# Antenna Selection with Power Adaptation in Interference-Constrained Cognitive Radios

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Abstract—The performance of an underlay cognitive radio (CR) system, which can transmit when the primary is on, is curtailed by tight constraints on the interference it can cause to the primary receiver. Transmit antenna selection (AS) improves the performance of underlay CR by exploiting spatial diversity but with less hardware. However, the selected antenna and its transmit power now both depend on the channel gains to the secondary and primary receivers. We develop a novel Chernoffbound based optimal AS and power adaptation (CBBOASPA) policy that minimizes an upper bound on the symbol error probability (SEP) at the secondary receiver, subject to constraints on the average transmit power and the average interference to the primary. The optimal antenna and its power are presented in an insightful closed form in terms of the channel gains. We then analyze the SEP of CBBOASPA. Extensive benchmarking shows that the SEP of CBBOASPA for both MPSK and MQAM is one to two orders of magnitude lower than several ad hoc AS policies and even optimal AS with on-off power control.

*Index Terms*—Cognitive radio, Underlay, Antenna selection, Multiple antennas, Diversity techniques, Fading channels, Symbol error probability, Interference constraint, Power constraint

## I. INTRODUCTION

**C** OGNITIVE radio (CR) is a technology that promises to significantly improve the availability and utilization of precious radio spectrum. Cognitive nodes intelligently adapt their transmissions and co-exist with other users. One common paradigm of CR divides the users into two classes, namely, primary users (PUs), which have unrestricted access to the spectrum, and secondary users (SUs), which can access the same spectrum but must ensure that their transmissions do not interfere excessively with the PUs.

Several modes of CR operation, such as interweave and underlay, have been considered in the literature for facilitating such hierarchical co-existence [2]. In the underlay mode, which is the focus of this paper, a secondary transmitter (STx) can transmit even when the primary is on, but under tight constraints on the interference it causes to the primary receiver (PRx). These constraints can significantly curtail the data rates at which the STx can transmit, and motivate the development of novel, interference-aware transmission techniques.

Among these, multiple input multiple output (MIMO) antenna techniques are very promising as they exploit spatial diversity to improve CR performance [3], [4]. However, in MIMO, each antenna requires a dedicated radio frequency (RF) chain. Single antenna selection (AS) is a popular technique that has been extensively studied in the literature in order to address this challenge [5], [6]. In it, one of the antennas is selected as a function of the channel conditions to transmit data. Thus, only one RF chain is now required at the transmitter. This reduces the hardware complexity, foot print, and cost of the device.

AS has also been considered for CR [7]–[14]. In underlay CR, the interference constraint imposed on the STx fundamentally alters the criteria used to select the transmit antenna. Intuitively, even a transmit antenna with a high channel power gain to the secondary receiver (SRx) should not be selected if it causes significant interference to the PRx. Similarly, the transmit power from the selected antenna should also be a function of the channel gains to the SRx and the PRx. This is unlike receive AS, in which choice of the antenna remains unaffected by the interference constraint [11].

# A. Literature Survey

Several transmit AS policies have been proposed for underlay CR. These include the minimum interference (MI) rule [8], which selects the antenna with the weakest channel power gain to the PRx; the maximum signal power to leak interference ratio (MSLIR) rule [8], which selects the antenna with the largest ratio of the channel power gains to the SRx and PRx; and the difference AS (DAS) rule [9], which selects the antenna with the largest weighted difference between the channel power gains to the SRx and PRx. While the STx transmits with a fixed power in [8], [9], AS with on-off power control is proposed in [10], in which the selected antenna transmits either with a fixed power or with zero power.

Joint AS and power control is considered in [15] with the goal of maximizing the SU capacity under a peak interference constraint. However, no constraints are placed on the transmit power. As a result, the transmit power is simply proportional to the reciprocal of the STx-PRx channel power gain. Optimal power control to maximize the SU capacity is considered in [12], which imposes a constraint on the average or peak transmit power in addition to a constraint on the average or peak interference power. However, AS is not considered. Optimal power control – but without AS – to maximize ergodic capacity or minimize SEP for underlay CR with

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average or peak interference power constraints is considered in [16]–[18].

In [13], the transmit and receive antenna subsets and the transmit covariance matrix are jointly optimized for a MIMO CR system to maximize an achievable rate of the STx subject to an instantaneous interference constraint and a peak transmit power constraint. A computationally efficient iterative algorithm based on a convex approximation is developed and a norm-based heuristic is also proposed. The channel gains of the transmit-receive antenna pairs in the STx-SRx, STx-PRx, and primary transmitter (PTx)-SRx links are assumed to be known to the STx. In [11], the STx simultaneously transmits using zero-forcing beamforming to  $N_t$  users, where  $N_t$  is the number of transmit antennas at the STx. First, all users whose channels from the STx have a correlation less than  $\delta_p$ to the STx-PRx channel are pre-selected so as to limit the interference caused to the PRx. Among these,  $N_t$  users with channels whose cross-correlation is less than  $\delta_c$  are selected for transmission so as to maximize the downlink sum rate. However, no interference constraint is explicitly specified. The number of users is assumed to be much greater than  $N_t$ . Further, single receive AS is considered.

# B. Contributions

In this paper, we consider an STx with  $N_t$  antennas that employs transmit AS and transmits data to an SRx with  $N_r$ antennas. It is subject to an average transmit power constraint and an average primary interference power constraint. For this problem, even the criteria for optimally selecting the transmit antenna and its power are not known; this open problem is solved in this paper. This problem is different from antenna subset selection in conventional MIMO systems, which has been studied in [19] and the references therein. Here, the optimal subset selection criterion is known, and the focus is on developing computationally efficient search algorithms.

We first derive for MPSK an SEP-optimal AS and power adaptation (ASPA) policy that minimizes a Chernoff upper bound on the SEP of the secondary system. We shall henceforth refer to it as the Chernoff bound-based optimal ASPA (CBBOASPA) policy. We focus on single transmit AS because it is practically and theoretically relevant [6], [8], [14], [15]. It requires only one RF chain at the transmitter. Yet, in conventional MIMO systems, it achieves the same diversity order as antenna subset selection [20]. The average interference constraint has been used in [12], [16], [18], [21].

CBBOASPA exploits the fact that the channel state information required to select an antenna can also help adapt the transmit power of the selected antenna. An elegant and insightful closed-form characterization of the optimal antenna and its transmit power in terms of the STx-SRx and STx-PRx channel power gains is developed. It turns out to be functionally different from the ad hoc AS policies proposed in the literature and even the optimal AS policy with on-off power control. As we shall see, the exponential form of the Chernoff bound, which arises for other constellations such as MQAM and is functionally similar to the approximations proposed in [22, Chap. 9], makes CBBOASPA applicable to other constellations as well. While the Chernoff bound is well known, its use to convert an intractable problem into a tractable problem that yields an insightful, non-obvious, and elegant solution is a contribution of this paper.

The second contribution is an analysis of the SEP of CBBOASPA. The SEP analysis is challenging and different from that for conventional AS because of the non-linear dependence of both the selected antenna and its transmit power on the various channel power gains. We first derive an exact and novel expression for its SEP, which is useful given that it is general and exact. To gain further insights, we then present a simpler SEP upper bound that tracks the exact SEP well over all regimes of interest. We gain further insights by studying the special case with one receive antenna. These results provide different trade-offs between accuracy and simplicity.

Thirdly, extensive numerical results are presented to quantify the effect of  $N_t$ ,  $N_r$ , constellation size, interference constraint, and power constraint on CBBOASPA. We also extensively benchmark its performance with the several aforementioned AS policies. We show that for both MPSK and MQAM it reduces the SEP by up to two orders of magnitude.

We note that this paper differs from [10] in its basic idea and model, analysis, and performance. In terms of idea and modeling, the manuscript jointly adapts the power and selected antenna, while [10] fixes the power. This leads to an optimal policy that is different from that in [10], and does not follow from it. The SEP analysis that ensues and the final SEP expressions are also different and novel. This is because the composite random variables (RVs) that drive AS are different from those in [10]. The ability to adapt the transmit power continuously adds a new and challenging dimension to the analysis. For example, while three regimes of operation have to be specified for CBBOASPA, the simpler policy in [10] operates in only one of two regimes. In terms of performance, CBBOASPA reduces the SEP by more than an order of magnitude over the policy in [10] for both MPSK and MQAM, which demonstrates the efficacy and significance of our approach.

# C. Organization and Notation

The paper is organized as follows. Section II sets up the system model and the optimization problem. The CBBOASPA policy and its SEP are derived in Sec. III. Numerical results and our conclusions are presented in Sec. IV and Sec. V, respectively.

We shall use the following notation henceforth. The probability of an event A is denoted by  $\Pr(A)$ , and the conditional probability of A given that an RV X takes the value x is denoted by  $\Pr(A|X = x)$ . The expectation with respect to the RV X is denoted by  $\mathbf{E}_X$  [.]. The sum  $\sum_{i=k_1}^{k_2}$  is identically zero if  $k_2 < k_1$ .

#### **II. SYSTEM MODEL AND PROBLEM STATEMENT**

As shown in Figure 1, we consider an underlay CR system in which an STx with  $N_t$  antennas transmits data to an SRx, which has  $N_r$  receive antennas. Its transmissions cause interference at a PRx, which has one antenna. The SRx and the STx constitute the secondary system. The STx needs to select one of its antennas for transmission. For  $i \in \{1, 2, ..., N_r\}$  and  $j \in \{1, 2, ..., N_t\}$ ,  $h_{ij}$  denotes the instantaneous baseband channel gain between the  $j^{\text{th}}$  transmit antenna of the STx and the  $i^{\text{th}}$  receive antenna of the SRx, and  $g_j$  denotes the instantaneous baseband channel gain between the  $j^{\text{th}}$  transmit antenna of the STx and the PRx. Let the corresponding channel power gains be denoted by

$$\Lambda_{ij} = |h_{ij}|^2 \quad \text{and} \quad \omega_j = |g_j|^2 \,. \tag{1}$$

Further, let  $\Lambda$  denote the matrix  $[\Lambda_{ij}]$  and let  $\omega = [\omega_1, \ldots, \omega_{N_t}]$ . The  $N_t N_r$  STx-SRx channels are assumed to be independent and identically distributed (i.i.d.) random variables (RVs), and so are the  $N_t$  STx-PRx channels. In the SEP analysis, we consider Rayleigh fading, in which case  $\Lambda_{ij}$  and  $\omega_j$  are exponential RVs with means  $\overline{\Lambda}$  and  $\overline{\omega}$ , respectively.

# A. Selection Options and Data Transmission

For ease of exposition, we first focus on MPSK. Our results generalize to other constellations such as MQAM, which is covered in Sec. III-C. The STx transmits a symbol z that is drawn with equal probability from an MPSK constellation of size M. For transmitting z, it selects one out of the  $N_t$  antennas. It also adapts its transmit power as a function of the channel power gains.

As the transmit power decreases, the SEP monotonically increases. Its maximum value occurs in the limiting case when the STx transmit power is zero. It equals  $m \triangleq 1 - \frac{1}{M}$  in this case because the optimal receiver just chooses any one of the M symbols as its decoded symbol with equal probability [18]. We treat the case where the STx transmits with zero power separately because it then does not matter which antenna is selected.<sup>1</sup> We treat it as a transmission from a virtual antenna 0 whose corresponding channel power gains and transmit power are all zero:  $\Lambda_{i0} = 0$ , for  $1 \le i \le N_r$ , and  $\omega_0 = 0$ .

Let  $s \in \{0, 1, ..., N_t\}$  be the antenna selected. In baseband, the interference signal  $i_p$  seen by the PRx and the signal  $r_i$ received by the *i*<sup>th</sup> receive antenna of the SRx are given by

$$r_i = \sqrt{P_s} h_{is} z + \varepsilon_i + \zeta_i, \tag{2}$$

$$i_p = \sqrt{P_s g_s z},\tag{3}$$

where z is the unit energy complex data symbol,  $P_s$  is the power transmitted from the selected antenna s, and  $\varepsilon_i$  is circular symmetric complex additive white Gaussian noise. The interference seen by the SRx due to primary transmissions is  $\zeta_i$ , and is assumed to be Gaussian. This corresponds to a worst case model for the interference and makes the problem of finding the selection rule tractable [8], [9], [23]. This model also covers the models considered in [18], [24], in which the interference from the primary transmitter (PTx) is assumed to be negligible. Therefore,  $\varepsilon_i + \zeta_i$  is a circularly symmetric complex Gaussian RV, whose variance is denoted by  $\sigma^2$ .

The STx is assumed to know  $\Lambda$  and  $\omega$ , i.e., its channel power gains to the SRx and to the PRx [8], [9], [15]. No knowledge of the phases of any complex baseband channel gain is required at the STx. In the time division duplex



Fig. 1. System model with one PRx and a secondary system consisting of an STx with  $N_t$  transmit antennas and one RF chain that communicates with an SRx with  $N_r$  receive antennas.

(TDD) mode of operation, the STx can exploit reciprocity and estimate  $\Lambda$  and  $\omega$  from the signals it overhears from the SRx and PRx when they transmit. Since phase information is not required, simple signal strength based techniques can be used for estimation. The SRx uses a coherent receiver, and only needs to know the complex channel gain from the selected antenna to the receive antennas [10]. In practice, this can be achieved by embedding pilots along with the data symbols in every coherence interval.

# B. Problem Statement

An ASPA policy  $\phi$  is a mapping  $\phi : (\mathbb{R}^+)^{N_r} \times (\mathbb{R}^+)^{N_t} \times (\mathbb{R}^+)^{N_t} \to \{0, 1, \dots, N_t\} \times \mathbb{R}^+$  that selects one of the  $N_t + 1$  antennas and the corresponding transmit power for every realization of  $\Lambda$  and  $\omega$ . For a policy  $\phi$ , we denote the selected antenna and transmit power as  $s_{\phi(\Lambda,\omega)}$  and  $P_{\phi(\Lambda,\omega)}$ , respectively. Our goal is to find the optimal policy  $\phi^*$  that minimizes the fading-averaged SEP of the secondary system subject to (s.t.) the following two constraints:

- The average interference the STx causes to the PRx must not exceed a threshold I<sub>ave</sub>. Therefore, from (3), E<sub>Λ,ω</sub> [P<sub>φ(Λ,ω)</sub>ω<sub>sφ(Λ,ω)</sub>] ≤ I<sub>ave</sub>.
   The average transmit power of the STx must not exceed
- The average transmit power of the STx must not exceed  $P_{\text{ave}}$ , i.e.,  $\mathbf{E}_{\mathbf{\Lambda},\boldsymbol{\omega}} \left[ P_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})} \right] \leq P_{\text{ave}}$ .

The SEP for MPSK conditioned on  $\Lambda$  and  $\omega$ , denoted by  $\Pr(\text{Err}|\Lambda, \omega)$ , is then [25, (8.23)]

$$\Pr\left(\operatorname{Err}|\mathbf{\Lambda},\boldsymbol{\omega}\right) = \frac{1}{\pi} \int_{0}^{m\pi} \exp\left(-\frac{P_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})} \sum_{i=1}^{N_{\tau}} \Lambda_{is_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})}}}{\eta m \sin^{2} \theta}\right) d\theta,$$
(4)

$$\leq m \exp\left(-\frac{P_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})} \sum_{i=1}^{N_r} \Lambda_{is_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})}}}{\eta m}\right), \quad (5)$$

where  $\eta = \frac{\sigma^2}{m \sin^2(\frac{\pi}{M})}$ , and the summation term  $\sum_{i=1}^{N_r} \Lambda_{is_{\phi(\Lambda,\omega)}}$  arises due to the use of maximal ratio combining (MRC) by the optimal receiver at the SRx. The exact SEP expression in (4) is in the form of a single integral, which is intractable for the purposes of optimization. To gain analytical insights, we minimize its integral-free Chernoff upper bound given in (5). This form is similar to

<sup>&</sup>lt;sup>1</sup>This case must be considered in the problem formulation since otherwise the solution to an interference-constrained problem will be the trivial policy in which the STx always transmits with zero power, which is not reasonable.

the integral-free SEP approximations proposed in [22]. It also applies to other constellations, as we discuss in Sec. III-C.

Our problem can be mathematically stated as the following mixed integer, non-linear, stochastic optimization problem over the space of all ASPA policies  $\phi$ :

$$\min_{\phi} \quad \mathbf{E}_{\mathbf{\Lambda},\boldsymbol{\omega}} \left[ m \exp\left( -\frac{P_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})} \sum_{i=1}^{N_r} \Lambda_{is_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})}}}{\eta m} \right) \right], \quad (6)$$

s.t. 
$$\mathbf{E}_{\mathbf{\Lambda},\boldsymbol{\omega}} \left[ P_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})} \omega_{s_{\phi(\mathbf{\Lambda},\boldsymbol{\omega})}} \right] \leq I_{\text{ave}},$$
 (7)

$$\begin{aligned} \mathbf{E}_{\mathbf{\Lambda},\omega} \left[ \Gamma_{\phi(\mathbf{\Lambda},\omega)} \right] &\geq \Gamma_{\text{ave}}, \\ s_{\phi(\mathbf{\Lambda},\omega)} &\in \{0, 1, \dots, N_t\} \text{ and } P_{\phi(\mathbf{\Lambda},\omega)} > 0, \ \forall \ \mathbf{\Lambda}, \omega. \end{aligned}$$

$$\delta_{\phi}(\mathbf{\Lambda}, \boldsymbol{\omega}) \subset \{0, 1, \dots, N_t\}$$
 and  $\Gamma_{\phi}(\mathbf{\Lambda}, \boldsymbol{\omega}) \geq 0, \forall \mathbf{\Pi}, \boldsymbol{\omega}$ . (9)

Unless mentioned otherwise, we no longer show the dependence of the selected antenna and power on  $\Lambda$  and  $\omega$ . This is done in order to keep the notation simple.

## C. Alternate Models and Extensions

The problem that we study can be generalized in several interesting ways. Firstly, it can also be posed under alternate models for interference. For example, the instantaneous interference power can be constrained along the lines of [12], [16], [21], [23]. In terms of the effect on the primary system, a primary outage based constraint becomes more relevant. Note, however, that it requires knowledge of the PTx-PRx link. For example, the mechanisms described in [26], [27] to acquire the above channel knowledge require the PRx to communicate with the STx. This may not be practically feasible when the primary operates oblivious to the secondary system. Secondly, the interference from the PTx to the SRx need not even be Gaussian. For example, the asynchronous turning on and off of the PTx can affect its statistics. The optimal ASPA policy for these alternate models is beyond the scope of this paper.

# III. CBBOASPA AND ITS PERFORMANCE ANALYSIS

At least one of the two constraints in (7) and (8) must be active in the optimal policy.<sup>2</sup> To present the general solution of CBBOASPA, we first define two ASPA policies  $\phi_I$  and  $\phi_P$  as follows. The proofs of their optimality are simpler versions of the proof for Theorem 1 below, and are not shown.

The optimal ASPA policy  $\phi_I$  that minimizes the SEP subject only to the average interference constraint in (7) and (9) is as follows. The optimal antenna  $s_{\phi_I}$  is

$$s_{\phi_I} = \begin{cases} 0, & \frac{\sum_{i=1}^{N_r} \Lambda_{i_1}}{\omega_1} \le \eta \lambda_I, \dots, \frac{\sum_{i=1}^{N_r} \Lambda_{i_N_t}}{\omega_{N_t}} \le \eta \lambda_I, \\ \arg\max_{j \in \{1,\dots,N_t\}} \left\{ \frac{\sum_{i=1}^{N_r} \Lambda_{i_j}}{\omega_j} \right\}, & \text{otherwise.} \end{cases}$$
(10)

The optimal power  $P_{\phi_I}$  transmitted from  $s_{\phi_I}$  is

$$P_{\phi_I} = \begin{cases} 0, & s_{\phi_I} = 0, \\ \frac{m\eta}{\sum_{i=1}^{N_r} \Lambda_{is_{\phi_I}}} \log_e\left(\frac{\sum_{i=1}^{N_r} \Lambda_{is_{\phi_I}}}{\lambda_I \eta \omega_{s_{\phi_I}}}\right), & \text{otherwise.} \end{cases}$$
(11)

<sup>2</sup>If both constraints are not met with equality, then the transmit power of the STx can be increased by a factor that is strictly greater than unity without violating the constraints. Doing so yields a new policy that has a lower SEP. Therefore, the original policy cannot be optimal.

Here,  $\lambda_I > 0$  is chosen such that the average interference constraint is satisfied with equality. To the best of our knowledge, this result itself is novel for an average interference constrained underlay CR system.

Similarly, the optimal ASPA policy  $\phi_P$  that minimizes the SEP when the STx is subject only to the average power constraint in (8) and to (9) is as follows. The optimal antenna  $s_{\phi_P}$  is

$$s_{\phi_P} = \begin{cases} 0, & \sum_{i=1}^{N_r} \Lambda_{i1} \le \eta \lambda_P, \dots, \sum_{i=1}^{N_r} \Lambda_{iN_t} \le \eta \lambda_P, \\ \arg\max_{j \in \{1,\dots,N_t\}} \left\{ \sum_{i=1}^{N_r} \Lambda_{ij} \right\}, & \text{otherwise.} \end{cases}$$
(12)

The optimal power  $P_{\phi_P}$  transmitted from  $s_{\phi_P}$  is

$$P_{\phi_P} = \begin{cases} 0, & s_{\phi_P} = 0, \\ \frac{m\eta}{\sum_{i=1}^{N_r} \Lambda_{is_{\phi_P}}} \log_e \left(\frac{\sum_{i=1}^{N_r} \Lambda_{is_{\phi_P}}}{\lambda_P \eta}\right), & \text{otherwise.} \end{cases}$$
(13)

Here,  $\lambda_P > 0$  is chosen such that the average power constraint is satisfied with equality. This result can be interpreted as an extension of the power control policy derived in [28] that minimizes the SEP of BPSK.

We now present the CBBOASPA policy for any given  $P_{\text{ave}}$ and  $I_{\text{ave}}$ . Define a *feasible policy* to be one that satisfies all the constraints (7), (8), and (9).

**Theorem** 1: Define

$$X_j \triangleq \frac{\sum_{i=1}^{N_r} \Lambda_{ij}}{\lambda_P + \lambda_I \omega_j}, \quad \text{for } j \in \{1, \dots, N_t\}.$$
(14)

If  $\phi_P$  is feasible, then  $\phi^* = \phi_P$ . Else, if  $\phi_I$  is feasible, then  $\phi^* = \phi_I$ . Else, the optimal antenna  $s_{\phi^*}$  is

$$s_{\phi^*} = \begin{cases} 0, & X_1 \le \eta, \dots, X_{N_t} \le \eta, \\ \arg \max_{j \in \{1,\dots,N_t\}} \{X_j\}, & \text{otherwise.} \end{cases}$$
(15)

The optimal power  $P_{\phi^*}$  transmitted from  $s_{\phi^*}$  is

$$P_{\phi^*} = \begin{cases} 0, & s_{\phi^*} = 0, \\ \frac{m\eta}{\sum_{i=1}^{N_r} \Lambda_{is_{\phi^*}}} \log_e\left(\frac{X_{s_{\phi^*}}}{\eta}\right), & \text{otherwise.} \end{cases}$$
(16)

Here,  $\lambda_I > 0$  and  $\lambda_P > 0$  are chosen so as to meet the average power and interference constraints with equality, and such a choice exists.

*Proof:* The proof is given in Appendix A.

Note that the above result holds regardless of the fading distribution of  $\Lambda_{ij}$  and  $\omega_j$ , and does not preclude spatially correlated antennas. These only manifest themselves through the two constants  $\lambda_P$  and  $\lambda_I$ , which are found numerically. This is typical in several constrained adaptation problems in wireless [22]. The functional form of  $X_j$  partially validates the intuition behind the ad hoc MSLIR rule, which makes its selection decisions based on the ratio  $\frac{\Lambda_j}{\omega_j}$ . What it missed is the affine form of the denominator in (14). It also did not consider power adaptation, which is when such a ratio matters.

To better understand CBBOASPA, we plot in Figure 2 the transmit power as a function of the channel power gain  $\Lambda_{1s}$  from the selected antenna s to the SRx for the powercum interference-constrained regime ( $\lambda_I > 0$  and  $\lambda_P > 0$ ). This is done for different values of  $\omega_s$  for  $N_t = 2$  and  $N_r = 1$ . The transmit power is a function of both  $\Lambda_{1s}$ 



Fig. 2. Transmit power from selected antenna in the power- cum interferenceconstrained regime ( $I_{\text{ave}} = 10 \text{ dB}$ ,  $\bar{\omega} = 1$ ,  $\bar{\Lambda} = 1$ ,  $\sigma^2 = 1$ , and  $P_{\text{ave}} = 12 \text{ dB}$ ).

and  $\omega_s$ . It decreases as  $\omega_s$  increases due to the interference constraint. The behavior in the interference-constrained regime and in the power-constrained regime is qualitatively similar except that in the latter regime, the transmit power  $P_s$  is independent of  $\omega_s$ . In all the regimes, the transmit power is zero for  $\Lambda_{1s} \leq \eta(\lambda_P + \lambda_I\omega_s)$ . Thereafter, as  $\Lambda_{1s}$  increases, the transmit power increases. However, for larger values of  $\Lambda_{1s}$ , it decreases towards zero. This decrease occurs because when the instantaneous SEP is small, increasing the transmit power to further reduce it does not influence the average SEP as much. Thus, the node instead conserves power in this case.

Complexity: Since the optimal solution is specified in a closed form, the complexity of computing it is quite minimal. Computing an  $X_j$  requires  $N_r + 1$  additions, one multiplication, and one division operation. Selecting the optimal antenna involves  $N_t$  comparisons. Determining the transmit power requires at most two multiplications and divisions, and a log function computation. Thus, the number of operations is linear in both  $N_t$  and  $N_r$ .

## A. SEP Analysis of CBBOASPA

Since CBBOASPA involves thresholding, ordering, and comparing the  $N_t$  i.i.d. RVs  $X_1, \ldots, X_{N_t}$ , we first characterize the cumulative distribution function (CDF) and probability distribution function (PDF) of  $X_j$ .

**Lemma** 1: The CDF  $Pr(X_j \le x)$  and PDF  $f_{X_j}(x)$  of  $X_j$  are given, for  $x \ge 0$ , by

$$\Pr\left(X_{j} \leq x\right) = 1 - \frac{\bar{\Lambda}e^{\frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}}}{\bar{\omega}\lambda_{I}} \sum_{k=0}^{N_{r}-1} \frac{x^{k}\Gamma\left(k+1,\frac{\lambda_{P}}{\bar{\Lambda}}x+\frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}\right)}{k!\left(x+\frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}}\right)^{k+1}},$$
(17)

and

$$f_{X_j}(x) = \sum_{k=0}^{N_r-1} \left[ \frac{\lambda_P \left(\frac{\lambda_P}{\Lambda}x\right)^k e^{-\frac{\lambda_P}{\Lambda}x}}{k! \left(\bar{\omega}\lambda_I x + \bar{\Lambda}\right)} - \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I} e^{\frac{\lambda_P}{\bar{\omega}\lambda_I}} \right] \times \frac{\left(k\frac{\bar{\Lambda}}{\bar{\omega}\lambda_I}x^{k-1} - x^k\right) \Gamma\left(k+1, \frac{\lambda_P}{\Lambda}x + \frac{\lambda_P}{\bar{\omega}\lambda_I}\right)}{k! \left(x + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I}\right)^{k+2}} \right]. \quad (18)$$

where  $\Gamma$  denotes the upper incomplete gamma function [29].

*Proof:* The proof is given in Appendix B. An exact expression for the SEP of CBBOASPA is as follows.

**Theorem** 2: The SEP of CBBOASPA for MPSK equals

$$\begin{split} \operatorname{SEP} &= m \left( 1 - \frac{\bar{\Lambda} e^{\frac{\lambda_P}{\bar{\omega}\lambda_I}}}{\bar{\omega}\lambda_I} \sum_{k=0}^{N_r - 1} \frac{\eta^k \Gamma \left(k + 1, \frac{\lambda_P}{\Lambda} \eta + \frac{\lambda_P}{\bar{\omega}\lambda_I}\right)}{k! \left(\eta + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I}\right)^{k+1}} \right)^{N_t} \\ &+ \frac{N_t}{\pi} \int_{\eta}^{\infty} \int_{0}^{m\pi} \left(\frac{\eta}{x}\right)^{\operatorname{csc}^2(\theta)} \sum_{k=0}^{N_r - 1} \left[ \frac{\lambda_P \left(\frac{\lambda_P}{\Lambda} x\right)^k e^{-\frac{\lambda_P}{\Lambda} x}}{k! \left(\bar{\omega}\lambda_I x + \bar{\Lambda}\right)} \right. \\ &- \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I} e^{\frac{\lambda_P}{\bar{\omega}\lambda_I}} \frac{\left(k \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I} x^{k-1} - x^k\right) \Gamma \left(k + 1, \frac{\lambda_P}{\Lambda} x + \frac{\lambda_P}{\bar{\omega}\lambda_I}\right)}{k! \left(x + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I}\right)^{k+2}} \right] \\ &\times \left[ 1 - \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I} e^{\frac{\lambda_P}{\bar{\omega}\lambda_I}} \sum_{k=0}^{N_r - 1} \frac{x^k \Gamma \left(k + 1, \frac{\lambda_P}{\Lambda} x + \frac{\lambda_P}{\bar{\omega}\lambda_I}\right)}{k! \left(x + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_I}\right)^{k+1}} \right]^{N_t - 1} d\theta dx. \end{split}$$
(19)

**Proof:** The derivation is given in Appendix C. While the expression is exact and general, it is in the form of a double integral and is quite involved. This is unavoidable because of the non-linear dependence of both the optimal AS rule in (15) and the optimal transmit power in (16) on the channel power gains to the PRx. In order to gain more insights, we now present simpler bounds. We also study the simpler  $N_t \times 1$  model, and show that for it the bounds reduce to a closed form that depends on the regime of operation.

The SEP expression can be simplified to a single integral form by writing its integral over  $\theta$  as a sum of integrals over two intervals  $[0, \frac{\pi}{4}]$  and  $[\frac{\pi}{4}, m\pi]$ , along the lines of [19], [30]. In the first and second integrals,  $\sin^2(\theta)$  is upper bounded by  $\frac{1}{2}$  and 1, respectively. The resultant upper bound SEP<sub>u</sub> is then

$$\begin{split} \operatorname{SEP}_{u} &= m \left( 1 - \frac{\bar{\Lambda} e^{\frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}}}{\bar{\omega}\lambda_{I}} \sum_{k=0}^{N_{r}-1} \frac{\eta^{k} \Gamma \left(k+1, \frac{\lambda_{P}}{\Lambda} \eta + \frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}\right)}{k! \left(\eta + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}}\right)^{k+1}} \right)^{N_{t}} \\ &+ N_{t} \int_{\eta}^{\infty} \frac{\eta}{x} \left(m - \frac{1}{4} + \frac{\eta}{4x}\right) \sum_{k=0}^{N_{r}-1} \left[ \frac{\lambda_{P} \left(\frac{\lambda_{P}}{\Lambda} x\right)^{k} e^{-\frac{\lambda_{P}}{\Lambda}x}}{k! \left(\bar{\omega}\lambda_{I}x + \bar{\Lambda}\right)} \right. \\ &\left. - \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}} e^{\frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}} \frac{\left(k \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}} x^{k-1} - x^{k}\right) \Gamma \left(k+1, \frac{\lambda_{P}}{\Lambda} x + \frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}\right)}{k! \left(x + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}}\right)^{k+2}} \right] \\ &\times \left[ 1 - \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}} e^{\frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}} \sum_{k=0}^{N_{r}-1} \frac{x^{k} \Gamma \left(k+1, \frac{\lambda_{P}}{\Lambda} x + \frac{\lambda_{P}}{\bar{\omega}\lambda_{I}}\right)}{k! \left(x + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}}\right)^{k+1}} \right]^{N_{t}-1} dx. \end{split}$$

$$(20)$$

The single integral(s) above can be easily evaluated numerically because of the exponentially decaying term in the integrand(s).

## B. Insightful Special Case: One Receive Antenna $(N_r = 1)$

We now show that the upper bound simplifies further to an insightful closed form when  $N_r = 1$ . The derivation is given

in Appendix D.

• *Power-constrained Regime:* When the interference constraint is inactive, i.e.,  $\lambda_I = 0$ ,

$$\begin{split} \mathtt{SEP}_u &= m \left( 1 - e^{-\frac{\lambda_P \eta}{\Lambda}} \right)^{N_t} + \frac{N_t m \eta \lambda_P}{\overline{\Lambda}} \\ &\times \sum_{l=0}^{N_t - 1} \binom{N_t - 1}{l} (-1)^l E_1 \left( \frac{(l+1)\eta \lambda_P}{\overline{\Lambda}} \right), \quad (21) \end{split}$$

where  $E_1(\cdot)$  is the standard exponential integral function [29, pp. xxxv].

• Interference-constrained Regime: When the power constraint is inactive, i.e.,  $\lambda_P = 0$ , we get the following simple bound, which is also a bound on the error floor:

$$\operatorname{SEP}_{u} = \frac{\bar{\omega}\lambda_{I}m\eta}{\bar{\Lambda}(N_{t}-1)} \left[ 1 - \left(\frac{\bar{\omega}\lambda_{I}\eta}{\bar{\omega}\lambda_{I}\eta + \bar{\Lambda}}\right)^{N_{t}-1} \right]. \quad (22)$$

• Power- cum Interference-constrained Regime: When both the constraints are active, i.e.,  $\lambda_I > 0$  and  $\lambda_P > 0$ , we get (23), which is given at the top of the next page.

#### C. Extension to Other Constellations

We now develop CBBOASPA for MQAM. It can be shown that when the selected antenna is  $s_{\phi}$  and the transmit power is  $P_{\phi}$ , the SEP of MQAM is upper bounded by [25, (8.12)]

$$\Pr\left(\operatorname{Err}|\boldsymbol{\Lambda},\boldsymbol{\omega}\right) \leq \left(\frac{a}{2} - \frac{a^2}{16}\right) \exp\left(-\frac{3P_{\phi}\sum_{i=1}^{N_r}\Lambda_{is_{\phi}}}{2(M-1)\sigma^2}\right),\tag{24}$$

where  $a = 4 - \frac{4}{\sqrt{M}}$ . This is functionally similar to the expression in (5), except that  $\frac{1}{\eta m}$  is replaced with  $\frac{3}{2(M-1)\sigma^2}$  inside the exponential. Thus, the structure of the optimal ASPA policy remains same for MQAM, except for scaling constants. The SEP analysis for MQAM is also similar to that for MPSK; the expressions are not shown here due to space constraints.

# IV. NUMERICAL RESULTS AND PERFORMANCE BENCHMARKING

We now present Monte Carlo simulations that use  $10^6$  samples to verify our analytical results and to benchmark the performance of CBBOASPA under different scenarios. Unless mentioned otherwise,  $\bar{\Lambda} = \bar{\omega} = \sigma^2 = 1$ .

# A. Verification of Analysis and Quantitative Insights

Figure 3 plots the SEP of CBBOASPA (cf. (19)) as a function of the average transmit power constraint  $P_{\text{ave}}$ . This is done for different values of the average interference threshold  $I_{\text{ave}}$  with  $N_t = 2$ ,  $N_r = 1$ , and 8PSK. We observe that the simulation and analysis results are in excellent agreement. Notice that the SEP decreases as  $I_{\text{ave}}$  increases because the STx can transmit at a higher average power. We also see the occurrence of an error floor because of the interference constraint. As expected, it decreases as  $I_{\text{ave}}$  increases.

Figure 4 plots the SEP (cf. (19)) and its upper bound (cf. (20)) as a function of  $P_{\text{ave}}$  for  $I_{\text{ave}} = 10$  dB. We see that upper bound tracks the SEP well for both QPSK and 8PSK over the entire range of values of  $P_{\text{ave}}$ . As shown, the



Fig. 3. SEP as a function of  $P_{\rm ave}$  for different values of  $I_{\rm ave}$  ( $N_t = 2$ ,  $N_r = 1$ , and 8PSK).



Fig. 4. SEP and its upper bound as a function of  $P_{\rm ave}$  for different constellation sizes ( $I_{\rm ave} = 10$  dB,  $N_t = 2$ , and  $N_r = 1$ ).

gap between the two is only 0.7 dB for QPSK and 1.0 dB for 8PSK at  $P_{\text{ave}} = 9$  dB. As expected, the SEP increases with the constellation size, and an error floor again occurs.

Figure 5 studies the effect of the number of receive antennas  $N_r$  at the SRx on the SEP. It plots the SEP as a function of  $P_{\text{ave}}$  for different  $N_r$  when  $I_{\text{ave}} = 2$  dB and  $N_t = 2$ . As expected, the SEP decreases as  $N_r$  increases. The error floor decreases markedly as  $N_r$  increases – it drops from 0.1 to 0.02 to 0.0005 when  $N_r$  increases from 1 to 2 to 4. The behavior when the number of transmit antennas is increased is similar.

# B. Performance Benchmarking

We now benchmark the performance of CBBOASPA with several AS policies considered in the literature for underlay CR, namely, MI, MSLIR, DAS, and SEP-optimal AS with onoff power control. This is done for both MPSK and MQAM. The MI and MSLIR policies, as originally proposed, require the STx to transmit with a fixed power  $P_t$  always. Hence, they may not even be feasible. We, therefore, enhance them by incorporating on-off power control in them, so that they are feasible for all system parameter settings and serve as useful benchmarks [10]. We also extend all the benchmark policies to handle multiple receive antennas. For single transmit AS, the MSLIR rule arises naturally for the instantaneous interference constraint used in [13] because it makes the transmit power of the selected antenna to be inversely proportional to the STx-

$$\begin{split} \text{SEP}_{u} &= m \left( 1 - \frac{\bar{\Lambda} e^{-\frac{\lambda_{P}\eta}{\bar{\Lambda}}}}{\bar{\omega}\lambda_{I}\eta + \bar{\Lambda}} \right)^{N_{t}} + N_{t}m\eta \sum_{l=0}^{N_{t}-1} \binom{N_{t}-1}{l} (-1)^{l} \left[ \frac{\lambda_{P} + \bar{\omega}\lambda_{I}}{\bar{\Lambda}} E_{1} \left( \frac{(l+1)\lambda_{P}\eta}{\bar{\Lambda}} \right) - \sum_{k=1}^{l} \frac{\lambda_{P} \left( \bar{\omega}\lambda_{I} \right)^{k-1-l} e^{-\frac{(l+1)\lambda_{P}\eta}{\bar{\Lambda}}}}{(\bar{\Lambda})^{k-l} \left( l+1-k \right)!} \\ &\times \sum_{n=1}^{l+1-k} \frac{(n-1)!(-1)^{l+1-k-n} \left[ \frac{(l+1)\lambda_{P}}{\bar{\Lambda}} \right]^{l+1-k-n}}{\left( \eta + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}} \right)^{n}} - \sum_{k=1}^{l+1} \left( \frac{\bar{\omega}\lambda_{I}}{\bar{\Lambda}} \right)^{k-1-l} e^{-\frac{(l+1)\lambda_{P}\eta}{\bar{\Lambda}}} \sum_{n=1}^{l+2-k} \frac{(n-1)! \left( -\frac{(l+1)\lambda_{P}}{\bar{\Lambda}} \right)^{l+2-k-n}}{(l+2-k)! \left( \eta + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}} \right)^{n}} \\ &+ \left( \frac{(l+1)\lambda_{P}}{\bar{\Lambda}} \left( 1 + \frac{\lambda_{P}}{\bar{\omega}\lambda_{I}} \sum_{k=1}^{l} \frac{\left( 1-k \right) \left( -\frac{(l+1)\lambda_{P}}{\bar{\omega}\lambda_{I}} \right)^{l-k}}{(l+2-k)!} \right) - \frac{\lambda_{P} + \bar{\omega}\lambda_{I}}{\bar{\Lambda}} \right) e^{\frac{(l+1)\lambda_{P}}{\bar{\omega}\lambda_{I}}} E_{1} \left( (l+1) \left( \frac{\lambda_{P}\eta}{\bar{\Lambda}} + \frac{\lambda_{P}}{\bar{\omega}\lambda_{I}} \right) \right) \right]. \end{aligned}$$
(23)



Fig. 5. Effect of number of receive antennas: SEP as a function of  $P_{\text{ave}}$  for different  $N_r$  ( $I_{\text{ave}} = 2$  dB,  $N_t = 2$ , and QPSK).

## PRx channel power gain.<sup>3</sup>

1) SEP-Optimal AS Rule with On-off Power Control [10]: In it, the STx either transmits with a fixed power  $P_t$  or with zero power. The selected antenna s is given by

$$s = \operatorname*{arg\,min}_{j \in \{0,...,N_t\}} \left\{ \varphi \omega_j + m \exp\left(-\frac{P_t \sum_{i=1}^{N_r} \Lambda_{ij}}{\eta m}\right) \right\}, \quad (25)$$

where  $\varphi = 0$  if  $P_t$  is small enough such that the interference caused to the PRx is always below  $I_{\text{ave}}$ . In this case, the selection rule reduces to the conventional AS rule [5]. Else,  $\varphi > 0$  is numerically set so that the interference constraint is met with equality.

2) Enhanced MI (EMI) Rule: In it, the selected antenna s is 0 if  $\omega_1 \geq \tau, \ldots, \omega_{N_t} \geq \tau$ , and is given by  $\arg\min_{j\in\{1,\ldots,N_t\}} \{\omega_j\}$ , otherwise. The threshold  $\tau$  is chosen to satisfy the interference constraint with equality. Setting  $\tau = \infty$  reduces the EMI rule to the MI rule proposed in [8]. 3) Enhanced MSLIR (EMSLIR) Rule: In it, the selected antenna s is 0 if  $\frac{\sum_{i=1}^{N_T} \Lambda_{i1}}{\omega_1} \leq \beta, \ldots, \frac{\sum_{i=1}^{N_T} \Lambda_{iN_t}}{\omega_{N_t}} \leq \beta$ , and is given by  $\arg\max_{j\in\{1,\ldots,N_t\}} \left\{ \frac{\sum_{i=1}^{N_T} \Lambda_{ij}}{\omega_j} \right\}$  otherwise. The threshold  $\beta$  is chosen such that the interference constraint is satisfied with equality, i.e.,  $P_t \mathbf{E}[\omega_s] = I_{\text{ave}}$ . Setting  $\beta = 0$  reduces the rule to the MSLIR rule proposed in [8].

4) DAS Rule: The selected antenna s is given by [9]  $\arg \max_{j \in \{1,...,N_t\}} \left\{ \delta \sum_{i=1}^{N_r} \Lambda_{ij} - (1-\delta)\omega_j \right\}, \text{ where } \delta \in$ 

[0,1] is a pre-specified constant. The DAS rule controls the average interference caused to the primary by suitably choosing  $\delta$ . Setting  $\delta = 1$  corresponds to the unconstrained conventional AS rule, while  $\delta = 0$  corresponds to the MI rule.

In all the above rules, STx transmits with a fixed power  $P_t$  when  $s \neq 0$ .

Figure 6 compares the SEPs of CBBOASPA, EMI rule, EMSLIR rule, DAS rule, and AS with on-off power control as a function of  $P_{\text{ave}}$  for BPSK with  $I_{\text{ave}} = 8$  dB,  $N_t = 2$ , and  $N_r = 1$ . For the EMI rule,  $P_t$  and  $\tau$  are jointly chosen as follows. For small values of  $P_{\text{ave}}$ , the interference constraint will be inactive. Therefore,  $P_t = P_{\text{ave}}$ , which means that the power constraint is met with equality, and  $\tau = \infty$ . For larger  $P_{\text{ave}}$ ,  $P_t$  and  $\tau$  are jointly chosen such that the power constraint and interference constraint are both met with equality. The values of  $P_t$  and  $\beta$  for the EMSLIR rule and  $P_t$ and  $\delta$  for the DAS rule are also chosen in a similar manner. The minimum SEP of CBBOASPA is lower by a factor of 140, 58, 51, and 33 than that of EMI, EMSLIR, DAS, and AS with on-off power control, respectively.

The figure also plots the SEP of the optimal ASPA policy that is based on the exact SEP expression in (4), which is in the form of a single integral. We shall refer to this as the *Exact ASPA* policy. It is the outcome of an intensive simulation effort. We see that the minimum SEP of CBBOASPA is within 10% of that of the Exact ASPA policy. This shows the effectiveness of the insightful approach used to arrive at CBBOASPA.

There are three different regimes of operation for CB-BOASPA:

- P<sub>ave</sub> ≤ 8 dB = I<sub>ave</sub>: In this regime, the interference constraint is not active, and only the average power constraint is active. Here, the average interference caused to the PRx is P<sub>ave</sub>. This is because E [ω<sub>j</sub>] = 1, and from (12), the choice of the selected antenna is determined only by Λ<sub>1j</sub>. The SEP decreases as P<sub>ave</sub> increases. In this regime, the interference constraint is inactive for all the other benchmark policies as well, and their SEPs also decrease as P<sub>ave</sub> increases.
- 8  $dB < P_{\rm ave} \leq 11.8 \ dB$ : In this regime, both the interference and power constraints are active. Here, the slope of the SEP curve changes, and the curve begins to flatten. In this regime, the SEPs of all the benchmark policies eventually start to increase as  $P_{\rm ave}$  increases. This is because the fraction of time the STx transmits

 $<sup>^3 \</sup>mathrm{Since}$  [11] uses receive AS, a quantitative comparison with it is not possible.



Fig. 6. MPSK: Comparison of the SEPs of CBBOASPA with Exact ASPA and benchmark policies ( $N_t = 2$ ,  $N_r = 1$ ,  $I_{ave} = 8$  dB, and BPSK).



Fig. 7. MQAM: Comparison of the SEPs of CBBOASPA and benchmark policies ( $N_t = 4$ ,  $N_r = 1$ ,  $I_{ave} = 13$  dB, and 16QAM).

with zero power correspondingly increases in order to satisfy the average interference constraint. Eventually, the zero transmit power option contributes the most to the SEP.

•  $P_{\rm ave} > 11.8 \ dB$ : In this regime, only the average interference constraint is active. Consequently, the SEP of CBBOASPA becomes independent of  $P_{\rm ave}$ , which leads to an error floor. As in the previous regime, the SEPs of the benchmark policies increase as  $P_{\rm ave}$  increases.

Figure 7 presents performance benchmarking results for 16QAM with  $N_t = 4$ . Now, the minimum SEP of CBBOASPA is lower by a factor of 162.0, 56.7, 56.0, and 13.4 than the minimum SEPs of the EMI, EMSLIR, DAS, and on-off power control, respectively.

# V. CONCLUSIONS

We developed an ASPA policy called CBBOASPA for a secondary transmitter that operates in the underlay CR mode and is subject to constraints on its average transmit power and the average interference power it causes to the primary. We saw that whether an antenna gets selected or not is determined by a simple ratio in which the numerator is the sum of the channel power gains from the antenna to the SRx and the denominator is an affine function of the channel power gain from the antenna to the PRx. For the general case with multiple transmit antennas and multiple receive antennas, we then derived the exact SEP of CBBOASPA for MPSK and

a simpler upper bound. We also saw that it applies to other constellations such as MQAM. Compared to the several ad hoc AS policies proposed in the literature, CBBOASPA reduced the SEP by up to two orders of magnitude for both MPSK and MQAM. The significant gains it achieved over AS with on-off power control demonstrated the effectiveness of jointly adapting the transmit power. Several interesting avenues for future work arise out of this work. These include incorporating subset selection and modeling multiple secondary and primary nodes.

## APPENDIX

# A. Proof of Theorem 1

The proof below consists of three key steps. In the first step, we check whether optimal policies for the two lesser constrained cases, in which one of the two inequalities is inactive, are feasible. If this is the case, then we are done. In the second step, which arises when the above two policies are not feasible, we define a policy  $\phi^*$  that minimizes an auxiliary function given two constants  $\lambda_P \ge 0$  and  $\lambda_I \ge 0$ , and show that it is the desired optimal policy. In the third step, we unravel the structure of  $\phi^*$ .

First, consider the case when  $\phi_P$  is feasible. In this case, it must clearly be optimal because it is the solution to a less constrained optimization problem. Similarly, when  $\phi_I$  is feasible, it is optimal. Else, consider the case where both  $\phi_P$ and  $\phi_I$  are not feasible, i.e., the average interference generated by  $\phi_P$  is greater than  $I_{\text{ave}}$  and the average power consumed by  $\phi_I$  is greater than  $P_{\text{ave}}$ .<sup>4</sup>

For  $j \in \{0, 1, ..., N_t\}$ , let

$$\Omega_{\mathbf{\Lambda},\boldsymbol{\omega}}^{\tilde{\lambda}_{I},\tilde{\lambda}_{P}}(j,P) = \zeta \left(\sum_{i=1}^{N_{r}} \Lambda_{ij}, P\right) + \tilde{\lambda}_{P}P + \tilde{\lambda}_{I}P\omega_{j}, \quad (26)$$

where  $\zeta(x, P) = m \exp\left(-\frac{Px}{m\eta}\right)$  is the SEP when the power is P and the sum of channel power gains to all the receive antennas is x, and  $\tilde{\lambda}_I > 0$  and  $\tilde{\lambda}_P > 0$  are constants. Given  $\tilde{\lambda}_I$  and  $\tilde{\lambda}_P$ , let  $\tilde{\phi}$  to be the ASPA policy that is defined as follows for each realization of  $\Lambda$  and  $\omega$ :

$$(s_{\tilde{\phi}(\Lambda,\omega)}, P_{\tilde{\phi}(\Lambda,\omega)}) = \operatorname*{arg\,min}_{\{(j,P):j=0,1,\dots,N_t, P \ge 0\}} \Omega_{\Lambda,\omega}^{\tilde{\lambda}_I, \tilde{\lambda}_P}(j, P).$$
(27)

Furthermore, for any ASPA policy  $\phi$ , let

$$L_{\phi}\left(\tilde{\lambda}_{P},\tilde{\lambda}_{I}\right) = \mathbf{E}_{\boldsymbol{\Lambda},\boldsymbol{\omega}}\left[\Omega_{\boldsymbol{\Lambda},\boldsymbol{\omega}}^{\tilde{\lambda}_{I},\tilde{\lambda}_{P}}(s_{\phi(\boldsymbol{\Lambda},\boldsymbol{\omega})},P_{\phi(\boldsymbol{\Lambda},\boldsymbol{\omega})})\right].$$
 (28)

Choose  $\tilde{\lambda}_P$  and  $\tilde{\lambda}_I$  so that  $\tilde{\phi}$  meets the two constraints with equality, i.e.,  $\mathbf{E}_{\Lambda,\omega} \left[ P_{\tilde{\phi}(\Lambda,\omega)} \right] = P_{\text{ave}}$  and

 $\mathbf{E}_{\boldsymbol{\Lambda},\boldsymbol{\omega}}\left[P_{\tilde{\phi}(\boldsymbol{\Lambda},\boldsymbol{\omega})}\omega_{s_{\tilde{\phi}(\boldsymbol{\Lambda},\boldsymbol{\omega})}}\right] = I_{\mathrm{ave}}.^{5} \text{ Let the corresponding val-}$ ues of  $\tilde{\lambda}_P$  and  $\tilde{\lambda}_I$  be  $\lambda_P$  and  $\lambda_I$ , respectively, and let the corresponding policy with these constants be denoted by  $\phi^*$ . Clearly,  $\phi^*$  is feasible. From the definition of  $\phi^*$ , it follows that  $L_{\phi^*}(\lambda_P, \lambda_I) \leq L_{\phi}(\lambda_P, \lambda_I)$ . Rearranging terms, we get

$$\mathbf{E}_{\Lambda,\omega} \left[ \zeta \left( \sum_{i=1}^{N_r} \Lambda_{is_{\phi^*(\Lambda,\omega)}}, P_{\phi^*(\Lambda,\omega)} \right) \right] \\
\leq \mathbf{E}_{\Lambda,\omega} \left[ \zeta \left( \sum_{i=1}^{N_r} \Lambda_{is_{\phi(\Lambda,\omega)}}, P_{\phi(\Lambda,\omega)} \right) \right] \\
+ \lambda_P \left( \mathbf{E}_{\Lambda,\omega} \left[ P_{\phi(\Lambda,\omega)} \right] - P_{\text{ave}} \right) \\
+ \lambda_I \left( \mathbf{E}_{\Lambda,\omega} \left[ P_{\phi(\Lambda,\omega)} \omega_{s_{\phi(\Lambda,\omega)}} \right] - I_{\text{ave}} \right). \quad (29)$$

For any feasible policy  $\phi$ , we know that  $\mathbf{E}_{\Lambda,\omega} \left[ P_{\phi(\Lambda,\omega)} \right] \leq$  $P_{\text{ave}}$  and  $\mathbf{E}_{\Lambda,\omega} \left[ P_{\phi(\Lambda,\omega)} \omega_{s_{\phi(\Lambda,\omega)}} \right] \leq I_{\text{ave}}$ . Therefore, (29) implies

$$\mathbf{E}_{\boldsymbol{\Lambda},\boldsymbol{\omega}}\left[\zeta\left(\sum_{i=1}^{N_{r}}\Lambda_{is_{\phi^{*}(\boldsymbol{\Lambda},\boldsymbol{\omega})}},P_{\phi^{*}(\boldsymbol{\Lambda},\boldsymbol{\omega})}\right)\right]$$
$$\leq \mathbf{E}_{\boldsymbol{\Lambda},\boldsymbol{\omega}}\left[\zeta\left(\sum_{i=1}^{N_{r}}\Lambda_{is_{\phi(\boldsymbol{\Lambda},\boldsymbol{\omega})}},P_{\phi(\boldsymbol{\Lambda},\boldsymbol{\omega})}\right)\right].$$
(30)

Hence, the SEP of  $\phi^*$  is less than or equal to the SEP of any

feasible policy. Therefore,  $\phi^*$  is optimal. 1) Minimizing  $\Omega_{\Lambda,\omega}^{\lambda_I,\lambda_P}(j,P)$ : Given  $\Lambda$  and  $\omega$ , we first determine the optimal power  $P_j^*$  when the selected antenna is  $j \in \{1, \ldots, N_t\}$ . Equating the derivative of  $\Omega_{\Lambda, \omega}^{\lambda_I, \lambda_P}(j, P)$ with respect to P to zero, we get

$$\exp\left(-\frac{P\sum_{i=1}^{N_r}\Lambda_{ij}}{m\eta}\right) = \eta\frac{(\lambda_P + \lambda_I\omega_j)}{\sum_{i=1}^{N_r}\Lambda_{ij}} = \frac{\eta}{X_j},\qquad(31)$$

where the last equality follows from the definition of  $X_j$ in (14).<sup>6</sup> Therefore, the optimal power  $P_i^*$  is equal to

$$P_j^* = \begin{cases} \frac{m\eta}{\sum_{i=1}^{N_r} \Lambda_{ij}} \log_e\left(\frac{X_j}{\eta}\right), & \text{if } X_j > \eta, \\ 0, & \text{otherwise.} \end{cases}$$
(32)

Substituting this in (26) and simplifying, we can show that, for  $P_i^* > 0$ ,

$$\Omega_{\mathbf{\Lambda},\boldsymbol{\omega}}^{\lambda_{I},\lambda_{P}}(j,P_{j}^{*}) = \frac{m\eta}{X_{j}} \left(1 + \log_{e}\left(\frac{X_{j}}{\eta}\right)\right) = \Psi(X_{j}), \quad (33)$$

where  $\Psi(x) = \frac{m\eta}{x} \left( 1 + \log_e \left( \frac{x}{\eta} \right) \right)$ . It can be shown that the maximum value of  $\Psi(x)$  is *m* and it occurs at  $x = \eta$ . Furthermore, it is monotonically decreasing for  $x \ge \eta$ .

<sup>5</sup>The existence of such  $\lambda_P$  and  $\lambda_I$  can be shown by means of the intermediate value theorem [31] using the following facts: (i) Let the two constants that define  $\phi_P$  be  $\lambda'_P > 0$  and  $\lambda'_I = 0$ . As  $\phi_P$  is not feasible, we know that for these two constants  $\mathbf{E}_{\Lambda, \omega} \left[ P_{s_{\phi_P(\Lambda, \omega)}} \right] = P_{\mathrm{ave}}$  and  $\mathbf{E}_{\boldsymbol{\Lambda},\boldsymbol{\omega}} \left[ P_{s_{\phi_P(\boldsymbol{\Lambda},\boldsymbol{\omega})}} \omega_{s_{\phi_P(\boldsymbol{\Lambda},\boldsymbol{\omega})}} \right] > I_{\text{ave.}} \text{ (ii) Similarly, let the two constants that define } \phi_I \text{ be } \lambda_P'' = 0 \text{ and } \lambda_I'' > 0. \text{ Since } \phi_I \text{ is also not feasible, }$ we know that for these two constants  $\mathbf{E}_{\mathbf{\Lambda}, \boldsymbol{\omega}} \left| P_{s_{\phi_I(\mathbf{\Lambda}, \boldsymbol{\omega})}} \right| > P_{\mathrm{ave}}$  and  $\mathbf{E}_{\Lambda,\omega}\left[P_{s_{\phi_I}(\Lambda,\omega)}\omega_{s_{\phi_I}(\Lambda,\omega)}\right] = I_{\text{ave.}}$  (iii) The policy in which the STx always transmits with zero power is a feasible policy; therefore, the set of all feasible policies is non-empty. (iv) Furthermore, the optimal power that minimizes  $\Omega_{\Lambda,\omega}^{\tilde{\lambda}_I,\tilde{\lambda}_P}(j,P)$  is a monotonically decreasing function of  $\tilde{\lambda}_I$  and  $\tilde{\lambda}_P$ .

<sup>6</sup>The dependence of  $X_i$  on  $\lambda_I$  and  $\lambda_P$  is not shown to avoid clutter.

2) Optimal Antenna: All that remains now is determining the optimal antenna. Recall that setting the transmit power to zero is equivalent to selecting antenna 0. Hence, if  $X_1 \leq$  $\eta, \ldots, X_{N_t} \leq \eta$ , then, from (32), it follows that  $s_{\phi^*} = 0$ . Furthermore, in this case,  $\Omega^{\lambda_I,\lambda_P}_{\Lambda,\omega}(0,P_{\phi^*})=m.$ 

Otherwise, let ] be the non-empty set of all transmit antennas j for which  $X_i > \eta$ :

$$\mathbf{I} = \{ j \in \{1, \dots, N_t\} : X_j > \eta \}.$$
(34)

From (33), it is clear that for minimizing  $\Omega_{\Lambda,\omega}^{\lambda_I,\lambda_P}$ , one of the antennas in I should be selected and its transmit power should be non-zero. This is because  $P_i^* > 0$  is equivalent to  $X_j > \eta$ , which, from (33), implies that  $\Omega_{\Lambda,\omega}^{\lambda_I,\lambda_P}(j,P_j^*) < m$ . Therefore, the optimal antenna is

$$s_{\phi^*} = \operatorname*{arg\,min}_{j \in \mathtt{J}} \Omega^{\lambda_I, \lambda_P}_{\boldsymbol{\Lambda}, \boldsymbol{\omega}}(j, P_j^*) = \operatorname*{arg\,min}_{j \in \mathtt{J}} \Psi(X_j), \quad (35)$$

where the last step again follows from (33). We know that  $\Psi(X_j)$  is a monotonically decreasing function in  $X_j$ , for  $X_j > 0$  $\eta$ , which is the case for all the antennas in ]. Therefore, the above equation is equivalent to  $s_{\phi^*} = \arg \max_{i \in \exists} X_i$ .

The above results together can be written in the compact form given in (15).

#### B. Proof of Lemma 1

The CDF of the RV  $X_j$  is given by

$$\Pr\left(X_j \le x\right) = \Pr\left(\frac{\sum_{i=1}^{N_r} \Lambda_{ij}}{\lambda_P + \lambda_I \omega_j} \le x\right).$$
(36)

Conditioning on  $\omega_i$ , which is an exponential RV with mean  $\bar{\omega}$ , we get

$$\Pr\left(X_{j} \leq x\right) = \int_{0}^{\infty} \Pr\left(\frac{\sum_{i=1}^{N_{r}} \Lambda_{ij}}{(\lambda_{P} + \lambda_{I}\omega_{j})} \leq x \middle| \omega_{j}\right) \frac{e^{-\frac{\omega_{j}}{\bar{\omega}}}}{\bar{\omega}} d\omega_{j}$$

The sum  $\sum_{i=1}^{N_r} \Lambda_{ij}$  of  $N_r$  i.i.d. exponential RVs is a gamma RV [25]. Substituting its CDF, we get

$$\Pr\left(X_{j} \leq x\right) = 1 - \sum_{k=0}^{N_{r}-1} \frac{e^{-\frac{x\lambda_{P}}{\Lambda}}}{k!\bar{\omega}}$$
$$\times \int_{0}^{\infty} \left(\frac{x\left(\lambda_{P} + \lambda_{I}\omega_{j}\right)}{\bar{\Lambda}}\right)^{k} e^{-\left(\frac{x\lambda_{I}}{\Lambda} + \frac{1}{\bar{\omega}}\right)\omega_{j}} d\omega_{j}. \quad (37)$$

Substituting  $y = \frac{x(\lambda_P + \lambda_I \omega_j)}{\bar{\lambda}}$  in the integrand above and simplifying yields (17). Differentiating (17) with respect to x yields (18).

## C. Proof of Theorem 2

Let  $\mathbf{X} \triangleq [X_1, \dots, X_{N_t}]$  and let  $\mathbf{x} = [x_1, \dots, x_{N_t}]$  denote a realization of X. The SEP conditioned on X = x, which we denote by  $\Pr(\operatorname{Err} | \mathbf{X} = \mathbf{x})$ , can be written using the chain rule in terms of the selected antenna as

$$\Pr\left(\operatorname{Err}|\mathbf{X}=\mathbf{x}\right) = \sum_{j=0}^{N_t} \Pr\left(s=j|\mathbf{X}=\mathbf{x}\right)$$
$$\times \Pr\left(\operatorname{Err}|s=j, \mathbf{X}=\mathbf{x}\right). \quad (38)$$

Averaging over  $\mathbf{X}$  and using the fact that the  $N_t$  transmit antennas see statistically identical channels, we get

$$SEP = \mathbf{E}_{\mathbf{X}} \left[ \Pr\left(s = 0 | \mathbf{X} = \mathbf{x}\right) \Pr\left(\operatorname{Err} | s = 0, \mathbf{X} = \mathbf{x}\right) \right] \\ + N_t \mathbf{E}_{\mathbf{X}} \left[ \Pr\left(s = 1 | \mathbf{X} = \mathbf{x}\right) \Pr\left(\operatorname{Err} | s = 1, \mathbf{X} = \mathbf{x}\right) \right].$$
(39)

Given s = 0, the SEP is equal to m since the STx transmits with zero power. It is, thus, independent of **X**. Furthermore, the SEP conditioned on s = 1 depends only on  $X_1$  because  $\sum_{i=1}^{N_r} \Lambda_{i1}$  and  $\omega_1$  are mutually independent of  $\Lambda_{ij}$ , for  $1 \le i \le N_r$  and  $2 \le j \le N_t$ . Hence, from (4) and (16), we get

$$\Pr\left(\operatorname{Err}|\mathbf{X}=\mathbf{x},s=1\right) = \frac{1}{\pi} \int_{0}^{m\pi} \left(\frac{\eta}{x_{1}}\right)^{\csc^{2}(\theta)} d\theta, \quad (40)$$

where csc denotes cosecant. Therefore, the SEP is given by

$$SEP = m\mathbf{E}_{\mathbf{X}} \left[ \Pr\left(s = 0 | \mathbf{X} = \mathbf{x}\right) \right] \\ + \frac{N_t}{\pi} \mathbf{E}_{\mathbf{X}} \left[ \Pr\left(s = 1 | \mathbf{X} = \mathbf{x}\right) \int_0^{m\pi} \left(\frac{\eta}{x_1}\right)^{\csc^2(\theta)} d\theta \right]. \quad (41)$$

From the fundamental theorem of expectation, we get  $\mathbf{E}_{\mathbf{X}} \left[ \Pr \left( s = 0 | \mathbf{X} = \mathbf{x} \right) \right] = \Pr \left( s = 0 \right)$  and

$$\mathbf{E}_{\mathbf{X}}\left[\Pr\left(s=1|\mathbf{X}=\mathbf{x}\right)\int_{0}^{m\pi} \left(\frac{\eta}{x_{1}}\right)^{\csc^{2}\left(\theta\right)} d\theta\right]$$
$$=\mathbf{E}_{X_{1}}\left[\Pr\left(s=1|X_{1}=x_{1}\right)\int_{0}^{m\pi} \left(\frac{\eta}{x_{1}}\right)^{\csc^{2}\left(\theta\right)} d\theta\right].$$
 (42)

Therefore, the SEP expression in (41) simplifies to

$$\begin{aligned} \mathbf{SEP} &= m \Pr\left(s = 0\right) \\ &+ N_t \mathbf{E}_{X_1} \left[ \Pr\left(s = 1 | X_1 = x_1\right) \frac{1}{\pi} \int_0^{m\pi} \left(\frac{\eta}{x_1}\right)^{\csc^2(\theta)} d\theta \right]. \end{aligned} \tag{43}$$

We now simplify the above two summation terms, which we denote by  $Q_0$  and  $Q_1$ , respectively.

Term  $Q_0$ : Virtual antenna 0 is selected if and only if (iff)  $X_1 \leq \eta, \ldots, X_{N_t} \leq \eta$ . Thus,

$$Q_0 = m \Pr(s = 0) = m \Pr(X_1 \le \eta, \dots, X_{N_t} \le \eta).$$
 (44)

Since  $X_1, \ldots, X_{N_t}$  are i.i.d. RVs, we get

$$Q_0 = m \left( \Pr\left( X_1 \le \eta \right) \right)^{N_t}.$$
(45)

Substituting the CDF of the RV  $X_1$  from (17) completes  $Q_0$ . *Term*  $Q_1$ : Antenna 1 will be selected iff  $X_1 > \eta$  and  $X_2 <$ 

$$X_1, \ldots, X_{N_t} < X_1$$
. Therefore,

$$\Pr(s = 1 | X_1 = x_1) = I_{\{x_1 > \eta\}} \\ \times \Pr(X_2 < x_1, \dots, X_{N_t} < x_1 | X_1 = x_1), \quad (46)$$

where  $I_{\{x\}}$  denotes the indicator function; it is equal to 1 if x is true, and is 0 otherwise. Conditioned on  $X_1 = x_1$ , the events  $X_2 < x_1, \ldots, X_{N_t} < x_1$  are i.i.d. Hence, we get

$$\Pr(s = 1 | X_1 = x_1) = I_{\{x_1 > \eta\}} \left[\Pr(X_2 < x_1)\right]^{N_t - 1}.$$
 (47)

Substituting (47) in  $Q_1$  and writing the expectation as an integral over  $x_1$ , we get

$$Q_1 = \frac{N_t}{\pi} \int_0^{m\pi} \int_{\eta}^{\infty} \left( \Pr\left(X_2 < x_1\right) \right)^{N_t - 1} \left(\frac{\eta}{x_1}\right)^{\csc^2(\theta)} \times f_{X_1}(x_1) dx_1 d\theta.$$

Substituting the CDF and PDF of  $X_2$  and  $X_1$ , respectively, from Lemma 1 yields the desired expression for  $Q_1$ . Adding this to the expression for  $Q_0$  in (45) results in (19).

## D. Derivation of Simpler SEP Upper Bound for $N_r = 1$

We treat the following three cases separately: (i)  $\lambda_I = 0$ ,  $\lambda_P > 0$ , (ii)  $\lambda_I > 0$ ,  $\lambda_P = 0$ , and (iii)  $\lambda_I > 0$ ,  $\lambda_P > 0$ . To gain insights, we consider a simpler version of (20) in which the term  $\frac{\eta}{x} \left(m - \frac{1}{4} + \frac{\eta}{4x}\right)$  in its integrand is replaced with its upper bound  $\frac{m\eta}{x}$ , for  $x \ge \eta$ . Further,  $N_r = 1$ .

1)  $\lambda_I = 0$ ,  $\tilde{\lambda}_P > 0$ : Substituting  $\lambda_I = 0$  in (20) yields

$$\begin{aligned} \text{SEP}_{u} &= m \left( 1 - e^{-\frac{\lambda_{P}\eta}{\Lambda}} \right)^{N_{t}} \\ &+ \frac{N_{t}m\eta\lambda_{P}}{\bar{\Lambda}} \int_{\eta}^{\infty} \frac{e^{-\frac{\lambda_{P}}{\Lambda}x_{1}}}{x_{1}} \left( 1 - e^{-\frac{\lambda_{P}}{\Lambda}x_{1}} \right)^{N_{t}-1} dx_{1}. \end{aligned}$$
(48)

Expanding  $\left(1 - e^{-\frac{\lambda_P}{\Lambda}x_1}\right)^{N_t-1}$  as a binomial series, and using the definition of the standard exponential integral function [29] we get (21).

2)  $\lambda_P = 0$ ,  $\lambda_I > 0$ : Substituting  $\lambda_P = 0$  in (20), we get

$$\operatorname{SEP}_{u} = m \left( \frac{\bar{\omega} \lambda_{I} \eta}{\bar{\Lambda} + \bar{\omega} \lambda_{I} \eta} \right)^{N_{t}} + \frac{N_{t} m \eta \bar{\Lambda}}{\bar{\omega} \lambda_{I}} \int_{\eta}^{\infty} \frac{x_{1}^{N_{t}-2}}{\left(x_{1} + \frac{\bar{\Lambda}}{\bar{\omega} \lambda_{I}}\right)^{N_{t}+1}} dx_{1}.$$

Writing  $x_1^{N_t-2}$  as  $\left(\left[x_1 + \frac{\overline{\Lambda}}{\overline{\omega}\lambda_I}\right] - \frac{\overline{\Lambda}}{\overline{\omega}\lambda_I}\right)^{N_t-2}$ , expanding it as a binomial series, and integrating, we get

$$SEP_{u} = m \left( \frac{\bar{\omega}\lambda_{I}\eta}{\bar{\Lambda} + \bar{\omega}\lambda_{I}\eta} \right)^{N_{t}} + \frac{N_{t}m\eta\bar{\omega}\lambda_{I}}{\bar{\Lambda}} \times \sum_{l=0}^{N_{t}-2} \binom{N_{t}-2}{l} \frac{(-1)^{N_{t}-l}}{N_{t}-l} \left( \frac{\frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}}}{\eta + \frac{\bar{\Lambda}}{\bar{\omega}\lambda_{I}}} \right)^{N_{t}-l}.$$
 (49)

By integrating the binomial identity  $x(1-x)^{N_t-2} = \sum_{l=0}^{N_t-2} {N_t-2 \choose l} (-1)^{N_t-l} x^{N_t-l-1}$ , it can be shown that  $\sum_{i=0}^{N_t-2} (-1)^{N_t-l} {N_t \choose i} \frac{x^{N_t-i}}{N_t-i} = \frac{(1-x)^{N_t}}{N_t} - \frac{(1-x)^{N_t-1}}{N_t-1} + \frac{1}{N_t(N_t-1)}$ . Using this identity, (49) simplifies to (22).

3)  $\lambda_P > 0$  and  $\lambda_I > 0$ : The second summand in (20), which we denote by R, is equal to

$$R = N_t m \eta \int_{\eta}^{\infty} \frac{1}{x_1} \left( 1 - \frac{\bar{\Lambda} e^{-\frac{\lambda_P}{\bar{\Lambda}} x_1}}{\bar{\omega} \lambda_I x_1 + \bar{\Lambda}} \right)^{N_t - 1} \\ \times \left( \frac{\lambda_P e^{-\frac{\lambda_P}{\bar{\Lambda}} x_1}}{x_1 \bar{\omega} \lambda_I + \bar{\Lambda}} + \frac{\bar{\Lambda} \bar{\omega} \lambda_I e^{-\frac{\lambda_P}{\bar{\Lambda}} x_1}}{\left(x_1 \bar{\omega} \lambda_I + \bar{\Lambda}\right)^2} \right) dx_1.$$
(50)

Expanding  $\left(1 - \frac{\bar{\Lambda}e^{-\frac{\lambda_P}{\Lambda}x_1}}{\bar{\omega}\lambda_I x_1 + \Lambda}\right)^{N_t - 1}$  as a binomial series, using partial fractions, and simplifying further yields (23).

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