ACM Transactions on Sensor Networks, accepted for publication

Measurement Based As-You-Go Deployment of Two-Connected Wireless Relay Networks

AVISHEK GHOSH, Indian Institute of Science
ARPAN CHATTOPADHYAY, Indian Institute of Science
ANISH ARORA, The Ohio State University
ANURAG KUMAR, Indian Institute of Science

Motivated by the need for impromptu or as-you-go deployment of wireless sensor networks in some situations, we study the problem of optimal sequential deployment of wireless sensors and relays along a line (e.g., a forest trail) of unknown length. Starting from the sink node (e.g., a base station), a "deployment agent" walks along the line, stops at equally spaced points ("potential" relay locations), placing relays at some of these points, until he reaches a location at which the source node (i.e., the sensor) needs to be placed, the objective being to create a multihop wireless relay network between the source and the sink. The deployment agent decides whether to place a relay or not at each of the potential locations, depending upon the link quality measurements to the previously placed relays.

In this paper, we seek to design efficient deployment algorithms for this class of problems, in order to achieve the objective of 2-connectivity in the deployed network. We ensure multi-connectivity by allowing each node to communicate with more than one neighbouring nodes. By proposing a network cost objective which is additive over the deployed relays, we formulate the relay placement problem as a Markov decision process. We provide structural results for the optimal policy, and evaluate the performance of the optimal policy via numerical exploration. Computation of such an optimal deployment policy requires a statistical model for radio propagation; we extract this model from the raw data collected via measurements in a forest-like environment. To validate the results obtained from the numerical study, we provide an experimental study of algorithms for 2-connected network deployment.

CCS Concepts: •**Networks** → **Mobile ad hoc networks;**

Additional Key Words and Phrases: Wireless sensor networks, Sequential relay placement, Measurement based impromptu deployment, As-you-go relay placement, Two-connected network, Markov Decision Process.

1. INTRODUCTION

Interconnection between wireless sensors or mobile devices and an infrastructure network (wireline) via wireless relay nodes is an important requirement, since a direct

This work was supported by (i) the Department of Electronics and Information Technology (India) and the National Science Foundation (USA) via an Indo-US collaborative project titled Wireless Sensor Networks for Protecting Wildlife and Humans, and (ii) the J.C. Bose National Fellowship (of the Govt. of India).
Avishek Ghosh and Anurag Kumar are with the Department of Electical Communication Engineering (ECE), Indian Institute of Science, Bangalore-560012, India; email: avishek.ghosh38@gmail.com, anurag@ece.iisc.ernet.in. Arpan Chattopadhyay is with INRIA-ENS, 2 rue Simone Iff, 75012 Paris; email: arpanc.ju@gmail.com. Anish Arora is with the Department of Computer Science and Engineering, The Ohio State University, Columbus, OH 43210, USA; email: anish@cse.ohio-state.edu. This work was done when Arpan Chattopadhyay was a Ph.D student in ECE Department, Indian Institute of Science, Bangalore.
This work is a significant extension of the paper, "As-you-go deployment of a 2-connected wireless relay network for sensor-sink interconnection", published in the Proceedings of IEEE International Conference on Signal Processing and Communications (SPCOM), 2014, DOI:http://dx.doi.org/10.1109/SPCOM.2014.6983982.

DOI:

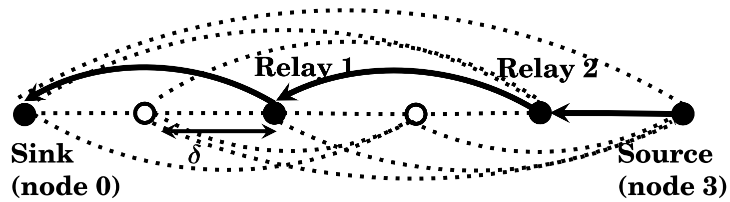

Fig. 1. Two wireless relays (shown as filled dots) deployed to connect a source to a sink along a line by a multihop path. The unfilled dots show the potential relay placement locations where measurements are made but the deployment agent decided not to place relays. The potential links between the potential placement locations are denoted by thin dashed lines, and the solid lines with arrow heads represents the actual links used in the network. The distance between two successive potential locations is the step size δ.

one-hop link from the source node to the infrastructure "base-station" may not always be feasible, either because of the distance between those two nodes or because of the poor channel quality in that particular link. This limitation of single-hop communication gives rise to the necessity of multihop communication via relay nodes. The relay nodes are battery operated and costly. Therefore, the resource constrained relays need to be placed *optimally* in the sense of placing a small number of relays in an energy conscious manner while achieving satisfactory packet transfer performance.

Ad hoc wireless networks, with static nodes, can be deployed by making exhaustive measurements between all pairs of potential node placement locations; with reference to Figure 1, this would require us to measure the qualities of all *potential* links (represented by all solid and dotted lines). Such an approach will provide the global optimal solution to the optimal relay placement problem, but, on the negative side, will require a large number of link quality measurements (of the order of the square of the number of potential node placement locations). This type of "planned" deployment, therefore, will take considerable time for carrying out a deployment.

In the light of the above difficulty in *planned* deployment, it is often desirable to deploy the network in an impromptu (or "as-you-go") fashion. One example is fast network deployment in emergency situations by firemen and commandos (see [**?**], [**?**] etc.). As-you-go deployment can be very useful for deployment over a large terrain without a precise radio map, such as a long forest trail (see [**?**], [**?**]), or if the deployment needs to be stealthy (e.g., to monitor poaching or fugitives in a forest).

Motivated by the above set of practical problems, we consider the problem of "as-you-go" deployment of relay nodes along a line of unknown length, between a sink node and a source node (see Figure 1). The transmit power required to establish a link (with a certain minimum quality) between any two nodes is modeled by a random variable capturing the effect of path-loss and shadowing. The placement decision at a point is purely based on link quality measurements from the current location to the previously placed relays (placement is done sequentially as the deployment agent walks along the line). In this work, we retain many of the assumptions made in prior literature (see [**?**] and its extended version [**?**]): (i) a single deployment agent walks along the line, starting from the sink, (ii) there are potential relay placement points at multiples of a fixed, given, distance δ (say, 10 meters) from the sink, (iii) based on link quality measurements to the already placed relays, the agent must decide whether to place a relay at a potential placement location or move on, (iv) a sensor has to be placed at an a priori unknown location that is discovered as the agent walks along the line, (experience does show that the source locations are often not precisely fixed a priori, for example, in forest monitoring application) (v) assuming a light packet rate regime, the objective of the deployment is to minimise an expected additive cost over the deployed nodes, where the cost of a deployed network is a linear combination of the sum power, sum outage and the number of relays placed.

The most relevant prior work reported in [**?**] assumes that after the line network is created, a relay is constrained to communicate only with its adjacent relays. But this assumption is a severe drawback when we consider the possibility of node or link failures in the deployed network (either due to physical damage of the nodes, or due to battery exhaustion, or due to long-term variation of link qualities in the network (such as seasonal variation of radio propagation characteristics)). Since the deployment algorithms reported in [**?**] do not consider these possibilities, any single node or link failure can turn out to be fatal to the performance of the entire network, and can even disconnect the source from the sink, causing complete network failure.

In order to alleviate these problems, in this paper we seek to design and verify measurement based as-you-go deployment algorithms that place relays in such a way that the network is K node-connected, with $K > 1$. The choice of K could be determined by a statistical characterization of the long term variations in the links. The goal, in this paper, is for the deployment agent to place nodes as he walks along a line, so as to ensure K node disjoint paths (to be formalised later) from the sensor (source) to the sink (destination). In this paper, we focus on the case where $K = 2$.

In the $K = 2$ case, while formulating the sequential decision problem for relay placement, we need to define the cost of placing a relay at a potential location. We do this by taking a linear combination of the costs of two links from the current location to two preceding nodes, and provide a method for determining the combining coefficients. Then the problem is formulated as a discounted cost or average cost Markov decision process (MDP), and structural results for the optimal policy are obtained. The techniques that we use easily extend to $K > 2$, albeit with the need to take more measurements at each decision step, and with the increase in the computational complexity of determining the optimal policy.

1.1. Related Work

Problems of "as-you-go" deployment of wireless networks are addressed by heuristic or experimental techniques in existing literature. Howard et al., in [**?**], describe an incremental deployment algorithm for a mobile sensor network. The proposed algorithm deploys nodes one at a time in an unknown environment. The deployment location is determined by using the information gathered by previously placed relays. In a somewhat similar setting, Loukas et al., [**?**], addressed the problem with dynamic localization of robots that can serve as wireless relays in emergency situations to connect wireless devices and the infrastructure network. Souryal et al., in [**?**], came up with a deployment algorithm based on an experimental study of RF link variation in an indoor setting. The heuristic algorithm proposed in the paper exploits on-site measurements that are made during the deployment process. In a survey article, Fisher et al., [**?**], describe various localization techniques for assisting emergency responders. Liu et al. ([**?**]) describe a *breadcrumbs* system (BCS) to aid fire-fighters inside buildings by communicating to the base station outside the building. [**?**] provide reliable multiuser breadcrumb system which exploits efficient and automatic co-ordination. Bao and Lee ([**?**]) consider the problem of multiple person walking in an unknown terrain and collaboratively placing relays. The objective is to maximize the area covered by them while staying connected. They propose a heuristic algorithm based on measurements between the deployed relays. Gao et al., [**?**] propose an architecture for an emergency response system relying on a self-configuring wireless mesh network for public safety. In [**?**], Naudts et al., describe the concept and implementation of a monitoring tool that helps an emergency team in deploying a network and also providing a real time overview of the status of the network.

In the literature referred to thus far in this section, many heuristic algorithms were proposed for relay placement and their performance was verified numerically or experimentally, without any optimality guarantee. There has been little effort to formulate the optimal relay placement problem rigorously, until the work by Mondal et al. ([**?**]). The authors of [**?**] took the first step towards addressing the as-you-go deployment problem on a line via an MDP formulation; they assumed a probability distribution for the unknown location of the source along the line and derived optimal relay placement policies under that. Sinha et al., in ([**?**]), extended this work by addressing the problem of impromptu relay placement along a random lattice path. Both of these papers assumed a deterministic mapping between the wireless link length and link quality; a conservative fade margin was used to account for spatial variation of link quality due to shadowing. This shortcoming was addressed by the authors of [**?**] and [**?**], who took a measurement based approach to decide the relay placement locations. The measured link quality (in terms of link *outage* probability) in their work takes care of shadowing in a wireless link, and the effect of fading is averaged out since a lot of data packets are transmitted over multiple coherence times in order to measure the quality of a link. This approach was later extended and implemented for creating a network along a forest trail (see [**?**]).

1.2. Our Contribution

1-connected networks might not be useful for a practical setting where the network deployment should be robust against node/ link failures (either due to physical damage of the nodes, or due to battery exhaustion, or due to long-term variation of link qualities in the network (such as seasonal variation of radio propagation characteristics)). In this paper, we address the problem of creating a 2-connected network that is robust to such failures. We view this paper as a significant extension of our previous papers [**?**] and [**?**]. The key differences of our current paper with [**?**] and [**?**] are the following:

— **Modeling of the wireless channel:** The radio propagation model, statistical modeling of the wireless channel, theory and experiments that yield the statistical model are explained in Appendix B. We utilize Gudmundson's model and concepts from hypothesis testing to obtain the channel parameters, namely the path loss exponent and shadowing variance. Also, experiments reported in Appendix B provide a way to fix the deployment step length used throughout the paper, and the number of packets to be transmitted for link evaluation. The radio propagation modeling with experimental validation was neither present in [**?**] nor in [**?**].

— **Formulation with sum outage, and implications for the optimal policy:** In [**?**], we consider a fixed target outage for deployment algorithms. But, as explained in Section 2.2.1, these algorithms often run into the trouble of "Deployment Failures". In order to fix this, we include outage cost in our formulation and the value iteration becomes much more complex computationally. Formulation with outage is considered in [**?**] for 1 connected networks, but again, extending it to 2 connected network brings additional complexity in value iteration and policy computation.

— **Routing on a 2-connected network:** Once the network is set up there is the issue of routing over this network, an issue that did not exist with the 1-connected design (for example in [**?**]. In the one connected network problem, every node communicates with its immediate neighbor. In two connected network, the presence of redundant links gives rise to multiple routes from the source to the sink. We formulate the net-

work optimization problem involving the co-efficients c and c_1 in Section 2.5, where we argue that, in general, c and c_1 are chosen to give relative importance to adding additional links, thereby creating redundant paths. One approach for the choice of c and c_1 could be the relative frequency of using the direct neighbor link (i.e., c) or the two-hop neighbor link (i.e., c_1) when a packet arrives at a node. As an example of such an approach, in Section 2.6, we provide a particular methodology for choosing c and c_1 in the case where routing over the resulting network is probabilistic. It turns out that, under probabilistic routing, c and c_1 can be parameterized by a single parameter p.
The simulations and experimental work described in the current paper are much more extensive because we study the effect of parameters c and c_1 on deployment; this aspect was absent in [**?**].

2. SYSTEM MODEL

Deployment is done by a *single* deployment agent along a line discretized in *steps* of length δ (see Figure 1); we call each such point on the line a *potential relay location*. The possibility of another person following the agent behind, who can learn from the measurements and actions of the first person, and supplement the actions of the preceding individual is not considered in this paper.

After the network is deployed, the sink node is denoted by Node 0, the relay closest to the sink is indexed as Node 1, and likewise the relays are enumerated by $\{1, 2, \ldots, N\}$, where N is a random number depending on the stochastic evolution of the shadowing encountered by the deployment agent. The source is called node $(N + 1)$. The link having transmitter node i and receiver node j is denoted by (i, j). Also we sometimes denote a generic link by e. The length of any link is an integer multiple of step size δ.

We first develop a channel model and define the outage probability based on the model. We then discuss the evolution of the deployment process and analyze different distance models from sink to source. After that, different network topologies for 2 connected network is discussed. Given a specific deployment process and network topology, we then formulated an objective function involving power cost outage cost and relay placement cost that the deployment algorithm minimizes.

After the deployment of the network, since 2-connected network topology provides the flexibility of redundant links and hence an opportunity of routing. Any routing could be used over this network. For example, if RPL (Routing Protocol for Low-Power and Lossy Networks, [**?**]) is used, then it would determine shortest path routes, based on whatever link metric it is programmed for. Only one special approach to routing over this network has been considered in this paper, that is probabilistic routing (Section 2.6), where, as the packet progresses from the source to the sink, at an intermediate node the one-hop previous neighbor is chosen with probability p, and the other downstream neighbor is chosen otherwise. With such a routing we suggest that coefficients c and c_1 (involved in total network cost, see Section 2.5 for details) can be taken to be the probabilities of the corresponding link being used; these probabilities are derived in terms of the parameter p.

Finally we discuss a traffic model, namely the lone packet model, which is motivated by many practical examples and argued that the deployment algorithm works under such traffic model.

2.1. Channel Model and Outage Probability

The received signal power of a packet (say k-th packet, where $k \geq 1$) for a particular link (i.e., a transmitter-receiver pair) of length r is given by:

$$P_{\mathrm{rcv,k}} = \gamma a \left(\frac{r}{r_0}\right)^{-\eta} H_k W \tag{1}$$

where γ is the transmit power, a corresponds to the path-loss at the reference distance r_0, η is the path-loss exponent, H_k denotes the realisation of the fading random variable seen by the kth packet, and W denotes the shadowing. For a given link the realization of W is fixed, whereas fading varies randomly over time. Thus, different links of length r will have different, but fixed, realisations of the shadowing random variable W, and, for each of them, their respective H_k sequences will model fading over time. As shadowing captures the spatial variation of link qualities, different links in a network observes different realizations of shadowing. The transmit power of a node is assumed to take values from a finite set $\mathcal{S}$, since practical radios can transmit only at a finite set of power levels.

Shadowing models the spatial variation of the mean path loss around the loss given by the basic power law model. The marginal distribution of the shadowing process is usually modelled as a multiplicative, log normal random variable with a typical standard deviation of $7-9$ dB. Also, shadowing is spatially uncorrelated over distances comparable to the sizes of the objects in propagation environment. Our measurements in a forest-like region inside Indian Institute of Science campus supported the log-normality of shadowing and gave a shadowing decorrelation distance of 6 meters (see Appendix B). *In this paper, i.i.d. shadowing across links is assumed*; the assumption is reasonable if the step size δ is chosen to be at least the shadowing decorrelation distance.

A link is said to be in *outage* if the received power (RSSI) for a packet falls below a given target P_{th} (e.g. $P_{th} = -88$ dBm for the popular TelosB mote to achieve 2% packet error rate (PER), see [**?**] for experimental validation of this assertion). Outage occurs because of random variation of packet RSSI values over time due to fading. Let us consider a generic wireless link (e.g., a Tx-Rx pair) of length r, shadowing realization $W = w$ and the transmit power γ. The outage probability $Q_{out}(r, \gamma, w)$ depends on fading statistics modelled as random variable H. Outage will correspond to the event $P_{rcv} \leq P_{th}$. The outage probability of the link is defined as,

$$Q_{out}(r, \gamma, w) = P(P_{rcv} \leq P_{th}) = P(\gamma.a.(\frac{r}{r_0})^{-\eta}.w.H_k \leq P_{th})$$

If we assume Rayleigh fading, H_k is exponentially distributed with parameter 1.

$$Q_{out}(r, \gamma, w) = 1 - e^{-\frac{P_{th}\cdot(\frac{r}{r_0})^{\eta}}{\gamma.a.w}} \tag{2}$$

$Q_{out}(r, \gamma, w)$ in a link can be measured by sending packets over multiple channel coherence times and measuring the fraction of packets having RSSI below P_{th}.

2.2. The Deployment Process

Starting from the sink node, the deployment agent, at each multiple of the basic "step-length" δ (i.e., at each potential relay location), measures the link outage probabilities (at all possible transmit power levels) to *some of the previous relays*. In Figure 2, the deployment agent measures link outage probabilities from its current location to the immediately previous 2 relays. The distances from the current location to the previ-

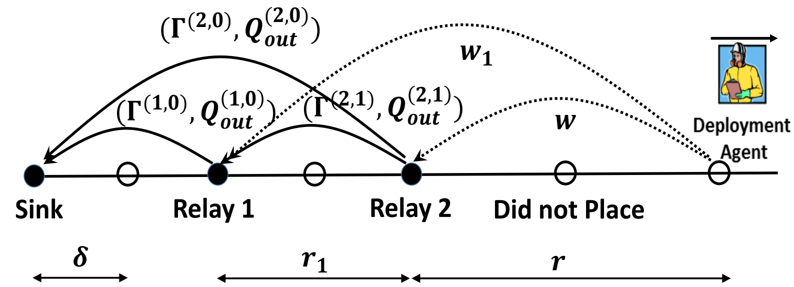

Fig. 2. From the current location (shown by the image of deployment agent), the agent measures outage probabilities at all transmit power levels to the previously placed 2 relays shown by filled circle. The circles (both filled and unfilled) represent potential locations which are δ distance apart. The unfilled circle represents potential placement locations where measurements have been made but relays are not placed. The deployment agent stops at all potential locations in order to take outage measurements. The distances from the current location to the previously placed 2 relays are r and $r + r_1$ steps respectively, with $r + r_1 \leq B$, where B is a constant. The shadowing in the corresponding links are w and w_1. The wireless links are shown using solid lines with arrow heads with transmit powers $\Gamma^{(i,i-1)}$ and $\Gamma^{(i,i-2)}$ and outage probabilities $Q_{out}^{(i,i-1)}$ and $Q_{out}^{(i,i-2)}$ for $i = \{1, 2\}$.

ously placed 2 relays are r and $r + r_1$ steps respectively with corresponding shadowing realization of w and w_1. Given his current location with respect to the relays already deployed, and given the measurements made from the current location to the previous nodes, the deployment agent decides whether or not to place a relay at that point. If the agent decides to place a relay, he also decided the power level to be used by the relay. In this process, if $r + r_1 = B$ steps (B is a parameter that is set to a particular value before deployment starts), he must place a node there. We assume that there is a single sensor (the source) that has to be placed at one of these locations; in the deployment process, if the deployment agent reaches the location where the source needs to be placed, he places the source there, and the deployment process terminates.

The distance of the source location from the sink node is random, and its realization is revealed to the deployment agent only after he discovers the source location in the process of deployment. Uncertainty in the source location would be a practical reality in applications where the need for placing a sensor at a location is realized only as the terrain is explored. As the deployment is based on on-line measurements of (random) channel qualities, the locations of the deployed relays and the number of relays placed between source and sink, N, are random.

2.2.1. Including outage in network cost. In [**?**], deployment algorithms have been developed for minimizing sum power only given a fixed outage constraint. This approach has a serious drawback. Suppose that the deployment agent has reached the B-th location (where B-th location denote the location that is $B\delta$ apart from the sink, and B denotes the transmission range of the relay under consideration) and from there the measured link quality to the two previous nodes is very poor. This can occur because log normal shadowing W has support $(0, \infty)$; it can take an arbitrarily small value. In this case, the power needed for that link to achieve the target outage may exceed the maximum available power in the mote. We call this *deployment failure*. It is interesting to compute the probability of such deployment failure in the 2-connected algorithm that we have derived. With Rayleigh fading, log-normal shadowing, target outage probability of 1%, $B = 5$, and step size $\delta = 11$ m, it turns out that the computed probability of deployment failure is 2.69%. By simulating 10,000 deployments, we found that the

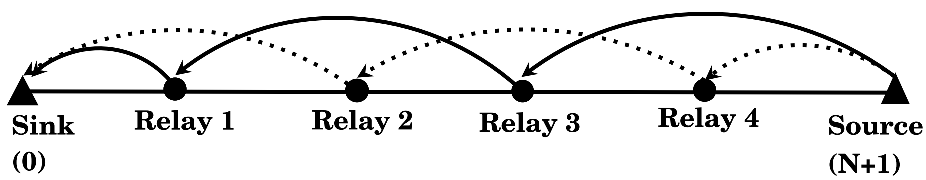

Fig. 3. The filled dots represent placed relays and they are separated by integer multiples of step size δ. The sink and source nodes are shown as node 0 and N+1 respectively, with N = 4. The figure shows a topology in which each relay, except the first, has a link to two (immediately) previous neighbors. There are 2 node disjoint paths shown with solid and thin dashed lines.

deployment failure probability for $\xi_{relay} = 0.001, 0.01,$ and 0.1 are $0.72\%, 1.24\%,$ and 1.76% respectively. To address this issue, in this paper, we have included outage as one of the objectives to minimize, and thus our algorithm is robust to deployment failure.

2.2.2. Choice of B. The upper limit B ensures that the deployment agent does not move away too far from the previous relays without placing a node. The choice of B will depend upon the statistical model parameters of the radio propagation environment and on the constraints in the deployment process. B must be chosen such that the outage probability $Q_{out}(B, \gamma, W)$ is within a tolerable limit with high probability, with the highest transmit power; otherwise the deployment algorithm might create a very long link having high outage probability.

2.3. Models for the Distance from the Sink to the Source

One possible model for the distance of the sink from the source is that the source (i.e., the sensor) is at an unknown distance $L \times \delta$ away, where $L \geq 1$ is an integer valued random variable with mean $\overline{L}$ and δ is the step length. It is well known that the geometric distribution is the maximum entropy discrete probability mass function with a given mean. Hence, one reasonable model is to take L to be geometrically distributed with mean $\frac{1}{\theta}$; i.e., $Prob(L = k) = (1-\theta)^{k-1}\theta, k \geq 1$. This means that if the line has not ended at the current location of the deployment agent, (i.e., the deployment agent has not reached the location where the source needs to be placed) it will end in the next step with probability θ and continue with probability $(1-\theta)$. If the line ends, then the source node has to be placed. Hence, given an estimate of the distance of the source from the sink, and given the value of δ, we can obtain $\overline{L}$. Then θ is obtained by setting $\frac{1}{\theta} = \overline{L}$. By using the geometric distribution, we are leaving the length of the line as uncertain as we can, given the prior knowledge of $\overline{L}$. In the analysis part of this paper, we assume $\delta = 1$ for simplicity.

It is to be noted that the mean distance $\bar{L}$ may not be known apriori. Also the length may be very large, in terms of the number of steps (typically this will be the case in forest deployment). Therefore an alternate model for L is to take the line to be of infinite length, i.e., $L = \infty$. $L = \infty$ is a mathematical realization of a long network, and permits us to get a tractable sequential decision formulation. Here the goal will be to deploy a string of relays so that the average cost of the network per unit distance is small. This model would also be useful in a situation where the line is long or when there is no information about $\overline{L}$. For multiple sources, if networks are to be deployed along multiple trails in a forest and if the trails are close together then a 2-dimensional approach would be better (though such an approach does not as yet exist). If, however, the trails are not close together and the propagation along them is homogeneous then the agent can successively deploy along them, assuming that it is one long trail.

2.4. 2-Connected Topologies

In this work we design deployment algorithm for 2 connected networks. An application of this design can be in forest monitoring, where the source to sink distance can be several hundreds of meters. In order to ensure a reasonable end-to-end packet error rate (PER), we need a network with a small number of hops (up to 5, say). Hence the hop lengths will be relatively large, and with typical transmit power levels of the radios used in these systems, it is unlikely that good links will exist between nodes that are more than two hops apart. Thus, in practice, $K = 2$ would suffice and this motivates us to design deployment algorithms only for $K = 2$.

Let us denote the set of potential locations by $V_p := \{0, 1, 2, \cdots\}$, with the sink at location 0. We assume that there is a *given* positive integer parameter B, such that there is a *potential link* between a pair of potential node locations only if the two locations are no more than B steps apart, i.e., the set of potential edges is $E_p := \{(i, j) : j < i, i - j \leq B, i \in V_p, j \in V_p\}$. The corresponding directed graph is denoted by $G_p = (V_p, E_p)$.

Given a deployment of N relays, indexed $1, 2, \cdots, N$, at the potential locations $\{\ell_1, \ell_2, \cdots, \ell_N\}$, we denote $V := \{0, \ell_1, \ell_2, \cdots, \ell_N, L\}$. Let $E \subset E_p$ denote the set of edges (on V) selected by the deployment algorithm. Consider the directed acyclic graph $G = (V, E)$. The deployment should be such that there are two node disjoint *and* edge disjoint directed paths on this graph, connecting the sensor to the sink (see Figure 3). After the deployment is over, the link whose transmitter is Node m (at location ℓ_m) and receiver is Node n (at location ℓ_n) is called link (m, n). Let $\Gamma^{(m,n)}$ denote the power used in the link (m, n). Due to random shadowing the links evaluated in the deployment process, $\Gamma^{(m,n)}$ is a random variable.

2.4.1. Two Neighbour (2N) Topologies. Consider a subgraph of G, in which for each $j, 2 \leq j \leq N$, we retain the links (j, i_1) and (j, i_2), such that $0 \leq i_2 < i_1 < j$, i.e., every node has a link with two of the earlier placed nodes. It is easy to see, and will be proved in Theorem 2.2, that each node $j, 2 \leq j \leq N + 1$, has two node disjoint and edge disjoint directed paths to the sink. The special case in which, with $j \geq 2$, it holds that $i_1 = j - 1$, and $i_2 = j - 2$ will be called *Two Nearest Neighbour (2NN) Topologies.* Figure 3 shows a 2NN topology with $N = 4$. In this paper, we select the 2NN topology. One of the motivations to choose 2NN topology over 2N topology is that in practical applications, 2N topology is rare. As explained earlier, in applications like forest monitoring, having a 3 or 4 hop wireless link is rare and thus 2NN topology is a more practical choice.

Definition 2.1. In a directed graph, a pair of nodes (s, t) is said to be K edge connected (resp., K relay connected) if the removal of any $K - 1$ arbitrary edges (resp., relays) ensures the existence of a directed (s, t) path.

THEOREM 2.2. *In a 2N topology with number of relays $N \geq 1$, the (source, sink) pair is 2 edge-connected as well as 2 relay-connected.*

PROOF. See Appendix A. □

Corollary. The results hold for a 2NN Topology which is a special case of the 2N topology.

2.5. Network Cost Structure

In this section, we develop the network cost to evaluate the performance of any policy. Let us denote the number of relays placed upto distance $x\delta$ by N_x, and $N_0 := 0$. Since the decision to place a relay is based on the measurements to the already placed relays and the path loss over a link is a random variable (owing to shadowing), $\{N_x, x \geq 1\}$ is a random process and the nodes are enumerated as $\{0, 1, 2, \ldots, N_x\}$. In 2NN topology, when a node i is placed, the deployment agent prescribes the transmit power this node should use, i.e., $\Gamma^{(i,i-1)}$ and $\Gamma^{(i,i-2)}$. The outage probabilities over link $(i, i-1)$ and $(i, i-2)$ are $Q_{out}^{(i,i-1)}$ and $Q_{out}^{(i,i-2)}$ (Figure (2)). Given two weighting coefficients c and c_1, the network cost up to distance $x\delta$ is a linear combination of three cost measures:

(1) The number of relays N_x.
(2) The weighted sum power over all links, $\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)}\right)$. This is the measure of energy required for network operation. The motivation for this is described later in Section 2.7.3.
(3) The weighted sum outage over all links, $\left(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)} + c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)}\right)$. The motivation for this measure is that, for small values of Q_{out}, the sum outage is approximately equal to the probability that a packet sent from distance $x\delta$ to the source encounters an outage along the path from the point x back to the sink (since we assume "lone packet model", there is no contention).

Note that when deciding on the placement of node i, the coefficient c multiplies the cost metric of the link to node $i-1$, whereas the coefficient c_1 multiplies the cost of the link to node $i-2$ (if $i \geq 2$). If $c = 1$ and $c_1 = 0$, the deployment objective does not care about the quality of $(i, i-2)$ link, and the problem degenerates into one in which routing is to the immediate previous relay. In such as situation, the relays might be placed too far apart for the $(i, i-2)$ links to be usable in the deployed network. On the other hand, if c_1 is positive the deployment objective seeks node placement so that the $(i, i-2)$ links are usable, and there is a reasonable compromise between the outage and power cost on these links. We provide a numerical and experimental study of the effect of the choice of these coefficients in Sections 3.7 and 4. In Section 2.6, we suggest how values of c and c_1 can be obtained if probabilistic routing is used on the deployed network.

Now, these three costs are combined linearly into one single cost measure with expectation (for policy π)

$$\min_{\pi\in\Pi}\mathbb{E}_{\pi}\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)} + \xi_{out}(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)} + c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)}) + \xi_{relay}N_x\right) \quad (3)$$

The multipliers $\xi_{relay} \geq 0$ and $\xi_{out} \geq 0$ can be interpreted as *Lagrange multipliers* for a constrained optimization problem (See Section 2.7). $\xi_{relay} \geq 0$ and $\xi_{out} \geq 0$ are viewed as an emphasis we give to outage and relay placement rate in our deployment policy. For example, if we want low outage; then we need to choose a high value of ξ_{out}. We now provide a choice of c and c_1 via probabilistic routing.

2.6. An approach for choosing c and c_1

We will provide a numerical study of the effect of choosing various values of c and c_1 in Section 3.7. Here we motivate a particular choice of c and c_1 is probabilistic routing is used on the deployed network, i.e., during network operation a relay forwards a packet to the one hop previous neighbour with probability p, and the two hop previous neighbour with probability $1-p$.

With probabilistic routing, in order to develop expressions for c and c_1 in terms of p, we consider an infinitely long network with a 2NN topology, and trace the path of a packet from the source to the sink. In this setup, consider the kth relay from the source, and define η_k to be the probability that the packet traverses this node. It can then be shown (Lemma (A.1) in Appendix A) that $\lim_{k\to\infty}\eta_k = \frac{1}{2-p}$. Thus, for large k, the probability that the link to the immediate neighbour towards the sink is used is $\frac{p}{2-p}$, whereas the probability that the other link is used is $\frac{1-p}{2-p}$. Based on this analysis we take $c = \frac{p}{2-p}$ and $c_1 = \frac{1-p}{2-p}$.

2.7. Formulation as a Sequential Decision Process

A sequential decision process is a process where at each step or iteration, based on past observation and current state, the decision maker chooses an action from the action set available to him. In the current setup, at each step from source to sink, based on the wireless channel condition, the decision maker decides whether to place a relay at that location and if he chooses to place a relay, he also decides the power level to be used by the agent. So, the deployment process is a sequential decision process. We now write the objective function that our algorithm minimizes. First, we mention the unconstrained problem, where total cost (consists of power cost, outage cost and relay cost) per step is minimized. We also show that, this is equivalent of solving a constraint optimization problem where total cost per step is minimized with a constraint on average outage cost and average relay cost.

2.7.1. The Unconstrained Problem. Motivated by the cost structure of (3), we seek to solve the following problem:

$$\inf_{\pi\in\Pi}\limsup_{x\to\infty}\frac{\mathbb{E}_\pi\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)}+c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)}+\xi_{out}(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)}+c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)})+\xi_{relay}N_x\right)}{x} \quad (4)$$

where Π is the set of stationary, deterministic policies. We formulate (4) as a long term average cost Markov decision process.

2.7.2. Connection to a Constrained Problem. We see that (4) is the relaxed version of the following constrained problem, where we seek to minimize the mean power per step subject to constraints on the mean number of relays per step and the mean outage per step:

$$\begin{aligned}
&\inf_{\pi\in\Pi}\limsup_{x\to\infty}\frac{\mathbb{E}_\pi(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)}+c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)})}{x}\\
\text{s.t.}\quad &\limsup_{x\to\infty}\frac{\mathbb{E}_\pi(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)}+c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)})}{x}\leq\overline{q}\\
\text{and}\quad &\limsup_{x\to\infty}\frac{\mathbb{E}_\pi N_x}{x}\leq\overline{N}
\end{aligned}\quad (5)$$

The following result tells us a way to choose the ξ_{out} and ξ_{relay} (see [**?**], Theorem 4.3):

THEOREM 2.3. *If there exists a pair $\xi_{out}^*\geq 0$, $\xi_{relay}^*\geq 0$ and a policy π^* for the constrained problem (5) such that π^* is an optimal policy of the unconstrained problem (4) given $(\xi_{out}^*,\xi_{relay}^*)$, and, the constraints in (5) are met with equality under π^*, then π^* will be an optimal policy for (5) also.*

The proof is provided in Appendix A.

2.7.3. A Motivation for Sum Power Objective. If all the nodes have wake-on radios, the nodes normally stay in sleep mode. A node in sleeping mode draws a very small current from the battery (see [**?**]). When a node has a packet, it sends a wake-up tone to the intended receiver and the receiver wakes up. The sender transmits the packet and the receiver sends an ACK packet in reply. Clearly, the energy spent in transmission and reception of data packets governs the lifetime of a node, because ACK size is negligible compared to the packet size.

Let t_p denote the transmission duration of a packet over a link, and node i $(1 \leq i \leq N_x)$ uses powers $\Gamma^{(i,i-1)}$ and $\Gamma^{(i,i-2)}$ during transmission to its immediate two neighbors. It is assumed that P_r is the power expended in the electronics at any receiving node for any packet. If the packet generation rate at the source, τ, is very small (so that there is no collision in the network), the lifetime of the k-th node $(2 \leq k \leq N_x)$ is $T_k := \frac{E}{\tau(c\Gamma^{(k,k-1)}+c_1\Gamma^{(k,k-2)}+P_r)t_p}$ seconds (the total energy of a fresh battery is E). For $k = 1$, the term $\Gamma^{(k,k-2)}$ is absent. Hence, the battery replacement rate in the network from the sink up to distance x steps is given by $\sum_{k=1}^{N_x} \frac{1}{T_k} = \sum_{k=1}^{N_x} \frac{\tau c \Gamma^{(k,k-1)} t_p}{E} + \sum_{k=2}^{N_x} \frac{\tau c_1 \Gamma^{(k,k-2)} t_p}{E} + \sum_{k=1}^{N_x} \frac{\tau P_r t_p}{E}$. We can absorb the term $\sum_{k=1}^{N_x} \frac{\tau P_r t_p}{E}$ into ξ_{relay}. Hence, the battery depletion rate between the sink and the point x is proportional to $c \sum_{k=1}^{N_x} \Gamma^{(k,k-1)} + c_1 \sum_{k=2}^{N_x} \Gamma^{(k,k-2)}$. Note that, this is the total transmit power to send a packet from node N_x to the sink node, since there is no collision among packets transmitted from various nodes. This is justified due to the lone packet model which we will describe in Section 2.8.

2.8. Traffic Model

In order to make the problem formulation tractable, we assume that the traffic is so light that there is only one packet in the network at a time. We call this the "lone packet model". As the traffic is very low, the transmit power over a link only depends on losses in the propagation environment, since there are no simultaneous transmissions and hence no interference. This permits us to write the total communication cost over the relays deployed as a linear combination of certain link costs (Section 2.5). Since the deployment takes account the stochastic fading and shadowing in wireless links and their effects on the number of deployed nodes and the powers they use, the assumption of "lone packet model" does not trivialize the deployment problem.

Very light traffic is a practical assumption for ad-hoc networks that carry occasional alarm packets. In [**?**], the authors designed passive infra-red (PIR) sensor platforms that can detect intrusion of a human or an animal and can classify whether a particular intrusion is a human or an animal. The data rate for this system is very low. Also, in [**?**], the authors use a duty cycle of 1.1% for a multi-hop sensor network for wildlife monitoring application. Lone packet is also realistic for industrial telemetry application ([**?**]), where successive measurements are done at large time intervals. In machine-to-machine communication, an infrequent data model is quite common also (see [**?**]). Table 1 and Table 3 of [**?**] illustrate sensors with very low sampling rate and small sized sampled data packets; it also shows data rate requirement as small as few bytes per second for habitat monitoring applications.

Although the designed network is formally designed to operate under the lone packet model, in practice, it will be able to carry some amount of positive traffic from the source to the sink while achieving acceptable quality of service. The experimental verification of this claim is found in [**?**], where a 1-connected network, deployed over a 500m long trail, in an as-you-go manner, under the assumption of the lone packet model, was able to carry 127 byte packets at a rate of 4 packets per second with end-to-end packet loss probability less than 1%.

The assumption of "lone packet model" is also valid when interference-free communication is achieved using multi-channel access (see [**?**], [**?**], [**?**], [**?**] for recent efforts to realize multi-channel access in 802.15.4 networks). It can be shown that under a certain CSMA MAC, in order to provide the desired QoS under positive traffic, it is necessary to achieve the target QoS under lone packet model (see [**?**] for proof). Our future research interest will be to provide a methodology for as-you-go deployment of relays in order to carry a given positive traffic intensity, with desired quality of service.

3. OPTIMAL DEPLOYMENT OF A 2-CONNECTED NETWORK; FORMULATION AS AN MDP

3.1. Markov Decision Process (MDP) Formulation

Here we seek to solve Problem (4). Let us recall the deployment procedure as described in Section 2.2. When the agent is r steps away from the previous node and the distance between the previous relay and the relay next to it is r_1 (see Figure (2)), ($1 \leq r + r_1 \leq B$),(B being a constant) he measures the outage probabilities from his current location to the mentioned two relays, where w, w_1 are the realizations of shadowing in the links. The agent uses the knowledge of r, r_1 and the outage probabilities to decide whether to place a node there, and the corresponding transmit power γ and γ_1 to be used. We formulate the problem as a Markov Decision Process with state space $\{1, 2, \cdots, B-1\} \times \{1, 2, \cdots, B-1\} \times \mathcal{W} \times \mathcal{W}$. Although the samples of shadowing might come from a continuous random variable, we assume that the cardinality of $\mathcal{W}$ is finite by discretizing the range. Thus the state space is finite. At state $(r, r_1, w, w_1), 1 \leq r + r_1 \leq B, w \in \mathcal{W}, w_1 \in \mathcal{W}$, the action is either to place a relay and select some transmit powers $\gamma, \gamma_1 \in \mathcal{S}$, or not to place. When $r + r_1 = B$, the only feasible action is to place and select transmit powers $\gamma, \gamma_1 \in \mathcal{S}$. When a relay is placed, a network cost of $c\gamma + c_1\gamma_1 + \xi_{out}(cQ_{out}(r, \gamma, w) + c_1 Q_{out}(r + r_1, \gamma_1, w_1)) + \xi_{relay}$ is incurred (see Section 2.5 for details). When the source is placed, the process terminates. The randomness in the system comes from the geometric distribution of the length of the line and the random shadowing in different links.

3.2. Formulation for $L \sim Geometric(\theta)$

We will first minimize the expected total cost for $L \sim Geometric(\theta)$. This formulation for $L \sim Geometric(\theta)$ is a precursor for analysis of the problem with $L = \infty$ ([**?**], Chapter 4).

Recall the definition of $\Gamma^{(m,n)}$ from Section 2.4. Consider the situation where the deployment agent placed N number of relays between the source and the sink, where the 0-th node and the $(N+1)$-st nodes are represented by sink and source respectively. The problem we seek to solve is:

$$\inf_{\pi \in \Pi} \mathbb{E}_{\pi} \left(c \sum_{i=1}^{N+1} \Gamma^{(i,i-1)} + c_1 \sum_{i=2}^{N+1} \Gamma^{(i,i-2)} + \xi_{out}(c \sum_{i=1}^{N+1} Q_{out}^{(i,i-1)} + c_1 \sum_{i=2}^{N+1} Q_{out}^{(i,i-2)}) + \xi_{relay} N \right) \quad (6)$$

where Π is the set of all stationary, deterministic, Markov policies.

Any deterministic Markov policy π is a sequence $\{\mu_k\}_{k \geq 1}$ of mappings from the state space to the action space. A deterministic Markov policy is called "stationary" if $\mu_k = \mu$ for all $k \geq 1$.

The assumption P of Chapter 3 in [**?**] is satisfied in our problem, since the single-stage costs are nonnegative (power, outage and relay costs are all nonnegative). Hence, by [**?**, Proposition 1.1.1], we can restrict ourselves to the class of stationary deterministic Markov policies.

3.3. Bellman Equation

Let us define $J(r, r_1, \gamma, \gamma_1)$ and $J(\mathbf{0})$ to be the optimal cost-to-go starting from state $(r, r_1, \gamma, \gamma_1)$ and $\mathbf{0}$ respectively. "Cost-to-go" from a state means the total expected cost incurred in the process of deployment of the remaining partial network from that state. State 0 represents the start or initial state, where the sink node is placed. As an example, the cost-to-go from the start state will be the total cost of the network. We also define $J(\mathbf{0};r)$ to be the optimal cost-to-go if a relay has been placed at the current step and the distance from the previous relay is r steps. Note that here we have an infinite horizon total cost MDP with a finite state space and finite action space. The optimal value function $J(\cdot)$ satisfies the Bellman equation ([**?**]) which is given by,

$$\begin{aligned} J(r,r_1,w,w_1) = \min\{c_p, c_{np}\}; 1 \leq r + r_1 \leq (B-1) \\ J(r, B-r, w, w_1) = c_p(r, B-r, w, w_1) \end{aligned} \tag{7}$$

where c_p and c_{np} denote the cost of placing and not placing a relay respectively. c_p and c_{np} are given by,

$$\begin{aligned} c_p(r,r_1,w,w_1) \;&=\; \min_{\gamma,\gamma_1\in\mathcal{S}}\Big(c\gamma + c_1\gamma_1 + \xi_{out}(cQ_{out}(r,\gamma,w) + c_1Q_{out}(r+r_1,\gamma_1,w_1))\Big) \\ &+\; \xi_{relay} + J(\mathbf{0};r) \end{aligned} \tag{8}$$

$$\begin{aligned} c_{np}(r,r_1,w,w_1) \;&=\; \theta\mathbb{E}_{W,W_1}\min_{\gamma,\gamma_1\in\mathcal{S}}\Big(c\gamma + c_1\gamma_1 + \xi_{out}(cQ_{out}(r+1,\gamma,W) \\ &+\; c_1Q_{out}(r+r_1+1,\gamma_1,W_1))\Big) + (1-\theta)\mathbb{E}_{W,W_1}J(r+1,r_1,W,W_1) \end{aligned} \tag{9}$$

$$\begin{aligned} J(\mathbf{0};r) = \theta\mathbb{E}_{W,W_1}\;\min\nolimits_{\gamma,\gamma_1\in\mathcal{S}}\;\Big(c\gamma + c_1\gamma_1 + \xi_{out}(cQ_{out}(1,\gamma,W) + c_1Q_{out}(r+1,\gamma_1,W_1))\Big) \\ +(1-\theta)\mathbb{E}_{W,W_1}J(1,r,W,W_1) \end{aligned} \tag{10}$$

The equations can be explained as follows. Consider that the current state is (r,r_1,w,w_1) and the line has not ended. We can either place a relay and set the power levels as γ and γ_1 or we may move on. If a relay is placed, a cost of $\min_{\gamma,\gamma_1\in S}\Big(c\gamma+c_1\gamma_1+\xi_{out}(cQ_{out}(r,\gamma,w)+c_1Q_{out}(r+r_1,\gamma_1,w_1))+\xi_{relay}\Big)$ is incurred at the current step, and the cost-to-go from the location is $J(\mathbf{0};r)$. If the relay is not placed and if the line does not end at the next step, the cost-to-go from there will be $\mathbb{E}_{W,W_1}J(r+1,r_1,W,W_1)$. If the line ends (with probability θ), a cost of $\theta\mathbb{E}_{W,W_1}\min_{\gamma,\gamma_1\in\mathcal{S}}\Big(c\gamma+c_1\gamma_1+\xi_{out}(cQ_{out}(r+1,\gamma,W)+c_1Q_{out}(r+r_1+1,\gamma_1,W_1))\Big)$ is incurred.

Unless the first relay is placed, there is only one downstream neighbour with respect to the current location and hence, the typical state in this situation is denoted by (r,w) and the "cost-to-go" from this state is denoted by $J(r,w)$.

$$J(r,w) = \min \Big\{ \min_{\gamma \in \mathcal{S}} (\gamma + \xi_{out} Q_{out}(r,\gamma,w)) + \xi_{relay} + J(\mathbf{0}; r), \tag{11}$$

$$\theta \mathbb{E}_W \min_{\gamma \in \mathcal{S}} (\gamma + \xi_{out} Q_{out}(r+1,\gamma,W)) + (1-\theta)\mathbb{E}_W J(r+1,W) \Big\}, \; r < B-1$$

$$J(B-1,w) \;=\; \min_{\gamma \in \mathcal{S}} (\xi_{relay} + \gamma + \xi_{out} Q_{out}(B,\gamma,w)) + J(\mathbf{0}; B-1)$$

$$J(\mathbf{0}) \;=\; \theta \mathbb{E}_W \min_{\gamma \in \mathcal{S}} (\gamma + \xi_{out} Q_{out}(1,\gamma,W)) + (1-\theta)\mathbb{E}_W J(1,W) \tag{12}$$

$J(\mathbf{0})$ denotes the total cost (cost to go from start state) of the discounted cost problem.

3.4. Value Iteration

The value iteration for (6) can be obtained as follows. Replace all $J(\cdot)$ in (7) to (12) by $J^{(k+1)}(\cdot)$ on the left hand side and by $J^{(k)}$ on the right hand side. Define $J^{(0)} = 0$ for all states. From standard MDP theory, $J^{(k)}(\cdot) \uparrow J(\cdot)$ as $k \to \infty$ for all states. In order to carry out the value iteration efficiently, we define,

$$V(r,r_1) = \mathbb{E}_{W,W_1} J(r,r_1,W,W_1) = \sum_{w \in \mathcal{W}} \sum_{w_1 \in \mathcal{W}} p_W(w) p_{W_1}(w_1) J(r,r_1,w,w_1)$$

where, $p_W(w)$ and $p_{W_1}(w_1)$ are probability mass function of the discretized shadowing random variable, and the product is due to the fact that the links have independent shadowing. Also for each stage, $V^k(r,r_1) = \mathbb{E}_{W,W_1} J^k(r,r_1,W,W_1)$. In the cost update equations, (Equation 7 to 12), we multiply both sides by $p_W(w)p_{W_1}(w_1)$ and sum over realizations of w and w_1. Since the sequence of $J^k(r,r_1,w,w_1)$ converges to $J(r,r_1,w,w_1)$, $V^k(r,r_1)$ also converges to some $V(r,r_1)$. Hence we need not have to iterate the value iteration over (r,r_1,w,w_1). It is sufficient to iterate over r and r_1 only, which is computationally efficient.

3.5. Policy Structure

THEOREM 3.1. *At state (r,r_1,w,w_1) ($1 \le r + r_1 \le B-1$), the optimal decision is to place a relay iff $\min_{\gamma,\gamma_1 \in \mathcal{S}}(c\gamma + c_1\gamma_1 + \xi_{out}(cQ_{out}(r,\gamma,w) + c_1 Q_{out}(r+r_1,\gamma_1,w_1))) \le c_{th}(r,r_1)$ where $c_{th}(r,r_1)$ is a threshold obtained from solving the value iteration. In this case if the decision is to place a relay, the optimal powers to be selected are given by $\arg\min_{\gamma,\gamma_1 \in \mathcal{S}}(c\gamma + c_1\gamma_1 + \xi_{out}(cQ_{out}(r,\gamma,w) + c_1 Q_{out}(r+r_1,\gamma_1,w_1))$. At state $(r, B-r, w, w_1)$, the optimal action is to place and select the powers $\arg\min_{\gamma,\gamma_1 \in \mathcal{S}}(c\gamma + c_1\gamma_1 + \xi_{out}(cQ_{out}(r,\gamma,w) + c_1 Q_{out}(B,\gamma_1,w_1)))$.*

PROOF. By Proposition 3.1.3 of [?], if we have a stationary policy such that for each state, the action chosen by the policy is the minimizer in the Bellman equation, then that stationary policy will be an optimal policy. When the state is (r,r_1,w,w_1) with $r + r_1 \le B-1$, it is optimal to place the relay if $c_p \le c_{np}$. From the definitions of c_p and c_{np} in Section 3.3, the policy structure follows. □

3.6. Formulation via Average Cost MDP: Sum Power and Sum Outage Objective

We can now proceed to solve (4). For any (ξ_{relay}, ξ_{out}), let the optimal value function of the problem (6) be denoted by $J_{(\xi_{relay},\xi_{out},\theta)}(\mathbf{0})$. By Proposition 4.1.7 of [?], the optimal policy for (4) is the same as that of (6) when θ is sufficiently close to 0 since problem (6) can be considered as infinite horizon discounted cost problem with discount factor $(1-\theta)$ and the state and action spaces are finite. Also, the optimal per-step cost λ^* of problem (4) is equal to $\lim_{\theta \to 0} \theta J_{(\xi_{relay},\xi_{out},\theta)}(\mathbf{0})$ (by Section 4.1.1 of Bertsekas [?]). As

Table I. Components of network cost, and average network cost with $c = 1/7, c_1 = 3/7$, for various values of relay cost ξ_{relay} and ξ_{out}. λ^* denotes average cost per step. Power is expressed in mW, and distance is measured in steps.

ξ_{relay}	ξ_{out}	$\overline{u}$	$(\overline{\gamma})$	$(\overline{Q}_{out})$	λ^*
0.001	1	1.1	0.0235	0.0192	0.0397
0.001	10	1.0	0.0530	0.0047	0.1010
0.01	1	1.1	0.0231	0.0195	0.0478
0.01	10	1.1	0.0760	0.0043	0.1170
0.1	1	2.0	0.0721	0.0912	0.1321
0.1	10	1.3	0.0661	0.0051	0.167

Table II. Components of network cost, and average network cost with $c = 1/3, c_1 = 1/3$, for various values of relay cost ξ_{relay} and ξ_{out}. λ^* denotes average cost per step. Power is expressed in mW, and distance is measured in steps.

ξ_{relay}	ξ_{out}	$\overline{u}$	$(\overline{\gamma})$	$(\overline{Q}_{out})$	λ^*
0.001	1	1.2	0.0225	0.0189	0.0344
0.001	10	1.1	0.0520	0.0045	0.0871
0.01	1	1.2	0.0213	0.0192	0.0417
0.01	10	1.2	0.0560	0.0041	0.0886
0.1	1	2.2	0.0702	0.0910	0.1172
0.1	10	1.6	0.0639	0.0049	0.1316

$\theta \downarrow 0$, a sequence of optimal policies are obtained, and a limit point of them will be average cost optimal policy.

3.7. Computational Examples (Deployment for minimum average cost per step)

To verify the performance of the optimal algorithm, we need a statistical modeling of the wireless channel. We obtain the channel model via extensive experiments reported in Appendix B. Motivated by the experimental results, we take the path loss factor $\eta = 4.7$, the shadowing random variable, W, to be log-normally distributed ($10\log_{10} W \sim \mathcal{N}(0, \sigma^2)$) with $\sigma = 7.7$ dB, $\delta = 11$ meters, $a = 10^{0.17}$ and $B = 5$ (i.e., the maximum length of a link is 5 steps, i.e., 55 meters; recall Section 2.1 for the channel model). The set of transmit power levels is $\{-25, -15, -10, -5, 0\}$ dBm. Since we assume deployment with TelosB motes (**[?]**), $P_{th} = -88$dBm (see **[?]** for experimental verification of this data). ξ_{out} and ξ_{relay} are varied and optimal mean power cost per relay, $\overline{\gamma}$, mean placement distance (in steps of δ) $\overline{u}$, mean outage incurred per relay ($\overline{Q}_{out}$) and the optimal average cost per step, λ^*, are computed for different combinations of c and c_1. We take $\theta = 0.00025$. We observed that $\theta J_{(\xi_{relay}, \xi_{out}, \theta)}(0)$ does not change significantly if we reduce θ further. So we used $\theta = 0.00025$ because smaller θ takes longer time for value iteration to converge. The results are tabulated in Tables I, II, III and IV.

Discussion of the Numerical Results:

(1) As one would expect, the relays are placed farther apart as the relay cost ξ_{relay} increases, and consequently the mean power per node increases.
(2) When ξ_{out} is high, we impose more importance on the outage in the link and thus the mean outage per node, $\overline{Q}_{out}$, decreases with an increase in ξ_{out}.
(3) Optimal average cost per step, λ^*, increases if ξ_{relay} and ξ_{out} are increased. This comes from the definition of λ^*.

Effect of c and c_1:

Table III. Components of network cost, and average network cost with $c = 3/5, c_1 = 1/5$, for various values of relay cost ξ_{relay} and ξ_{out}. λ^* denotes average cost per step. Power is expressed in mW, and distance is measured in steps.

ξ_{relay}	ξ_{out}	$\overline{u}$	$(\overline{\gamma})$	$(\overline{Q}_{out})$	λ^*
0.001	1	1.2	0.0212	0.0185	0.0339
0.001	10	1.2	0.0517	0.0042	0.0789
0.01	1	1.3	0.0216	0.0189	0.0388
0.01	10	1.3	0.0548	0.0041	0.0814
0.1	1	2.4	0.0702	0.0876	0.1074
0.1	10	1.6	0.0637	0.0048	0.1323

Table IV. Components of network cost, and average network cost: $c = 1, c_1 = 0$, for various values of relay cost ξ_{relay} and ξ_{out}. λ^* denotes average cost per step. Power is expressed in mW, and distance is measured in steps.

ξ_{relay}	ξ_{out}	$\overline{u}$	$(\overline{\gamma})$	$(\overline{Q}_{out})$	λ^*
0.001	1	1.3	0.0210	0.0182	0.0287
0.001	10	1.3	0.0512	0.0041	0.0693
0.01	1	1.3	0.0216	0.0187	0.0379
0.01	10	1.4	0.0497	0.0041	0.0709
0.1	1	2.6	0.0701	0.0835	0.0962
0.1	10	1.7	0.0632	0.0048	0.1187

(1) Evidently, a relatively larger value of c_1 helps to promote a network in which each deployed node $i, i \geq 2$, has a good link to the node $i-2$. Comparing across the multiple cases with different c and c_1, we observe that with relatively higher values of c_1, the relays are closer (see Tables I, II,III and IV for comparison) in order to enable workable links to two previously placed nodes.
(2) A further comparison across the two cases is made with respect to the mean power cost per link. With relatively higher values of c_1 (Table I, II), we see that there is an increase in mean power cost. In order to make the two hop link workable more power is needed, thus raising the average power cost.

4. EXPERIMENTAL RESULTS

A total of 22 TelosB motes ([**?**]) were deployed in the forest-like Jubilee Garden of the Indian Institute of Science. 11 motes were placed on each side of a trail (see Figure 4). The distance between successive motes along the trail edge (i.e., step size δ) is 11m. Each relay broadcasts 2000 packets, at each power level, while the others are quiet and take measurements to assess their link qualities *from* the transmitting node. In this manner one by one, each relay gets a turn to broadcast 2000 packets. For each transmit power level, the average received power and link outage at every other node are measured. Thus we obtained the mean RSSI (averaged over fading) for various potential links (having different lengths) at various power levels.

We apply the optimal policy for infinite horizon problem to the collected data. Given the field data, we have all measurements that can be possibly made during an actual deployment. Thus, we can use the measurements to determine the actual network that will be deployed if an agent was to walk along the trail starting from sink at location 1 (Figure 4) and the source at location 11 (110 meters). We call this "virtual" deployment of relay nodes.

From the radio propagation modeling experiment presented in Appendix B, we found that shadowing, W could be modeled as a log-normal random variable with standard

Fig. 4. A segment of the trail in the Jubilee Gardens in the Indian Institute of Science campus. Motes were mounted on trees along each side of the trail at a height of about 2 meters. The right panel shows a depiction of the deployment of 22 motes along a stretch of the trail. Several network deployments were made with this setup. All the nodes in each such network were among nodes $1, 2, \cdots, 11$ on one side of the trail (say, the "left" side) or nodes $1, 2, \cdots, 11$ on the right side of the trail (say, the "right" side). Thus, radio propagation was always "through" the foliage.

Table V. Results from experimental data: Network realization for the right side of the trail under consideration for different values of c and c_1 with $\xi_{relay} = 0.1$ and $\xi_{out} = 10$.

c	c_1	Placement Locations (Location no.)	Total Outage Cost (in mW)	Total Power Cost (in mW)	Total Cost (in mW)
1	0	2,3,5,7,9	0.0184	0.577	1.261
3/5	1/5	2,3,5,6,7,9	0.0185	0.579	1.264
1/3	1/3	2,4,5,6,7,9,10	0.0186	0.581	1.467
1/7	3/7	2,3,4,5,7,9,10	0.0189	0.583	1.472

Table VI. Results from experimental data: Network realization for the left side of the trail under consideration for different values of c and c_1 with $\xi_{relay} = 0.1$ and $\xi_{out} = 10$.

c	c_1	Placement Locations (Location no.)	Total Outage Cost (in mW)	Total Power Cost (in mW)	Total Cost (in mW)
1	0	2,3,5,7,9,10	0.0138	0.417	1.155
3/5	1/5	2,3,5,6,7,9,10	0.0139	0.418	1.257
1/3	1/3	2,3,5,7,8,9,10	0.0140	0.419	1.259
1/7	3/7	2,3,4,5,7,8,9,10	0.0141	0.421	1.362

deviation of 7.7 dB, the path-loss exponent, η, is 4.7, and the spatial de-correlation distance of W as 6m. We take the step size $\delta = 11$m and B is taken to be 5. The set of possible power transmit levels is $\mathcal{S} = \{-25, -15, -10, -5, 0\}$ (in dBm). The experiments via which the statistical parameters of the channel is obtained do not require any deployment algorithm. In Appendix B, we see that the relays are placed at regular intervals to obtain the channel parameters. Channel modeling experiment can be done along a small part of the trail and then the deployment can use the obtained channel parameters to deploy over a very long trail.

We take relay cost $\xi_{relay} = 0.1$. The choice is motivated by the fact that for lower values of ξ_{relay} such as 0.01, and 0.001, the relays are placed very often and the algorithm places relays in almost all potential locations. From Tables I to IV, we can see that the mean outage per relay is very high for $\xi_{out} = 1$ and $\xi_{relay} = 0.1$; thus leading to a network with very high end-to-end outage. On the other hand, $\xi_{out} = 10$ gives a very small and thus practical end-to-end outage. Hence we choose $(\xi_{relay}, \xi_{out}) = (0.1, 10)$ for experimental purposes.

Table VII. Comparison of optimal cost per step (λ^*, in mW) obtained from simulations based on theoretical policy computation and experiments performed in the Jubilee gardens (Fig 4). Network realization for the right side of the trail under consideration for different values of c and c_1 with $\xi_{relay} = 0.1$ and $\xi_{out} = 10$.

c	c_1	λ^* (Theoretical)	λ^* (Experimental)
1	0	0.1187	0.1261
3/5	1/5	0.1323	0.1264
1/3	1/3	0.1316	0.1467
1/7	3/7	0.1670	0.1472

In Tables V, VI we report the virtual deployment results obtained with different values of c and c_1. In Table V, we see that with a higher value of c_1 (last 2 rows of Tables V, VI) the number of nodes placed is higher over a 110m trail whereas with a relatively low value of c_1 (first 2 rows of Tables V, VI), the agent places less number of relays. In order to make the two hop link workable, the relays are placed more often when c_1 is higher. The "Total Power Cost" columns show the sum of the weighted transmitter powers over all the deployed nodes. We notice that, with an increase of c_1, the number of deployed relays is increased as well as the total power. This is expected because the inter-relay distances are less for deployment with higher c_1. That, in essence, is the additional operational cost we pay for the increase in path redundancy.

We now compare the experimental average cost per step with the simulated average cost per step to show that the experimental results are consistent with theoretical findings. We observe that under different c and c_1, both the cost components are close. The results are tabulated in Table VII.

5. CONCLUSION AND FUTURE WORK

We have provided an approach for measurement-based as-you-go deployment of a 2-connected wireless relay network along a line, to connect a sensor with a sink, so as to carry very light traffic. The deployment problem was formulated as a Markov decision process and policy structures were obtained. Computational and experimental experience was reported to illustrate the performance of such networks; we found that at an expense of a small increase in network cost, path redundancy can be incorporated in the deployed network, thus rendering the network robust to link failures.

We propose to take care of the following issues as a part of our future work. (i) The computation of the optimal deployment policy involves solving an MDP; the network propagation parameters (e.g., η, σ etc.) are required to obtain the transition structure of this MDP. But, in practice these parameters might not be accurately known to the deployment agent. Hence, we need to design a learning algorithm for the impromptu deployment of 2-connected network (in [**?**], a learning algorithm is developed for the one connected problem). (ii) Also developing a model for long-term variation in the wireless propagation environment (e.g., seasonal variations in the foliage in a forest setting) could be used in the deployment algorithm itself, thereby providing robustness to long term variations in the propagation characteristics of the deployment environment. (iii) Innovative ways to use the redundant downlink neighbors, perhaps using *physical layer* techniques would also be of interest. (iv) The network was designed to carry lone packet traffic; while the networks so obtained do permit the carrying of some positive traffic rate without violating QoS, a design technique with given positive traffic carrying capability is also of interest.

APPENDIX

A. SYSTEM MODEL

Proof of Theorem 2.2:
Two edge connectivity is immediate, as the minimum (source, sink) edge cut is of size two. The max-flow-min-cut theorem provides the conclusion that the (source, sink) pair is two edge-connected.

We turn to establishing 2 relay-connectivity between the source and the sink. For $N \geq 1$, a relay $i \in V$ and its corresponding edges are removed from the directed acyclic graph G. Take any one of the paths that pass through node i and, on this path, let node j be the node previous to node i, i.e., $i < j$. We will argue that there exists a path from j to the sink that bypasses i.

If j has a downstream neighbor i' such that $i' < i$, we are done. Suppose $i' > i$, and thus $i < i' < j$. We can argue similarly for the node i' instead of the node j and end up with a node i'' such that $i < i'' < i'$. Proceeding in this manner, we either end up with a path bypassing i or with the node i^* such that $i^* = i + 1$. Since i^* has 2 previous (downstream) neighbors, it is guaranteed that there exists a path that bypasses i. Hence the (source,sink) pair is 2 relay-connected.

Proof of Theorem 2.3:
By the hypotheses about π^*, for all $\pi \in \Pi$,

$$\limsup_{x\to\infty} \frac{\mathbb{E}_{\pi^*}\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)} + \xi^*_{out}(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)} + c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)}) + \xi^*_{relay}N_x\right)}{x}$$

$$\leq \limsup_{x\to\infty} \frac{\mathbb{E}_{\pi}\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)} + \xi^*_{out}(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)} + c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)}) + \xi^*_{relay}N_x\right)}{x}$$

$$\Rightarrow \limsup_{x\to\infty} \frac{\mathbb{E}_{\pi^*}\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)}\right)}{x} \leq \limsup_{x\to\infty} \frac{\mathbb{E}_{\pi}\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)}\right)}{x}$$
$$+\xi^*_{out}\left(\limsup_{x\to\infty} \frac{\mathbb{E}_{\pi}\left(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)} + c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)}\right)}{x} - \bar{q}\right) + \xi^*_{relay}\left(\limsup_{x\to\infty}\frac{\mathbb{E}_{\pi}N_x}{x} - \bar{N}\right)$$

Now we restrict ourselves to π such that,

$$\limsup_{x\to\infty} \frac{\mathbb{E}_{\pi}(c\sum_{i=1}^{N_x}Q_{out}^{(i,i-1)} + c_1\sum_{i=2}^{N_x}Q_{out}^{(i,i-2)})}{x} \leq \overline{q}$$
$$\limsup_{x\to\infty}\frac{\mathbb{E}_{\pi}N_x}{x} \leq \overline{N}$$

It follows that,

$$\limsup_{x\to\infty} \frac{\mathbb{E}_{\pi^*}\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)}\right)}{x} \leq \limsup_{x\to\infty} \frac{\mathbb{E}_{\pi}\left(c\sum_{i=1}^{N_x}\Gamma^{(i,i-1)} + c_1\sum_{i=2}^{N_x}\Gamma^{(i,i-2)}\right)}{x}$$

Thus we conclude that π^* is optimal for constrained problem as well.

LEMMA A.1. *For probabilistic routing in an infinite node 2NN network,* $\lim_{k\to\infty}\eta_k = \frac{1}{2-p}$.

PROOF. Once the network is deployed, let us reverse the point of view and consider the source as being at the origin. In Figure 3, the sink node is enumerated as 0. The deployed relay nodes are denoted by $1, 2, \ldots$ and for a long network the source will be at infinity. The routing will be from source to sink. When we say, "reverse the point of view", we mean to alter the enumeration of source and sink. In the reverse view, the source (which was at location infinity earlier) will be at location 0. The node immediately next (left) to the source will be enumerated by 1 and so on.

Consider a packet being launched from the source, and let η_k denote the probability that the packet "hits" the kth node (indexed from the source). It follows that

$$\eta_k = p\eta_{k-1} + (1-p)\eta_{k-2} \tag{13}$$

Clearly, $\eta_0 = 1$, $\eta_1 = p$. We take z-transform on both sides of (13). After rearranging, taking the inverse z-transform and taking the limit $k \to \infty$, $\lim_{k\to\infty} \eta_k = \frac{1}{2-p}$. □

B. RADIO PROPAGATION MODELING

All our experiments were conducted on a trail in the forest-like Jubilee Gardens in the Indian Institute of Science campus (see Figure 4(a)). Our experiments were conducted by placing the wireless devices on the edge of the trail so that the line-of-sight between the nodes passed through the foliage.

B.1. Modeling of Path-Loss and Shadowing

We kept the transmitter fixed and placed 9 receivers along the trail at distances $50, 53, 56, \ldots, 74$ meters respectively from the transmitter, and measured the mean received power (averaged over fading) at all receiving nodes. We repeated this with varying the transmitter location 25 times, thereby obtaining 25 realizations of the network with 9 links of length $50, 53, 56, \cdots, 74$ meters in each realization (we chose the link lengths at least 50 meters because in reality the step size will be at least tens of meters). Under a given network realization, for the i-th link of length r_i meters and shadowing realization ν_i dB, the mean received power in dBm (averaged over fading; see Section 2 for channel model):

$$\phi_i = \phi_0 - 10\eta \log\left(\frac{r_i}{r_0}\right) + \nu_i \,, 1 \le i \le 9 \tag{14}$$

where ϕ_0 is the mean received power (in dBm) at distance r_0.

B.1.1. Estimation of η, σ and the Shadowing Decorrelation Distance. According to Gudmundson's model [**?**], covariance between shadowing in two different links with one end fixed and the other ends on the same line at a distance d from each other can be modeled by $R_X(r_i, r_j) = \sigma^2 \exp(-d/D)$ where σ denotes standard deviation (in dB) of shadowing random variables, and D is a constant. Let $\theta := [\phi_0 \;\; \eta \;\; D \;\; \sigma^2]$. Define ν_i^k to be the shadowing random variable for the link from the transmitter to node i for the k-th realization of the network, where $1 \le i \le 9$ and $1 \le k \le 25$. Assuming that $\nu^{\mathbf{k}} := [\nu_1^k \;\; \nu_2^k \ldots \nu_M^k]'$ is jointly Gaussian with covariance matrix denoted by $\mathbf{C}(\theta)$ (elements of this matrix are determined by Gudmundson's model), and $\nu^{\mathbf{k}}$ is i.i.d. across k, we calculate the maximum likelihood estimate $\hat{\theta}_{MLE}$: $\hat{D}_{MLE} = 2.6$ meters, $\hat{\sigma}_{MLE} = 7.7$ dB, $\hat{\eta}_{MLE} = 4.7$. The correlation coefficient of shadowing between two links is less than 0.1 beyond $2.3D$ distance, which implies that beyond 5.98 meters the shadowing can be safely assumed to be independent. Hence, we need $\delta \ge 6$ m.

B.1.2. Binary Hypothesis Based Approach to find the Shadowing Decorrelation Distance. The sample correlation coefficient $\hat{\rho}(r)$ between shadowing of all pairs of links whose transmitter is common and the receivers are r distance apart from each other is computed as a function of r. We want to decide whether the shadowing losses over two links with a common receiver but whose transmitters are separated by distance r are correlated. Define the null Hypotheses $H_0 : \rho = 0$ and the alternate Hypotheses $H_1 : \rho \neq 0$. For a target false alarm probability $\alpha = 0.05$ (called the significance level of the test), it turns out that we need $\hat{\rho}(r) \leq 0.34$, which requires $r \geq 3$ meters. *Hence, under the jointly lognormal shadowing assumption, shadowing is independent beyond* 3 *meters.*

If we take the step size δ to be 6m, it satisfies the condition of Appendix B.1.1 and B.1.2 for independent shadowing across links. Motivated by the virtual deployment experiment reported in Section 4, we take $\delta = 11$m during numerical work (Section 3.7). In practical scenarios, the step size will typically be $20 - 50$m which is much greater than the shadowing decorrelation distance, thus ensuring independent shadowing across links.

B.1.3. Testing Normality of Shadowing Random Variable via Non-Parametric Tests. We picked 25 links from 25 independent network realizations, and calculated their shadowing gains $\nu_i, 1 \leq i \leq 25$ from (14). Then we applied Kolmogorov-Smirnov One Sample test (see [**?**]): define the null hypothesis $\mathcal{H}_0$ to be the event that the samples are coming from $\mathcal{N}(0, \hat{\sigma}^2_{MLE})$ distribution, and $\mathcal{H}_1$ to be the event that they do not. The test accepted H_0 with level of significance 0.05. *Hence, lognormal shadowing is a good model in our setting.*

B.2. Number of packets to be transmitted for link evaluation

In the experiments, in order to measure the outage probability of a link, at a given transmit power a certain number of packets are sent and their RSSI values recorded. To arrive at the required number of packets we conducted the following experiment. Over several links in the field, 5000 packets were sent at intervals of 50 ms, and their RSSI values were recorded. We then characterize the coherence time of the fading process by modeling it as a two state process. We say that the channel is in “Bad” state when the packet RSSI falls below the mean RSSI (over packets) of the link by 20 dB, otherwise the link is in “Good” state. From the per-packet RSSI values in the 5000 packet experiment, we observed that the mean number of packet duration over which a channel remains in “Good” state is 56, i.e., 2.8 seconds, and that the mean duration of the “Bad” state is 100 ms. *Hence, we conclude that sending* 2000 *packets (*100 *seconds duration, approximately* 33 *Good-Bad cycles) is sufficient for the fading to be averaged out.*