(\beta=0|h_{SD},\mathbf{h})\Pr(\text{Err}|\beta=0,h_{SD},\mathbf{h})]$ and $T_2 = \mathbb{E}_{h_{SD},\mathbf{h}}[\Pr(\beta=1|h_{SD},\mathbf{h})\Pr(\text{Err}|\beta=1,h_{SD},\mathbf{h})]$. The factor L arises because the L relays are statistically identical.

1) Evaluating T_1: As $\Pr(\beta=0|h_{SD},\mathbf{h})$ is function of $\mathbf{h}$ only, and $\Pr(\text{Err}|\beta=0,\mathbf{h})$ is a function of h_{SD} only, we get

$$T_1 = \Pr(\beta=0)\mathbb{E}_{h_{SD}}\left[\frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q\gamma_{SD}}{\sin^2\theta}}\,d\theta\right] = \Pr(\beta=0)\text{SEP}_0. \tag{34}$$

Here, $\text{SEP}_0 = \frac{1}{\pi}\int_0^{m\pi}\left(1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)^{-1}d\theta$, and is given in closed-form in the result statement.

Let $y_i \triangleq 1/(1+q\gamma_i)$ and $g_i \triangleq E_r|h_{iX}|^2, 1\le i\le L$. Then, $\{g_i\}_{i=1}^L$ are i.i.d. exponential RVs with mean $E_r\mu_{RX}$, and $\{y_i\}_{i=1}^L$ are also i.i.d. RVs. Therefore, from (13), we get

$$\begin{aligned}\Pr(\beta=0) &= \Pr(y_1+\lambda g_1>1, y_2+\lambda g_2>1,\ldots,y_L+\lambda g_L>1),\\ &= \left[\Pr\left(y_1+\lambda g_1>1\right)\right]^L.\end{aligned} \tag{35}$$

Let $y_{l_i} \triangleq 1/(1+q\gamma_{u_i}), 1\le i\le L$. Then $\{y_{l_i}\}_{i=1}^L$ are i.i.d. RVs, whose PDF can be shown to be $f_{y_{l_i}}(y) = \frac{1}{q\overline{\gamma}_u y^2}e^{-\frac{1}{q\overline{\gamma}_u}\left(\frac{1}{y}-1\right)}, 0\le y\le 1$. Since $y_{l_i}\le y_i$, (35) implies that $\Pr(\beta=0)\ge[\Pr(y_{l_1}+\lambda g_1>1)]^L$. Substituting the PDFs of g_1 and y_{l_1}, integrating over g_1, and using $t=1/y_{l_1}$, we get

$$\Pr(\beta=0)\ge\left(\frac{e^{\frac{1}{q\overline{\gamma}_u}-\frac{1}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}_u}\int_1^\infty e^{\frac{1}{\lambda E_r\mu_{RX}t}-\frac{t}{q\overline{\gamma}_u}}\,dt\right)^L. \tag{36}$$

Expanding $e^{\frac{1}{\lambda E_r\mu_{RX}t}}$ as $\sum_{k=0}^\infty\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}t}\right)^k$, interchanging the order of integration and summation using the dominated convergence theorem (DCT) [32], and substituting (36) into (34) yields the first term of the SEP lower bound in (17).

2) Evaluating T_2: Since $\Pr(\beta=1|h_{SD},\mathbf{h})$ is only a function of $\mathbf{h}$, we get

$$T_2 = \mathbb{E}_{h_{SD},\mathbf{h}}\left[\Pr(\beta=1|\mathbf{h})\frac{1}{\pi}\int_0^{m\pi}e^{-\frac{q(\gamma_{SD}+\gamma_1)}{\sin^2\theta}}\,d\theta\right]. \tag{37}$$

Let $\mathbf{y}\triangleq[y_1,y_2,\ldots,y_L]$ and $\mathbf{g}\triangleq[g_1,g_2,\ldots,g_L]$. Interchanging the order of the finite integral and expectation, averaging over h_{SD}, and from the law of total expectation, we get

$$T_2 = \int_0^{m\pi}\frac{\mathbb{E}_{y_1,g_1}\left[e^{-\left(\frac{1}{y_1}-1\right)\csc^2\theta}\Pr(\beta=1|y_1,g_1)\right]}{\pi\left(1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)}\,d\theta. \tag{38}$$

Now, the conditional probability that Relay 1 is selected is

$$\begin{aligned}\Pr(\beta=1|y_1,g_1) &= \Pr\left(1>y_1+\lambda g_1, y_2+\lambda g_2>y_1+\lambda g_1,\right.\\ &\qquad \left.\ldots,y_L+\lambda g_L>y_1+\lambda g_1|y_1,g_1\right),\\ &= \left[1-F_{y_2+\lambda g_2}\left(y_1+\lambda g_1\right)\right]^{L-1}1_{\{y_1+\lambda g_1<1\}}.\end{aligned}$$

Let $y_{u_i}\triangleq 1/(1+q\gamma_{l_i}), 1\le i\le L$. Clearly, $y_{u_i}\ge y_i$. Since the CDF is a monotonically non-decreasing function, we get

$$\Pr(\beta=1|y_1,g_1)\ge\left[1-F_{y_{l_2}+\lambda g_2}\left(y_{u_1}+\lambda g_1\right)\right]^{L-1}1_{\{y_{u_1}+\lambda g_1<1\}}. \tag{39}$$

Substituting the PDFs of g_2 and y_{l_2}, and simplifying further,

$$\begin{aligned}F_{y_{l_2}+\lambda g_2}(y_{u_1}+\lambda g_1) &= e^{-\frac{1-(y_{u_1}+\lambda g_1)}{q\overline{\gamma}_u(y_{u_1}+\lambda g_1)}} - \frac{e^{\frac{1}{q\overline{\gamma}_u}-\frac{y_{u_1}+\lambda g_1}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}_u(y_{u_1}+\lambda_{g_1})}\\ &\quad\times\sum_{k=0}^\infty\frac{1}{k!}\left(\frac{y_{u_1}+\lambda g_1}{\lambda E_r\mu_{RX}}\right)^k\text{E}_k\left(\frac{1}{q\overline{\gamma}_u(y_{u_1}+\lambda g_1)}\right).\end{aligned} \tag{40}$$

From (16), it can be shown that $y_{u_1}=2y_{l_1}/(y_{l_1}+1)$. Substituting (40) into (39) and simplifying further, we get the following lower bound from (38):

$$\begin{aligned}T_2 &\ge \frac{2e^{\frac{1}{q\overline{\gamma}_u}}}{\pi\lambda E_r\mu_{RX}q\overline{\gamma}_u}\int_0^{m\pi}\int_0^1\int_{\frac{1}{x}}^\infty\frac{\kappa(x)^{L-1}}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}e^{-\left(\frac{x}{\lambda E_r\mu_{RX}}-\csc^2\theta\right)}\\ &\quad\times e^{\frac{1}{\lambda E_r\mu_{RX}t}}e^{-(2t-1)\left(\csc^2\theta+\frac{1}{q\overline{\gamma}_u}\right)}\,dt\,dx\,d\theta,\end{aligned} \tag{41}$$

where $\kappa(x) = 1-e^{-\frac{1}{q\overline{\gamma}_u}\left(\frac{1}{x}-1\right)}+\frac{e^{\frac{1}{q\overline{\gamma}_u}-\frac{x}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}_u}\sum_{k=0}^\infty\frac{1}{k!(\lambda E_r\mu_{RX})^k}\times x^{k-1}\text{E}_k\left(\frac{1}{q\overline{\gamma}_u x}\right)$. As before, expanding $e^{\frac{1}{\lambda E_r\mu_{RX}t}}$ in (41) as $\sum_{k=0}^\infty\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}t}\right)^k$, interchanging the order of summation and integral over t using DCT, integrating over t, and, finally, interchanging the order of summation and finite integrals over x and θ, we get

$$\begin{aligned}T_2 &\ge \frac{2e^{\frac{2}{q\overline{\gamma}_u}}}{\pi q\overline{\gamma}_u}\sum_{k=0}^\infty\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^{k+1}\int_0^{m\pi}\int_0^1\text{E}_k\left(\frac{2\csc^2\theta+\frac{2}{q\overline{\gamma}_u}}{x}\right)\\ &\quad\times\frac{x^{k-1}\kappa(x)^{L-1}e^{-\left(\frac{x}{\lambda E_r\mu_{RX}}-2\csc^2\theta\right)}}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}\,dx\,d\theta.\end{aligned} \tag{42}$$

The double integral above can be lower bounded by partitioning the region of integration over θ into sub-intervals $[\theta_0,\theta_1],\ldots,[\theta_3,\theta_4]$, where $\theta_0=0,\theta_1=\pi/4,\theta_2=\pi/2,\theta_3=3\pi/4$, and $\theta_4=m\pi$. In each sub-interval $[\theta_{i-1},\theta_i]$, we replace θ in the term $e^{2\csc^2\theta}\text{E}_k\left(\frac{2\csc^2\theta+\frac{2}{q\overline{\gamma}_u}}{x}\right)$ in (42) by θ_{i-1} for $1\le i\le 2$, and by θ_i for $3\le i\le 4$, to get a lower bound, and finally integrate $\left(1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)^{-1}$ over θ. This yields the second term of the SEP lower bound in (17).

The derivation of the SEP upper bound is similar except that $\overline{\gamma}_u$ is replaced by $\overline{\gamma}_l=\overline{\gamma}_u/2$.

C. Derivation of SEP Approximation for IOA Rule

Let $y_i'\triangleq 1/(1+q\gamma_i'), 1\le i\le L$. Clearly, $\{y_i'\}_{i=1}^L$ are i.i.d. RVs. Their PDF can be shown to be $f_{y_i'}(y)=e^{-\frac{1}{q\overline{\gamma}'}\left(\frac{1}{y}-1\right)}/(q\overline{\gamma}'y^2)$, for $0\le y\le 1$.

The SEP of the IOA rule is then approximately equal to

$$\mathrm{SEP}^{\mathrm{IOA}} \approx \widetilde{T}_1 + L\widetilde{T}_2, \tag{43}$$

where $\widetilde{T}_1 = \mathrm{SEP}_0[\Pr(y_1' + \lambda g_1 > 1)]^L$ and $\widetilde{T}_2 = \frac{1}{\pi}\int_0^{m\pi} \frac{1}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}} \mathbb{E}_{y_1',g_1}\left[e^{-\left(\frac{1}{y_1'}-1\right)\csc^2\theta}\Pr(\beta=1|y_1',g_1)\right] d\theta$.

1) Evaluating $\widetilde{T}_1$: Substituting the PDFs of g_1 and y_1' in the expression for $\widetilde{T}_1$, integrating over g_1, and using $t = 1/y_1'$,

$$\widetilde{T}_1 = \mathrm{SEP}_0\left[\frac{1}{q\overline{\gamma}'}e^{\frac{1}{q\overline{\gamma}'}-\frac{1}{\lambda E_r\mu_{RX}}}\int_1^\infty e^{\frac{1}{\lambda E_r\mu_{RX}t}-\frac{t}{q\overline{\gamma}'}}\,dt\right]^L. \tag{44}$$

As before, expanding $e^{\frac{1}{\lambda E_r\mu_{RX}t}}$ in (44) and interchanging the order of integration and summation yields the first term in (21).

2) Evaluating $\widetilde{T}_2$: As in Appendix B, $\Pr(\beta=1|y_1',g_1) = \left[1 - F_{y_2'+\lambda g_2}(y_1'+\lambda g_1)\right]^{L-1} 1_{\{y_1'+\lambda g_1<1\}}$. Substituting the PDFs of g_2 and y_2', and simplifying further, we get $F_{y_2'+\lambda g_2}(y_1'+\lambda g_1) = e^{-\frac{1-(y_1'+\lambda g_1)}{q\overline{\gamma}'(y_1'+\lambda g_1)}} - \frac{e^{\frac{1}{q\overline{\gamma}'}-\frac{(y_1'+\lambda g_1)}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}'(y_1'+\lambda g_1)} \times \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{y_1'+\lambda g_1}{\lambda E_r\mu_{RX}}\right)^k \mathrm{E}_k\left(\frac{1}{q\overline{\gamma}'(y_1'+\lambda g_1)}\right)$. Substituting this into $\Pr(\beta=1|y_1',g_1)$ and simplifying further, $\widetilde{T}_2$ is given by

$$\widetilde{T}_2 = \frac{e^{\frac{1}{q\overline{\gamma}'}}}{\pi\lambda E_r\mu_{RX}q\overline{\gamma}'}\int_0^{m\pi}\int_0^1\int_{\frac{1}{x}}^\infty \frac{\zeta(x)^{L-1}}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}e^{-\left(\frac{x}{\lambda E_r\mu_{RX}}-\csc^2\theta\right)} \times e^{\frac{1}{\lambda E_r\mu_{RX}t}}e^{-t\left(\csc^2\theta+\frac{1}{q\overline{\gamma}'}\right)}\,dt\,dx\,d\theta, \tag{45}$$

where $\zeta(x) = 1 - e^{-\frac{1}{q\overline{\gamma}'}\left(\frac{1}{x}-1\right)} + \frac{e^{\frac{1}{q\overline{\gamma}'}-\frac{x}{\lambda E_r\mu_{RX}}}}{q\overline{\gamma}'}\sum_{k=0}^{\infty}\frac{1}{k!(\lambda E_r\mu_{RX})^k} \times x^{k-1}\mathrm{E}_k\left(\frac{1}{q\overline{\gamma}'x}\right)$. Expanding $e^{\frac{1}{\lambda E_r\mu_{RX}t}}$ in (45), interchanging the order of summation and integral over t, integrating over t, and simplifying further yields

$$\widetilde{T}_2 = \frac{e^{\frac{1}{q\overline{\gamma}'}}}{\pi q\overline{\gamma}'}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda E_r\mu_{RX}}\right)^{k+1}\int_0^{m\pi}\int_0^1 \mathrm{E}_k\left(\frac{\csc^2\theta+\frac{1}{q\overline{\gamma}'}}{x}\right) \times \frac{x^{k-1}\zeta(x)^{L-1}e^{-\left(\frac{x}{\lambda E_r\mu_{RX}}-\csc^2\theta\right)}}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}\,dx\,d\theta. \tag{46}$$

As in Appendix B, we integrate θ over sub-intervals $[\theta_0,\theta_1],\ldots,[\theta_3,\theta_4]$, after replacing θ by θ_i for $1 \le i \le 2$, and by θ_{i-1} for $3 \le i \le 4$, in the term $e^{\csc^2\theta}\mathrm{E}_k\left(\frac{\csc^2\theta+\frac{1}{q\overline{\gamma}'}}{x}\right)$ in (46). This yields the second term in (21).

D. Proof of Result 3

We first derive the lower bound of the SEP. As before, the SEP is equal to $T_1' + LT_2'$, where $T_1' = \mathrm{SEP}_0\Pr(\beta=0)$ and $T_2' = \frac{1}{\pi}\int_0^{m\pi}\frac{1}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}\mathbb{E}_{\mathbf{h}}\left[e^{-\frac{q\gamma_1}{\sin^2\theta}}\Pr(\beta=1|\mathbf{h})\right] d\theta$.

1) Evaluating T_1': From the linear rule in (14), we get

$$T_1' = \mathrm{SEP}_0\left[\Pr(\gamma_1 - \lambda g_1 < 0)\right]^L \ge \mathrm{SEP}_0\left[\Pr(\gamma_{u_1} - \lambda g_1 < 0)\right]^L.$$

Substituting the PDFs of γ_{u_1} and g_1 in the above inequality, we get the first term of the SEP lower bound in (23).

2) Evaluating T_2': Using the law of total expectation,

$$T_2' = \frac{1}{\pi}\int_0^{m\pi}\frac{1}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}\mathbb{E}_{\gamma_1,g_1}\left[e^{-\frac{q\gamma_1}{\sin^2\theta}}\Pr(\beta=1|\gamma_1,g_1)\right] d\theta. \tag{47}$$

Now,

$$\begin{aligned}\Pr(\beta=1|\gamma_1,g_1) &= \left[F_{\gamma_2-\lambda g_2}(\gamma_1-\lambda g_1)\right]^{L-1} 1_{\{\gamma_1-\lambda g_1>0\}},\\ &\ge \left[F_{\gamma_{u_2}-\lambda g_2}(\gamma_{l_1}-\lambda g_1)\right]^{L-1} 1_{\{\gamma_{l_1}-\lambda g_1>0\}}.\end{aligned}$$

Since $\gamma_{l_1} = \gamma_{u_1}/2$, it can be shown that $\Pr(\beta=1|\gamma_1,g_1) \ge \left(1 - \frac{\overline{\gamma}_u e^{-\frac{\left(\frac{\gamma_{u_1}}{2}-\lambda g_1\right)}{\overline{\gamma}_u}}}{\overline{\gamma}_u+\lambda E_r\mu_{RX}}\right)^{L-1} 1_{\{\frac{\gamma_{u_1}}{2}-\lambda g_1>0\}}$. Substituting this inequality into (47) and using $\gamma_{u_1} \ge \gamma_1$, we get

$$T_2' \ge \frac{1}{\pi E_r\mu_{RX}\overline{\gamma}_u}\int_0^{m\pi}\int_0^\infty\int_0^{\frac{\gamma_{u_1}}{2\lambda}}\left[1 - \frac{\overline{\gamma}_u e^{-\frac{\left(\frac{\gamma_{u_1}}{2}-\lambda g_1\right)}{\overline{\gamma}_u}}}{\overline{\gamma}_u+\lambda E_r\mu_{RX}}\right]^{L-1} \times \frac{e^{-\left(\frac{q\gamma_{u_1}}{\sin^2\theta}+\frac{g_1}{E_r\mu_{RX}}+\frac{\gamma_{u_1}}{\overline{\gamma}_u}\right)}}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}\,dg_1\,d\gamma_{u_1}\,d\theta. \tag{48}$$

Substituting $\gamma_{u_1}/2 - \lambda g_1 = x$ in (48), integrating over g_1, and integrating θ over sub-intervals $[\theta_0,\theta_1],\ldots,[\theta_3,\theta_4]$ yields the second term of the SEP lower bound in (23).

The derivation of the upper bound is along similar lines.

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