Efficient Computation of Multivariate Rayleigh and Exponential Distributions

Reneeta Sara Isaac and Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—We propose an efficient approach for the computation of cumulative distribution functions of *N* correlated Rayleigh or exponential random variables (RVs) for arbitrary covariance matrices, which arise in the design and analysis of many wireless systems. Compared to the approaches in the literature, it employs a fast and accurate randomized quasi-Monte Carlo method that markedly reduces the computational complexity by several orders of magnitude as *N* or the correlation among the RVs increases. Numerical results show that an order of magnitude larger values of *N* can now be computed for. Its application to the performance analysis of selection combining is also shown.**

***Index Terms*—Multivariate, Rayleigh, exponential, correlated fading, cumulative distribution function.**

I. Introduction

MULTIVARIATE or correlated Rayleigh and exponential distributions arise in the design and analysis of many wireless systems. An example of this is a multiple input multiple output (MIMO) system when the transmit or receive antennas are spaced close together, which leads to the channel gains of the different transmit-receive antenna pairs being correlated [1, Ch. 6], [2, Ch. 13]. Another example is a sensor network, in which the measurements of different sensor nodes are correlated. Alternately, when the sensor nodes or the relay nodes in a cooperative relay system are close to each other or when there is a common scatterer in the propagation environment, the channel gains seen by the different nodes are correlated [3], [4]. These distributions also arise in orthogonal frequency division multiplexing (OFDM) systems, where the subchannel gains of a user are correlated [2, Ch. 19].

Performance analyses of communication techniques employed in these systems often lead to expressions that involve the multivariate cumulative distributive function (CDF) of such correlated channel gains. For example, the outage probability of an *N*-branch selection combiner (SC) in a multi-antenna system [1, Ch. 9] or of an *N*-relay cooperative system [4] is written in terms of the multivariate CDF of the channel gains.

Considerable effort has been made in the literature to derive tractable mathematical representations for the multivariate probability density functions (PDFs) and CDFs of these distributions. One class of approaches considers arbitrary covariance matrices, but severely limits the number of correlated random variables (RVs) *N*. For example, infinite series representations are derived for $N = 2$ in [5] and $N = 3$ and 4 in [6]–[8] for the arbitrary correlation model.

A second class of approaches limits itself to special forms of the covariance matrix. For example, for the constant correlation model, the CDF $F_{\mathbf{E}}(\mathbf{a})$ of *N* exponential RVs $\mathbf{E} = (E_1, \ldots, E_N)$ evaluated at $\mathbf{a} = (a_1, \ldots, a_N)$ is written in terms of an infinite series as follows [9]:

$$F_{\mathbf{E}}(\mathbf{a}) = \frac{1-\rho}{1+(N-1)\rho} \sum_{n=0}^{\infty} \left(\frac{\rho}{1+(N-1)\rho}\right)^n \times \sum_{\substack{(l_1,\ldots,l_N) \\ l_1 \geq 0,\ldots,l_N \geq 0 \\ l_1+\cdots+l_N=n}} \binom{n}{l_1,\ldots,l_N} \prod_{i=1}^{N} \gamma\left(\frac{a_i}{c(1-\rho)}, l_i+1\right), \quad (1)$$

where ρ^2 is the correlation coefficient between E_i and E_j, for $i \neq j$, c is the mean of E_i, for $1 \leq i \leq N$, and $\gamma(x,k)$ is the lower incomplete gamma function [10, eq. (6.5.2)]. Computing the n^{th} term of this series requires enumerating all $\binom{N+n-1}{N-1}$ combinations of the vector of non-negative integers $(l_1, \ldots, l_N)$ that satisfy $\sum_{i=1}^{N} l_i = n$. This entails a computational complexity of $\mathcal{O}(n^{N-1})$, which increases exponentially with *N*. This significant complexity challenge also arises while evaluating the CDF of correlated Nakagami-*m* RVs that is derived in [11].

When the inverse of the covariance matrix of the underlying Gaussian RVs has a tridiagonal structure, the multivariate Rayleigh CDF simplifies [12]. However, it is still in the form of an infinite series, evaluating which entails $\mathcal{O}(n^{N-1})$ complexity. The one exception is the specialized correlation model of [13] in which the multivariate PDF and CDF are given in single-integral forms. These can be numerically integrated using Gauss-Laguerre quadrature [10, eq. (25.4.45)].

A third class of approaches considers arbitrary covariance matrices and applies to any *N*. In [14], the multivariate Nakagami-*m* CDF for an arbitrary correlation matrix is given as a multi-dimensional integration of the PDF, which itself is in the form of an infinite series. In [15], the Green's approach is used to approximate any arbitrary covariance matrix to a matrix whose inverse has a tridiagonal form. However, the multivariate Rayleigh CDF again involves $(N-1)$ nested infinite series, which entails a computational complexity of $\mathcal{O}(n^{N-1})$ if the series is truncated after n terms. Therefore, results for $N > 5$ are seldom available in the literature for general correlation models. However, larger values of *N* are of interest in practice. For example, more antennas for next generation systems and wireless systems with more relays and

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The authors are with the Department of Electrical Communication Engineering, Indian Institute of Science, Bengaluru 560012, India (e-mail: reneetaisaac@gmail.com; nbmehta@iisc.ac.in).

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sensors are being considered. In many systems, the covariance matrix need not take a specialized form.

Contributions: In this letter, we first propose a novel approach to numerically compute the multivariate CDF of N Rayleigh or exponential RVs with an arbitrary covariance matrix. Its key ideas are as follows. It employs a series of transformations to first convert the multivariate PDF of Rayleigh or exponential RVs to the multivariate PDF of correlated Gaussian RVs. The CDF expression is then accurately and quickly computed using the randomized quasi-Monte Carlo (RQMC) method [16, Ch. 4]. In contrast to the above approaches, we show that it can compute the CDF easily even for $N = 50$, which is much larger than the values considered in [5]–[9], [11], [12], [14], and [15]. Notably, it can do so even when the RVs are highly correlated. This is unlike the above approaches, which require more terms to be computed as the RVs become more correlated since the rate of convergence of the series decreases. We then illustrate the utility of our approach by presenting results for the performance analysis of an N-branch SC, which is a popular receive diversity combining technique [1, Ch. 9], for a range of values of N.

Outline: In Section II, we present the proposed approach. In Section III, we compare its accuracy and computational complexity with existing approaches. In Section IV, we analyze the outage probability of SC. Our conclusions follow in Section V.

Notations: The probability of an event A is denoted by Pr(A). The expectation of an RV X is denoted by $\mathbb{E}[X]$ and its variance is denoted by var(X). The joint CDF of a random vector $\mathbf{X} = (X_1, \ldots, X_N)$ evaluated at $\mathbf{x} = (x_1, \ldots, x_N)$ is denoted by $F_{\mathbf{X}}(\mathbf{x}) = \Pr(X_1 \leq x_1, \ldots, X_N \leq x_N)$. The joint PDF of $\mathbf{X}$ is denoted by $f_{\mathbf{X}}(\cdot)$. The transpose of a vector $\mathbf{x}$ is denoted by $\mathbf{x}^T$.

II. Computation of Multivariate Exponential and Rayleigh CDF

We first consider the multivariate exponential distribution and then the multivariate Rayleigh distribution.

A. Multivariate Exponential CDF

Consider two real Gaussian random vectors $\mathbf{X} = (X_1, \ldots, X_N)^T$ and $\mathbf{Y} = (Y_1, \ldots, Y_N)^T$ with zero mean whose second-order moments are given in general by

$$\begin{aligned} \mathbb{E}\left[X_k^2\right] &= \mathbb{E}\left[Y_k^2\right] = \sigma_k^2, \text{ for } 1 \leq k \leq N, \\ \mathbb{E}\left[X_k X_j\right] &= \mathbb{E}\left[Y_k Y_j\right] = \rho_{kj}\sigma_k\sigma_j, \text{ for } k \neq j, \\ \mathbb{E}\left[X_k Y_j\right] &= 0, \text{ for } 1 \leq k, j \leq N. \end{aligned} \quad (2)$$

Thus, ρ_{kj} is the correlation coefficient between X_k and X_j or Y_k and Y_j. The multivariate Gaussian PDF $f_{\mathbf{X},\mathbf{Y}}(\cdot,\cdot)$ of $\mathbf{X}$ and $\mathbf{Y}$ is

$$f_{\mathbf{X},\mathbf{Y}}(\mathbf{x},\mathbf{y}) = \frac{1}{\sqrt{|\boldsymbol{\Sigma}|(2\pi)^{2N}}} \exp\left(-\frac{1}{2}\left[\mathbf{x}^T \;\; \mathbf{y}^T\right]\boldsymbol{\Sigma}^{-1}\begin{bmatrix}\mathbf{x}\\ \mathbf{y}\end{bmatrix}\right), \quad (3)$$

where $\mathbf{x} = (x_1, \ldots, x_N)^T$, $\mathbf{y} = (y_1, \ldots, y_N)^T$, $\boldsymbol{\Sigma} = \mathbb{E}\left[\begin{bmatrix}\mathbf{X}\\ \mathbf{Y}\end{bmatrix}\left[\mathbf{X}^T \; \mathbf{Y}^T\right]\right]$, and $|\cdot|$ denotes determinant.

Converting to polar coordinates, let

$$E_i = X_i^2 + Y_i^2 \quad \text{and} \quad \phi_i = \tan^{-1}(Y_i/X_i). \quad (4)$$

It can be seen that E_i is an exponential RV with mean $2\sigma_i^2$ and ϕ_i is uniformly distributed between 0 and 2π. Furthermore, E_i and ϕ_j are mutually independent for all i and j (including $i = j$) and $(\mathbb{E}[E_i E_j] - 4\sigma_i^2\sigma_j^2)/(\sqrt{\mathrm{var}(E_i)\mathrm{var}(E_j)}) = \rho_{kj}^2$.

The multivariate CDF $F_{\mathbf{E}}(\cdot)$ of $\mathbf{E} = (E_1, \ldots, E_N)^T$ evaluated at $\mathbf{a} = (a_1, \ldots, a_N)^T$ is then

$$F_{\mathbf{E}}(\mathbf{a}) = \int_0^{2\pi} \cdots \int_0^{2\pi} \int_0^{a_1} \cdots \int_0^{a_N} f_{\mathbf{E},\boldsymbol{\Phi}}(\mathbf{e},\boldsymbol{\theta}) \; d\mathbf{e} \; d\boldsymbol{\theta}, \quad (5)$$

where $\boldsymbol{\Phi} = (\phi_1, \ldots, \phi_N)^T$ and $f_{\mathbf{E},\boldsymbol{\Phi}}(\cdot)$ is the joint CDF of $\mathbf{E}$ and $\boldsymbol{\Phi}$.

Using the rules governing transformation of variables, we get the following from (3) and (4):

$$F_{\mathbf{E}}(\mathbf{a}) = \int_{l_1}^{u_1} \cdots \int_{l_{2N}}^{u_{2N}} \frac{f_{\mathbf{X},\mathbf{Y}}(\mathbf{x},\mathbf{y})}{\sqrt{x_1^2 + y_1^2} \cdots \sqrt{x_N^2 + y_N^2}} \, d\mathbf{x} d\mathbf{y}, \quad (6)$$

where $l_1 = -\sqrt{a_1}, \ldots, l_N = -\sqrt{a_N}$, $l_{N+1} = -\sqrt{a_1 - x_1^2}, \ldots, l_{2N} = -\sqrt{a_N - x_N^2}$, $u_1 = \sqrt{a_1}, \ldots, u_N = \sqrt{a_N}$, $u_{N+1} = \sqrt{a_1 - x_1^2}, \ldots, u_{2N} = \sqrt{a_N - x_N^2}$.

Next, we show how to efficiently compute the integral in (6) using the RQMC method of [16, Ch. 4].

B. RQMC Method

It transforms the integral in (6) into an integral over a unit hyper-cube. This is then numerically computed using quasi-Monte Carlo techniques. It is as follows.

1) Take the Cholesky decomposition of $\boldsymbol{\Sigma} = \mathbf{C}\mathbf{C}^{\mathbf{T}}$, where $\mathbf{C} = [c_{ij}]$ is a lower triangular matrix with $c_{ii} > 0$. Using the variable transformation $\mathbf{z} = \mathbf{C}^{-1}\left[\mathbf{x}^T\mathbf{y}^T\right]^T$ changes (6) to

$$F_{\mathbf{E}}(\mathbf{a}) = \frac{1}{\sqrt{(2\pi)^{2N}}} \times \int_{l_1'}^{u_1'} \cdots \int_{l_{2N}'}^{u_{2N}'} e^{-\frac{z_1^2}{2}} \cdots e^{-\frac{z_{2N}^2}{2}} g(\mathbf{z}) \, d\mathbf{z}, \quad (7)$$

where $l_i' = (l_i - \sum_{j=1}^{i-1} c_{ij} z_j)/c_{ii}$ and $u_i' = (u_i - \sum_{j=1}^{i-1} c_{ij} z_j)/c_{ii}$, for $1 \leq i \leq 2N$, and $g(\mathbf{z}) = \prod_{i=1}^{N} [(\sum_{j=1}^{i} c_{ij} z_j)^2 + (\sum_{j=1}^{N+i} c_{(N+i)j} z_j)^2]^{-1/2}$.

2) Using $v_i = \Psi(z_i)$, where $\Psi(z) \triangleq \int_{-\infty}^{z} e^{-\frac{\theta^2}{2}} d\theta/(\sqrt{2\pi})$, yields

$$F_{\mathbf{E}}(\mathbf{a}) = \int_{b_1}^{e_1} \cdots \int_{b_{2N}}^{e_{2N}} \prod_{i=1}^{N} \left[\left(\sum_{j=1}^{i} c_{ij}\Psi^{-1}(v_j)\right)^2 + \left(\sum_{j=1}^{N+i} c_{(N+i)j}\Psi^{-1}(v_j)\right)^2\right]^{-\frac{1}{2}} d\mathbf{v}, \quad (8)$$

where $b_i = \Psi([l_i - \sum_{j=1}^{i-1} c_{ij}\Psi^{-1}(v_j)]/c_{ii})$ and $e_i = \Psi([u_i - \sum_{j=1}^{i-1} c_{ij}\Psi^{-1}(v_j)]/c_{ii})$, for $1 \leq i \leq 2N$.

3) Applying the transformation $w_i = (v_i - d_i)/(e_i - d_i)$ changes (8) to the following integral over a unit hyper-cube:

$$F_{\mathbf{E}}(\mathbf{a}) = \int_0^1 \cdots \int_0^1 f(\mathbf{w})\, d\mathbf{w}, \tag{9}$$

where

$$f(\mathbf{w}) = \left[\prod_{k=1}^{2N} (e_k - b_k)\right] \times \prod_{i=1}^{N} \left[\left(\sum_{j=1}^{i} c_{ij} \Psi^{-1}(b_j + w_j(e_j - b_j))\right)^2 + \left(\sum_{j=1}^{N+i} c_{(N+i)j} \Psi^{-1}(b_j + w_j(e_j - b_j))\right)^2\right]^{-\frac{1}{2}}. \tag{10}$$

This integral is computed using the RQMC method that uses a carefully selected, deterministic sequence of P samples $\mathbf{s}_1, \ldots, \mathbf{s}_P$ for the vector $\mathbf{w}$, where the q^{th} sample $\mathbf{s}_q$ is the vector $(s_{q,1}, \ldots, s_{q,2N})$. An example of this is the Kronecker sequence, which is given by $s_{q,i} = \{q\sqrt{p_i}\}$, where p_i is the i^{th} prime number and $\{\cdot\}$ denotes the remainder modulo 1. To each sample $\mathbf{s}_q$, a random shift $\mathbf{u} = (u_1, \ldots, u_{2N})$ is added, where $u_1, \ldots, u_{2N}$ are independent and identically distributed (i.i.d.) RVs that are uniformly distributed between 0 and 1, to get the shifted sample $\mathbf{s}_q^{\text{shift}} = \{\mathbf{s}_q + \mathbf{u}\}$. The integrand is averaged over M such random shifts, where M is typically between 8 and 12. To speed up convergence, $f(\mathbf{w})$ is replaced with $(f(\mathbf{w}) + f(1-\mathbf{w}))/2$ and $\mathbf{w}$ with $|2\mathbf{w} - 1|$ in the integrand in (9). Thus, we get

$$F_{\mathbf{E}}(\mathbf{a}) \approx \frac{1}{2MP} \sum_{i=1}^{M} \sum_{q=1}^{P} \Big(f(|2\{\mathbf{s}_q + \mathbf{u}_i\} - 1|) + f(1 - |2\{\mathbf{s}_q + \mathbf{u}_i\} - 1|) \Big). \tag{11}$$

The RQMC method has a convergence rate of $\mathcal{O}(1/P)$ [16, Ch. 4], which is better than the convergence rate of $\mathcal{O}(1/\sqrt{P})$ of the conventional Monte Carlo method that takes an average over any P random samples of $\mathbf{w}$. The computational complexity of the proposed approach is $\mathcal{O}(MP)$.

C. Multivariate Rayleigh Distribution

Let $\mathbf{R} = (R_1, \ldots, R_N)^T$ be the vector of correlated Rayleigh RVs generated from $\mathbf{X}$ and $\mathbf{Y}$, such that $R_i = (X_i^2 + Y_i^2)^{1/2}$, for $1 \leq i \leq N$. Then, the multivariate CDF $F_{\mathbf{R}}(\mathbf{r})$ of $\mathbf{R}$ evaluated at $\mathbf{r} = (r_1, \ldots, r_N)^T$ can be expressed in terms of the exponential CDF $F_{\mathbf{E}}(\cdot)$ as follows:

$$\begin{aligned} F_{\mathbf{R}}(\mathbf{r}) &= \Pr(R_1 \leq r_1, \ldots, R_N \leq r_N), \\ &= \Pr\Big(R_1^2 \leq r_1^2, \ldots, R_N^2 \leq r_N^2\Big), \\ &= F_{\mathbf{E}}(r_1^2, \ldots, r_N^2). \end{aligned} \tag{12}$$

Therefore, the multivariate Rayleigh CDF can also be easily computed using the above approach.

TABLE I
COMPARISON OF CDFS COMPUTED AT PRE-SPECIFIED POINTS (GIVEN IN [11, TBL. 1]) USING PROPOSED APPROACH AND ISA, AND COMPUTATIONAL COMPLEXITY OF ISA

N	CDF evaluated at $(r_1, \ldots, r_N)$	ρ	CDF value using proposed approach	CDF value using ISA	Number of terms summed in ISA
3	$(2, 3, 5)$	0.3	0.8564	0.8563	16
		0.9	0.8646	0.8643	576
4	$(1, 3, 4, 5)$	0.3	0.3904	0.3902	27
		0.8	0.3934	0.3934	2197
5	$(1, 1, 3, 4, 5)$	0.3	0.1622	0.1622	81
		0.9	0.2866	0.2864	1.9×10^5

III. COMPUTATIONAL COMPLEXITY AND COMPARISON

We now compare our approach with the infinite series approach (ISA) of [11] and [15] given that it applies to arbitrary correlation models. Consider the following correlation model for Gaussian random vectors $\mathbf{X}$ and $\mathbf{Y}$, in which $\rho_{kj} = \rho^{|k-j|}$ and $\sigma_k = 1$, for $1 \leq k, j \leq N$. For it, the multivariate Rayleigh CDF of $\mathbf{R}$ is given in the form of the following infinite series [11], [15]:

$$\begin{aligned} F_{\mathbf{R}}(r_1, \ldots, r_N) &= (1 - \rho^2) \sum_{i_1, \ldots, i_{N-1}=0}^{\infty} \frac{G_N \rho^{2(i_1 + \cdots + i_{N-1})}}{\prod_{j=1}^{N-1} (i_j!)^2} \\ &\times \gamma\left(i_1 + 1, \frac{r_1^2}{2(1-\rho^2)}\right) \gamma\left(i_1 + i_2 + 1, \frac{r_2^2(1+\rho^2)}{2(1-\rho^2)}\right) \\ &\times \cdots \times \gamma\left(i_{N-2} + i_{N-1} + 1, \frac{r_{N-1}^2(1+\rho^2)}{2(1-\rho^2)}\right) \\ &\times \gamma\left(i_{N-1} + 1, \frac{r_N^2}{2(1-\rho^2)}\right), \end{aligned} \tag{13}$$

where $G_N = (1 + \rho^2)^{-[i_1 + 2i_2 + \cdots + 2i_{N-2} + i_{N-1} + N - 2]}$, for $N \geq 3$, and $G_N = 1$, for $N = 2$.

Table I tabulates the number of terms required by ISA and the CDFs of ISA and the proposed approach to an accuracy of 4 decimal places at pre-specified points. The pre-specified points and correlation coefficients are chosen because they were used in [11, Table 1] for $N = 3$ and 4. For $N = 5$, r_1 is set as 1 and $r_2, \ldots, r_5$ are set to be the same as $r_1, \ldots, r_4$ for $N = 4$. For ISA, the number of terms that need to be summed over increases exponentially as N or ρ increases. Even for N as small as 5, it exceeds 10^5 for $\rho = 0.9$. The number of terms required by the proposed approach for $M = 8$ and $P = 1000$, which are sufficient for all the considered values of N and ρ, is only $MP = 8000$. A comparison for larger N is not shown since ISA becomes computationally infeasible.

IV. ILLUSTRATIVE APPLICATION TO N-BRANCH SC

Consider a wireless communication system that consists of a transmitter with one antenna and a receiver with N antennas. Let $\gamma_k = R_k^2 \omega_s / N_0$ be the instantaneous signal-to-noise ratio (SNR) of the signal received at the k^{th} antenna, where R_k is its channel amplitude, which is a Rayleigh RV, ω_s is the transmitted symbol energy, and N_0 is the additive white Gaussian noise power spectral density. After selection combining, the

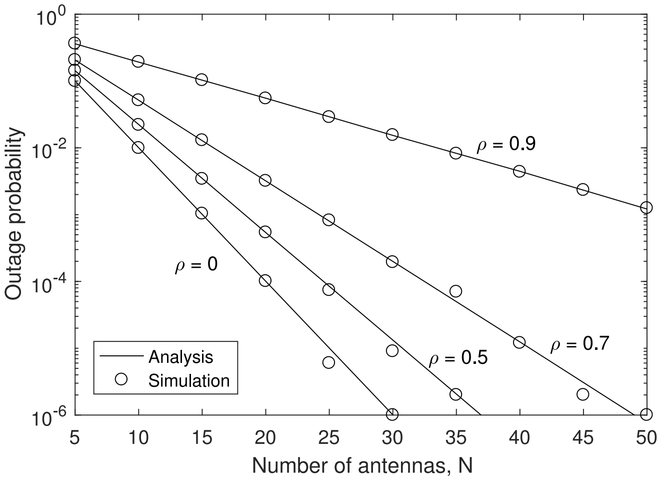

Fig. 1. Exponential correlation model: Outage probability of SC as a function of *N* for different ρ ($\gamma_{\rm th} = 3$ dB and $\omega_s/N_0 = 1$).

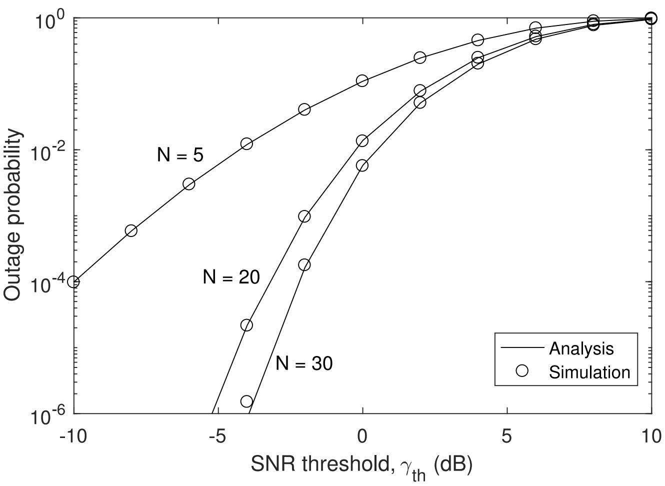

Fig. 2. Arbitrary correlation model: Outage probability of SC as a function of $\gamma_{\rm th}$ for different *N*.

output SNR $\gamma_{\rm SC}$ is given by $\gamma_{\rm SC} = \max\{\gamma_1, \ldots, \gamma_N\}$. The outage probability of SC for an SNR threshold $\gamma_{\rm th}$ is given by

$$P_{\rm out} = \Pr(\gamma_{\rm SC} \le \gamma_{\rm th}) = F_{\mathbf{R}}\left(\sqrt{\frac{\gamma_{\rm th} N_0}{\omega_s}}, \ldots, \sqrt{\frac{\gamma_{\rm th} N_0}{\omega_s}}\right). \quad (14)$$

Exponential Correlation Model: In this model, $\rho_{kj} = \rho^{|k-j|}$ and $\sigma_k = 1$, for $1 \le k, j \le N$. It arises in equi-spaced diversity antennas [1]. Fig. 1 plots the outage probability of SC as a function of *N* for different ρ. To compute the multivariate CDF in (11), we use $M = 8$ and $P = 1000$. Also shown are results from simulations, in which 10^6 realizations of the vector of N correlated Rayleigh RVs $\mathbf{R} = (R_1, \ldots, R_N)$ are generated and the average number of outage occurrences is measured. The analysis and simulation results match well even for *N* as large as 50. For a given ρ, $P_{\rm out}$ decreases as *N* increases. However, as ρ increases, $P_{\rm out}$ increases.

Arbitrary Correlation Model: To evaluate the utility for an arbitrary correlation model, the correlation coefficients are taken to be linearly spaced between $\rho_{\min}$ and $\rho_{\max}$ with $\sigma_k = 1$, for $1 \le k \le N$. Specifically, $\rho_{kj} = \rho_{\max} - (|k - j| - 1)(\rho_{\max} - \rho_{\min})/(N-2)$. It applies to linear arrays with antennas spaced unevenly [11], [17].

Fig. 2 plots $P_{\rm out}$ as a function of $\gamma_{\rm th}$ for this model when $\rho_{\min} = 0.5$ and $\rho_{\max} = 0.9$. The analysis and simulation results are in good agreement even for *N* as large as 30. Similar to Fig. 1, $P_{\rm out}$ decreases as *N* increases. Compared to the exponential correlation model, $P_{\rm out}$ of this model is higher for a given $\gamma_{\rm th}$ and *N*. Intuitively, this is because the RVs are more correlated in the arbitrary correlation model than in the exponential correlation model for the parameters used.

V. Conclusion

The complexity of computing the multivariate CDF with an arbitrary covariance matrix using conventional approaches increased exponentially as *N* increased. We presented a novel approach for computing the multivariate Rayleigh and exponential CDFs for any *N* and for an arbitrary correlation structure. It had a much lower computational complexity that no longer increased as *N* or ρ increased. We then demonstrated the utility of our approach by analyzing the outage probability of an *N*-branch SC receiver.

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