Online Reinforcement Learning in Structured Environments

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Motivation

Reinforcement Learning is concerned with learning to take actions to maximize rewards, by trial and error, in environments that can evolve in response to actions

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- Optimization Goal: Maximize total reward / Minimize regret (shortfall) in total reward compared to an optimal policy
  - **Applications:** Recommendation systems/ Sequential investment/ Dynamic resource allocation in communication systems ....
  - No separate budget to purely exploring the unknown environment
  - Exploration and Exploitation must be carefully balanced

Intended Goal of Thesis

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  - Nonparametric model: Infinite number of actions
  - Generalization across arms: Kernelized structure

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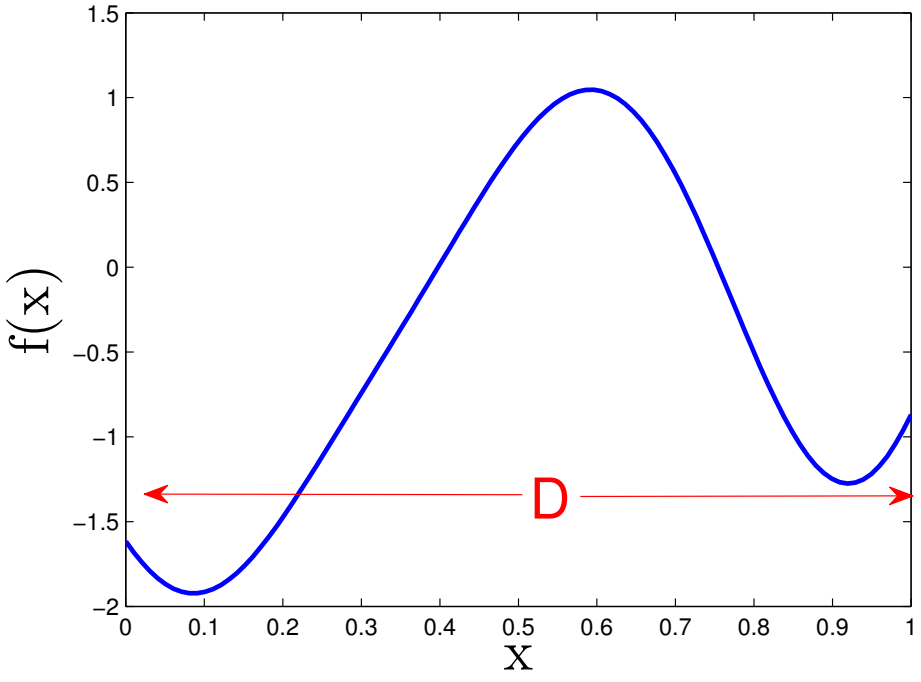
- **Part 1:** Online learning in large scale **Multi-armed Bandits**
  - Nonparametric model: Infinite number of actions
  - Generalization across arms: Kernelized structure
- **Part 2:** Online learning in large unknown **Markov Decision Processes**
  - Nonparametric MDP model: Uncertainty is represented over Infinite dimensional function classes via Kernelized structure

Part 1: Online Learning in Kernelized Multi-armed Bandits[1]

[1]S. R. Chowdhury and A. Gopalan, *On kernelized multi-armed bandits*, In Proceedings of the 34th International Conference on Machine Learning (**ICML**), pp. 844853, 2017.

Problem Statement

Sequentially Maximize $f : D \to \mathbb{R}$



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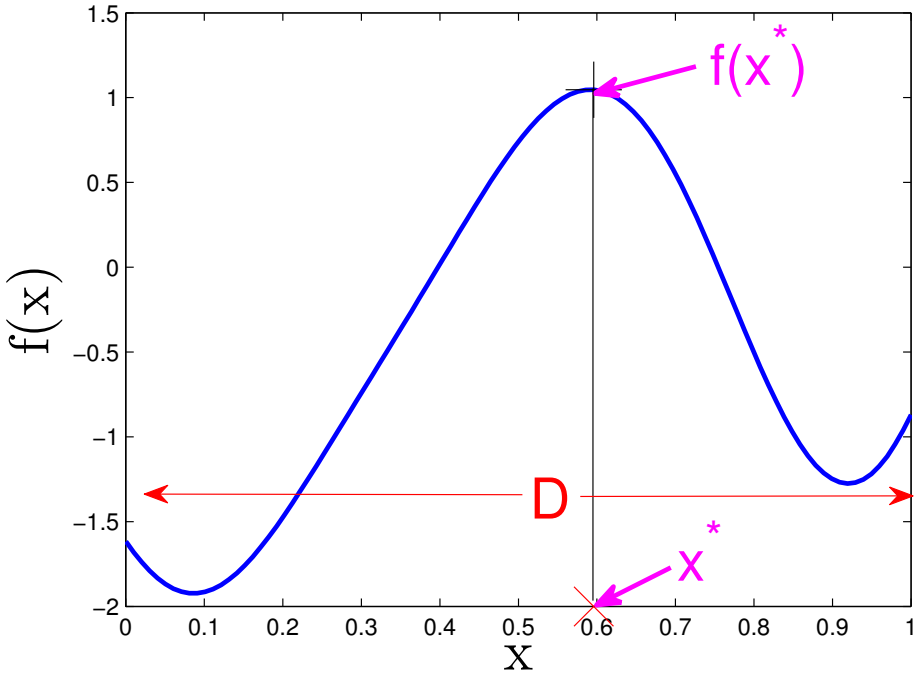

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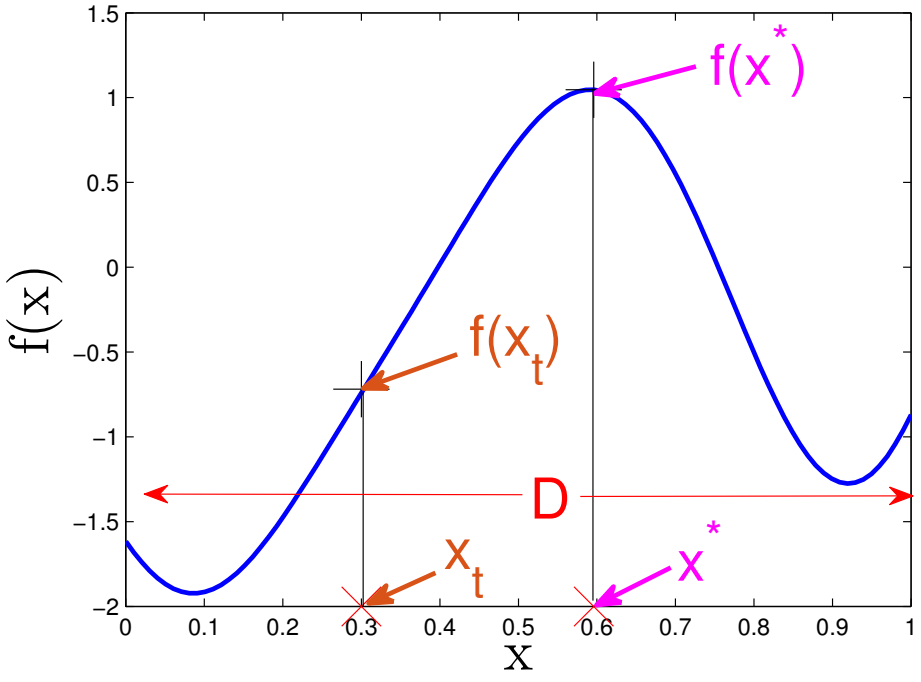

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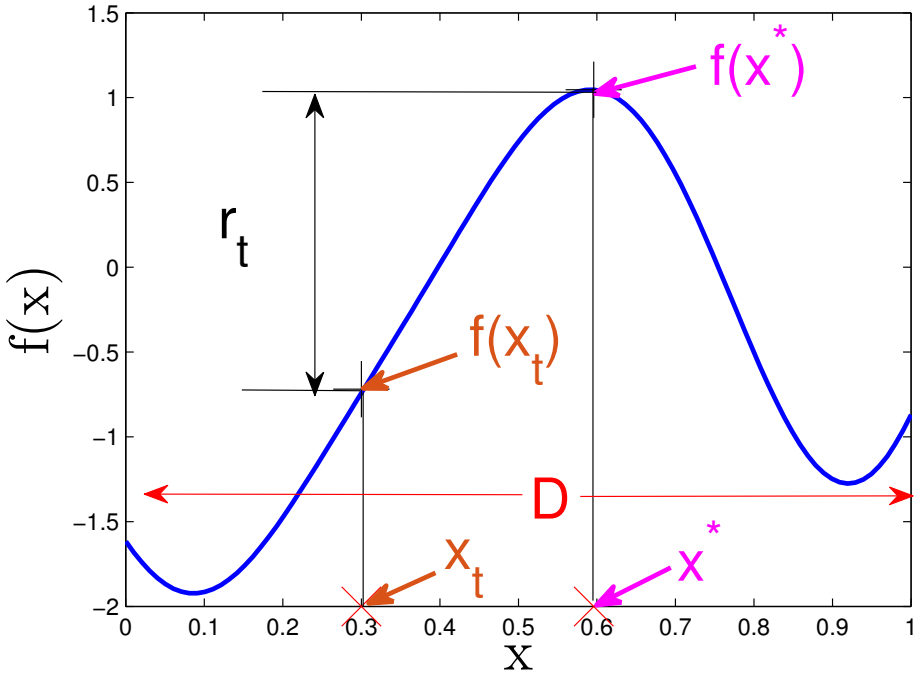

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Performance Metric

- Minimize Cumulative Regret: $\sum_{t=1}^{T} \Big(f(x^\star) - f(x_t) \Big)$

Assumptions

- Smoothness: f lies in **R**eproducing **K**ernel **H**ilbert **S**pace (**RKHS**) of functions $D \to \mathbb{R}$
  - Positive semi-definite kernel function $k : D \times D \to \mathbb{R}$ (known)
  - Reproducing property: $f(x) = \langle f, k(x, \cdot) \rangle_k$
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- Example Kernels
  - Squared Exponential kernel: $k(x, y) = \exp\left(\frac{-\|x-y\|_2^2}{2l^2}\right)$
  - Linear Kernel: $k(x, y) = x^T y$
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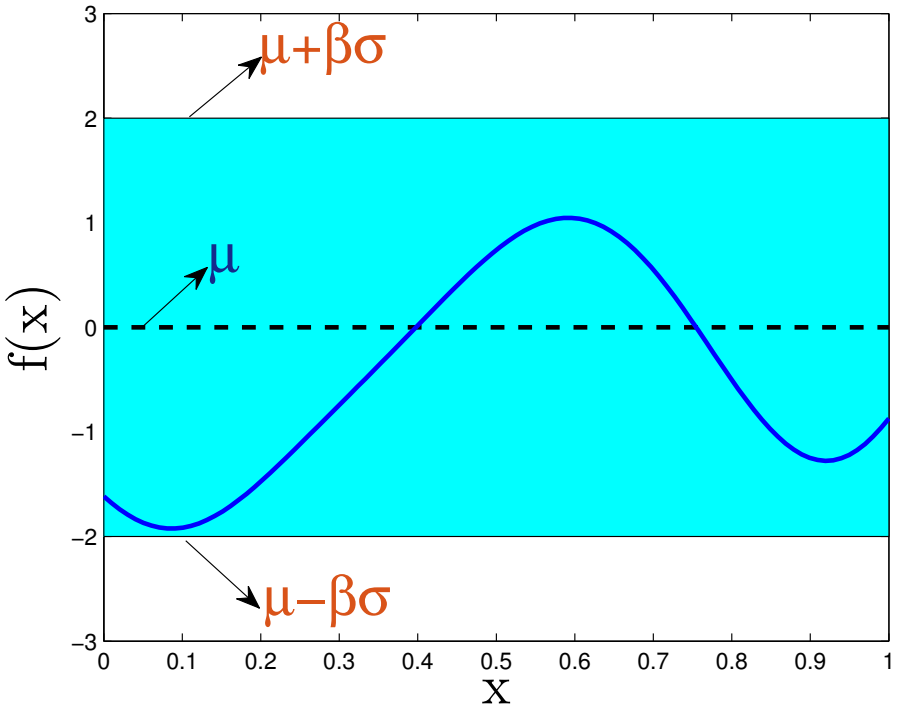
- Noise ε_t is zero mean, Sub-Gaussian

Algorithm Design Philosophy

Key Idea: Represent uncertainty over f using **G**aussian **P**rocess (**GP**)



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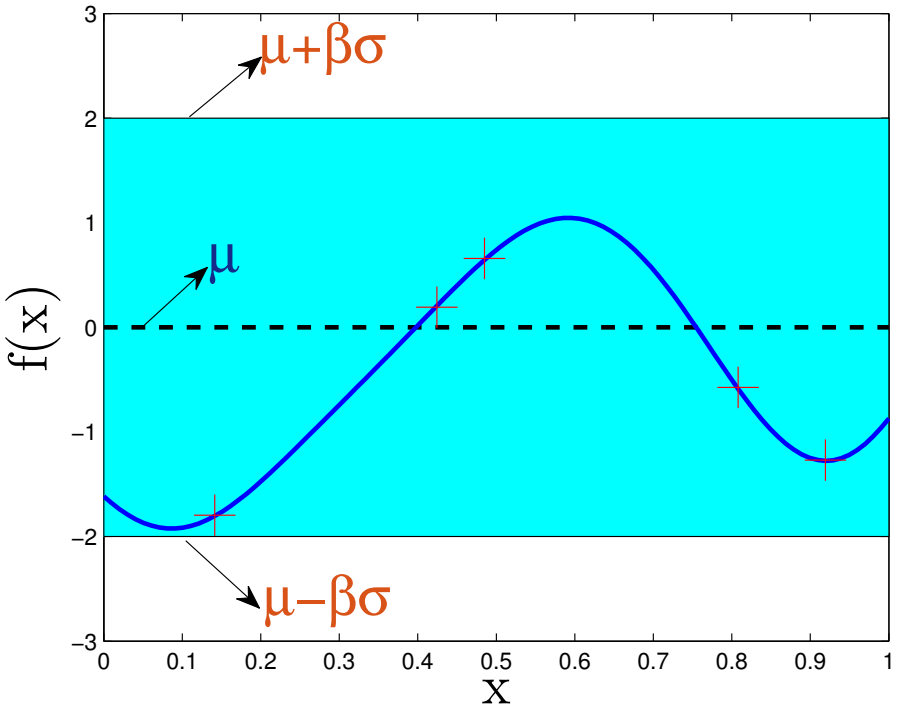

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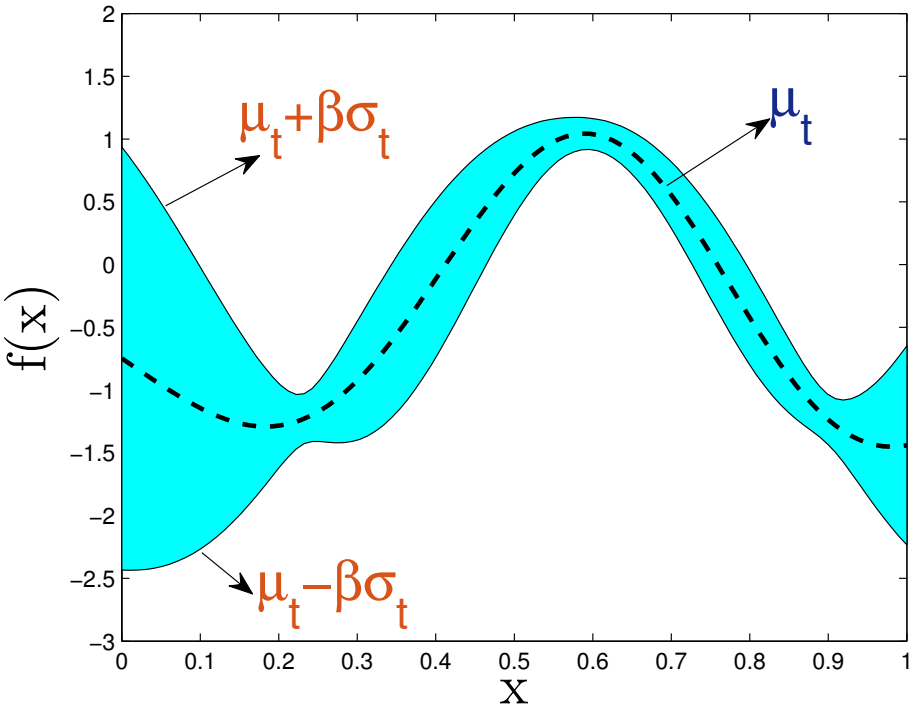

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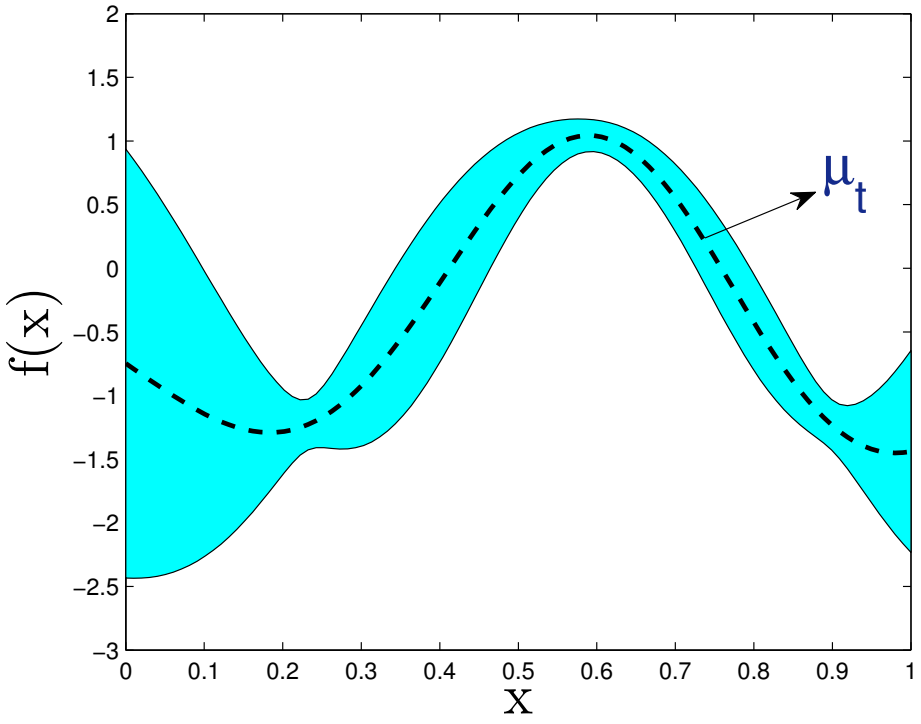
Posterior of f after t rounds: $GP(\mu_t(x), k_t(x,y))$

$$\begin{aligned} \mu_t(x) &= k_t(x)^T (K_t + \lambda I)^{-1} y_{1:t} \\ k_t(x,y) &= k(x,y) - k_t(x)^T (K_t + \lambda I)^{-1} k_t(y) \end{aligned}$$

Algorithm 1: Improved GP-UCB (IGP-UCB)

Key Idea: Play the arm with highest Upper Confidence Bound (UCB)



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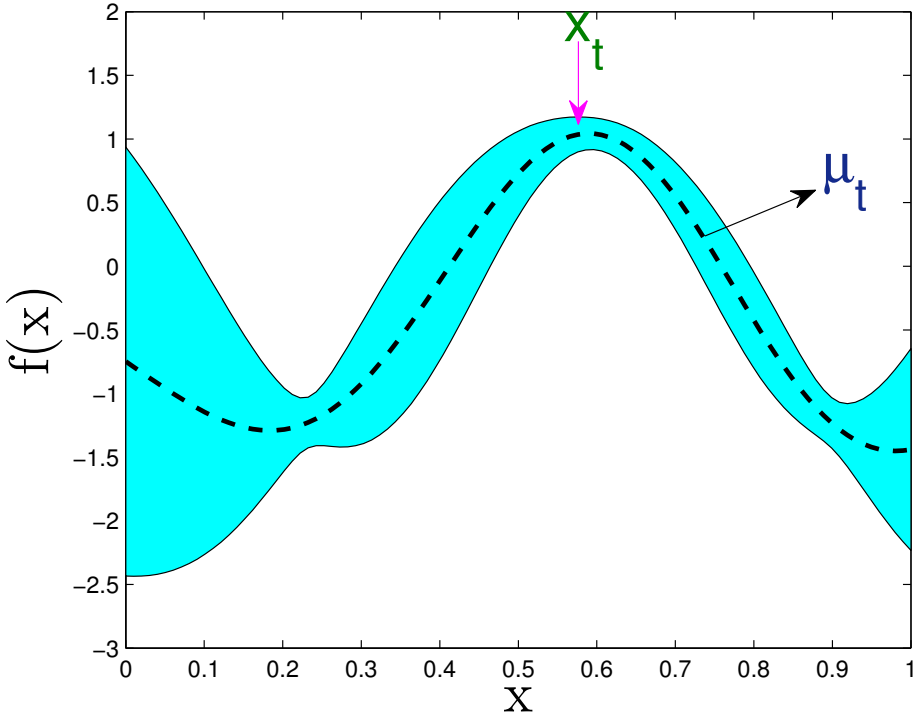

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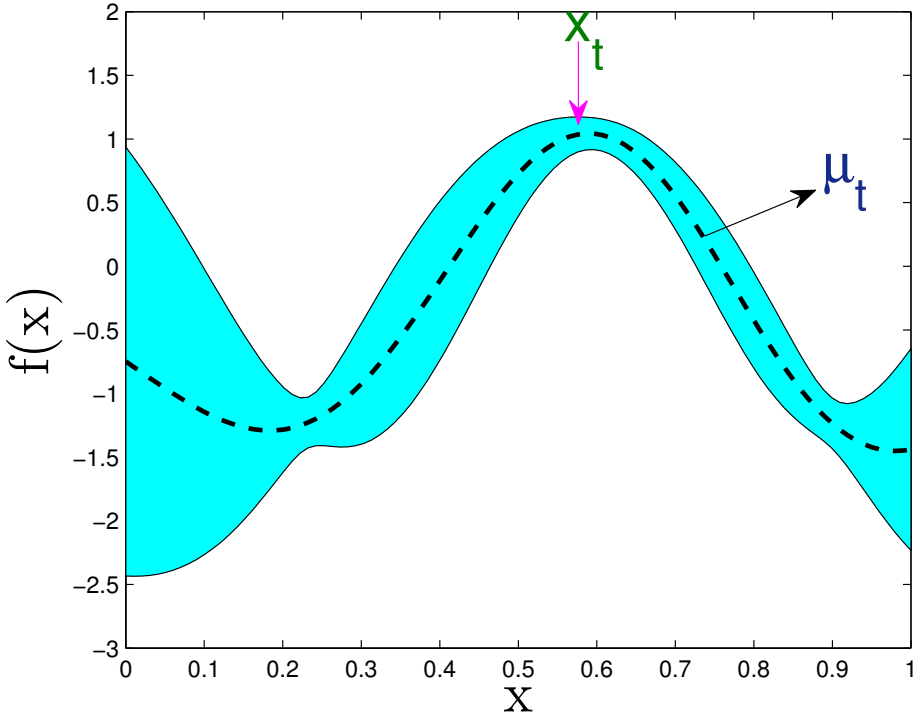

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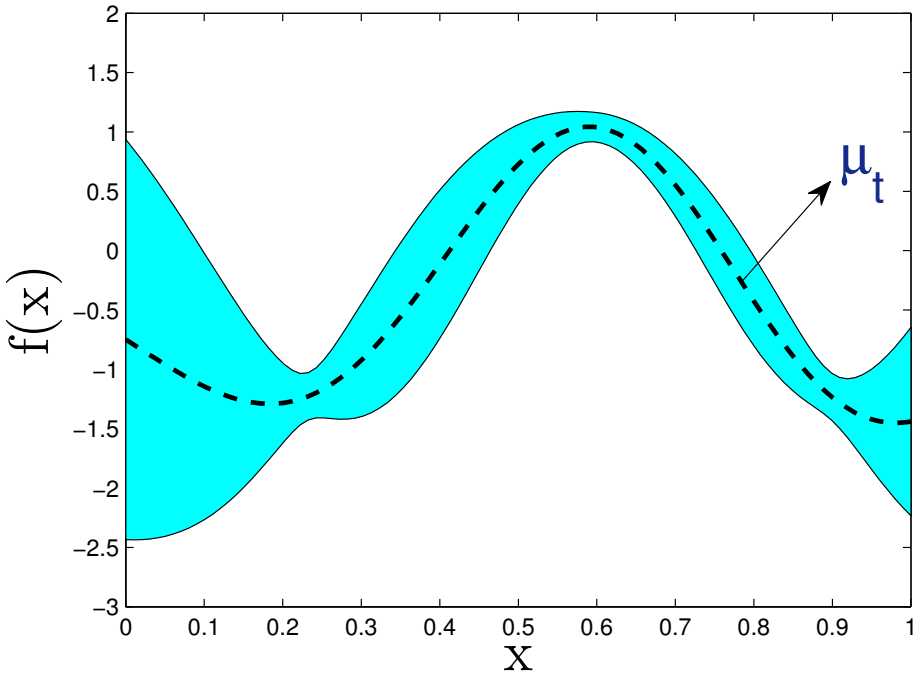
- β_t trades off b/w **exploration** and **exploitation**
- First appeared as **GP-UCB** (Srinivas et al., ICML 2010) $\rightarrow$ We Reduced (Improved) width (β_t) of confidence interval

Algorithm 2: Gaussian Process Thompson Sampling (GP-TS)

Key Idea: Sample a random function and play its maximizer



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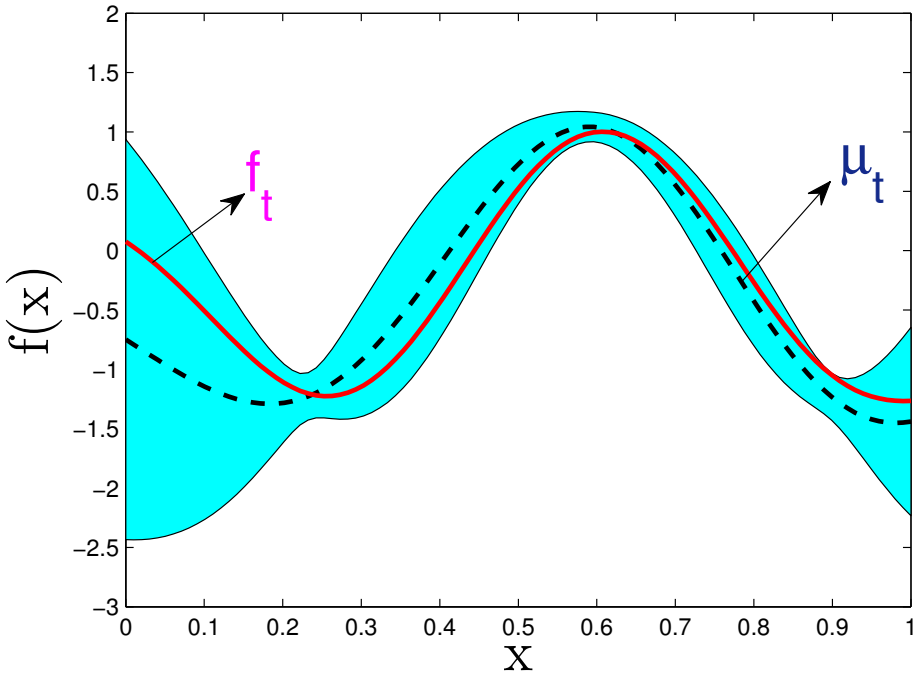

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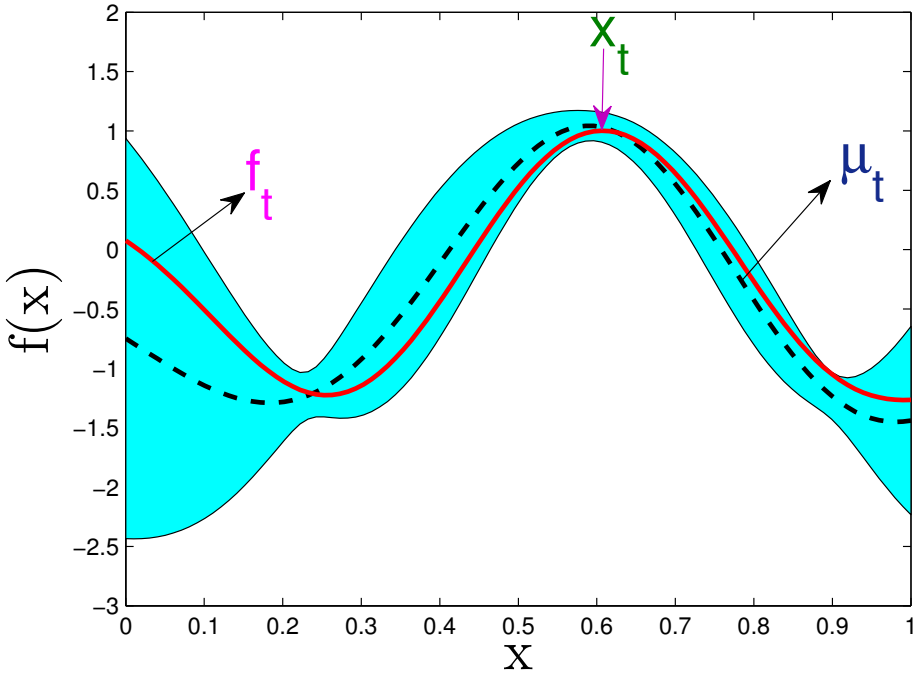

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$D_t \subset D$: suitably chosen Discretization sets

Regret Bounds [ICML, 2017]

Result 1

Cumulative regret of **IGP-UCB** is $O\left(\sqrt{T}(\sqrt{\gamma_T} + \gamma_T)\right)$ *with high probability (whp)*

- γ_T is **Maximum Information Gain** about f after T rounds and quantifies Reduction in uncertainty after observing rewards
- Squared Exponential kernel: $\gamma_T = O((\ln T)^{d+1}) \rightarrow$ **Sublinear** regret

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Result 2

- Cumulative regret of **GP-TS** is $O\left(\sqrt{Td\ln(dT)}(\sqrt{\gamma_T} + \gamma_T)\right)$ *whp*
- First **frequentist** regret guarantee of TS in the non-parametric setting of infinite action spaces

Recovering Regret Bounds for Linear Bandits

Linear Kernel

- $k(x, y) = x^T y$
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Exactly recovers regrets of **Linear Bandit** algorithms (Abbasi-Yadkori et al., NIPS 2011, Agrawal and Goyal, ICML 2013)

Numerical Results

Algorithms Compared:

1. GP-Expected Improvement (Močkus, 1975)
2. GP-Probabilistic Improvement (Kushner, 1964)
3. GP-UCB (Srinivas et al., 2010)
4. IGP-UCB (this work)
5. GP-TS (this work)

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f sampled from RKHS
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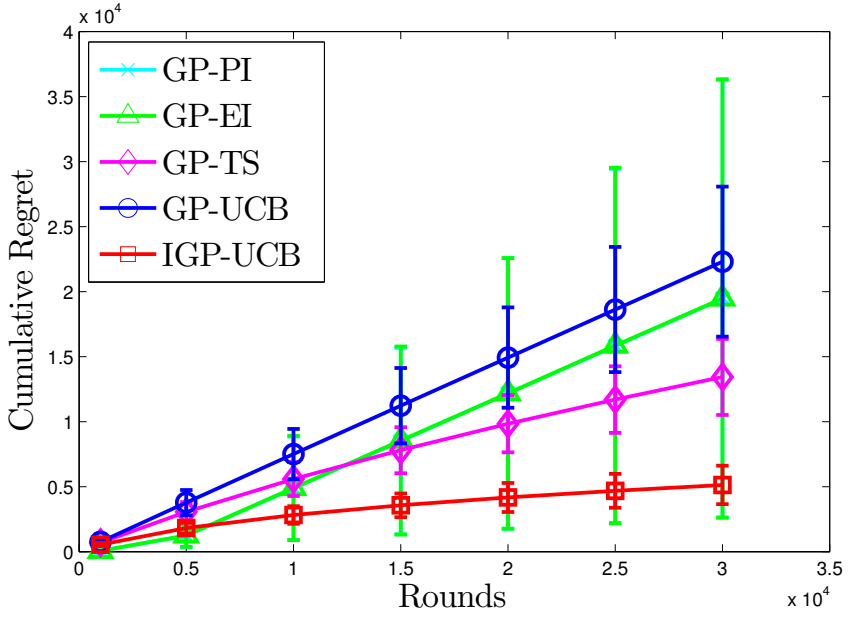

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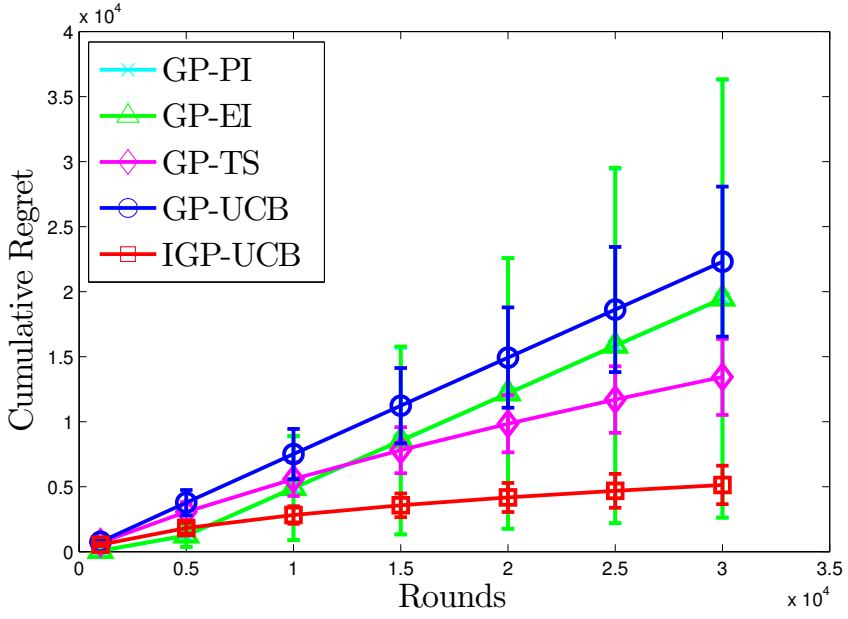

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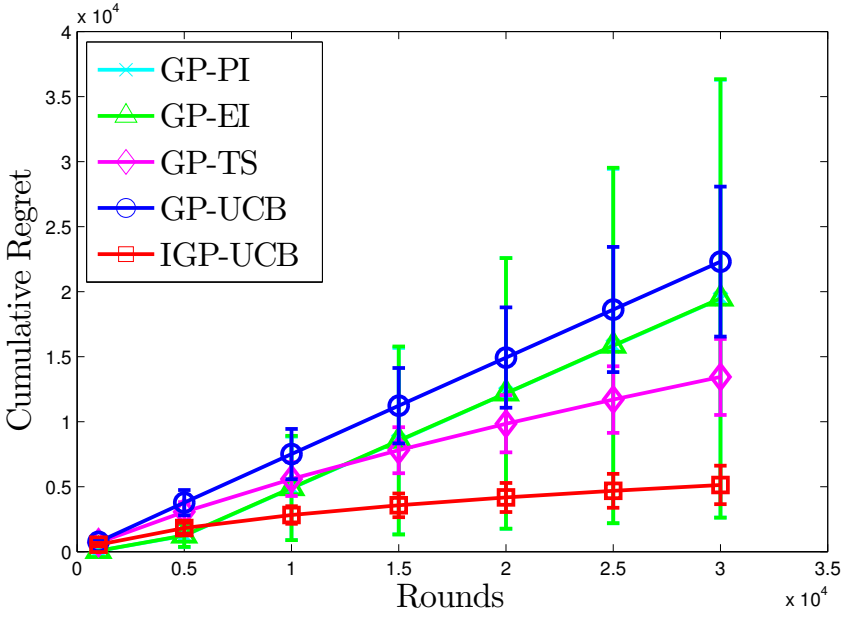

Temperature Sensor Data
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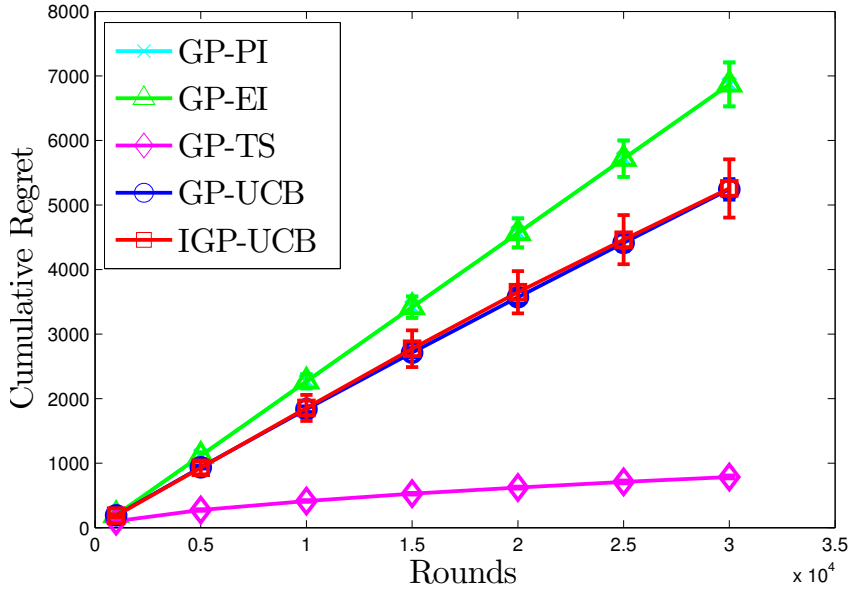

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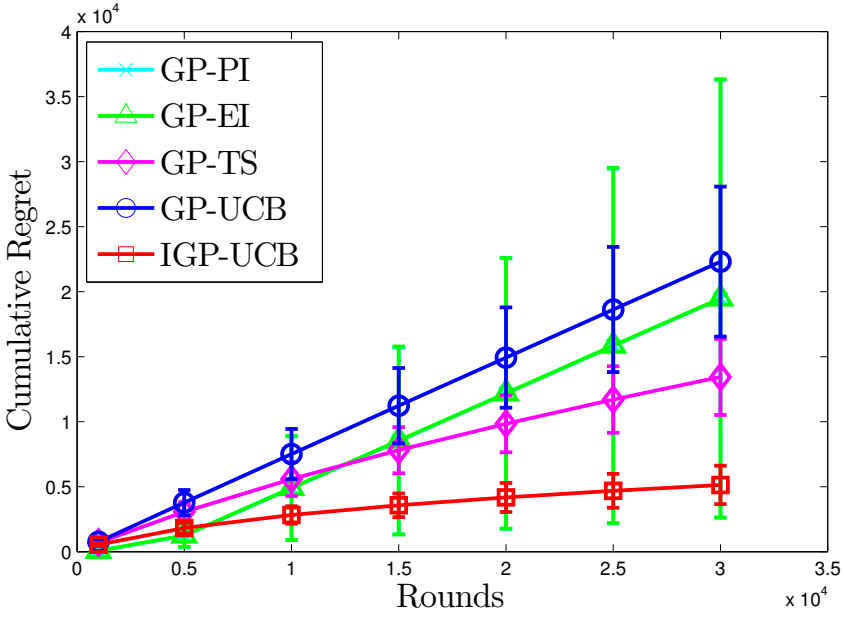

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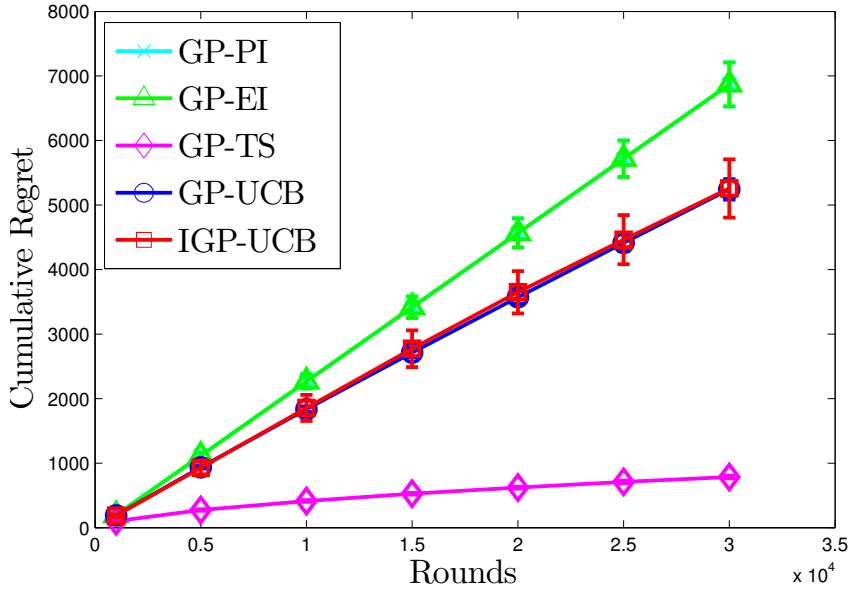

- IGP-UCB performs similar to GP-UCB ✓
- GP-TS performs the best ☺

Key Technique: Posterior Concentration

Lemma: Concentration of Posterior Distribution

For all t and for all $x \in D$:

$$\mu_t(x) - \beta_t \sigma_t(x) \leq f(x) \leq \mu_t(x) + \beta_t \sigma_t(x) \qquad \text{whp}$$

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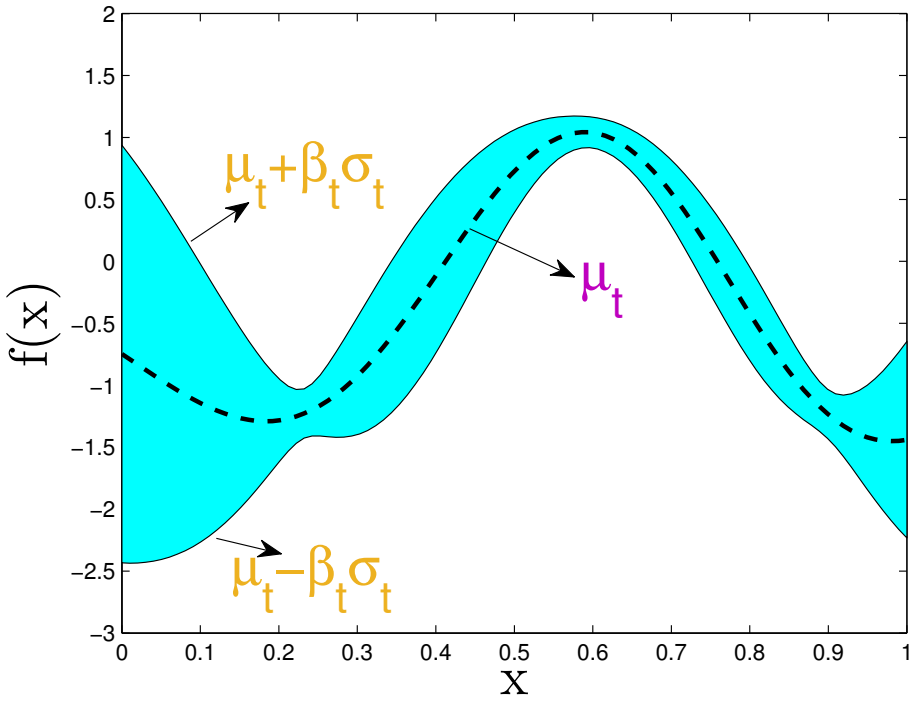

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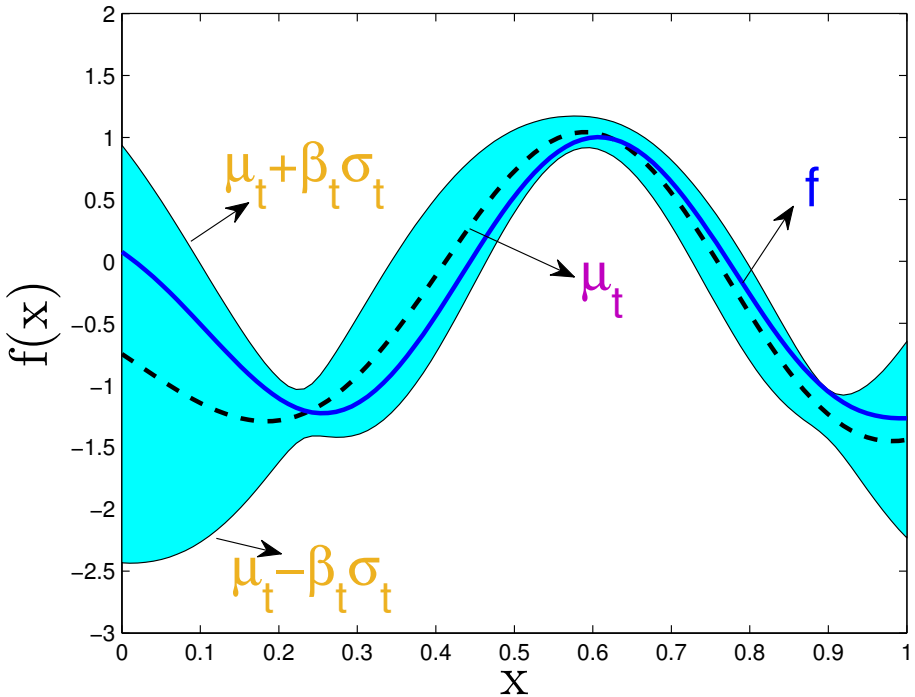

At every round, the unknown original function lies within properly constructed confidence interval of shrinking width

Key Technique: New Concentration Inequality (CI)

Result 3: Self-Normalized CI for RKHS-valued Martingales

- For all t: $\|S_t\|^2_{V_t^{-1}} \leq 2R^2 \ln(\frac{\sqrt{\det(K_t+I)}}{\delta})$ with probability at least $1-\delta$ if K_t is positive-definite
- **Generalizes** finite-dimensional Inequality for vector-valued Martingales (Abbasi-Yadkori et al., NIPS 2011) to infinite dimensions

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Example: Let $\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots$ be a sequence of independent random variables and $x_1, x_2, x_3, \ldots$ be a predictable sequence of random variables. Define $S_t = \sum_{s=1}^{t} \varepsilon_s x_s$ and $V_t = 1 + \sum_{s=1}^{t} x_s^2$. Then $S_t/\sqrt{V_t}$ is a Self-normalized process and its fluctuations are $O(\ln t)$

Summary

For Non-parametric Bandits, we

- **Improved** existing UCB based algorithm
- **Introduced** new Thompson Sampling based algorithm
- **Developed** new self-normalized concentration inequality for RKHS-valued martingales

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Possible Extensions:

- Kernel function not known to the learner
- Time varying functions from RKHS

Part 2: Online Learning in Kernelized Markov Decision Processes[2]

[2]S. R. Chowdhury and A. Gopalan, *Online Learning in Kernelized Markov Decision Processes*, ArXiv e-prints, May 2018. (Under review in **NIPS**)

Problem Statement

Episodically maximize reward in (unknown) MDP $M = \{\mathcal{S}, \mathcal{A}, R, P, H, \rho\}$

- State space $\mathcal{S} \subseteq \mathbb{R}^m$, known
- Action space $\mathcal{A} \subseteq \mathbb{R}^n$, known
- Reward distribution $R(s, a)$, unknown
- Transition distribution $P(s, a)$, unknown
- Episode length H, known
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At each period h within an episode:

- Learner takes action $a_h \in \mathcal{A}$ based on current state s_h and past observations
- Receives reward $r_h \sim R(s_h, a_h)$
- Observes next state $s_{h+1} \sim P(s_h, a_h)$

Definitions

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  - Mean reward $\overline{R}(s, a)$

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- Optimal policy $\pi_\star \in \operatorname{argmax}_\pi V_{\pi,h}(s) \;\; \forall s, \; \forall h$
- Cumulative Regret $= \sum_I \mathbb{E}\Big[V_{\pi_\star,1}(s) - V_{\pi_I,1}(s)\Big]$

Goal: Minimize the loss incurred in Value function due to not knowing the optimal policy $\pi_\star$ of the unknown MDP M and instead using any other policy π_I at episode I

Assumptions

- View $r \sim R(s,a) \Longrightarrow r = \overline{R}(s,a) + \varepsilon_R$
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- Additional Regularity condition on Value function $\to$ Transition distributions with same means are identical

Algorithm 1: Gaussian Process-UCRL (GP-UCRL)

Same philosophy as earlier: Put *separate* Gaussian process priors over mean reward and mean transition functions and update posteriors at the end of every episode

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3. Choose the optimistic policy π_l for the **set of MDPs** $\mathcal{M}_l$ and execute it for the entire episode

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At each episode I:

1. Build an MDP M_I such that mean reward and mean transition functions are samples from respective posterior distributions
2. Choose the optimal policy π_I for the **sampled MDP** M_I and execute it for the entire episode

GP-TSRL: For Gaussian Process prior and Gaussian likelihood model, the posteriors admit nice closed form of Gaussian processes

Regret Bounds

Result 1

Cumulative Regret of **GP-UCRL** is $\tilde{O}\left((\gamma_{R,T} + \gamma_{P,T})\sqrt{T}\right)$ *whp*

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Expected (over prior distribution of MDPs) cumulative Regret of **GP-TSRL** is $\tilde{O}\left((\gamma_{R,T} + \gamma_{P,T})\sqrt{T}\right)$

- $\gamma_{R,T}$ and $\gamma_{P,T}$ are **Maximum Information Gain** about (unknown) mean reward and mean transition functions after T rounds
- Grow sub-linearly with T for common kernels (e.g. Squared Exponential) and for their compositions (products, sums)

Summary

Proved First regret guarantees of UCRL and TSRL in kernel based MDPs, where

- Mean reward and transition functions are elements from Reproducing Kernel Hilbert Spaces
- Mean reward and transition functions are samples from Gaussian Processes (Bayesian setup, not presented here)

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Proved First regret guarantees of UCRL and TSRL in kernel based MDPs, where

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Future work:

- Frequentist regret bound of GP-PSRL
- Online learning in Model free MDPs (obviate complicated planning step)

Thank You