Low latency storage of replicated fragments for memory constrained servers

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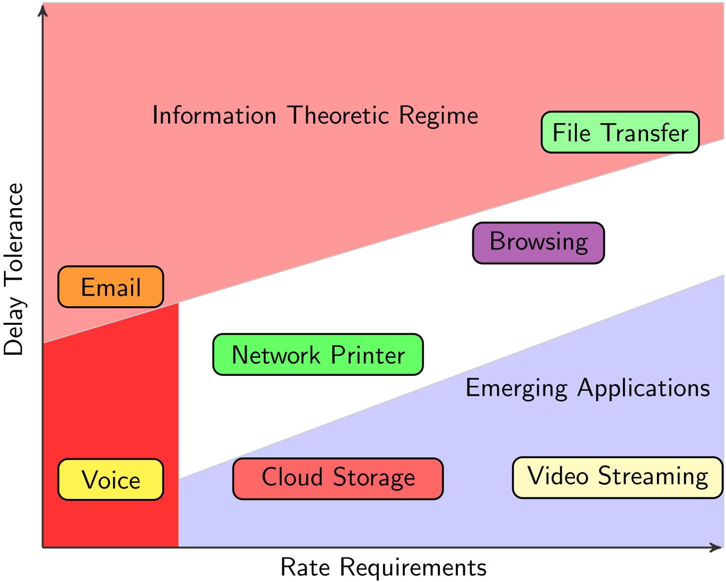
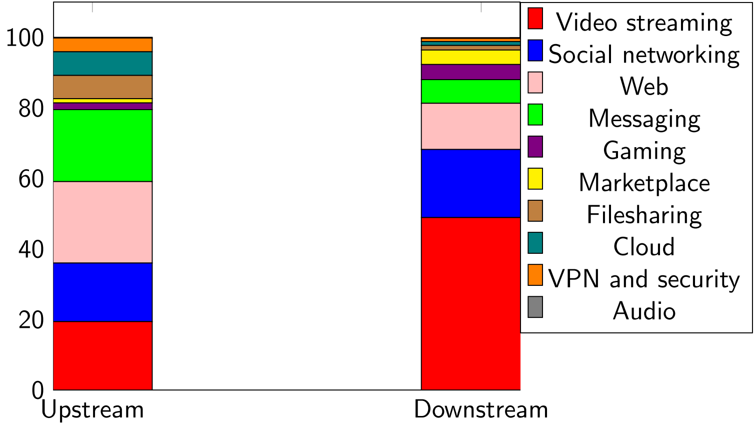
Evolving Digital Landscape

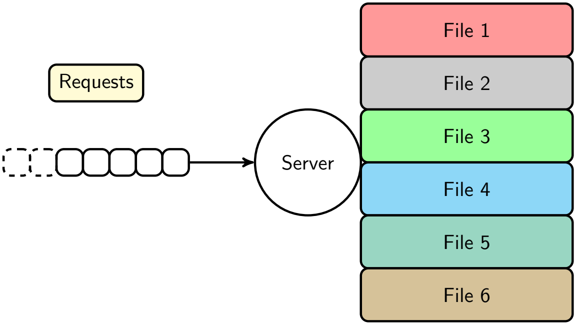


Global application traffic share 2021 [1]



[1] https://www.sandvine.com/hubfs/Sandvine_Redesign_2019/Downloads/2021/Phenomena/MIPR%20Q1%202021%2020

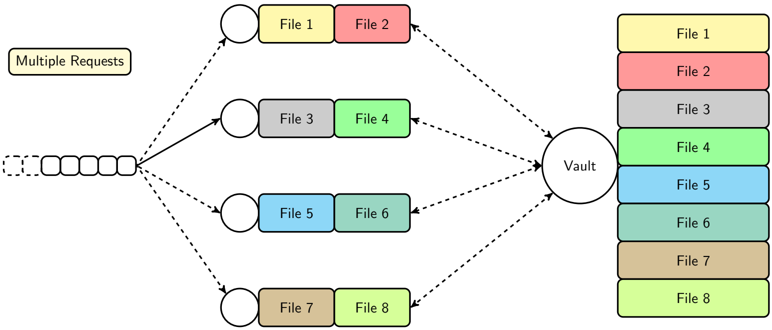
Centralized Paradigm



Potential Issues

- Not scalable with traffic load
- Susceptible to hardware failures and attacks

Distributed Paradigm



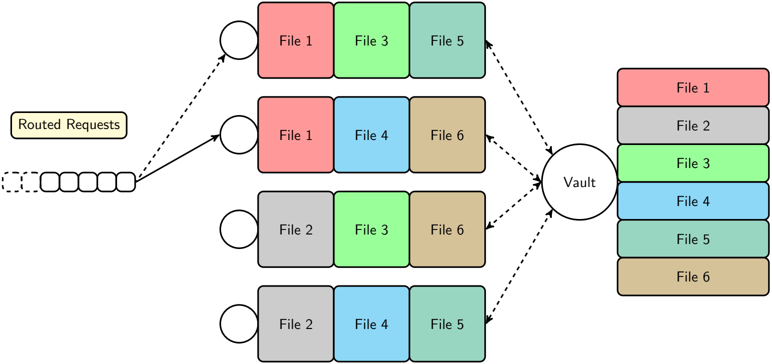
Potential Issues

- Susceptible to hardware failures and attacks

Large-scale distributed systems

- Distributed Computation
- Distributed Storage

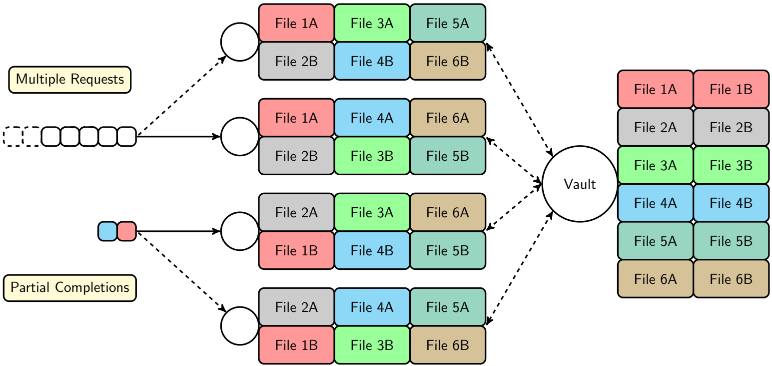
Resilience though redundancy



Latency redundancy tradeoff

- Download speedup due to parallel access
- Increased load due to redundant access

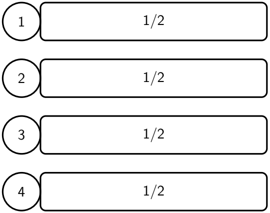
Load balancing through file fragmentation



Shared coherent access

- Availability and better content distribution
- File segments on multiple servers

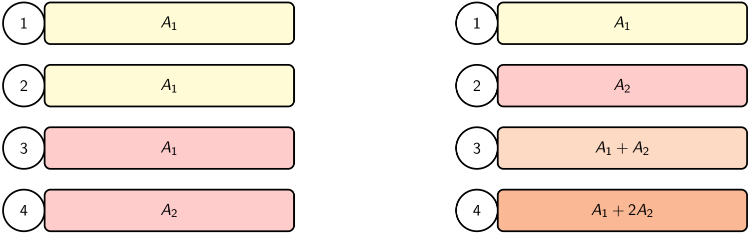
Memory constrained system



What are latency reducing storage and access schemes for replicated fragments?

- parallel access from all B servers
- α-fragment of message stored at each server

Coded Storage for single file



Single file divided into V fragments

- encoded into VR fragments
- each coded fragment stored over $B = VR$ servers
- reconstruction by set of V coded symbols

Prior Work

MDS codes

Outperform replication codes in file access delay

- Huang et al(2012), Li et al(2016), Badita et al(2019)

Rateless codes

Offers near optimal performance

- Mallick et al(2019)

Staircase codes

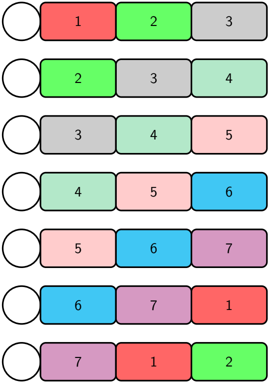
Subfragmentation improves latency performance

- Bitar et al(2020)

Our model

Replication codes for a file with equal sized fragmentation over multiple servers where each can store multiple file fragments

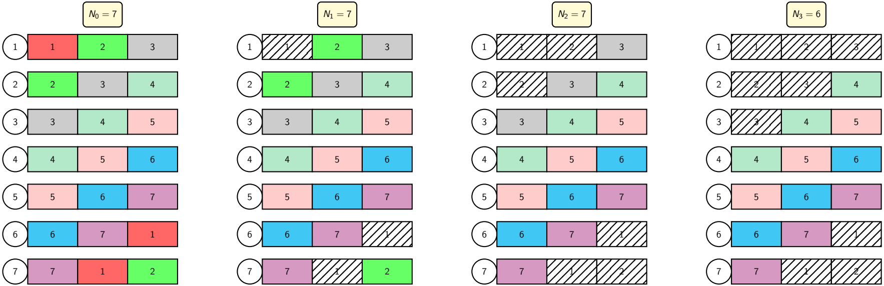
Latency optimal storage and access



A unit size divisible message $m = (m_1, \ldots, m_V)$

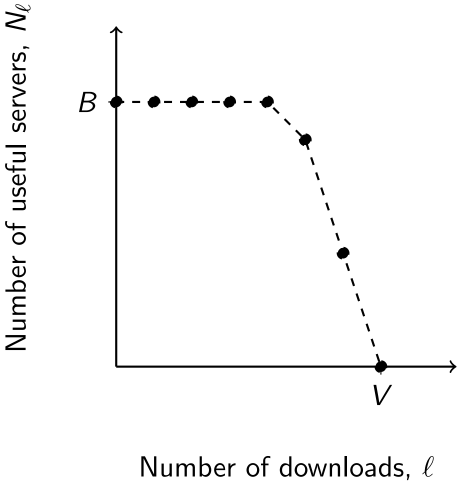
- replicated $R = \alpha B / V$ times
- **storage:** for each fragment, where to store each replica?
- **access:** for each server, sequence of access for replicas?

File download time



- Number of useful servers after ℓth download, N_ℓ
- Fragment download times are *i.i.d.* exponential with unit rate
- Rate of download at ℓth stage is N_ℓ
- The mean download time is $\mathbb{E}\sum_{\ell=0}^{V-1}\frac{1}{N_\ell}$

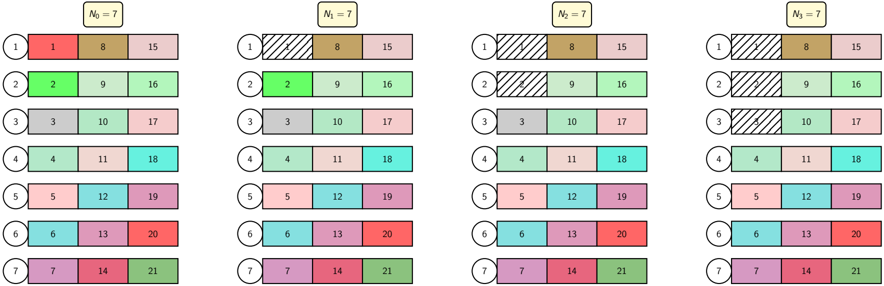
Optimality criterion



Optimality condition for storage scheme

Maximize the number of useful servers sequence

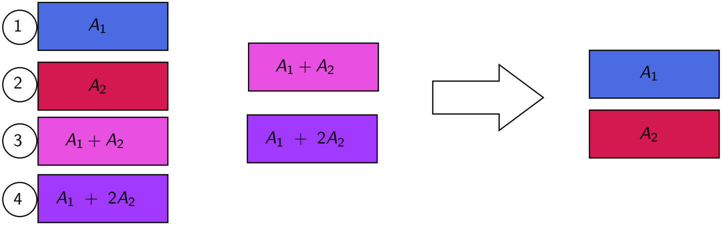
(VR, V) MDS code on α-B system



Optimality of MDS code

Reduction in useful servers is the least

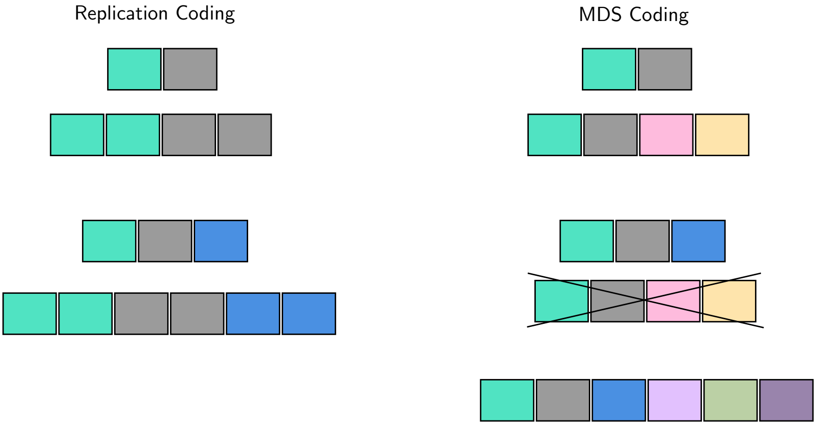
Decoding complexity



Implementation challenges

- Requires sufficiently large alphabet or large fragment sizes
- Polynomial decoding complexity that can't be parallelized

Scaling issues of MDS coding



Encoding growing data or redundancy

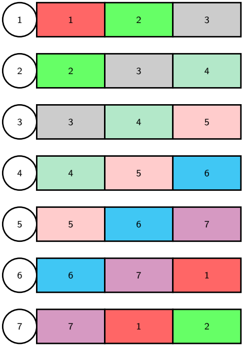
- Complete re-encoding of data blocks
- Potential data loss waiting for sufficient data blocks

Replication coded storage

α-(V, R) replication coded storage over B servers

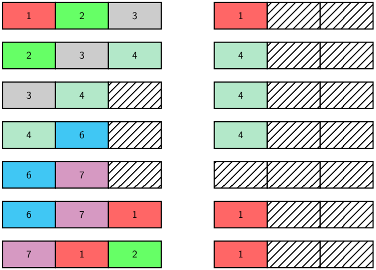
$$\mathcal{S} \triangleq \{(S_1, S_2, \ldots, S_B) : |S_b| = \alpha V \text{ for all } b, \alpha = R/B\}.$$

$\frac{3}{7} - (7, 3)$ replicated storage



- Fragment sets $S_1 = \{1, 2, 3\}, S_2 = \{2, 3, 4\}, \ldots$
- Occupancy sets $\Phi_1 = \{1, 6, 7\}, \Phi_2 = \{1, 2, 7\}, \ldots$

Upper bound on number of useful servers N_ℓ

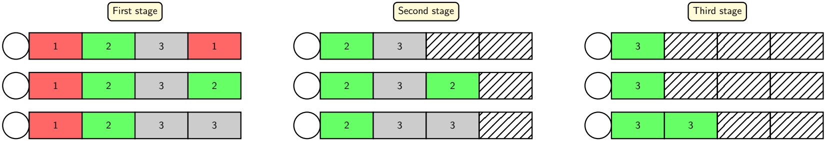


Upper bound

- For $m \triangleq \lceil B/R \rceil$, we have $N_\ell \leqslant B\mathbb{1}_{\{\ell \leqslant V-m\}} + (V-\ell)R\mathbb{1}_{\{\ell > V-m\}}$
- Normalized average of number of useful servers is upper bounded as

$$\frac{1}{BV}\sum_{\ell=0}^{V-1} N_\ell \leqslant 1 - \frac{(m+1)}{2V}.$$

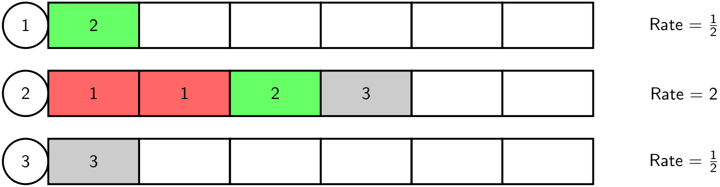
Trivial case: $\alpha \geqslant 1$



Replication as good as MDS without memory constraint

- Each server can store all the fragments
- All servers remain useful throughout
- What if $\alpha < 1$?

Randomized (B, V, R) replication coded storage

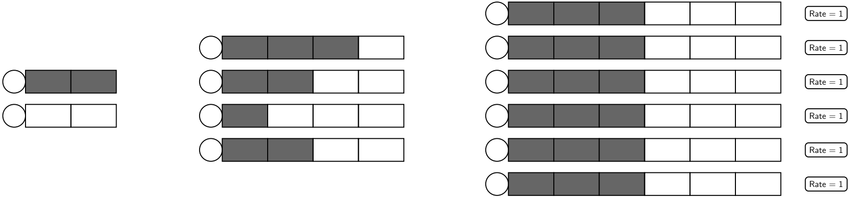


Place the fragments on randomly chosen servers

- Each server can store all coded VR fragments
- Exponential download rate $\propto$ the number of stored fragments

Asymptotically an α-(V, R) storage

- As V is increased with R/B fixed
- normalized storage at any server converges to $\alpha = R/B$
- service rate of servers converge to unity for almost all downloads



Asymptotic optimality

The randomized (B, V, R) storage scheme is an α-(V, R) storage scheme asymptotically in V.

Performance of Random Replication Storage

i.i.d. random storage vector Θ where $P\{\Theta_{vr} \neq b\} = (1 - 1/B)$

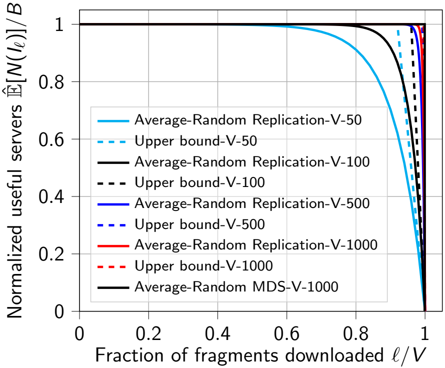
- $N_\ell = B - \sum_{b \in [B]} \prod_{v \notin I_\ell} \prod_{r \in [R]} \mathbb{1}_{\{\Theta_{vr} \neq b\}}$.
- $\frac{1}{BV} \mathbb{E} N_\ell = \frac{1}{V} \left(1 - \left(1 - \frac{1}{B}\right)^{R(V-\ell)}\right)$

Mean number of useful servers

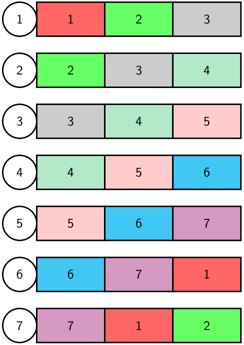
For the random (B, V, R) replication storage ensemble,

$$\frac{1}{BV} \sum_{\ell=0}^{V-1} \mathbb{E} N_\ell = 1 - \frac{\left(1 - \frac{1}{B}\right)\left(1 - (1 - \frac{1}{B})^{RV}\right)}{V\left(1 - (1 - \frac{1}{B})^{R}\right)}$$

Numerical Results



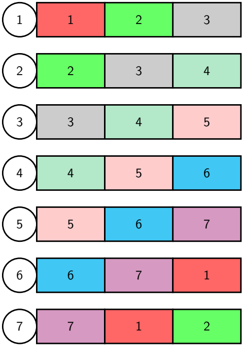
Bounding the number of useful servers



Maximum overlaps

- Between fragment sets $\tau_M \triangleq \max|S_a \cap S_b|$
- Between occupancy sets $\lambda_M \triangleq \max|\Phi_v \cap \Phi_w|$

Bounding the number of useful servers

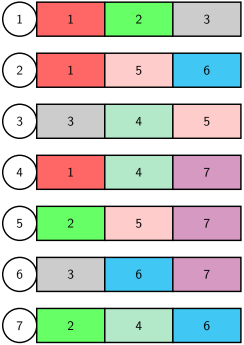


Universal bounds

- For $i \in \{0, \ldots, \lfloor \frac{K}{\tau_M} \rfloor\}$ and $\ell_i \triangleq iK - i(i-1)\frac{\tau_M}{2}$

$$N_\ell \geqslant \begin{cases} B - i, & \ell_i \leqslant \ell < \ell_{i+1}, \\ (V-\ell)(R - (V - \ell - 1)\frac{\lambda_M}{2}), & \ell \geqslant V - \lfloor \frac{R}{\lambda_M} \rfloor - 1 \end{cases}$$

Optimal Storage



Tightest lower bounds

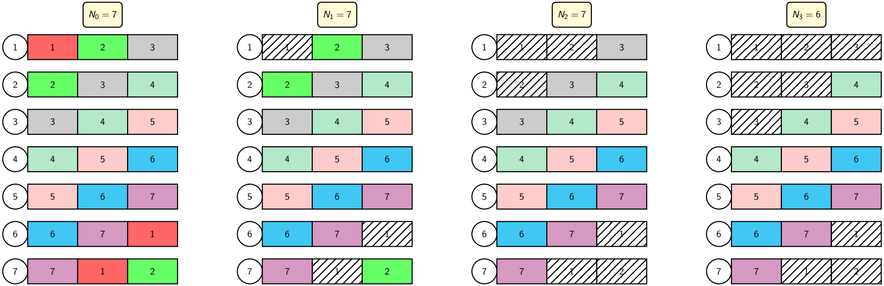
- The lower bounds are maximized for $\lambda_M = \tau_M = 1$
- Less overlaps are better

How to find the good storage schemes?

Table: Correspondence between designs and storage codes

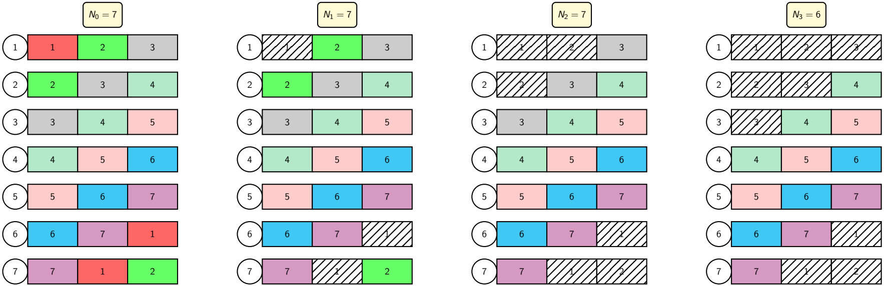
t-(V, K, λ) designs to codes	
Design parameter	**Storage parameter**
$\mathcal{P}$: Points	$[V]$: File fragments
$\mathcal{B}$: Blocks	$(S_b : b \in [B])$: Fragment sets at servers
$\lvert\mathcal{P}\rvert$: Number of points	V: Number of file fragments
$\lvert\mathcal{B}\rvert$: Number of blocks	B: Number of servers
K: Size of each block	K: Storage capacity at each server
R: Replication factor for each point	R: Replication factor for each fragment

Optimal Access



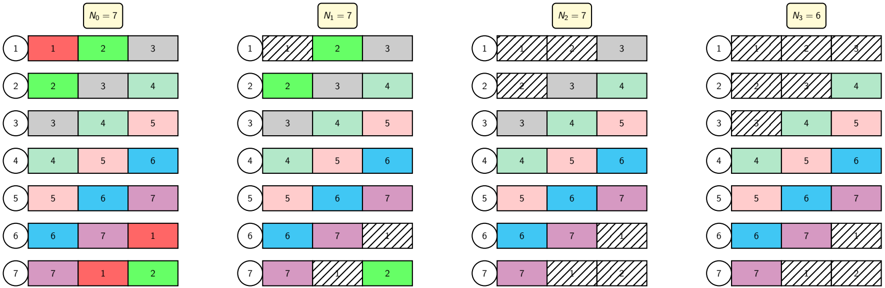
- The set of useful servers evolve as a Markov chain
- Given a storage scheme, optimal access is a Markov decision process that maximizes $\mathbb{E}\left[\sum_{\ell=0}^{V-1} N_\ell\right]$

Greedy Scheduler



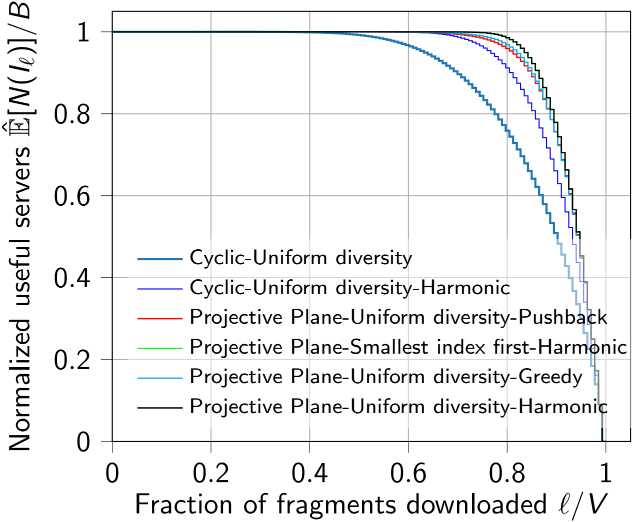
- Greedy scheduler rank $\rho_\ell^g(v) \triangleq \sum_{b \in \Phi_v} \mathbb{1}_{\{|S_b^\ell|=1\}}$ for every fragment
- $\mathbb{E}\left[N_{\ell+1} - N_\ell \mid I_\ell\right] = \sum_{v \notin I_\ell} p_{I_\ell, I_\ell \cup \{v\}} \rho_\ell^g(v)$

Ranked Scheduler



- For each useful server schedule the fragment with highest rank $\rho_\ell : I_\ell^c \to \mathbb{R}$
- Harmonic rank $\rho_\ell^h(v) \triangleq \sum_{b \in \Phi_v} \frac{1}{|S_b^\ell|}$ for every fragment

Numerical Studies



Conclusion

- We studied codes for distributed storage system with storage constraints and file subfragmentation for achieving low latency
- For exponential download times, we proposed to maximize mean number of useful servers instead of minimizing latency
- We show that MDS codes are optimal
- When there are no memory constraints at the server, replication coded file can be optimally placed
- When servers have memory constraints, we show that replication coding combined with probabilistic placement are optimal asymptotically
- Placement of coded fragments depends on overlap properties of storage codes
- Optimal access sequence is a Markov decision process

Acknowledgements

References

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