Statistical CSI Driven Transmit Antenna Selection and Power Adaptation in Underlay Spectrum Sharing Systems

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***Abstract*— In underlay spectrum sharing, transmit antenna selection (TAS) improves the performance of a secondary system and helps it control the interference it causes to a primary system. TAS does so with a hardware complexity and cost comparable to a single antenna system. We present a novel and optimal joint TAS and continuous power adaptation rule for a practically relevant, less explored model in which the secondary transmitter knows only the statistics of channel gains from itself to one or more primary receivers. The rule minimizes the average symbol error probability (SEP) of the secondary system for an entire class of stochastic interference constraints. This general class subsumes the average interference constraint and its novel generalization, and the interference-outage constraint. We derive closed-form expressions for the transmit power and selected antenna. We then develop a general analysis of the optimal average SEP that applies to several widely-used fading models. We also present computationally-efficient approaches to determine the parameters that specify the optimal rule. Our comprehensive numerical results characterize the very different impacts of the interference constraint on both secondary and primary systems. They show that the optimal rule reduces the average SEP by two orders of magnitude compared to conventional approaches.**

***Index Terms*— Spectrum sharing, underlay, antenna selection, power adaptation, channel state information, interference.**

I. Introduction

SPECTRUM sharing is crucial for accommodating the ever-increasing number of users and their demand for data in the limited wireless spectrum that is available. For example, the sub-6 GHz spectrum, which has favorable propagation characteristics, is already crowded and yet under-utilized. The under-utilization of the scarce spectrum has motivated regulators to make spectrum available for sharing. For example, the Federal Communications Commission has opened up the 3.5 GHz and 6 GHz bands for shared use [2], [3]. Spectrum sharing has been adopted in contemporary wireless standards such as IEEE 802.11af, long term evolution (LTE)-license assisted access, MulteFire, and citizen's broadband radio service [4], [5]. It is also part of next-generation wireless standards such as 5G new radio unlicensed and IEEE 802.11be [6]. In these, the secondary users share the spectrum allocated to higher priority, incumbent primary users, such as satellite services or public safety services [6].

Different spectrum sharing modes are being considered to enable the secondary and primary systems to co-exist [7]. In the interweave mode, the secondary transmitter (STx) transmits only if it senses that the spectrum is idle. In the underlay mode, which is the focus of our work, the STx can transmit concurrently with the primary. At the same time, it must adhere to constraints on the interference it causes to the primary receiver (PRx) [8]. While the interference constraint protects the primary from excessive interference, it limits the secondary's performance. This has led to the development of advanced, multiple antenna-based interference-aware transmission techniques to improve the secondary's performance [9], [10].

Transmit antenna selection (TAS) is one such technique. In it, the STx employs one radio frequency (RF) chain, which is dynamically switched to one of the transmit antennas. TAS exploits the diversity benefits of multiple antennas at a hardware cost and complexity comparable to a single antenna system [11]. This is because the RF chain consists of several components such as digital-to-analog converter, mixer, filter, and amplifier. Having a dedicated RF chain for each antenna element significantly increases the cost and complexity of a multi-antenna system. On the other hand, the antenna elements are relatively inexpensive. For these reasons, TAS has been incorporated in LTE and IEEE 802.11n [12].

The choice of the antenna depends on the channel state of the STx to secondary receiver (SRx) links and also of the STx-PRx links in order to control the interference caused to the PRx [13]. However, acquiring the instantaneous channel state information (CSI) of the STx-PRx links in a timely manner is challenging. This is because transmissions by the primary system, which the STx uses to acquire the CSI of the STx-PRx links, are not under the control of the secondary system. Consequently, TAS that is done only on the basis of the statistical CSI of the STx-PRx links, which changes at a much slower time scale, is practically appealing. This is even more so when multiple PRxs are present. It frees the STx from having to wait for all the PRxs to transmit before it can estimate all the STx-PRx links and transmit. Prototypes of underlay spectrum sharing with statistical CSI have also been demonstrated [14].

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The interference constraints themselves can be instantaneous or stochastic in nature, and affect the choice of the transmit antenna. Instantaneous constraints, such as the peak interference constraint, limit the instantaneous interference at the PRx [15]–[17]. However, to comply with them, the STx needs perfect and instantaneous STx-PRx CSI. Stochastic constraints, on the other hand, limit a statistical measure of the interference at the PRx. For example, the average interference constraint limits the fading-averaged interference power [18], while the interference-outage constraint limits the probability that the interference power exceeds a threshold [8]. These constraints can be complied with even if the STx has imperfect or statistical STx-PRx CSI.

Several TAS rules have been studied to address the many possible combinations of the CSI and interference constraints discussed above. We categorize and summarize them below.

1) *With Instantaneous STx-PRx CSI at the STx [8], [13], [15], [18]–[23]:* For *on-off* power adaptation, in which the STx transmits with either a fixed power or zero power, [15] develops a TAS rule for the peak interference constraint. Instead, [8], [19] develop optimal TAS rules that minimize the symbol error probability (SEP) of a secondary system that is subject to the average interference and interference-outage constraints, respectively. For an STx that transmits with a *fixed power*, [18] studies TAS for the average interference constraint. For *continuous power adaptation* (CPA), in which the STx can transmit with any power, [21]–[23] study different joint TAS-CPA rules for the peak interference constraint. Instead, [20] and [13] develop SEP-optimal joint TAS-CPA rules for the average interference and interference-outage constraints, respectively. While the above references focus on a single PRx scenario, [16], [17] study TAS in the presence of multiple PRxs with the peak interference constraint for each of them.
2) *With Statistical STx-PRx CSI at the STx [23], [24]:* For an STx that transmits with a fixed power, [23] studies TAS for the average interference constraint when the SRx employs maximal ratio combining (MRC). Selection combining (SC) is instead considered in [24].

A. Focus and Contributions

We see that TAS for the practically appealing scenario in which the STx has only statistical CSI of the STx-PRx link(s) has been less explored in the literature. We present a novel and optimal design of the TAS-CPA rule for a general model in which there are one or more PRxs and the interference constraint is stochastic in nature. The STx has instantaneous STx-SRx CSI, since the STx and SRx belong to the same secondary system, and only statistical STx-PRx CSI, since the STx cannot control the primaries' transmissions. We make the following contributions:

1) *Optimal TAS-CPA Rule and Its Structure:* We derive a novel, SEP-optimal TAS-CPA rule for an STx that shares the spectrum with one or more PRxs. We first prove that for any stochastic constraint that satisfies a mild technical condition, the optimal TAS-CPA rule is unique and the optimal antenna is the one with the highest instantaneous STx-SRx channel power gain. It is the optimal transmit power that depends on the interference constraint.[1]
2) *Generality:* One strength of the above result is its generality. It applies to a general class of stochastic interference constraints. This class includes the average interference constraint and its generalization, and the interference-outage constraint. The optimal rule applies to many fading models such as Rayleigh, Nakagami-m, and Weibull [25].
3) *Characterization of Optimal Transmit Power:* We derive novel closed-form expressions for the optimal transmit power for the generalized average interference constraint. For the interference-outage constraint, such a characterization turns out to be intractable. We present an alternate design in which the penalty function is replaced with a logarithmic function of the transmit power that mimics its behavior in several respects. It leads to an insightful closed-form characterization of the transmit power.
4) *Performance Analysis:* We derive novel, general, and insightful expressions for the average SEP for any stochastic interference constraint. These apply to any fading distribution and to both MRC and SC. We also present an analytical approach to determine the parameters of the optimal rule with less computational effort.
5) *Multiple and Non-Identical PRxs:* Our solution applies to the multiple PRx model, which has received less attention in the literature compared to the single PRx model. It covers the general, practically important scenario in which the STx-PRx links are statistically non-identical.
6) *Impact on Primary and Secondary Systems and Efficacy:* We present a comprehensive evaluation of the impact of the different interference constraints on both secondary and primary systems. We show that the impacts on the two systems are markedly different. We observe that the optimal rule lowers the average SEP by up to two orders of magnitude compared to conventional approaches.

B. Comparison With Literature

Our approach differs from the literature in its model, design, and analysis. First, our statistical CSI model is practically appealing and more realistic than the instantaneous STx-PRx CSI model [13], [15]–[18], [20], [22]. Second, the continuous power adaptation that we consider makes better use of the CSI compared to fixed power transmission [18], [23], [24] and on-off power adaptation [8], [15], [19], [26]. Third, our approach applies to a general class of stochastic interference constraints, whereas [8], [13], [15], [18]–[20], [22], [23] apply to only one specific interference constraint. Fourth, we consider the more general case of multiple PRxs with statistically non-identical STx-PRx links. It generalizes the single PRx model prevalent in the literature [8], [13], [15], [18]–[20], [22], [23]. While [16], [17] consider multiple PRxs, they assume that the STx-PRx channel gains are i.i.d. and

[1]We note that such a decoupling need not always occur. For example, the transmit power and antenna are coupled in the optimal rule in [13], in which the STx has instantaneous CSI of the STx-PRx links.

focus on the peak interference power constraint. Fifth, due to the above fundamental differences in the models, our derivation of the optimal TAS-CPA rule is more involved and different from those in [8], [13], [15], [19], [20], [22]. Finally, we study the impact of different interference constraints and their parameters on both secondary and primary systems. This is more comprehensive compared to [8], [13], [15], [18]–[20], [22], [26], which focus only on the secondary system and a specific constraint.

C. Outline and Notation

Section II presents the system model and the problem statement. The optimal TAS-CPA rule is derived in Section III. Section IV analyzes the proposed rule. Numerical results are presented in Section V. Our conclusions follow in Section VI.

Notation: Scalar variables are written in normal font, vector variables in bold font, and sets in calligraphic font. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. $\mathbb{E}_X[\cdot]$ denotes expectation with respect to a random variable (RV) X. The indicator function $I_{\{a\}}$ is 1 if an event a is true and is 0 otherwise.

II. System Model and Problem Statement

The STx is equipped with N_t antennas. It transmits to an SRx with N_r antennas. It interferes with M PRxs, each of which has one antenna. The STx is equipped with one RF chain, which is dynamically switched to one of the N_t antennas. The SRx can employ either MRC, which requires N_r RF chains at the SRx, or SC, which is analogous to TAS and requires only one RF chain at the SRx [22], [26]. The system model is shown in Figure 1. Let h_{nk} denote the instantaneous channel power gain from the k^{th} antenna of the STx to the n^{th} antenna of the SRx, and g_{ik} denote the instantaneous channel power gain from the k^{th} antenna of the STx to the i^{th} PRx. For the i^{th} PRx, the channel power gains $g_{i1}, \ldots, g_{iN_t}$ from the STx to it are independent and identically distributed (i.i.d.). For antenna k of the STx, the channel power gains $g_{1k}, \ldots, g_{Mk}$ from it to the M PRxs are independent. However, they need not be identically distributed. Let $\mu_i = \mathbb{E}[g_{ik}]$, for $1 \leq i \leq M$. This models the practical scenario in which the PRxs are at different distances from the STx and have different path-losses.

A. Data Transmission and CSI Model

The STx employs continuous power adaptation and varies its transmit power as a function of the antenna selected. Let $s \in \{1, 2, \ldots, N_t\}$ denote the index of the antenna selected and P_s its transmit power when the STx transmits a data symbol d. The SRx receives a signal R_n at the n^{th} receive antenna. Let I_i denote the interference at the i^{th} PRx due to the STx transmissions. Then, R_n and I_i are given by

$$R_n = \sqrt{P_s}\sqrt{h_{ns}}e^{j\theta_{ns}}d + w_n + z_n, \tag{1}$$

$$I_i = \sqrt{P_s}\sqrt{g_{is}}e^{j\varphi_{is}}d, \tag{2}$$

where $\mathbb{E}\left[|d|^2\right] = 1$, θ_{ns} and φ_{is} are the phases of the complex baseband channel gains of the STx-SRx and STx-PRx links,

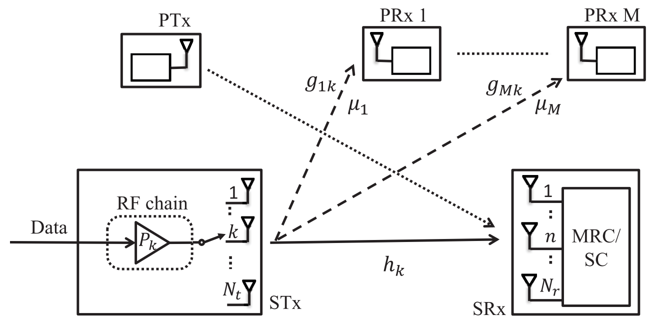


Fig. 1. System model that consists of an STx with N_t transmit antennas and one RF chain. It transmits data to an SRx with N_r antennas, which causes interference to M single antenna PRxs.

respectively, w_n is the additive white Gaussian noise, and z_n is the interference at the SRx from primary transmitters (PTxs). z_n is a circular symmetric complex Gaussian RV [8], [18], [27].[2] Let $\sigma^2 = \mathbb{E}\left[|w_n|^2\right] + \mathbb{E}\left[|z_n|^2\right]$ denote the sum of the thermal noise power and interference power from primary transmissions.

CSI Model: We discuss the STx-SRx and STx-PRx links separately since the manner in which the CSI about them is acquired is inherently different.

- *STx-SRx Links:* The STx knows the instantaneous STx-SRx channel power gains. This is a classical assumption in the TAS literature [8], [13], [15], [18]–[20], [22], [23], [26]. It is justifiable because the STx and the SRx belong to the same system. The STx can know the channel power gains of the N_t links to the SRx even though it has one RF chain as follows. In a time division duplexing (TDD) system, the SRx can transmit a reference signal and the different antennas of the STx receive it one by one. The STx then estimates the links using reciprocity [25]. In a frequency division duplexing (FDD) system, the STx can send reference signals from its antennas one by one. The SRx then estimates the links and feeds back their quantized versions to the STx [12]. The STx does not need to know the phase of any STx-SRx channel.
  To perform coherent demodulation, the SRx only needs to know the complex channel gains from the selected transmit antenna s to its N_r receive antennas. It can obtain them by using pilot symbols that are inserted along with data [8], [19].[3]
- *STx-PRx Links:* The STx only knows the *statistics* of the channel power gains from it to the different PRxs as it does not have any control over the primary transmissions that are needed to estimate these links. This model is practically appealing because the channel statistics changes at a much slower timescale than the instantaneous channel gains. This is even more so when the

[2]This assumption is widely-used in the literature due to its tractability. We refer the reader to [8] for a detailed discussion of the conditions under which it is applicable.

[3]This model is different from the statistical CSI model considered in [28], in which the transmitter has only statistical CSI of the links from itself to its receiver.

number of PRxs increases. Thus, channel statistics is easier to obtain than the instantaneous channel gains.

B. Stochastic Interference Constraint and Problem Statement

Our objective is to minimize the average SEP of the secondary system, which is an important measure of the reliability of a communication system [13], [22], [23]. Let $\mathsf{S}(P_k, h_k)$ denote the instantaneous SEP when the STx transmits with power P_k using antenna k. It is given by [25, (9.7)]

$$\mathsf{S}(P_k, h_k) = c_1 \exp\left(-c_2 \frac{P_k h_k}{\sigma^2}\right), \text{ for } 1 \leq k \leq N_t. \quad (3)$$

Here, $h_k = \sum_{n=1}^{N_r} h_{nk}$ when the SRx employs MRC and $h_k = \max\{h_{1k}, \ldots, h_{N_r k}\}$ when the SRx employs SC. The constants c_1 and c_2 depend on the constellation. Let $g_k \triangleq \sum_{i=1}^{M} g_{ik}$ denote the sum of the channel power gains from the k^{th} antenna of the STx to all the PRxs, $\mathbf{h} \triangleq [h_1, \ldots, h_{N_t}]$, and $\mathbf{g} \triangleq [g_1, \ldots, g_{N_t}]$.

Selection Rule: A TAS-CPA rule ϕ is a mapping from $(\mathbb{R}^+)^{N_t}$ to $\{1, 2, \ldots, N_t\} \times [0, P_{\max}]$ because it maps every realization $\mathbf{h}$ to an antenna $s \in \{1, 2, \ldots, N_t\}$ and a transmit power $P_s \in [0, P_{\max}]$. Thus, $(s, P_s) = \phi(\mathbf{h})$.[4] Here, $P_{\max}$ is the peak transmit power of the STx. The *peak transmit power* constraint $P_s \leq P_{\max}$ is motivated by practical limitations of the various components in the RF chain of the transmitter [13], [29]. Note that s and P_s cannot depend on $\mathbf{g}$ since the STx does not know it.

Stochastic Interference Constraint: It takes the following general form:

$$\mathbb{E}_{\mathbf{h},\mathbf{g}}\left[c\left(P_s, g_s\right)\right] \leq G_t, \quad (4)$$

where $c\left(P_s, g_s\right)$ is an instantaneous interference penalty function and G_t is a threshold. $c\left(P_s, g_s\right)$ is a monotonically increasing function of P_s. Since s and P_s are functions of only $\mathbf{h}$, (4) can be recast as

$$\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}\left(P_s\right)\right] \leq G_t, \quad (5)$$

where $\bar{\mathsf{C}}\left(p\right) = \mathbb{E}_{\mathbf{g}}\left[c\left(p, g_s\right)\right]$ is the average penalty function. $c\left(p, g_s\right)$ and $\bar{\mathsf{C}}\left(p\right)$ take the following specific forms:

1) *Generalized Average Interference Constraint:* From (2), the total instantaneous interference power at all the PRxs is $P_s \sum_{i=1}^{M} g_{is} = P_s g_s$. This constraint limits the fading-averaged value of $(P_s g_s)^m$ to be below a threshold τ. It can be written as

$$\mathbb{E}_{\mathbf{h},\mathbf{g}}\left[(P_s g_s)^m\right] \leq \tau. \quad (6)$$

Here, $c\left(P_s, g_s\right) = (P_s g_s)^m$, $G_t = \tau$, and $m \geq 1$ denotes the *interference-penalty exponent*. Intuitively, a larger m penalizes more a larger interference from the STx. Thus,

$$\bar{\mathsf{C}}\left(P_s\right) = \mathbb{E}_{\mathbf{g}}\left[(P_s g_s)^m\right] = P_s^m \mathbb{E}_{\mathbf{g}}\left[(g_1)^m\right], \quad (7)$$

where the second equality follows because s and P_s are independent of $\mathbf{g}$ and the RVs $g_1, \ldots, g_{N_t}$ are identically distributed. For example, for Rayleigh fading and an integer-valued m, we have $\bar{\mathsf{C}}\left(P_s\right) = M(M+1)\ldots(M+m-1)\left(P_s \mu_g\right)^m$ when $\mu_1 = \cdots = \mu_M = \mu_g$.

The average interference constraint is a special case of this constraint with $m = 1$ and $\bar{\mathsf{C}}\left(P_s\right) = P_s \mu$, where $\mu = \sum_{i=1}^{M} \mu_i$. It requires that $\sum_{i=1}^{M} \mathbb{E}\left[P_s g_{is}\right] \leq \tau$. This automatically implies a constraint on the average interference power at each PRx: $\mathbb{E}\left[P_s g_{is}\right] \leq \tau$, for $1 \leq i \leq M$. This holds even when the PRxs are at different locations and the ones closer to the STx experience more interference on average.

2) *Interference-Outage Constraint [8], [27]:* An interference-outage is an event in which the total instantaneous interference power $P_s g_s$ at all the PRxs exceeds a threshold τ. This constraint limits its probability to be below $O_{\max}$, i.e., $\Pr\left(P_s g_s > \tau\right) \leq O_{\max}$. It can be written as

$$\mathbb{E}_{\mathbf{h},\mathbf{g}}\left[I_{\{P_s g_s > \tau\}}\right] \leq O_{\max}. \quad (8)$$

Here, $c\left(P_s, g_s\right) = I_{\{P_s g_s > \tau\}}$ and $G_t = O_{\max}$. Hence,

$$\bar{\mathsf{C}}\left(P_s\right) = \mathbb{E}_{\mathbf{g}}\left[I_{\{P_s g_s > \tau\}}\right] = \Pr\left(P_s g_s > \tau\right), \quad (9)$$

$$= \Pr\left(g_1 > \frac{\tau}{P_s}\right) = F_g^c\left(\frac{\tau}{P_s}\right), \quad (10)$$

where $F_g^c(\cdot)$ denotes the complementary CDF (CCDF) of the i.i.d. RVs $g_1, \ldots, g_{N_t}$.

Problem Statement: Our goal is to find an optimal TAS-CPA rule $\phi^*(\mathbf{h})$ that minimizes the average SEP of the secondary system from the set $\mathcal{F}$ of all TAS-CPA rules. Our problem can be mathematically stated as the following constrained stochastic optimization problem:

$$\mathcal{P}: \min_{\phi \in \mathcal{F}} \ \mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_s, h_s)\right] \quad (11)$$

$$\text{s.t.} \ \mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}\left(P_s\right)\right] \leq G_t, \quad (12)$$

$$0 \leq P_s \leq P_{\max}, \quad (13)$$

$$(s, P_s) = \phi(\mathbf{h}). \quad (14)$$

The constraint in (12) remains the same for a single PRx with M antennas. Hence, all our results apply to this multi-antenna model as well.

III. Optimal TAS-CPA Rule

We now develop an optimal TAS-CPA rule that solves $\mathcal{P}$ for a general class of stochastic interference constraints. First, consider an interference unconstrained system in which (12) is inactive. Since $\mathsf{S}(P_s, h_s)$ is a monotonically decreasing function of $P_s h_s$, the following TAS-CPA rule, which we shall refer to as the *unconstrained rule*, minimizes it:

$$s = \underset{k \in \{1,2,\ldots,N_t\}}{\arg\max} \{h_k\} \text{ and } P_s = P_{\max}. \quad (15)$$

Hence, it also minimizes the average SEP. The average interference penalty of this rule is equal to $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}\left(P_{\max}\right)\right] = \bar{\mathsf{C}}\left(P_{\max}\right)$. Thus, this rule is optimal when $\bar{\mathsf{C}}\left(P_{\max}\right) \leq G_t$. We shall refer to this regime as the *unconstrained regime*.

However, when $\bar{\mathsf{C}}\left(P_{\max}\right) > G_t$, which we shall refer to as the *constrained regime*, the unconstrained rule does not satisfy the interference constraint. The following two lemmas solve $\mathcal{P}$ in this regime. Lemma 1 provides an explicit characterization of the optimal transmit antenna and its power in terms of a constant λ. Lemma 2 proves that such a λ always exists and is

[4] To keep the notation simple, we do not explicitly show the dependence of s and P_s on $\mathbf{h}$.

unique. The lemmas prove this for the following mild technical condition on $\bar{\mathsf{C}}'(p)$, which is the first derivative of $\bar{\mathsf{C}}(p)$:

A1: $\bar{\mathsf{C}}'(p)$ is a continuous and monotonically non-decreasing function of p, or it is a continuous and uni-modal function of p and $\bar{\mathsf{C}}'(0) = 0$.

It can be verified that A1 is satisfied by the generalized average interference constraint for any $m \geq 1$, and by the interference-outage constraint for several fading models such as Rayleigh, Nakagami-m, and Weibull.

Lemma 1: The optimal antenna s^* and its optimal transmit power $P_{s^*}(\lambda)$ in the constrained regime are given by

$$s^* = \underset{k\in\{1,2,\ldots,N_t\}}{\arg\max}\ \{h_k\}, \tag{16}$$

$$P_{s^*}(\lambda) = \underset{p\in[0,P_{\max}]}{\arg\min}\ \{\mathsf{SM}_{s^*}(\lambda,p)\}, \tag{17}$$

where $\mathsf{SM}_k(\lambda,p)$ is called the selection metric of antenna k and is defined as

$$\mathsf{SM}_k(\lambda,p) \triangleq \mathsf{S}(p,h_k) + \lambda\bar{\mathsf{C}}(p), \quad \text{for } p\in[0,P_{\max}]. \tag{18}$$

Here, λ is set to $\lambda^* > 0$ such that the interference constraint in (12) is met with equality, i.e., $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\lambda^*))\right] = G_t$.

Proof: The proof is given in Appendix A. ■

Notice that s^* is not a function of λ, and only $P_{s^*}(\lambda)$ depends on λ. Both s^* and $P_{s^*}(\lambda)$ depend on $\mathbf{h}$. We provide a closed-form characterization of the optimal power in Section III-A.

Let the penalty function $\bar{\mathsf{C}}(P_{s^*}(\lambda))$ averaged over the STx-SRx channel power gains be denoted by

$$G(\lambda) \triangleq \mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\lambda))\right]. \tag{19}$$

Lemma 2: $G(\lambda)$ is a continuous and monotonically decreasing function of λ that decreases from $\bar{\mathsf{C}}(P_{\max}) > G_t$ to 0 as λ increases from 0 to ∞. In the interference constrained regime, there exists a unique $\lambda = \lambda^*$ such that the interference constraint is met with equality for any stochastic interference constraint that satisfies the condition A1.

Proof: We first prove the monotonicity of $G(\lambda)$. Let $\lambda_2 > \lambda_1 > 0$. For an antenna s^*, from the definition of optimality, we know that $\mathsf{SM}_{s^*}(\lambda_1, P_{s^*}(\lambda_1)) < \mathsf{SM}_{s^*}(\lambda_1, P_{s^*}(\lambda_2))$ and $\mathsf{SM}_{s^*}(\lambda_2, P_{s^*}(\lambda_2)) < \mathsf{SM}_{s^*}(\lambda_2, P_{s^*}(\lambda_1))$. Summing these two inequalities and simplifying, we get $\bar{\mathsf{C}}(P_{s^*}(\lambda_2)) < \bar{\mathsf{C}}(P_{s^*}(\lambda_1))$. Taking expectation with respect to $\mathbf{h}$, we get $G(\lambda_2) < G(\lambda_1)$.

For $\lambda = 0$, $P_{s^*}(\lambda) = P_{\max}$ since the unconstrained rule is optimal. Hence, $G(0) = \bar{\mathsf{C}}(P_{\max}) > G_t$. As $\lambda \to \infty$, it can be shown that $P_{s^*}(\lambda) \to 0$, which implies that $G(\infty) = \mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\infty))\right] = 0$. Thus, $G(\lambda)$ decreases from $\bar{\mathsf{C}}(P_{\max})$ to 0 as λ increases from 0 to ∞.

We relegate the proof of continuity of $G(\lambda)$, which is involved, to Appendix B. It then follows from the intermediate value theorem that a unique $\lambda = \lambda^* > 0$ exists at which $G(\lambda^*) = G_t$. ■

Comments: Lemmas 1 and 2 together prove that the optimal TAS-CPA rule is unique and has a decoupled structure. To the best of our knowledge, this is the first time this result has been rigorously proved for a general class of stochastic interference constraints and channel fading models. Since the optimal rule is specified in terms of the instantaneous STx-SRx channel gains, it holds even when they are statistically correlated. The optimal rule differs from the TAS rules in [8], [13], [18], [22], in which the optimal antenna depends on the interference constraint and its parameters.

A. Optimal Transmit Power and Its Behavior

We now derive the optimal transmit power $P_{s^*}(\lambda)$ for different stochastic interference constraints. To keep the notation simple, we do not show its dependence on h_{s^*}.

1) Generalized Average Interference Constraint: We first consider the $m > 1$ case. Here, $\bar{\mathsf{C}}(p) = p^m\psi_m$, where $\psi_m = \mathbb{E}_{\mathbf{g}}[(g_1)^m]$ denotes the m^{th} moment of g_1. Substituting this along with (3) in (18) yields $\mathsf{SM}_{s^*}(\lambda,p) = c_1\exp\left(-c_2 p h_{s^*}/\sigma^2\right) + \lambda p^m\psi_m \geq 0$. Its minimum occurs at

$$p = \frac{(m-1)\sigma^2}{c_2 h_{s^*}} W_0\left(\left[\frac{c_1 c_2 h_{s^*}}{\lambda m \psi_m \sigma^2}\right]^{\frac{1}{m-1}} \frac{c_2 h_{s^*}}{(m-1)\,\sigma^2}\right), \tag{20}$$

where $W_l(\cdot)$ denotes the l^{th} branch of the Lambert-W function [30]. Therefore, the optimal transmit power is $P_{s^*}(\lambda) = \min\{p, P_{\max}\}$.

Next, we consider the boundary case of $m = 1$. Substituting (3) and $\bar{\mathsf{C}}(p) = p\mu$ in (18), we get $\mathsf{SM}_{s^*}(\lambda,p) = c_1\exp\left(-c_2 p h_{s^*}/\sigma^2\right) + \lambda p\mu$. As shown in Appendix C, the optimal power is as follows. When $P_{\max} > c_1/(e\lambda\mu)$,

$$P_{s^*}(\lambda) = \begin{cases} 0, & \text{if } h_{s^*} \leq A_\lambda, \\ \frac{\sigma^2}{c_2 h_{s^*}} \ln\left(\frac{h_{s^*}}{A_\lambda}\right), & \text{else}, \end{cases} \tag{21}$$

where $A_\lambda = \left(\lambda\sigma^2/(c_1 c_2)\right)\mu$. Otherwise,

$$P_{s^*}(\lambda) = \begin{cases} 0, & \text{if } h_{s^*} \leq A_\lambda, \\ P_{\max}, & \text{if } h_{\min} \leq h_{s^*} \leq h_{\max}, \\ \frac{\sigma^2}{c_2 h_{s^*}} \ln\left(\frac{h_{s^*}}{A_\lambda}\right), & \text{else}, \end{cases} \tag{22}$$

where

$$h_{\min} = -\frac{\sigma^2}{c_2 P_{\max}} W_0\left(-\frac{c_2 P_{\max} A_\lambda}{\sigma^2}\right), \tag{23}$$

$$h_{\max} = -\frac{\sigma^2}{c_2 P_{\max}} W_{-1}\left(-\frac{c_2 P_{\max} A_\lambda}{\sigma^2}\right). \tag{24}$$

Figure 2a illustrates the optimal transmit power $P_{s^*}(\lambda)$ as a function of h_{s^*} for the above two cases. Note that when $P_{\max} > c_1/(e\lambda\mu)$, the STx never transmits with power $P_{\max}$ and the peak transmit power constraint in (13) is inactive. Since the optimal power is computed explicitly, the optimal TAS-CPA rule involves comparing only N_t quantities to determine the best antenna.

2) Interference-Outage Constraint: Substituting $\bar{\mathsf{C}}(p) = F_g^c(\tau/p)$ in (17) yields

$$P_{s^*}(\lambda) = \underset{p\in[0,P_{\max}]}{\arg\min}\left\{c_1\exp\left(-\frac{c_2 p h_{s^*}}{\sigma^2}\right) + \lambda F_g^c\left(\frac{\tau}{p}\right)\right\}. \tag{25}$$

No closed-form solution is known for (25). The optimal power is found numerically using a one-dimensional search. However, this needs to be done for each h_{s^*}. We shall refer to it as *numerically-determined power*. It depends on the CCDF of the STx-PRx channel power gains, unlike the average interference constraint in which it depends only on their moments.

Log-Penalty Based Design: To avoid the computationally expensive numerical search, we now present a novel alternate design for the average penalty function $\bar{\mathsf{C}}(p)$. We replace $F_g^c(\tau/p)$ with $F_g^c(\tau/P_{\max})\ln(p)/\ln(P_{\max})$, which we refer to as the *log-penalty* function. It mimics the monotonic behavior of $F_g^c(\tau/p)$ in two key respects. First, both $F_g^c(\tau/p)$ and log-penalty function increase rapidly for small values of p, and increase slowly for large values of p. Second, both functions achieve the same maximum value $F_g^c(\tau/P_{\max})$ at $p = P_{\max}$. Substituting it in (18) yields

$$\mathsf{SM}_{s^*}(\lambda, p) = c_1 \exp\left(-\frac{c_2 h_{s^*} p}{\sigma^2}\right) + \lambda \frac{F_g^c(\tau/P_{\max})}{\ln(P_{\max})} \ln(p). \tag{26}$$

It can be shown that $\mathsf{SM}_{s^*}(\lambda, p)$ attains a finite minimum at $p = L_\lambda \sigma^2 / (c_2 h_{s^*}) > 0$, where

$$L_\lambda \triangleq -\mathrm{W}_{-1}\left(-\frac{\lambda F_g^c(\tau/P_{\max})}{c_1 \ln(P_{\max})}\right). \tag{27}$$

Therefore, the transmit power $P_{s^*}(\lambda)$ that minimizes $\mathsf{SM}_{s^*}(\lambda, p)$ is given in closed-form as

$$P_{s^*}(\lambda) = \begin{cases} P_{\max}, & \text{if } h_{s^*} \leq \dfrac{L_\lambda \sigma^2}{c_2 P_{\max}}, \\ \dfrac{L_\lambda \sigma^2}{c_2 h_{s^*}}, & \text{else.} \end{cases} \tag{28}$$

We observe that the STx transmit power is $P_{\max}$ for small values of h_{s^*}, and it is inversely proportional to h_{s^*} for large values of h_{s^*}. We shall refer to it as the *log-penalty based power*.

The appeal of this design is four-fold. First, from a practical point of view, it leads to a closed-form expression for the transmit power, which reduces the computational complexity. Second, it provides new analytical insights about the transmit power. Third, as illustrated in Figure 2b, the log-penalty based power in (28) tracks the optimal transmit power, which is obtained by solving (25) numerically, well and matches with it for larger values of h_{s^*}. Fourth, as we shall see in Section V, it incurs only a minor degradation in performance.

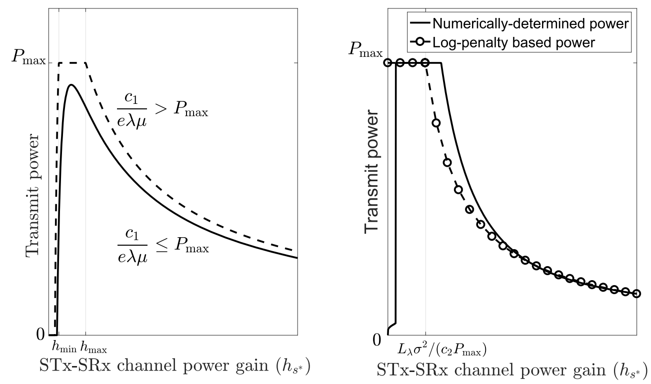
IV. Performance Analysis

We first derive a general expression for the average SEP, which is denoted by $\overline{\mathsf{SEP}}$, for the optimal rule for any penalty function. We then specialize it for specific interference constraints. For tractability, we assume that $h_1, \ldots, h_{N_t}$ are i.i.d. Let $F_h(\cdot)$ and $f_h(\cdot)$ denote their CDF and probability density function, respectively. Let $\mu_h = \mathbb{E}[h_{nk}]$ and $\Omega_s = P_{\max}\mu_h/\sigma^2$ denote the average channel power gain of the STx-SRx links and peak fading-averaged signal-to-interference-plus-noise ratio (SINR), respectively.

A. Average SEP

Result 1: For any stochastic interference constraint, the average SEP of the secondary system when the optimal rule is employed is given in general by

$$\overline{\mathsf{SEP}} = N_t \int_0^\infty [F_h(h_1)]^{N_t-1} \mathsf{S}(P_1(\lambda), h_1) f_h(h_1)\, dh_1, \tag{29}$$



(a) Average interference constraint (b) Interference-outage constraint

Fig. 2. Transmit power as a function of the STx-SRx channel power gain h_{s^*} for the average interference and interference-outage constraints ($m = 1$, $M = 1$, $\mu_1 = 1$, $\sigma^2 = 1$, $P_{\max}/\sigma^2 = 10$ dB, $\tau/\sigma^2 = 3$ dB, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).

where $P_1(\lambda)$ is the transmit power when the STx selects antenna 1.

Proof: The proof is given in Appendix D. ∎

For example, for Rayleigh fading and MRC, we have

$$F_h(x) = 1 - e^{-\frac{x}{\mu_h}} \sum_{n=0}^{N_r-1} \frac{1}{n!}\left(\frac{x}{\mu_h}\right)^n, \quad \text{for } x \in [0, \infty). \tag{30}$$

Instead, for Rayleigh fading and SC, we have $F_h(x) = (1 - \exp(-x/\mu_h))^{N_r}$, for $x \in [0, \infty)$.

We now specialize (29) to the different interference constraints and present cases with closed-form expressions that provide additional insights.

1) Generalized Average Interference Constraint: To gain further insights, we focus on $m = 1$. The solution depends on whether $P_{\max} > c_1/(e\lambda\mu)$ or $P_{\max} \leq c_1/(e\lambda\mu)$.

a) When $P_{\max} > c_1/(e\lambda\mu)$: As per (21), for antenna 1, the optimal transmit power is $P_1(\lambda) = 0$ and the SEP equals c_1 when $h_1 \leq A_\lambda$. Else, $P_1(\lambda) = \sigma^2 \ln(h_1/A_\lambda)/(c_2 h_1)$ and $\mathsf{S}(P_1(\lambda), h_1) = A_\lambda/h_1$. Substituting these in (29) and simplifying yields

$$\overline{\mathsf{SEP}} = T_1 + T_2, \tag{31}$$

where

$$T_1 = c_1 [F_h(A_\lambda)]^{N_t}, \tag{32}$$

$$T_2 = N_t c_1 A_\lambda \int_{A_\lambda}^\infty \frac{1}{h_1} [F_h(h_1)]^{N_t-1} f_h(h_1)\, dh_1. \tag{33}$$

Here, T_1 and T_2 correspond to the average SEP when the STx transmits with zero power and non-zero power, respectively. T_1 decreases as N_t or N_r increases. On the other hand, T_1 and T_2 both increase as A_λ increases. Also, they do not depend on $P_{\max}$. For example, for Rayleigh fading and SC, we get $T_1 = c_1(1 - \exp(-A_\lambda/\mu_h))^{N_t N_r}$ and

$$T_2 = \frac{N_t N_r c_1 A_\lambda}{\mu_h} \sum_{k=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{k} (-1)^k \times \mathrm{Z}(k+1, A_\lambda), \tag{34}$$

where $\mathrm{Z}(x, y) \triangleq \mathbf{E}_1(xy/\mu_h)$ and $\mathbf{E}_1(\cdot)$ denotes the exponential integral [31, pp. xxxv].

b) $P_{\max} \leq c_1/(e\lambda\mu)$: Substituting the optimal transmit power from (22) in (29), we get $\overline{\mathsf{SEP}} = T_1 + \widehat{T}_2$, where T_1 is given in (32) and

$$\widehat{T}_2 = N_t c_1 A_\lambda \int_{A_\lambda}^{h_{\min}} \frac{1}{h_1} \left[F_h(h_1)\right]^{N_t-1} f_h(h_1)\, dh_1 + N_t c_1 \int_{h_{\min}}^{h_{\max}} \left[F_h(h_1)\right]^{N_t-1} e^{-\frac{c_2 P_{\max} h_1}{\sigma^2}} f_h(h_1)\, dh_1 + N_t c_1 A_\lambda \int_{h_{\max}}^{\infty} \frac{1}{h_1} \left[F_h(h_1)\right]^{N_t-1} f_h(h_1)\, dh_1. \quad (35)$$

$\widehat{T}_2$ decreases as $P_{\max}$ increases. For Rayleigh fading and SC, it can be shown to simplify to

$$\widehat{T}_2 = N_t N_r c_1 \sum_{k=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{k} (-1)^k \times \left[\frac{e^{-(k+1+c_2\Omega_s)\frac{h_{\min}}{\mu_h}} - e^{-(k+1+c_2\Omega_s)\frac{h_{\max}}{\mu_h}}}{k+1+c_2\Omega_s} + \frac{A_\lambda}{\mu_h} \mathrm{Z}(k+1, A_\lambda) - \frac{A_\lambda}{\mu_h} \mathrm{Z}(k+1, h_{\min}) + \frac{A_\lambda}{\mu_h} \mathrm{Z}(k+1, h_{\max}) \right]. \quad (36)$$

2) Interference-Outage Constraint: For the log-penalty based power given in (28), we see that if $h_1 \leq L_\lambda \sigma^2/(c_2 P_{\max})$, the STx transmits with power $P_{\max}$ and the SEP is equal to $c_1 \exp\left(-c_2 P_{\max} h_1/\sigma^2\right)$. Else, it transmits with power $L_\lambda \sigma^2/(c_2 h_1)$ and the SEP is equal to $c_1 \exp(-L_\lambda)$. Substituting these in (29), we get $\overline{\mathsf{SEP}} = T_1' + T_2'$, where

$$T_1' = N_t c_1 \int_0^{\frac{L_\lambda \sigma^2}{c_2 P_{\max}}} \left[F_h(h_1)\right]^{N_t-1} e^{-\frac{c_2 P_{\max} h_1}{\sigma^2}} f_h(h_1)\, dh_1, \quad (37)$$

$$T_2' = c_1 e^{-L_\lambda} \left(1 - \left[F_h\left(L_\lambda \sigma^2/(c_2 P_{\max})\right)\right]^{N_t}\right). \quad (38)$$

For example, for Rayleigh fading and SC, these terms can be shown to simplify to

$$T_1' = N_t N_r c_1 \sum_{k=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{k} \frac{1 - e^{-L_\lambda - \frac{(k+1)L_\lambda}{c_2 \Omega_s}}}{(-1)^k (k+1+c_2\Omega_s)}, \quad (39)$$

$$T_2' = c_1 e^{-L_\lambda} \left(1 - \left[1 - e^{-\frac{L_\lambda}{c_2 \Omega_s}}\right]^{N_t N_r}\right). \quad (40)$$

Given τ, T_1' decreases as $P_{\max}$ increases. On the other hand, T_2' increases as $P_{\max}$ increases and eventually saturates. Given $P_{\max}$, T_1' increases and then saturates as τ increases. On the other hand, T_2' decreases as τ increases because the STx transmits with the peak power $P_{\max}$ more often.

B. Determining λ

We now derive analytical expressions for the average interference power and the interference-outage probability as a function of λ. In addition to giving insights about the influence of λ, these expressions also can be used to quickly compute the value of λ at which the interference constraint is met with equality. This makes it easier to implement the optimal TAS-CPA rule.

1) Average Interference Power: It is given as follows.

Result 2: The average interference power $\bar{I}_\lambda$ is given as follows:

a) When $P_{\max} > c_1/(e\lambda\mu)$:

$$\bar{I}_\lambda = \frac{N_t \sigma^2 \mu}{c_2} \int_{A_\lambda}^{\infty} \left[F_h(h_1)\right]^{N_t-1} \frac{1}{h_1} \ln\left(\frac{h_1}{A_\lambda}\right) f_h(h_1)\, dh_1. \quad (41)$$

b) When $P_{\max} \leq c_1/(e\lambda\mu)$:

$$\bar{I}_\lambda = P_{\max}\left(\left[F_h(h_{\max})\right]^{N_t} - \left[F_h(h_{\min})\right]^{N_t}\right)\mu + \frac{N_t \sigma^2 \mu}{c_2} \times \left(\int_{A_\lambda}^{h_{\min}} \left[F_h(h_1)\right]^{N_t-1} \frac{1}{h_1} \ln\left(\frac{h_1}{A_\lambda}\right) f_h(h_1)\, dh_1 + \int_{h_{\max}}^{\infty} \left[F_h(h_1)\right]^{N_t-1} \frac{1}{h_1} \ln\left(\frac{h_1}{A_\lambda}\right) f_h(h_1)\, dh_1\right). \quad (42)$$

Proof: The proof is given in Appendix E. ■

We see that $\bar{I}_\lambda$ increases as the number of PRxs increases. The first term in (42) corresponds to the average interference when the STx transmits with power $P_{\max}$. The other two terms correspond to the average interference when the STx transmits with power $\sigma^2 \ln(h_{s^*}/A_\lambda)/(c_2 h_{s^*})$.

2) Interference-Outage Probability for Log-Penalty Based Power: The interference-outage probability of the TAS-CPA rule when the STx uses the log-penalty based power is as follows.

Result 3: The interference-outage probability O_λ is given by

$$O_\lambda = F_g^c(\tau/P_{\max}) \left[F_h\left(L_\lambda \sigma^2/(c_2 P_{\max})\right)\right]^{N_t} + N_t \int_{\frac{L_\lambda \sigma^2}{c_2 P_{\max}}}^{\infty} \left[F_h(h_1)\right]^{N_t-1} F_g^c\left(\frac{c_2 \tau h_1}{L_\lambda \sigma^2}\right) f_h(h_1)\, dh_1. \quad (43)$$

Proof: The proof is given in Appendix F. ■

For Rayleigh fading, when $\mu_1 = \cdots = \mu_M = \mu_g$, the CCDF of the STx-PRx channel power gain $g_k = \sum_{i=1}^{M} g_{ik}$ is given by

$$F_g^c(x) = e^{-\frac{x}{\mu_g}} \sum_{n=0}^{M-1} \frac{1}{n!} \left(\frac{x}{\mu_g}\right)^n, \quad \text{for } x \in [0, \infty). \quad (44)$$

With SC and $M = 1$, (43) simplifies to

$$O_\lambda = e^{-\frac{\tau}{\mu_1 P_{\max}}} \left(1 - e^{-\frac{L_\lambda}{c_2 \Omega_s}}\right)^{N_t N_r} + N_t N_r \sum_{k=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{k} \frac{e^{-\frac{(k+1)}{c_2 \Omega_s} L_\lambda - \frac{\tau}{\mu_1 P_{\max}}}}{(-1)^k \left(k + 1 + \frac{c_2 \tau \mu_h}{L_\lambda \sigma^2 \mu_1}\right)}. \quad (45)$$

The first term corresponds to the interference-outage probability when the STx transmits with power $P_{\max}$. It decreases as $P_{\max}$ increases. It increases as τ increases because the STx transmits with power $P_{\max}$ more often. The second term corresponds to the interference-outage probability when the

STx transmits with power $L_\lambda \sigma^2/(c_2 h_{s^*})$. It increases as $P_{\max}$ increases. It decreases as τ increases because the STx transmits with power $L_\lambda \sigma^2/(c_2 h_{s^*})$ less often.

V. Numerical Results and Performance Comparison

We now numerically study the proposed TAS-CPA rules. We first assess the efficacy of the log-penalty based design for the interference-outage constraint. Next, we compare the performance of CPA with other power adaptation techniques. We then evaluate the impact of the different interference constraints and their parameters on the secondary and primary systems.

We show results for Rayleigh fading with $\mu_h = -114$ dB, $\mu_1 = \cdots = \mu_M = -125$ dB, interference plus noise power σ^2 of -109 dBm, and an average channel power gain of the PTx to PRx link of -103 dB. These values lead to a secondary peak fading-averaged SINR $\Omega_s = P_{\max}\mu_h/\sigma^2$ of 10 dB when $P_{\max}$ is 15 dBm and a primary average signal-to-noise ratio (SNR) of 21 dB when the PTx transmits with a fixed power of 15 dBm.[5]

Efficacy of Log-Penalty Based Design: Figure 3 plots the average SEP of the secondary system that is subject to the interference-outage constraint for different values of τ and $O_{\max}$. It shows $\overline{\text{SEP}}$ for the numerically-determined power in (25) and the log-penalty based power in (28). Both analysis and simulation results are shown for the latter, and they match well with each other. For small Ω_s, the average SEPs of the numerically-determined power and the log-penalty based power match exactly for all values of τ and $O_{\max}$. This is because the system is in the unconstrained regime and transmits with power $P_{\max}$. In the constrained regime, which occurs for $\Omega_s > \tau\mu_h/\left(\mu_1\sigma^2 \ln\left(1/O_{\max}\right)\right)$, the average SEPs of both approaches reach an error floor due to the interference constraint. The error floor decreases as τ or $O_{\max}$ increases because the interference constraint is relaxed. The average SEP with the log-penalty based power is only marginally more than that with numerically-determined power despite its considerably lower computational complexity. This demonstrates the efficacy of the log-penalty based power.

Performance Benchmarking: We now compare optimal CPA with the following transmit power adaptation techniques considered in the literature. In all cases, the transmit antenna is selected as per (16).

- *Fixed Power Transmission [23], [24]:* Here, the STx transmits with a fixed power $P_t \leq P_{\max}$, whose value is chosen such that the interference constraint is satisfied with equality. Since the interference-outage probability is equal to $F_g^c\left(\tau/P_t\right)$, we get $P_t = \tau/\left(\left(F_g^c\right)^{-1}\left(O_{\max}\right)\right)$, where $\left(F_g^c\right)^{-1}(\cdot)$ denotes the inverse function of the CCDF $F_g^c(\cdot)$. Accounting for the peak transmit power

[5]This corresponds to a carrier frequency of 2.4 GHz, bandwidth of 1 MHz, and 300 K temperature. For the simplified path-loss model of [25, Ch. 2.6], the parameters correspond to a path-loss exponent of 3.7, a reference distance of 1 m, and distances of 100 m between the STx and SRx, 50 m between the PTx and PRx, and 200 m between the STx and PRx.

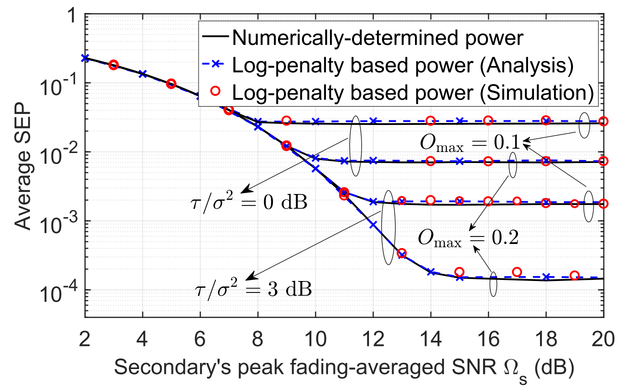


Fig. 3. Comparison of numerically-determined power and log-penalty based power: Average SEP of the secondary system as a function of Ω_s for different $O_{\max}$ and τ ($N_t = 4$, $N_r = 4$, $M = 1$, SC, and 8-PSK with $c_1 = 0.5$ and $c_2 = 0.17$).

constraint, we get

$$P_t = \min\left\{P_{\max}, \frac{\tau}{\left(F_g^c\right)^{-1}\left(O_{\max}\right)}\right\}. \quad (46)$$

For example, for $M = 1$ and Rayleigh fading, $P_t = \min\{P_{\max}, \tau/\left(\mu_1 \ln(1/O_{\max})\right)\}$. Similarly, for the average interference constraint, we get $P_t = \min\{P_{\max}, \tau/\mu_1\}$, which is the same as the transmit power employed in [23], [24].

- *On-Off Power Adaptation [8], [19]:* Here, the STx transmits with either zero power or peak power $P_{\max}$ as follows:

$$P_s = \begin{cases} 0, & \text{if } h_s \leq \beta, \\ P_{\max}, & \text{else}, \end{cases} \quad (47)$$

where the parameter $\beta > 0$ is set such that the interference constraint is met with equality in the constrained regime. For example, for $M = 1$, Rayleigh fading, and SC, $\beta = -\mu_h \ln\left(1 - \left[1 - \left(O_{\max}/\exp\left(-\tau/\left(\mu_1 P_{\max}\right)\right)\right)\right]^{\frac{1}{N_t N_r}}\right)$ for the interference-outage constraint, and $\beta = -\mu_h \ln\left(1 - \left[1 - \left(\tau/\left(\mu_1 P_{\max}\right)\right)\right]^{\frac{1}{N_t N_r}}\right)$ for the average interference constraint. In the unconstrained regime, $\beta = 0$ and the STx transmits with peak power $P_{\max}$.

Figure 4 plots the average SEP of an average interference constrained secondary system as a function of the interference power threshold τ.[6] It compares the above power adaptation schemes for different values of N_t and N_r. i) For $\tau/\sigma^2 \leq P_{\max}(\mu)/\sigma^2 = 1.0$ dB, the system is in the constrained regime for all N_t and N_r. For both CPA and fixed power transmission, $\overline{\text{SEP}}$ decreases as τ increases. It also decreases significantly as N_t and N_r increase. However, for on-off power adaptation, $\overline{\text{SEP}}$ is insensitive to τ, N_t, and N_r. CPA markedly reduces $\overline{\text{SEP}}$. For example, when $N_t = 3$, $N_r = 3$, and $\tau/\sigma^2 = -1$ dB, $\overline{\text{SEP}}$ of CPA is one order of magnitude lower compared to fixed power transmission and 2 to 3 orders of magnitude lower compared to on-off power adaptation. On-off power adaptation does much worse because the STx

[6]To avoid clutter, only simulation results are shown. We shall compare the analysis and simulation results in Figure 6a.

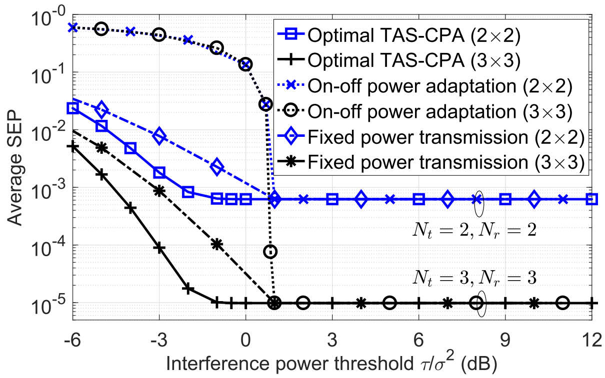
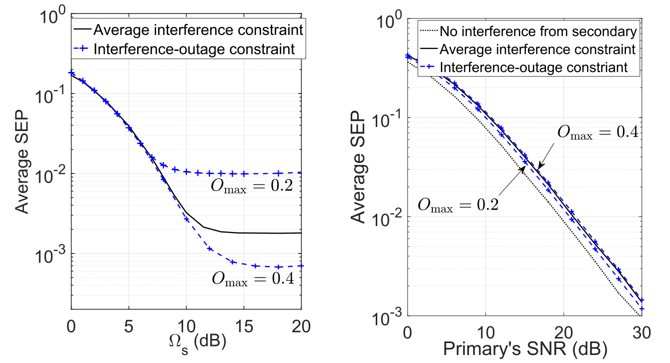
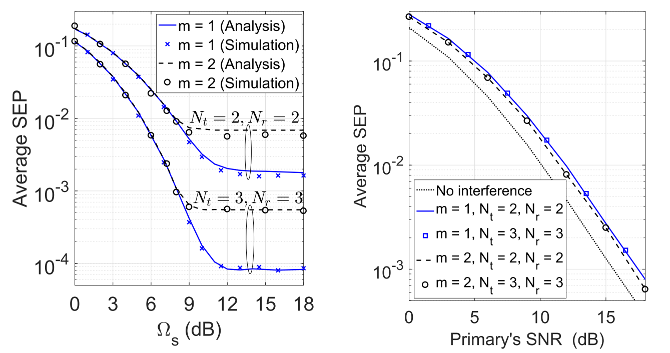


Fig. 4. Impact of power adaptation: Average SEP of the secondary system as a function of τ for different values of N_t and N_r with the average interference constraint ($m = 1$, $\Omega_s = 12$ dB, $M = 1$, SC, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).



(a) Secondary's performance (b) Primary's performance (Ω_s = 20 dB)

Fig. 5. Impact of interference constraint: Average SEP of the secondary system and primary system as a function of Ω_s and primary's SNR, respectively. ($N_t = 2$, $N_r = 2$, $M = 1$, $\tau/\sigma^2 = -3$ dB, SC, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).



(a) Secondary's performance (b) Primary's performance (Ω_s = 18 dB)

Fig. 6. Impact of interference penalty exponent and multiple antennas: Average SEPs of the secondary and primary system as a function of Ω_s and primary's SNR, respectively ($M = 2$, $\tau/\sigma^2 = 1$, SC, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).

transmits with zero power often in order to compensate for the interference it causes when it transmits with power $P_{\max}$. ii) For $\tau/\sigma^2 > 1.0$ dB, the average SEPs of the different power adaptation techniques become the same and do not depend on τ as the system is in the unconstrained regime. Note that $\overline{\mathsf{SEP}}$ of on-off power adaptation decreases rapidly as it enters the unconstrained regime. The trends are similar for the interference-outage constraint and are not shown.

Impact of Interference Constraint on Secondary and Primary Systems: Figure 5a plots the average SEP as a function of the peak fading-averaged SINR Ω_s of the secondary system for the interference-outage and average interference constraints when $\tau/\sigma^2 = -3$ dB. Similarly, Figure 5b plots the corresponding average SEP of the primary system as a function of its SNR when $\Omega_s = 20$ dB. We use the same performance measure for both primary and secondary systems. Since the interference-outage constraint is defined by the probability $O_{\max}$ in addition to τ, we show results for it for different $O_{\max}$. For $\Omega_s \leq 6$ dB, the average SEP of the secondary system is the same for both constraints and for all $O_{\max}$. This is because it is in the unconstrained regime. For larger Ω_s, the secondary system transitions to the constrained regime. For $O_{\max} = 0.2$, the average SEP of the secondary system with the average interference constraint is significantly lower than that with the interference-outage constraint, while the additional degradation in the primary's performance is small. For $O_{\max} = 0.4$, the average SEP of the secondary system with the interference-outage constraint is lower, while the average SEPs of the primary system for the two constraints are very close to each other. Thus, the two interference constraints have different impacts on the two systems.

Impact of Multiple Antennas and the Interference Penalty Exponent m: Figure 6a plots the average SEP of the secondary system, from simulations and analysis, as a function of Ω_s. Figure 6b plots the average SEP of the primary system, from simulations, as a function of its SNR. This is done for different values of m, N_t, and N_r. Given N_t and N_r, the average SEP of the secondary system initially decreases as Ω_s increases and eventually reaches an error floor. The constrained regime occurs for $\Omega_s > \tau\mu_h/(\sigma^2\mu)$ for $m = 1$ and $\Omega_s > \tau\mu_h \Big/ \left(\sigma^2\sqrt{\sum_{i=1}^{M} {\mu_i}^2 + \mu^2}\right)$ for $m = 2$. The error floor for $m = 1$ is significantly lower than that for $m = 2$. The average SEP of the primary system decreases as m increases, albeit marginally. Given m, the average SEP of the primary system is not a function of N_t and N_r, but the average SEP of the secondary system decreases significantly when N_t and N_r increase. Lastly, we see that analysis and simulation results match well in Figure 6a.

VI. Conclusion

We derived an optimal TAS-CPA rule for an underlay secondary system that had only statistical CSI of the STx-PRx links to multiple PRxs. This practical, less-explored model addressed the challenges faced by the secondary system in estimating the links to the primary receivers. Our approach applied to several widely-used fading models and to a general class of stochastic interference constraints. The structure of the optimal rule showed how the CSI available at the STx fundamentally affected TAS. For the average interference constraint, the optimal transmit power increased for smaller values of the secondary channel power gain and then decreased. On the other hand, for the interference-outage constraint, the log-penalty based power was inversely proportional to the secondary channel power gain. The performance gap between the tractable log-penalty based design and the computationally-intensive optimal solution was negligible. We showed that continuous power adaptation improved the secondary performance significantly compared to fixed power transmission and on-off power adaptation. Increasing the number of antennas at the secondary system improved its performance without degrading the primary's performance. The interference-outage constraint impacted the secondary and primary systems differently compared to the average interference constraint.

Appendix

A. Proof of Lemma 1

We shall call a rule that satisfies the interference constraint in (12) and the peak transmit power constraint in (13) as a feasible rule. Consider the joint TAS-CPA rule $\phi^*(\mathbf{h}) = (s^*, P_{s^*}(\lambda))$ that is defined by

$$(s^*, P_{s^*}(\lambda)) \triangleq \underset{k\in\{1,2,\ldots,N_t\},\ p\in[0,P_{\max}]}{\arg\min} \{\mathsf{SM}_k(\lambda, p)\}, \quad (48)$$

where $\mathsf{SM}_k(\lambda,p) \triangleq \mathsf{S}(p, h_k) + \lambda\bar{\mathsf{C}}(p)$. Here, $\lambda = \lambda^* > 0$ is set such that the interference constraint is met with equality, i.e., $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\lambda^*))\right] = G_t$. Clearly, $\phi^*(\mathbf{h})$ is a feasible rule.

Consider any other feasible rule ϕ such that $(s, P_s) = \phi(\mathbf{h})$. From the definition of $\phi^*(\mathbf{h})$ in (48), it is clear that

$$\mathbb{E}_{\mathbf{h}}\left[\mathsf{SM}_{s^*}(\lambda^*, P_{s^*}(\lambda^*))\right] \le \mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_s, h_s) + \lambda^*\bar{\mathsf{C}}(P_s)\right]. \quad (49)$$

Expanding $\mathsf{SM}_{s^*}(\lambda^*, P_{s^*}(\lambda^*))$ and rearranging the terms, we get

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_{s^*}(\lambda^*), h_{s^*})\right] &\le \mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_s, h_s)\right] \\ &\quad + \lambda^*\left(\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_s)\right] - \mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\lambda^*))\right]\right).\end{aligned} \quad (50)$$

For $\phi^*(\mathbf{h})$, we have $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\lambda^*))\right] = G_t$ by its very design. Thus,

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_{s^*}(\lambda^*), h_{s^*})\right] &\le \mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_s, h_s)\right] \\ &\quad + \lambda^*\left(\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_s)\right] - G_t\right).\end{aligned} \quad (51)$$

Since ϕ is feasible, it satisfies $\mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_s)\right] - G_t \le 0$ (c.f. (12)). Since $\lambda^* > 0$, the above inequality implies that $\mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_{s^*}(\lambda^*), h_{s^*})\right] \le \mathbb{E}_{\mathbf{h}}\left[\mathsf{S}(P_s, h_s)\right]$. Thus, $\phi^*(\mathbf{h})$ is SEP-optimal.

Consider two antennas l and q with channel power gains h_l and h_q such that $h_l > h_q$. Let $P_l(\lambda^*)$ and $P_q(\lambda^*)$ minimize $\mathsf{SM}_l(\lambda^*, p)$ and $\mathsf{SM}_q(\lambda^*, p)$, respectively. As $P_l(\lambda^*)$ minimizes the selection metric of antenna l, it follows that

$$\begin{aligned}&\mathsf{S}(P_l(\lambda^*), h_l) + \lambda^*\bar{\mathsf{C}}(P_l(\lambda^*)) \\ &\qquad \le \mathsf{S}(P_q(\lambda^*), h_l) + \lambda^*\bar{\mathsf{C}}(P_q(\lambda^*)).\end{aligned} \quad (52)$$

From (3), we know that $\mathsf{S}(P_q(\lambda^*), h_l) < \mathsf{S}(P_q(\lambda^*), h_q)$ because $h_l > h_q$. Combining this with (52), we get $\mathsf{S}(P_l(\lambda^*), h_l)+\lambda^*\bar{\mathsf{C}}(P_l(\lambda^*)) < \mathsf{S}(P_q(\lambda^*), h_q)+\lambda^*\bar{\mathsf{C}}(P_q(\lambda^*))$. This implies that $\mathsf{SM}_l(\lambda^*, P_l(\lambda^*)) < \mathsf{SM}_q(\lambda^*, P_q(\lambda^*))$. Thus, the antenna with the largest STx-SRx channel power gain – when it transmits at its optimal transmit power – has the smallest selection metric. Therefore, $s^* = \arg\max_{k\in\{1,2,\ldots,N_t\}}\{h_k\}$ is optimal. And, by definition of optimality, its transmit power is given by $P_{s^*}(\lambda^*) = \arg\min_{p\in[0,P_{\max}]}\{\mathsf{SM}_{s^*}(\lambda^*, p)\}$.

B. Brief Proof of Continuity of $G(\lambda)$

The partial derivative of $\mathsf{SM}_{s^*}(\lambda, p)$ with respect to p is given by

$$\frac{\partial}{\partial p}\mathsf{SM}_{s^*}(\lambda, p) = -\frac{c_2 h_{s^*}}{\sigma^2} c_1 e^{-\frac{c_2 h_{s^*} p}{\sigma^2}} + \lambda\bar{\mathsf{C}}'(p), \quad (53)$$

where $\bar{\mathsf{C}}'(p) = d\bar{\mathsf{C}}(p)/dp$. Equating (53) to 0, we get

$$c_1 e^{-\frac{c_2 h_{s^*} p}{\sigma^2}} = \frac{\lambda\sigma^2}{c_2 h_{s^*}}\bar{\mathsf{C}}'(p). \quad (54)$$

The following two cases arise for $\bar{\mathsf{C}}'(p)$ in the condition A1 about the interference constraint:

Case 1: $\bar{\mathsf{C}}'(p)$ is a continuous and monotonically non-decreasing function of p. Hence, its derivative is non-negative, i.e., $d^2\bar{\mathsf{C}}(p)/dp^2 \ge 0$. Thus,

$$\frac{\partial^2}{\partial p^2}\mathsf{SM}_{s^*}(\lambda, p) = \left(\frac{c_2 h_{s^*}}{\sigma^2}\right)^2 c_1 e^{-\frac{c_2 h_{s^*} p}{\sigma^2}} + \lambda\frac{d^2\bar{\mathsf{C}}(p)}{dp^2} > 0. \quad (55)$$

Hence, the solution of (54) minimizes the selection metric. Moreover, it is a continuous function of λ as both sides of (54) are continuous. Thus, $P_{s^*}(\lambda)$ and $\bar{\mathsf{C}}(P_{s^*}(\lambda))$ are continuous functions of λ. Hence, $G(\lambda) = \mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\lambda))\right]$ is a continuous function of λ.

Case 2: $\bar{\mathsf{C}}'(p)$ is a continuous uni-modal function of p and $\bar{\mathsf{C}}'(0) = 0$. Here,

$$G(\lambda) = \mathbb{E}_{\mathbf{h}}\left[\bar{\mathsf{C}}(P_{s^*}(\lambda))\right] = N_t\mathbb{E}_{\mathbf{h}}\left[\Pr(s^* = 1|\mathbf{h})\,\bar{\mathsf{C}}(P_1(\lambda))\right], \quad (56)$$

where the second equality follows by symmetry. Using the law of total expectation, we get $G(\lambda) = N_t\mathbb{E}_{h_1}\left[\Pr(s^* = 1|h_1)\,\bar{\mathsf{C}}(P_1(\lambda))\right]$.

To prove that $G(\lambda)$ is continuous in λ, we shall show that $|G(\lambda) - G(\lambda+\epsilon)| = \mathcal{O}(\epsilon)$, where ϵ is small and $\mathcal{O}(\cdot)$ is as per the Bachmann-Landau notation. From above,

$$\begin{aligned}&|G(\lambda) - G(\lambda+\epsilon)| \\ &\quad = \left|N_t\mathbb{E}_{h_1}\left[\mathcal{P}_{s1}\left(\bar{\mathsf{C}}(P_1(\lambda)) - \bar{\mathsf{C}}(P_1(\lambda+\epsilon))\right)\right]\right|, \quad (57)\\ &\quad \le N_t\mathbb{E}_{h_1}\left[\mathcal{P}_{s1}\left|\bar{\mathsf{C}}(P_1(\lambda)) - \bar{\mathsf{C}}(P_1(\lambda+\epsilon))\right|\right], \quad (58)\end{aligned}$$

where $\mathcal{P}_{s1} = \Pr(s^* = 1|h_1)$. Here, $P_1(\lambda)$ is the solution of (54) with $s^* = 1$.

The solutions of (54) are the minima or maxima of $\mathsf{SM}_{s^*}(\lambda, p)$. Since $\bar{\mathsf{C}}'(p)$ is a uni-modal function, it can be shown that the monotonically decreasing function $c_1\exp\left(-c_2 h_1 p/\sigma^2\right)$ intersects $\bar{\mathsf{C}}'(p)$ at one point or three points. In (54), since both $c_1\exp\left(-c_2 h_1 p/\sigma^2\right)$ and $\bar{\mathsf{C}}'(p)$ are continuous functions of λ and h_1, so are these points. Since $\bar{\mathsf{C}}'(0) = 0$, it follows from (53) that $\frac{\partial}{\partial p}\mathsf{SM}_{s^*}(\lambda, p) < 0$ at $p = 0$. Thus, the smallest positive solution of (54) is a minima. We now delve into these two possibilities separately below.

1) One Point of Intersection p_1: Since p_1 is a continuous function of λ, so are $P_1(\lambda) = \min\{P_{\max}, p_1\}$ and $\bar{\mathsf{C}}(P_1(\lambda))$. Hence, $\left|\bar{\mathsf{C}}(P_1(\lambda)) - \bar{\mathsf{C}}(P_1(\lambda+\epsilon))\right| = \mathcal{O}(\epsilon)$. From (58), it follows that $|G(\lambda) - G(\lambda+\epsilon)| = \mathcal{O}(\epsilon)$.

2) *Three Points of Intersection p_1, p_2, and p_3:* Let $p_1 < p_2 < p_3$. Here, p_1 is a minima, p_2 is a maxima, and p_3 is a minima. Thus, $P_1(\lambda)$ is either p_1 or p_3 or $P_{\max}$. Here, the global minimum can jump from p_1 or p_3 to $P_{\max}$ as h_1 varies. Hence, $P_1(\lambda)$ can be a discontinuous function of h_1. However, from (54), it can shown that for a given λ, there can be at most one $h_1 \triangleq d_\lambda$ at which $P_1(\lambda)$ jumps from p_1 to $P_{\max}$, i.e., once the optimal power becomes $P_{\max}$, it can never become p_1 for larger h_1. Furthermore, it can be shown d_λ is a continuous function of λ.

In summary, $P_1(\lambda)$ is a continuous function of λ for $h_1 \in [0, d_\lambda)$ and $h_1 \in (d_\lambda, \infty)$ and $P_1(\lambda+\epsilon)$ is continuous of λ for $h_1 \in [0, d_{\lambda+\epsilon})$ and $h_1 \in (d_{\lambda+\epsilon}, \infty)$. Using these, the integral in (58) can be written as

$$\begin{aligned} &|G(\lambda) - G(\lambda+\epsilon)| \\ &\leq N_t \int_0^{d_\lambda} \mathcal{P}_{s1} \left| \bar{\mathsf{C}}\left(P_1(\lambda)\right) - \bar{\mathsf{C}}\left(P_1(\lambda+\epsilon)\right) \right| f_h\left(h_1\right) dh_1 \\ &+ N_t \int_{d_\lambda}^{d_{\lambda+\epsilon}} \mathcal{P}_{s1} \left| \bar{\mathsf{C}}\left(P_1(\lambda)\right) - \bar{\mathsf{C}}\left(P_1(\lambda+\epsilon)\right) \right| f_h\left(h_1\right) dh_1 \\ &+ N_t \int_{d_{\lambda+\epsilon}}^{\infty} \mathcal{P}_{s1} \left| \bar{\mathsf{C}}\left(P_1(\lambda)\right) - \bar{\mathsf{C}}\left(P_1(\lambda+\epsilon)\right) \right| f_h\left(h_1\right) dh_1. \end{aligned}$$

Here, the first and third terms are clearly $\mathcal{O}(\epsilon)$ because $|\bar{\mathsf{C}}(P_1(\lambda)) - \bar{\mathsf{C}}(P_1(\lambda+\epsilon))| = \mathcal{O}(\epsilon)$. The second term is also $\mathcal{O}(\epsilon)$ because $|d_{\lambda+\epsilon} - d_\lambda| = \mathcal{O}(\epsilon)$. Therefore, $|G(\lambda) - G(\lambda+\epsilon)| = \mathcal{O}(\epsilon)$.

C. Optimal Transmit Power for Average Interference Constraint

The selection metric $\mathsf{SM}_{s^*}(\lambda, p) = c_1 \exp\left(-c_2 p h_{s^*}/\sigma^2\right) + \lambda p \mu$ attains its minimum value at $p = \sigma^2 \ln\left(h_{s^*}/A_\lambda\right) / \left(c_2 h_{s^*}\right)$, where $A_\lambda = \left(\lambda\sigma^2 / \left(c_1 c_2\right)\right) \mu$. When $h_{s^*} \leq A_\lambda$, we get $p \leq 0$. In this case $P_{s^*}(\lambda) = 0$.

Now consider $h_{s^*} > A_\lambda$. In this case, p attains a maximum value of $c_1/(e\lambda\mu)$ when $h_{s^*} = eA_\lambda$. Thus, when $P_{\max} > c_1/(e\lambda\mu)$, we have $P_{s^*}(\lambda) = p$. This gives (21). Else, $P_{s^*}(\lambda) = \min\{P_{\max}, p\}$. Equating p with $P_{\max}$ and solving, we get $P_{s^*}(\lambda) = P_{\max}$ for $h_{\min} \leq h_{s^*} \leq h_{\max}$, where $h_{\min}$ and $h_{\max}$ are defined in (23) and (24), respectively. Thus, $P_{s^*}(\lambda) = 0$, for $h_{s^*} < A_\lambda$, and $P_{s^*}(\lambda) = p$, for $A_\lambda \leq h_{s^*} < h_{\min}$ and $h_{s^*} > h_{\max}$. Combining these yields (22).

D. Proof of Result 1

Let Err denote the event in which a symbol is decoded incorrectly. Using the law of total probability, we get

$$\Pr\left(\text{Err}|\mathbf{h}\right) = \sum_{k=1}^{N_t} \Pr\left(s=k, \text{Err}|\mathbf{h}\right). \quad (59)$$

Averaging over $\mathbf{h}$ and exploiting symmetry, $\overline{\mathsf{SEP}}$ can then be written as

$$\overline{\mathsf{SEP}} = \mathbb{E}_{\mathbf{h}}\left[\Pr\left(\text{Err}|\mathbf{h}\right)\right] = N_t \mathbb{E}_{\mathbf{h}}\left[\Pr\left(s=1, \text{Err}|\mathbf{h}\right)\right]. \quad (60)$$

Also, $\Pr(s=1, \text{Err}|\mathbf{h}) = \Pr(s=1|\mathbf{h}) \Pr(\text{Err}|\mathbf{h}, s=1)$. We know that $\Pr(\text{Err}|\mathbf{h}, s=1) = \mathsf{S}(P_1, h_1)$. Hence,

$$\overline{\mathsf{SEP}} = N_t \mathbb{E}_{\mathbf{h}}\left[\Pr\left(s=1|\mathbf{h}\right) \mathsf{S}(P_1, h_1)\right], \quad (61)$$

$$= N_t \mathbb{E}_{h_1}\left[\Pr\left(s=1|h_1\right) \mathsf{S}(P_1, h_1)\right], \quad (62)$$

where the second equality follows from the law of total expectation.

Expression for $Pr(s=1|h_1)$: From (16), we know that antenna 1 is selected when $h_2 < h_1, \ldots, h_{N_t} < h_1$. Hence, $\Pr(s=1|h_1) = \Pr(h_2 < h_1, \ldots, h_{N_t} < h_1|h_1)$. Conditioned on h_1, the events $h_2 < h_1, \ldots, h_{N_t} < h_1$ are mutually independent. Hence, we get

$$\Pr\left(s=1|h_1\right) = \left[\Pr\left(h_2 < h_1|h_1\right)\right]^{N_t-1} = \left[F_h\left(h_1\right)\right]^{N_t-1}. \quad (63)$$

Substituting this in (62) and averaging over h_1 yields (29).

E. Proof of Result 2

From (7), the average interference is equal to $\bar{I}_\lambda = \mathbb{E}_{\mathbf{h},\mathbf{g}}\left[P_s g_s\right]$. Using the law of total probability, we get

$$\bar{I}_\lambda = \mathbb{E}_{\mathbf{h},\mathbf{g}}\left[P_s\right] \mu = \mathbb{E}_{\mathbf{h}}\left[\sum_{k=1}^{N_t} P_k \Pr\left(s=k|\mathbf{h}\right)\right] \mu. \quad (64)$$

The last step follows because P_s does not depend on $\mathbf{g}$. As above, this reduces to

$$\bar{I}_\lambda = N_t \mathbb{E}_{\mathbf{h}}\left[P_1 \Pr\left(s=1|\mathbf{h}\right)\right] \mu = N_t \mathbb{E}_{h_1}\left[P_1 \Pr\left(s=1|h_1\right)\right] \mu. \quad (65)$$

Substituting $\Pr(s=1|h_1) = \left[F_h(h_1)\right]^{N_t-1}$ in (65) and averaging over h_1 yields

$$\bar{I}_\lambda = N_t\left(\mu\right) \int_0^\infty \left[F_h\left(h_1\right)\right]^{N_t-1} P_1 f_h\left(h_1\right) dh_1. \quad (66)$$

Substituting the optimal power expressions from (21) and (22) yields (41) and (42), respectively.

F. Proof of Result 3

Using (9), the interference-outage probability O_λ can be written as $O_\lambda = \mathbb{E}_{\mathbf{h}}\left[F_g^c\left(\tau/P_s(\lambda)\right)\right]$. Conditioning on the antenna that gets selected and using symmetry, we get

$$O_\lambda = \mathbb{E}_{\mathbf{h}}\left[\sum_{k=1}^{N_t} \Pr\left(s=k|\mathbf{h}\right) F_g^c\left(\tau/P_k(\lambda)\right)\right], \quad (67)$$

$$= N_t \mathbb{E}_{\mathbf{h}}\left[\Pr\left(s=1|\mathbf{h}\right) F_g^c\left(\tau/P_1(\lambda)\right)\right]. \quad (68)$$

Since$P_1(\lambda)$ depends only on h_1, it follows from the law of total expectation that

$$O_\lambda = N_t \mathbb{E}_{h_1}\left[\Pr\left(s=1|h_1\right) F_g^c\left(\tau/P_1(\lambda)\right)\right]. \quad (69)$$

Substituting $P_1(\lambda)$ given in (28) and $\Pr(s=1|h_1) = \left[F_h(h_1)\right]^{N_t-1}$ in (69) and averaging over h_1 yields (43).

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