# Optimal Deployment of Impromptu Wireless Sensor Networks 

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#### Abstract

The need for impromptu wireless networks arises in emergency situations where the team responding to the emergency, needs to deploy sensors (such as motion sensors, or even imaging sensors) and a wireless interconnection network, without any prior planning or knowledge of the terrain. In this paper, we consider a simple model for the sequential deployment of wireless relays as a person steps along a "corridor" of unknown length, so as to create a multihop network for interconnecting a sensor to be placed at the end of the corridor with a control truck standing near the entry to the corridor. Assuming low traffic and simple link-by-link scheduling, we consider the problem of minimising an end-to-end cost metric (e.g., delay or power from the sensor to the control centre) subject to a constraint on the number of relays. Two kinds of constraints are considered: the expected number of relays is bounded, or the actual number of relays is bounded. In each case, the problem is formulated as a Markov decision process. The problem of deciding whether or not to place a relay at each step is shown to be equivalent to a certain stochastic shortest path problem embedded at relay placement points. Numerical results are provided to illustrate the performance trade-offs.


## I. Introduction

Emergency teams (such as fire-fighters or commandos) often need to enter large buildings in extremely dangerous situations. Their operation could be facilitated and their own safety could be improved if these teams had at their disposal a sensor network that could be placed in the building as they move through it (see Fig. 1).

Such a network could alert them about the situation in various parts of the building (the spread of the fire, the availability of escape routes, the movement of terrorists, and hostages, etc.), and could also serve to facilitate communication between team members and between the team members and the situation management vehicles outside the building. It is impossible to have a pre-planned network in this kind of situation, and the building plans may not be readily available. So a rapidly deployable wireless network (in an unknown terrain) is necessary to handle the situation where the wireless nodes are placed in a spontaneous fashion. We refer to such networks as impromptu wireless networks.

While the concept of an impromptu wireless network for first-responders has been around at least since 2001 ([1], [2],

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Fig. 1. Depiction of an emergency team conducting an impromptu deployment of a wireless sensor network in a building.


Fig. 2. Problem studied in this paper: Impromptu placement of wireless relays along a corridor at the end of which a sensor needs to be placed.
[3], [4]), the literature comprises mainly system architectures, ad hoc algorithms, and deployment experiences. In our work, we take the first steps towards rigorously formulating and addressing the problem of optimal deployment of impromptu wireless sensor networks. The particular situation we consider is depicted in Fig. 2 where the problem is of optimal sequential relay placement $(O S R P)$, as the deployment operative walks along a corridor of unknown length, in order to provide a multi-hop wireless path for a sensor to be placed at the end of the corridor. In this paper, the traffic generated by the sensor is assumed to be so light that each packet exits the network before the next packet is generated, thus obviating the need to consider link scheduling in the problem. The corridor is modeled as being of random length, with known distribution. Hence, the number of relays that are deployed is a random variable, $N$. The objective is to minimize the average end-toend packet delay subject to a constraint on (i) $\mathbb{E} N$, called the OSRP-EN problem, and (ii) on $N$, the OSRP-N problem.

We formally describe the system model and both the formulations in Section II. Sections III and IV are devoted towards obtaining the optimal placement policies for OSRP-EN and OSRP-N, respectively, where a decision can be made at each step as the operative walks along the corridor. Analysis of the problems reveals that both the problems can be considered as stochastic shortest path problems, with decisions needing to be made only at relay placement points. This alternate approach is provided in Section V. Numerical results are presented in

Section VI and finally we end by presenting future directions in Section VII. Due to space constraints we do not provide any proofs here. Formal proofs are available in the Master's thesis of the first author [5].

## II. System Model

In the situation shown in Fig. 2, let the interval $[0, L]$ represent the corridor, with the corridor entrance being located at 0 . The control truck's location is at $-x$. $L$ is unknown (to the crew deploying the network), which can be attributed to the building plans being unavailable, smoke filled corridors (in the case of fire fighting), etc. The person deploying the network enters the corridor (at 0 ), and is assumed to take equal size steps as he walks. At each step he has to decide whether or not place a relay node. Finally at the end of the corridor a sensor has to be placed, which is then expected to send measurements to the truck (or base-station (BS)) via the multihop network created by the relays.

In this work, we assume the "lone-packet" traffic model, i.e., there can be only one packet traversing the network. In practice, this could model low rate telemetry information being generated by the sensor, at the rate of a packet every few seconds. The network deployment objective is to minimize the total delay of a packet or the total power consumed by a packet. Due to the lone-packet model, it suffices to consider a one-hop mean delay function $f_{d}(r)$ or an average power function $f_{p}(r)$ between two neighbouring relays separated by distance $r$, with the end-to-end mean delay or power being the sum of these quantities over the successive hops from the sensor to the BS. We generalize our study by considering, instead of $f_{d}(\cdot)$ or $f_{p}(\cdot)$, a general one-hop cost function $f_{c}(\cdot)$ satisfying the following conditions, (1) $f_{c}(0)>0$, (2) $f_{c}(r)$ is strictly convex and strictly increasing in $r$, and (3) for any $r$ and $\delta>0$ the difference, $f_{c}(r+\delta)-f_{c}(r)$, is strictly increasing in $r$.

Condition (1) is a natural requirement since any transmission requires at least a minimum power or incurs at least a minimum delay even if two relays are very close to each other. Condition (2) is a natural property found in the above mentioned cost functions. Condition (3) is essential for the proofs of Theorem 1 and Theorem 2.

Now, suppose $N$ relays are placed at locations $0 \leq \ell_{1}<$ $\cdots<\ell_{N}<L$. Let $\mathbf{r}=\left(r_{1}, \cdots, r_{N+1}\right)$ denote the vector of consecutive hop lengths, from left to right, i.e., $r_{1}=x+\ell_{1}$, $r_{N+1}=L-\ell_{N}$ and for $i=2,3, \cdots, N, r_{i}=\ell_{i}-\ell_{i-1}$. Then the total cost, $C(\mathbf{r})$, is the sum of one-hop costs, i.e., $C(\mathbf{r})=\sum_{i=1}^{N+1} f_{c}\left(r_{i}\right)$.

## A. Deployment Policy $\pi$ and Problem Formulation

The deployment operative moves in steps of length $\delta>0$, and $L$ is an integral multiple of $\delta$, with $L$ being unknown. In the present work we assume that $L$ is a geometric random variable with the probability of the corridor ending at a step being $p$, i.e., $\mathbb{P}(L=k \delta)=(1-p)^{k-1} p$, for $k=1,2, \cdots$.

A deployment policy $\pi$ is a mapping, from the current state, that allows the operative to decide, at the $k$-th step, whether to
place or not place a relay node, where, in general, randomization over these two actions is allowed. Let $\Pi$ represent the set of all policies. For a given policy $\pi \in \Pi$, let $\mathbb{E}_{\pi}[\cdot]$ represent the expectation operator conditioned on using policy $\pi$. Let $C$ denote the total cost incurred using policy $\pi$ and $N$ the total number of relays used. Based on the nature of the constraint on the number relays, we have considered two different problem formulations.

OSRP-EN: Optimal Sequential Relay Placement - Expected Number of Relays Constraint

$$
\begin{array}{rc}
\min _{\pi \in \Pi} & \mathbb{E}_{\pi} C \\
\text { Subject to: } & \mathbb{E}_{\pi} N \leq \rho_{\text {avg }} . \tag{1}
\end{array}
$$

Having an average relay constraint is justified when (practically) a large number of relays are available for deployment. Thus, OSRP-EN would be a useful formulation when a truck carrying a large number of relays has to deploy them successively along several stretches of road (or several trails in a forest). However, in situations such as deployment along a building corridor in an emergency situation, carrying a large number of relays is not practical, which motivates us to further consider OSRP-N where we introduce an absolute relay constraint.

OSRP-N: Optimal Sequential Relay Placement - (Absolute) Number of Relays Constraint

$$
\begin{array}{rc}
\min _{\pi \in \Pi} & \mathbb{E}_{\pi} C \\
\text { Subject to: } & N \leq \rho_{a b s} . \tag{2}
\end{array}
$$

## III. OSRP-EN

Recall OSPR-EN from (1). We will introduce a "Lagrange" multiplier, $\lambda>0$, to obtain the following unconstrained problem,

$$
\begin{equation*}
\min _{\pi \in \Pi} \mathbb{E}_{\pi} C+\lambda \mathbb{E}_{\pi} N \tag{3}
\end{equation*}
$$

We also refer to $\lambda$ as the relay price. It can be shown that a Markov deterministic policy is optimal for the unconstrained problem in (3). In general, there can be multiple such optimal policies. From such Markov deterministic policies we can generate mixing policies by first picking a deterministic optimal policy with some probability and then using that policy for the entire deployment. All such mixing policies are also optimal for (3). The following lemma relates the main problem in (1) and the unconstrained problem in (3).
Lemma 1: Given a $\rho_{a v g}$, there exists a $\lambda_{\rho_{a v g}}$ and a policy $\pi_{\lambda_{\rho_{\text {avg }}}}^{*}$, possibly mixing, such that $\mathbb{E}_{\pi_{\lambda_{\rho_{\text {avg }}}}} N=\rho_{\text {avg }}$. Such a policy is optimal for the main problem in (1).

With Lemma 1 in mind, in the remainder of this section we develop optimal Markov deterministic policies for (3). It can be seen that for each $\rho_{a v g}$ the $\lambda_{\rho_{a v g}}$ and $\pi_{\lambda_{\rho_{\text {avg }}}}$ as required by Lemma 1 can be obtained, thus solving (1).

We formulate the unconstrained problem (3) as a Markov decision process (MDP) [6]. Each step within the corridor constitutes a decision instant (or stage) where, provided the current step is not the end of the corridor, the deployment
person (henceforth referred to as the DM (decision maker)) has to decide whether to place or not place a relay node. The decision at the $k$-th step should be based on the distance between the $k$-th step and the location of the previous relay. Let this distance be denoted as $r$. Then $r$ is referred to as the state of the system. Note that $r$ is either of the form $i \delta$ (for $i=1,2, \cdots, k$, corresponding to the case where the previous relay is $i$ steps behind $)$ or $x+k \delta(x+k \delta$ is the distance of the $k$-th step to the control center and hence corresponds to the case where still no relay is placed along the corridor). The decision process ends once the DM reaches the end of the corridor (i.e., location $L$ ). Without further details about the MDP framework we straight away proceed to the Bellman equation, the solution of which yields the optimal policy.

## A. Bellman Equation

Let $J(r)$ represent the optimal cost-to-go when the state at stage $k$ is $r$ (since the corridor length is geometric the cost-to-go does not depend on the stage index $k$ ). Then the average cost of, placing a relay at the $k$-th step and proceeding optimally from the next step onwards, is

$$
\begin{equation*}
c_{p}(r)=\lambda+f_{c}(r)+p f_{c}(\delta)+(1-p) J(\delta) \tag{4}
\end{equation*}
$$

In the above expression, $\lambda+f_{c}(r)$, represents the immediate cost of placing a relay at $k . \lambda$ is the relay price while $f_{c}(r)$ is the cost of the link established between the relay placed at $k$ and the previous relay which is $r$ units behind the $k$-th step. After placing a relay at $k$ the state at step $k+1$ is $\delta$ (recall that state is the distance from the previous relay). Now, when the DM proceeds to $k+1$ the corridor could end at $k+1$ (with probability $p$ ) in which case the DM has to install the sensor node at $k+1$ and the cost of this last link is $f_{c}(\delta)$. With probability $(1-p)$ the corridor does not end at $k+1$ in which case $J(\delta)$ represents the optimal value obtained if optimal action is taken from $k+1$ onwards. Thus the term, $p f_{c}(\delta)+(1-p) J(\delta)$, in (4) represents the average future cost.

Similarly the cost of not placing a relay at the $k$-th step, $c_{n p}(r)$, can be written as

$$
\begin{equation*}
c_{n p}(r)=p f_{c}(r+\delta)+(1-p) J(r+\delta) \tag{5}
\end{equation*}
$$

There is no immediate cost incurred for not placing a relay at $k$. Thus $c_{n p}(r)$ entirely is the average future cost. When the DM proceeds to $k+1$ without placing a relay at $k$ then the state at $k+1$ is $r+\delta$. Thus, if the corridor ends at $k+1$ (with probability $p$ ) then the DM places the sensor node at $k+1$ incurring a cost of $f_{c}(r+\delta)$ (which is the cost of the last link). Otherwise (i.e., if the corridor does not end at $k+1$, the probability of which is $1-p$ ) the optimal cost-to-go from $k+1$ is $J(r+\delta)$.

Now the optimal cost-to-go, $J(r)$, can be expressed as a min of both the costs, i.e.,

$$
\begin{equation*}
J(r)=\min \left\{c_{p}(r), c_{n p}(r)\right\} \tag{6}
\end{equation*}
$$

From the above expression it is clear that, at any step $k$, if the previous relay is $r$ units away from the $k$-th step, it is optimal to place a relay at $k$ if and only if $c_{p}(r) \leq c_{n p}(r)$.

## B. Threshold Structure of the Optimal Policy

For a given relay price $\lambda$, we characterize the optimal policy in terms of an optimal placement set,

$$
\begin{equation*}
\mathcal{S}_{\lambda}=\left\{r: c_{p}(r) \leq c_{n p}(r)\right\} \tag{7}
\end{equation*}
$$

Denote the minimum element in $\mathcal{S}_{\lambda}$ as $r_{\lambda}^{*}$, i.e., $r_{\lambda}^{*}=\min \mathcal{S}_{\lambda}$. For any $r<r_{\lambda}^{*}$ it is optimal to not place a relay. However, still for any $r \geq r_{\lambda}^{*}$ it is not immediately clear if it is optimal to place a relay or not. It may be possible to have gaps (i.e., sets of the form $\left\{r_{1}, r_{1}+\delta, \cdots, r_{2}\right\}$ where $r_{\lambda}^{*}<r_{1} \leq r_{2}$ ) where it is optimal to not place. In the following theorem we prove that such gaps are not possible and hence $r_{\lambda}^{*}$ completely characterizes the placement set $\mathcal{S}_{\lambda}$.

Theorem 1: $\mathcal{S}_{\lambda}=\left\{r_{\lambda}^{*}, r_{\lambda}^{*}+\delta, \cdots\right\}$. Thus, at any step $k$, it is optimal to place a relay if and only if $r \geq r_{\lambda}^{*}$, where $r$ is the distance of the $k$-th step from the previous relay location.

Remarks: The optimal policy can be implemented as follows. At step 0 if the distance to the control centre $x<r_{\lambda}^{*}$ then move a distance of $r_{\lambda}^{*}-x$ units before placing the first relay (if the corridor ends before covering this length then terminate the decision process by placing the sensor node at the step where the corridor ends). If $x \geq r_{\lambda}^{*}$ then place the first relay at the entrance (i.e., at step 0). Until the corridor ends, keep placing subsequent relays by proceeding a distance of $r_{\lambda}^{*}$ units from the previous relay.

The following result corresponds to the intuition that if relays are more expensive then we place them farther apart.

Lemma 2: $r_{\lambda}^{*}$ is non-decreasing in $\lambda$.

## IV. OSRP-N

We again use the MDP framework to solve the absolute relay constraint problem in (2). However, here we do not consider an unconstrained problem as in (3). Instead, at step $k$, along with $r$ (distance from previous relay) we use $n$, the number of remaining relays, to decide whether to place or not place a relay. Thus, at any step $k$ the state is of the form $(n, r)$ with the initial state (i.e., state at step 0 ) being $\left(\left\lfloor\rho_{a b s}\right\rfloor, x\right)$.

Note that this approach cannot be applied to solve OSRPEN, since a policy satisfying the constraint can use any number of relays (more or less than $\rho_{a v g}$ depending on the particular realization of $L$ ), only ensuring that the average $\mathbb{E}_{\pi} N$ is less than $\rho_{a v g}$. Thus, in OSRP-EN, one cannot begin at step 0 with any fixed number of given relays (e.g., $\left\lfloor\rho_{a b s}\right\rfloor$ in this case).

Let us proceed to write the Bellman equation. Let $J_{n}(r)$ represent the optimal-cost-to-go at any step $k$ when the state is $(n, r)$. When $n=0$ (i.e., no relays remaining) the DM has no options but to walk until the end of the corridor and install the sensor node so that $J_{0}(r)=\mathbb{E}_{L^{\prime}}\left[f_{c}\left(r+L^{\prime}\right)\right]$ where $L^{\prime}$ is the remaining corridor length conditioned on the fact that the corridor has not ended until the $k$-th step. Because of the memoryless property of $L$, the distribution of $L^{\prime}$ is same as that of $L$. Next, for $n \geq 1$

$$
\begin{equation*}
J_{n}(r)=\min \left\{c_{p}(n, r), c_{n p}(n, r)\right\} \tag{8}
\end{equation*}
$$

where as before $c_{p}(n, r)$ and $c_{n p}(n, r)$ are the cost of placing and not placing a relay, respectively, when the state at any step $k$ is $(n, r)$. The expressions for these costs are,

$$
\begin{align*}
c_{p}(n, r) & =f_{c}(r)+p f_{c}(\delta)+(1-p) J_{n-1}(\delta)  \tag{9}\\
c_{n p}(n, r) & =p f_{c}(r+\delta)+(1-p) J_{n}(r+\delta) \tag{10}
\end{align*}
$$

Again, as in (7), we characterize the optimal policy in terms of the optimal placement sets, for $n \geq 1$,

$$
\begin{equation*}
\mathcal{S}_{n}=\left\{r: c_{p}(n, r) \leq c_{n p}(n, r)\right\} \tag{11}
\end{equation*}
$$

We have a theorem analogous to Theorem 1,
Theorem 2: For each $n>0$, define $r_{n}^{*}=\min \mathcal{S}_{n}$. Then $\mathcal{S}_{n}=\left\{r_{n}^{*}, r_{n}^{*}+\delta, \cdots\right\}$ 。
Remarks: Thus the optimal policy for OSRP-N is characterized by a sequence of thresholds $\left\{r_{n}^{*}: n \geq 1\right\}$, which can be used as follows. At step 0 if $x<r_{\left\lfloor\rho_{a b s}\right\rfloor}^{*}\left(\left\lfloor\rho_{a b s}\right\rfloor\right.$ is the smallest integer $\left.\leq \rho_{a b s}\right)$ then move a distance of $r_{\left\lfloor\rho_{a b s}\right\rfloor}^{*}-x$ to place the first relay (if the corridor ends at any step before this, place the sensor and terminate the decision process). If $x \geq r_{\left\lfloor\rho_{a b s}\right\rfloor}^{*}$ then place a relay at 0 . After placing a relay suppose $n \geq 1$ is the number of relays left, then proceed a distance of $r_{n}^{*}$ (provided the corridor does not end in between) to place the next relay. When $n=0$ the only option is to proceed until the corridor ends at which location the sensor is placed.

## V. Stochastic Shortest Path View

The remarks following Theorem 1 and 2 suggest that the relay placement points could be thought of as the decision epochs where the DM has to decide how many steps to proceed before placing the next relay. This alternate formulation can be viewed as a stochastic shortest path problem where the current location at which the DM has placed a relay constitutes a node. From this current node several "paths" are available to the DM, each corresponding to the number of steps the DM chooses to move before placing the next relay (which is the action set available to the DM in this formulation). Suppose the DM chooses to move $i$ steps before placing the next relay, then with probability $(1-p)^{2}$ (which is the probability that the corridor does not end within the next $i$ steps) the DM reaches the next node (i.e., the step which is $i$ steps ahead of the current one) where a relay has to be placed. With the remaining probability, $1-(1-p)^{i}$, the DM reaches the corridor end (referred to as the terminating node) where the sensor has to be placed. The objective is to find the average shortest path from the corridor entrance (referred to as the source node) to the terminating node.

Formally, considering OSRP-EN first, let $J_{\lambda}$ represent the optimal cost-to-go from any step $k$ where a relay is placed. Then, $c_{i}$, the average cost of choosing path $i$ (corresponding to the action of moving $i$ steps before placing the next relay) can be written as
$c_{i}=(1-p)^{i}\left(\lambda+f_{c}(i \delta)+J_{\lambda}\right)+\sum_{j=1}^{i}(1-p)^{j-1} p f_{c}(j \delta)$.

The first term in the above expression, $\left(\lambda+f_{c}(i \delta)+J_{\lambda}\right)$, is the cost incurred if the corridor does not end in the next $i$ steps the probability of which is $(1-p)^{i}$. Each of the remaining terms in the summation contains the cost incurred if the corridor ends in $j \leq i$ steps. The DM can also choose to not place a relay in which case the average cost is, $c_{\infty}=\sum_{j=1}^{\infty}(1-p)^{j-1} p f_{c}(j \delta)$. Finally, the Bellman equation can be written as,

$$
\begin{equation*}
J_{\lambda}=\min \left\{c_{i}: i=1,2, \cdots,=\infty\right\} \tag{12}
\end{equation*}
$$

and the optimal policy is given by

$$
\begin{equation*}
i_{\lambda}^{*}=\operatorname{argmin}\left\{c_{i}: i=1,2, \cdots,=\infty\right\} \tag{13}
\end{equation*}
$$

Thus $i_{\lambda}^{*}$ represents the optimal number of steps that the DM should move from the current relay location, before placing the next one. The following is the key result obtained through this line of analysis.

Theorem 3: The optimal policy, $i_{\lambda}^{*}$, can be alternatively characterized using the increments of the cost function $f_{c}$ as

$$
i_{\lambda}^{*}=\min \left\{i \geq 1: f_{c}((i+1) \delta)-f_{c}(i \delta)>p\left(\lambda+J_{\lambda}\right)\right\}
$$

However, the optimal policy at the corridor entrance (denoted as $i_{\lambda}^{*}(x)$ ) could be different from $i_{\lambda}^{*}$ since the previous relay (i.e., the control centre) is already $x$ units away. We have shown the following relation between $i_{\lambda}^{*}(x)$ and $i_{\lambda}^{*}$,

$$
i_{\lambda}^{*}(x)= \begin{cases}0 & \text { if }\left\lfloor\frac{x}{\delta}\right\rfloor \geq i_{\lambda}^{*}  \tag{14}\\ i_{\lambda}^{*}-\left\lfloor\frac{x}{\delta}\right\rfloor & \text { otherwise }\end{cases}
$$

where $\left\lfloor\frac{x}{\delta}\right\rfloor$ represents the distance, in number of steps, from the control centre to the entrance. Thus, if the initial distance is already greater than $i_{\lambda}^{*}$ then place a relay at the entrance before proceeding futher. Otherwise, move $i_{\lambda}^{*}-\left\lfloor\frac{x}{\delta}\right\rfloor$ number of steps before placing the first relay. Subsequent relays are placed $i_{\lambda}^{*}$ steps apart.

Finally, we have performed similar analysis for OSRP-N as well. Here, one needs to begin by defining $J_{n}$ as the optimal cost-to-go when the DM, after placing a relay, has $n$ more relays remaining with him. Let $i_{n}^{*}$ represent the corresponding optimal policy. Then, similar to Theorem 3 we have,

Theorem 4: For $n \geq 1$,

$$
i_{n}^{*}=\min \left\{i \geq 1: f_{c}((i+1) \delta)-f_{c}(i \delta)>p J_{n-1}\right\}
$$

Analogous to (14), we also have

$$
i_{n}^{*}(x)= \begin{cases}0 & \text { if }\left\lfloor\frac{x}{\delta}\right\rfloor \geq i_{n}^{*}  \tag{15}\\ i_{n}^{*}-\left\lfloor\frac{x}{\delta}\right\rfloor & \text { otherwise }\end{cases}
$$

## VI. Numerical Results

For our numerical work we consider total power minimisation, and work with a cost function of the form, $f_{c}(r)=$ $P_{m}+\gamma r^{\eta}$, where $P_{m}>0$ is the minimum power required for any transmission, $\gamma>0$ is a constant (containing the noise variance and an SNR threshold) and $\eta$ is the pathloss attenuation factor usually in the range 2 to 5 [7]. Thus $f_{c}(r)$


Fig. 3. Results for OSRP-EN problem, (a) Thresholds, $r_{\lambda}^{*}$ and $i_{\lambda}^{*}$, as functions of $\lambda$, (b) Average power vs. $\lambda$, (c) Average No. of relays used vs. $\lambda$, and (d) Performance trade-off curve showing the variation between power and number of relays used. Also depicted is the power incurred by the optimal policy for an average relay constraint of $\rho_{\text {avg }}$.
represents the power required for a transmission between two relays, separated by a distance $r$, to be successful. We have fixed $P_{m}=0.1, \gamma=10^{-2}$ and $\eta=2$. The distance to the control centre is $x=20 \mathrm{mts}$ and the step size $\delta$ is 0.5 mts . The corridor length $L$ is geometric with $p=0.002$ being the probability that the corridor will end at the next step. Thus the average corridor length is 250 mts (equivalently, 500 steps).

The results corresponding to OSRP-EN are shown in Fig. 3. We have independently solved the primary formulation in Section III and the shortest path formulation in Section V to obtain the thresholds $r_{\lambda}^{*}$ and $i_{\lambda}^{*}$, respectively. These thresholds, as functions of $\lambda$, is shown in Fig. 3(a). As expected we observe that $r_{\lambda}^{*}$ (which is in meters) is $\delta$ times $i_{\lambda}^{*}$ (which is measured in number of steps). The thresholds are increasing with $\lambda$ since at higher values of $\lambda$, valuing the number of relays used more, we tend to place fewer of them separated by larger distance. For the same reason we see that the average total power, in Fig. 3(b), is increasing in $\lambda$ while the average number of relays used, in Fig. 3(c), is decreasing. Finally in Fig. 3(d) we have shown the trade-off between average power and average number of relays used. In Fig. 3(d) we have also depicted the minimum power obtained (which is approximately around 70) for an average relay constraint of $\rho_{a v g}=10$. Although not visible on the scale of Fig. 3(a), the plot is piecewise flat. Recalling Lemma 1, the optimal policy can be obtained from Fig. 3(c) as follows. Given $\rho_{a v g}$, if there is a $\lambda_{\rho_{\text {avg }}}$ such that $\mathbb{E}_{\pi_{\lambda_{\rho_{a v g}}}} N=\rho_{\text {avg }}$ then the corresponding $r_{\lambda_{\rho_{a v g}}}^{*}$, obtained from Fig. 3(a), is optimal. On the other hand, if the value $\rho_{\text {avg }}$ falls within a jump, then an optimal policy is obtained by mixing between the two deterministic policies corresponding to the neighboring flat parts of the plot.

Similarly, for OSRP-N we have obtained the thresholds $r_{n}^{*}$


Fig. 4. Results for OSRP-N problem, (a) Thresholds, $r_{n}^{*}$ and $i_{n}^{*}$, as function of $n$ (number of remaining relays), and (b) Total power incurred by the optimal policy as a function of $\rho_{a b s}$ (absolute relay constraint).
and $i_{n}^{*}$ independently from, Section IV and the shortest path formulation in Section V. In Fig. 4(a) we have plotted these thresholds as functions of $n$ (number of remaining relays). Again as expected $r_{n}^{*}$ is $\delta$ times $i_{n}^{*}$. Interestingly we observe that for $n=1$ the threshold $r_{1}^{*}=250 \mathrm{mts}$ (equivalently, $i_{1}^{*}=$ 500 steps) which is simply the average length of the corridor. Thus, when only one relay is left, the deployment person has to move a distance of 250 mts from the previous relay location (provided the corridor does not end) before placing the last relay and then move until the corridor ends to place the sensor.

The thresholds are decreasing with $n$ implying that with more relays remaining one has to place the subsequent relays close to each other. In Fig. 4(b) we have plotted the average total power as a function of $\rho_{a b s}$ which is the number of relays the deployment person begins with from the corridor entrance. The total power decreases with $\rho_{a b s}$ so that, beginning with a larger number of relays one can expect to obtain a deployment with better performance.

## VII. Future Directions

In our ongoing and future work we propose to extend the impromptu relay placement problem from a straight line corridor to placement over a two dimensional lattice and other more general regions. We will also go beyond the lone-packet traffic model, consider radio link scheduling, and also the impromptu deployment of mesh networks (as shown in Fig. 1).

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