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The Gelfand-Pinsker Channel: Strong Converse and Upper Bound for the Reliability Function

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Abstract—**We consider a Gelfand-Pinsker discrete memoryless channel (DMC) model and provide a strong converse for its capacity. The strong converse is then used to obtain an upper bound on the reliability function. Instrumental in our proofs is a new technical lemma which provides an upper bound for the rate of codes with codewords that are conditionally typical over large *message dependent subsets* of a typical set of state sequences. This technical result is a nonstraightforward analog of a known result for a DMC without states that provides an upper bound on the rate of a good code with codewords of a fixed type (to be found in, for instance, the Csiszár-Körner book).**

I. Introduction

We consider a state dependent discrete memoryless channel (DMC), in which the underlying state process is independent and identically distributed (i.i.d.) with known probability mass function (pmf). The transmitter is provided access at the outset to the entire state sequence prevailing during the transmission of a codeword. The capacity of this DMC with noncausal channel state information (CSI) at the transmitter was determined in [1]. Known popularly as the Gelfand-Pinsker channel, it has been widely studied for a broad range of applications which include fingerprinting, watermarking, broadcast communication, etc.

In this paper, we are concerned with the *strong converse* for this channel as well as its *reliability function*, i.e., the largest exponential rate of decay, with block codeword length, of the decoding error probability. Even for a DMC without states, the reliability function is not fully characterized for all rates below channel capacity. Our main contributions are the following. First, we provide a strong converse for the capacity of the Gelfand-Pinsker DMC model, that is of independent interest. Second, using this strong converse, we obtain an upper bound for the reliability function; the later constitutes a line of attack described earlier (see, for instance, [2]). Instrumental in the proofs of both is a new technical result which provides an upper bound on the rate of codes with codewords that are conditionally typical over large *message dependent* subsets of a typical set of state sequences. This technical result is a nonstraightforward analog of [2, Lemma 2.1.4] for a DMC without states; the latter provides a bound on the rate of a good code with codewords of a fixed type.

II. Preliminaries

Consider a state dependent DMC $W : \mathcal{X} \times \mathcal{S} \to \mathcal{Y}$ with finite input, state and output alphabets $\mathcal{X}$, $\mathcal{S}$ and $\mathcal{Y}$, respectively. The $\mathcal{S}$-valued state process $\{S_t\}_{t=1}^{\infty}$ is i.i.d. with known pmf P_S. The probability law of the DMC is specified by

$$W^n(\mathbf{y} \mid \mathbf{x}, \mathbf{s}) = \prod_{t=1}^{n} W(y_t \mid x_t, s_t),$$
$$\mathbf{x} \in \mathcal{X}^n, \mathbf{s} \in \mathcal{S}^n, \mathbf{y} \in \mathcal{Y}^n.$$

We consider the Gelfand-Pinsker model [1] in which the encoder possesses perfect CSI in a noncausal manner, i.e., the entire state sequence prior to transmission. A (M, n)-code is a pair of mappings (f, ϕ) where the encoder f is a mapping

$$f : \mathcal{M} \times \mathcal{S}^n \to \mathcal{X}^n$$

with $\mathcal{M} = \{1, \ldots, M\}$ being the set of messages, while the decoder ϕ is a mapping

$$\phi : \mathcal{Y}^n \to \mathcal{M}.$$

The rate of the code is $(1/n) \log M$. The corresponding (maximum) probability of error is

$$e(f, \phi) = \max_{m \in \mathcal{M}} \sum_{\mathbf{s} \in \mathcal{S}^n} \mathrm{P}_S(\mathbf{s}) \times W^n((\phi^{-1}(m))^c \mid f(m, \mathbf{s}), \mathbf{s}) \qquad (1)$$

where $\phi^{-1}(m) = \{\mathbf{y} \in \mathcal{Y}^n : \phi(\mathbf{y}) = m\}$ and $(\cdot)^c$ denotes complement.

We restrict ourselves to the situation where the receiver has no CSI. When the receiver, too, has (full) CSI, our results apply in a standard manner by considering an associated DMC with augmented output alphabet $\mathcal{Y} \times \mathcal{S}$.

Definition 1. Given $0 < \epsilon < 1$, a number $R > 0$ is ϵ-achievable if for every $\delta > 0$ and for all n sufficiently large, there exist (M, n)-codes (f, ϕ) with $(1/n) \log M > R - \delta$ and $e(f, \phi) < \epsilon$; R is an achievable rate if it is ϵ-achievable for all $0 < \epsilon < 1$. The supremum of all achievable rates is the capacity C of DMC.

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For a random variable U with values in a finite set $\mathcal{U}$, let $\mathcal{P}$ denote the set of all pmfs P_{USXY} on $\mathcal{U} \times \mathcal{S} \times \mathcal{X} \times \mathcal{Y}$ with

$$X = h(U, S) \tag{2}$$

for some mapping h,

$$U -\!\!\circ\!\!- S, X -\!\!\circ\!\!- Y, \tag{3}$$

$$P_{Y|X,S} = W. \tag{4}$$

As is well-known [1]

$$C = \max_{\mathcal{P}} I(U \wedge Y) - I(U \wedge S).$$

When the receiver, too, has (full) CSI it is known [3] that

$$C = \max_{P_{X|S}} I(X \wedge Y \mid S).$$

Definition 2. The *reliability function* $E(R)$, $R \geq 0$, of the DMC W with noncausal CSI, is the largest number $E \geq 0$ such that for every $\delta > 0$ and for all sufficiently large n, there exist n-length block codes (f, ϕ) as above of rate greater than $R - \delta$ and $e(f, \phi) \leq \exp\left[-n(E - \delta)\right]$ (see for instance [2]).

For a given pmf $\tilde{\mathrm{P}}_{SX}$ on $\mathcal{S} \times \mathcal{X}$, denote by $\mathcal{P}(\tilde{\mathrm{P}}_{SX}, W)$ the subset of $\mathcal{P}$ with $\mathrm{P}_{SX} = \tilde{\mathrm{P}}_{SX}$.

III. Statement of Results

An upper bound for the reliability function $E(R)$, $0 < R < C$, of a DMC without states, is derived in [2] using a strong converse for codes with codewords of a fixed type. For a state dependent DMC with *causal* CSI at the transmitter and no receiver CSI, a strong converse is given in [3]. An analogous result is not available for the case of noncausal transmitter CSI. For the latter situation, the following key lemma serves, in effect, as an analog of [2, Corollary 2.1.4] and gives an upper bound on the rate of codes with codewords that are conditionally typical over large *message dependent* subsets of the typical set of state sequences. We note that a direct extension of [2, Corollary 2.1.4] would have entailed a claim over a subset of typical state sequences *not depending* on the transmitted message; however, its validity is unclear.

For a DMC without states, the result in [2, Corollary 2.1.4] provides, in effect, an image size characterization of a good codeword set; this does not involve any auxiliary rv. In the same spirit, our key technical lemma below provides an image size characterization for good codeword sets for the noncausal DMC model, which now involves an auxiliary rv.

Lemma 1. *Let $\epsilon, \tau > 0$ be such that $\epsilon + \tau < 1$. Given a pmf $\tilde{\mathrm{P}}_S$ on S and conditional pmf $\tilde{P}_{X|S}$, let (f, ϕ) be a (M, n)-code as above. For each $m \in \mathcal{M}$, let $A(m)$ be a subset of $\mathcal{S}^n$ which satisfies the following conditions*

$$A(m) \subseteq \mathcal{T}^n_{[\tilde{\mathrm{P}}_S]}, \tag{5}$$

$$\|A(m)\| \geq \exp\left[n\left(H(\tilde{\mathrm{P}}_S) - \frac{\tau}{6}\right)\right], \tag{6}$$

$$f(m, \mathbf{s}) \in \mathcal{T}^n_{[\tilde{\mathrm{P}}_{X|S}]}(\mathbf{s}), \quad \mathbf{s} \in A(m). \tag{7}$$

Furthermore, let (f, ϕ) satisfy one of the following two conditions

$$W^n(\phi^{-1}(m) \mid f(m, \mathbf{s}), \mathbf{s}) \geq 1 - \epsilon, \quad \mathbf{s} \in A(m), \tag{8a}$$

$$\frac{1}{\|A(m)\|} \sum_{\mathbf{s} \in A(m)} W^n(\phi^{-1}(m) \mid f(m, \mathbf{s}), \mathbf{s}) \geq 1 - \epsilon. \tag{8b}$$

Then, for[1] $n \geq N(\|\mathcal{X}\|, \|\mathcal{S}\|, \|\mathcal{Y}\|, \tau, \epsilon)$*, it holds that*

$$\frac{1}{n} \log M \leq I(U \wedge Y) - I(U \wedge S)$$

where $\mathrm{P}_{USXY} \in \mathcal{P}(\tilde{\mathrm{P}}_S \tilde{\mathrm{P}}_{X|S}, W)$.

This lemma plays an instrumental role in proving the following two main results.

Theorem 2. *(Strong Converse) Given $0 < \epsilon < 1$ and a sequence of (M_n, n) codes (f_n, ϕ_n) with $e(f_n, \phi_n) < \epsilon$, it holds that*

$$\limsup_n \frac{1}{n} \log M_n \leq C.$$

Theorem 3. *(Sphere Packing Bound) Given $\delta > 0$, for $0 < R < C$, it holds that*

$$E(R) \leq E_{SP}(1 + \delta) + \delta,$$

where

$$E_{SP} = \min_{\tilde{\mathrm{P}}_S} \max_{\tilde{\mathrm{P}}_{X|S}} \min_{V \in \mathcal{V}(R, \tilde{\mathrm{P}}_S \tilde{\mathrm{P}}_{X|S})} \left[D(\tilde{\mathrm{P}}_S \| \mathrm{P}_S) + D(V \| W \mid \tilde{\mathrm{P}}_S \tilde{\mathrm{P}}_{X|S})\right] \tag{9}$$

with

$$\mathcal{V}(R, \tilde{\mathrm{P}}_{SX}) = \left\{V : \mathcal{X} \times \mathcal{S} \to \mathcal{Y} : \max_{P_{USXY} \in \mathcal{P}(\tilde{\mathrm{P}}_{SX}, V)} I(U \wedge Y) - I(U \wedge S) < R\right\}.$$

Remark 1. For the case when the receiver, too, possesses (full) CSI, the sphere packing bound above coincides with that obtained earlier in [4] for this case.

Remark 2. In (9), the terms $D(\tilde{\mathrm{P}}_S \| \mathrm{P}_S)$ and $D(V \| W \mid \tilde{\mathrm{P}}_S \tilde{\mathrm{P}}_{X|S})$ account, respectively, for the shortcomings of a given code for corresponding "bad" state pmf and "bad" channel.

IV. Proofs of Results

We provide below the proofs of Lemma 1 and Theorems 2 and 3.

Proof of Lemma 1:

Our proof below is for the case when (8a) holds; the case when (8b) holds can be proved similarly with minor modifications. Specifically, in the latter case, we can find

[1] In our assertions, we indicate the validity of a statement "for all $n \geq N(.)$" by showing the explicit dependency of N; however the standard picking of the "largest such N" from (finitely-many) such Ns is not indicated.

subsets $A'(m)$ of $A(m)$, $m \in \mathcal{M}$, that satisfy (5)-(7) and (8a) for some $\epsilon', \tau' > 0$ with $\epsilon' + \tau' < 1$ for all n sufficiently large.

Set

$$B(m) = \{(f(m,\mathbf{s}), \mathbf{s}) \in \mathcal{X}^n \times \mathcal{S}^n : \mathbf{s} \in A(m)\}, \; m \in \mathcal{M}.$$

Let $\tilde{\mathrm{P}}_Y = \tilde{\mathrm{P}}_{SX} \circ W$ be a pmf on $\mathcal{Y}$ defined by

$$\tilde{\mathrm{P}}_Y(y) = \sum_{s,x} \tilde{\mathrm{P}}_{SX}(s,x) W(y \mid x, s), \;\; y \in \mathcal{Y}.$$

Consequently,

$$W^n(\mathcal{T}^n_{[\tilde{\mathrm{P}}_Y]} \mid f(m,\mathbf{s}), \mathbf{s}) > \epsilon + \tau, \qquad \mathbf{s} \in A(m), \tag{10}$$

for all $n \geq N(\|\mathcal{X}\|, |\mathcal{S}\|, |\mathcal{Y}\|, \tau, \epsilon)$ (not depending on m and $\mathbf{s}$ in $A(m)$). Denoting

$$C(m) = \phi^{-1}(m) \cap \mathcal{T}^n_{[\tilde{\mathrm{P}}_Y]},$$

we see from (8a) and (10) that

$$W^n(C(m) \mid f(m,\mathbf{s}), \mathbf{s}) > \tau > 0, \qquad (f(m,\mathbf{s}), \mathbf{s}) \in B(m),$$

so that

$$\|C(m)\| \geq g_{W^n}(B(m), \tau),$$

where $g_{W^n}(B(m), \tau)$ denotes the smallest cardinality of a subset D of $\mathcal{Y}^n$ with

$$W^n(D \mid (f(m,\mathbf{s}), \mathbf{s})) > \tau, \qquad (f(m,\mathbf{s}), \mathbf{s}) \in B(m). \tag{11}$$

With $m_0 = \arg\min_{1 \leq m \leq M} \|C(m)\|$, we have

$$M\|C(m_0)\| \leq \sum_{m=1}^{M} \|C(m)\| = \|\mathcal{T}^n_{[\tilde{\mathrm{P}}_Y]}\| \leq \exp n\bigg(H(\tilde{\mathrm{P}}_Y) + \frac{\tau}{6}\bigg).$$

Consequently,

$$\frac{1}{n}\log M \leq H(\tilde{\mathrm{P}}_Y) + \frac{\tau}{6} - \frac{1}{n}\log \mathsf{g}_{\mathsf{W}^n}(B(m_0), \tau). \tag{12}$$

Define a stochastic matrix $V : \mathcal{X} \times \mathcal{S} \to \mathcal{S}$ with

$$V(s' \mid x, s) = \mathbf{1}(s' = s),$$

and let $\mathsf{g}_{\mathsf{V}^n}$ be defined in a manner analogous to $\mathsf{g}_{\mathsf{W}^n}$ above with $\mathcal{S}^n$ in the role of $\mathcal{Y}^n$ in (11). For any $m \in \mathcal{M}$ and subset E of $\mathcal{S}^n$, observe that

$$V^n(E \mid f(m,\mathbf{s}), \mathbf{s}) = \mathbf{1}(s \in E), \qquad \mathbf{s} \in \mathcal{S}^n.$$

In particular, if E satisfies

$$V^n(E \mid f(m,\mathbf{s}), \mathbf{s}) > \tau, \qquad \mathbf{s} \in A(m), \tag{13}$$

it must be that $A(m) \subseteq E$, and since $E = A(m)$ satisfies (13), we get that

$$\|A(m)\| = \mathsf{g}_{\mathsf{V}^n}(B(m), \tau) \tag{14}$$

using the definition of $B(m)$. Using the image size characterization [2, Theorem 3.3.11], there exists an auxiliary rv U and associated pmf $\mathrm{P}_{USXY} = \mathrm{P}_{U|SX}\tilde{\mathrm{P}}_{SX}W$ such that

$$\left|\frac{1}{n}\log \mathsf{g}_{\mathsf{V}^n}(B(m_0), \tau) - H(S|U) - t\right| < \frac{\tau}{6},$$
$$\left|\frac{1}{n}\log \mathsf{g}_{\mathsf{W}^n}(B(m_0), \tau) - H(Y|U) - t\right| < \frac{\tau}{6}, \tag{15}$$

where $0 \leq t \leq \min\{I(U \wedge Y), I(U \wedge S)\}$. Then, using (12), (14), (15) we get

$$\frac{1}{n}\log M \leq I(U \wedge Y) + H(S \mid U) - \frac{1}{n}\log \|A(m_0)\| + \frac{\tau}{2},$$

which by (6) yields

$$\frac{1}{n}\log M \leq I(U \wedge Y) - I(U \wedge S) + \tau.$$

In (15), P_{USXY} belongs to $\mathcal{P}(\tilde{\mathrm{P}}_S\tilde{\mathrm{P}}_{X|S}, W)$ but need not satisfy (2). Finally, the asserted restriction to $\mathrm{P}_{USXY} \in \mathcal{P}(\tilde{\mathrm{P}}_S\tilde{\mathrm{P}}_{X|S}, W)$ follows from the convexity of $I(U \wedge Y) - I(U \wedge S)$ in $\mathrm{P}_{X|US}$ for a fixed P_{US} (as observed in [1]). ■

Proof of Theorem 2:

Given $0 < \epsilon < 1$ and a (M,n)-code (f, ϕ) with $e(f, \phi) \leq \epsilon$, the proof involves the identification of sets $A(m)$, $m \in \mathcal{M}$, satisfying (5)-(7) and (8a). The assertion then follows from Lemma 1. Note that $e(f, \phi) \leq \epsilon$ implies

$$\sum_{\mathbf{s} \in \mathcal{S}^n} \mathrm{P}_S(\mathbf{s}) W^n(\phi^{-1}(m) \mid f(m,\mathbf{s}), \mathbf{s}) \geq 1 - \epsilon$$

for all $m \in \mathcal{M}$. Since $\mathrm{P}_S\left(\mathcal{T}^n_{[\mathrm{P}_S]}\right) \to 1$ as $n \to \infty$, we get that for every $m \in \mathcal{M}$,

$$\mathrm{P}_S\left(\left\{\mathbf{s} \in \mathcal{T}^n_{[\mathrm{P}_S]} : W^n(\phi^{-1}(m) \mid f(m,\mathbf{s}), \mathbf{s}) > \frac{1-\epsilon}{2}\right\}\right) \geq \frac{1-\epsilon}{3} \tag{16}$$

for all $n \geq N(\|\mathcal{S}\|, \epsilon)$. Denoting the set $\{\cdot\}$ in (16) by $\hat{A}(m)$, clearly for every $m \in \mathcal{M}$,

$$W^n(\phi^{-1}(m) \mid f(m,\mathbf{s}), \mathbf{s}) \geq \frac{1-\epsilon}{2}, \qquad \mathbf{s} \in \hat{A}(m),$$

and

$$\mathrm{P}_S\left(\hat{A}(m)\right) \geq \frac{1-\epsilon}{3}$$

for $n \geq N(\|\mathcal{S}\|, \epsilon)$, whereby for an arbitrary $\delta > 0$, we get

$$\|\hat{A}(m)\| \geq \exp\left[n(H(\mathrm{P}_S) - \delta)\right]$$

for $n \geq N(\|\mathcal{S}\|, \delta)$. Partitioning $\hat{A}(m)$, $m \in \mathcal{M}$, into sets according to the (polynomially many) conditional types of $f(m,\mathbf{s})$ given $\mathbf{s}$ in $\hat{A}(m)$, we obtain a subset $A(m)$ of $\hat{A}(m)$ for which

$$f(m,\mathbf{s}) \in \mathcal{T}^n_m(\mathbf{s}), \qquad \mathbf{s} \in A(m),$$
$$\|A(m)\| \geq \exp\left[n(H(\mathrm{P}_S) - 2\delta)\right],$$

for $n \geq N(\|\mathcal{S}\|, \|\mathcal{X}\|, \delta)$, where $\mathcal{T}_m^n(\mathbf{s})$ represents a set of those sequences in $\mathcal{X}^n$ that have the same conditional type (depending only on m).

Once again, the polynomial size of such conditional types yields a subset $\mathcal{M}'$ of $\mathcal{M}$ such that $f(m, \mathbf{s})$ has a fixed conditional type (not depending on m) given $\mathbf{s}$ in $A(m)$, and with

$$\frac{1}{n} \log \|\mathcal{M}'\| \geq \frac{1}{n} \log M - \delta$$

for all $n \geq N(\|\mathcal{S}\|, \|\mathcal{X}\|, \delta)$. Finally, the strong converse follows by applying Lemma 1 to the subcode corresponding to $\mathcal{M}'$ and noting that $\delta > 0$ is arbitrary. ■

Proof of Theorem 3:

Consider sequences of type $\tilde{\mathrm{P}}_S$ in $\mathcal{S}^n$. Picking $\hat{A}(m) = \mathcal{T}^n_{\tilde{\mathrm{P}}_S}$, $m \in \mathcal{M}$, in the proof of Theorem 2, and following the arguments therein to extract the subset $A(m)$ of $\hat{A}(m)$, we have for a given $\delta > 0$ that for $n \geq N(\|\mathcal{S}\|, \|\mathcal{X}\|, \delta)$, there exists a subset $\mathcal{M}'$ of $\mathcal{M}$ and a fixed conditional type, say $\tilde{\mathrm{P}}_{X|S}$ (not depending on m), such that for every $m \in \mathcal{M}'$,

$$\begin{aligned} A(m) &\subseteq \hat{A}(m) = \mathcal{T}^n_{\tilde{\mathrm{P}}_S}, \\ \|A(m)\| &\geq \exp\left[n(H(\tilde{\mathrm{P}}_S) - \delta)\right], \\ f(m, \mathbf{s}) &\in \mathcal{T}^n_{\tilde{\mathrm{P}}_{X|S}}(\mathbf{s}), \qquad \mathbf{s} \in A(m), \\ \frac{1}{n} \log \|\mathcal{M}'\| &\geq R - \delta. \end{aligned}$$

Then for every $V \in \mathcal{V}(R, \tilde{\mathrm{P}}_S \tilde{\mathrm{P}}_{X|S})$, we obtain using Lemma 1 (in its version with condition (8b)), that for every $\delta' > 0$, there exists $m \in \mathcal{M}'$ (possibly depending on δ' and V) with

$$\frac{1}{\|A(m)\|} \sum_{\mathbf{s} \in A(m)} V^n((\phi^{-1}(m))^c \mid f(m, \mathbf{s}), \mathbf{s}) \geq 1 - \delta'$$

for all $n \geq N(\|\mathcal{S}\|, \|\mathcal{X}\|, \|\mathcal{Y}\|, \delta')$. For this m, apply [2, Theorem 2.5.3, (5.21)] with the choices

$$\begin{aligned} Z &= \mathcal{Y}^n \times A(m), \\ S &= (\phi^{-1}(m))^c \times A(m), \\ Q_1(\mathbf{y}, \mathbf{s}) &= \frac{V^n(\mathbf{y} \mid f(m, \mathbf{s}), \mathbf{s})}{\|A(m)\|}, \\ Q_2(\mathbf{y}, \mathbf{s}) &= \frac{W^n(\mathbf{y} \mid f(m, \mathbf{s}), \mathbf{s})}{\|A(m)\|}, \end{aligned}$$

for $(\mathbf{y}, \mathbf{s}) \in Z$, to obtain

$$\begin{aligned} \frac{1}{\|A(m)\|} &\sum_{\mathbf{s} \in A(m)} W^n((\phi^{-1}(m))^c \mid f(m, \mathbf{s}), \mathbf{s}) \\ &\geq \exp\left[-\frac{nD(V\|W \mid \tilde{\mathrm{P}}_{X|S} \tilde{\mathrm{P}}_S) + 1}{1 - \delta'} \right]. \end{aligned}$$

Finally,

$$\begin{aligned} e(f, \phi) &\geq \sum_{\mathbf{s} \in A(m)} \mathrm{P}_S(\mathbf{s})\, W^n\left((\phi^{-1}(m))^c \mid f(m, \mathbf{s}), \mathbf{s}\right) \\ &\geq \exp[-n(D(\tilde{\mathrm{P}}_S\|\mathrm{P}_S) \\ &\qquad + D(V\|W \mid \tilde{\mathrm{P}}_{X|S}\tilde{\mathrm{P}}_S)(1 + \delta) + \delta)] \end{aligned}$$

for $n \geq N(\|\mathcal{S}\|, \|\mathcal{X}\|, \|\mathcal{Y}\|, \delta, \delta')$, whereby it follows that

$$\begin{aligned} \limsup_n &-\frac{1}{n} \log e(f, \phi) \\ &\leq \min_{\tilde{\mathrm{P}}_S} \max_{\tilde{\mathrm{P}}_{X|S}} \min_{V \in \mathcal{V}(R, \tilde{\mathrm{P}}_S \tilde{\mathrm{P}}_{X|S})} [D(\tilde{\mathrm{P}}_S\|\mathrm{P}_S) \\ &\qquad + D(V\|W \mid \tilde{\mathrm{P}}_{X|S}\tilde{\mathrm{P}}_S)(1 + \delta) + \delta] \end{aligned}$$

for every $\delta > 0$. ■

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