Optimal Hop Distance and Power Control for a Single Cell, Dense, Ad Hoc Wireless Network

Venkatesh Ramaiyan, *Member*, *IEEE*, Anurag Kumar, *Fellow*, *IEEE*, and Eitan Altman, *Fellow*, *IEEE*

Abstract—We consider a dense, ad hoc wireless network, confined to a small region. The wireless network is operated as a single cell, i.e., only one successful transmission is supported at a time. Data packets are sent between source-destination pairs by multihop relaying. We assume that nodes self-organize into a multihop network such that all hops are of length d meters, where d is a design parameter. There is a contention-based multiaccess scheme, and it is assumed that every node always has data to send, either originated from it or a transit packet (saturation assumption). In this scenario, we seek to maximize a measure of the transport capacity of the network (measured in bit-meters per second) over power controls (in a fading environment) and over the hop distance d, subject to an average power constraint. We first motivate that for a dense collection of nodes confined to a small region, single cell operation is efficient for single user decoding transceivers. Then, operating the dense ad hoc wireless network (described above) as a single cell, we study the hop length and power control that maximizes the transport capacity for a given network power constraint. More specifically, for a fading channel and for a fixed transmission time strategy (akin to the IEEE 802.11 TXOP), we find that there exists an intrinsic aggregate bit rate (Θ_{opt} bits per second, depending on the contention mechanism and the channel fading characteristics) carried by the network, when operating at the optimal hop length and power control. The optimal transport capacity is of the form $d_{opt}(\bar{P}_t) \times \Theta_{opt}$ with d_{opt} scaling as $\bar{P}_t^{\frac{1}{\eta}}$, where $\bar{P}_t$ is the available time average transmit power and η is the path loss exponent. Under certain conditions on the fading distribution, we then provide a simple characterization of the optimal operating point. Simulation results are provided comparing the performance of the optimal strategy derived here with some simple strategies for operating the network.

Index Terms—Multihop relaying, cross-layer optimization

1 Introduction

We consider a scenario in which there is a large number of stationary nodes (e.g., hundreds of nodes) confined to a small area (e.g., spatial diameter 30 m), and organized into a multihop ad hoc wireless network. We assume that traffic in the network is homogeneous and data packets are sent between source-destination pairs by multihop relaying with single user decoding and forwarding of packets, i.e., signals received from nodes other than the intended transmitter are treated as interference. A distributed multiaccess contention scheme is used in order to schedule transmissions; for example, the CSMA/CA-based distributed coordination function (DCF) of the IEEE 802.11 standard for wireless local area networks (WLANs). We assume that all nodes can decode all the contention control transmissions (i.e., there are no hidden nodes), and only one successful transmission takes place at any time in the network. In this sense, we say that we are dealing with a *single cell* scenario. We further assume that, during the exchange of contention control packets, pairs of communicating nodes are able to estimate the channel fade between them and are thus able to perform power control per transmission.

There is a natural tradeoff between using high power and long hop lengths (single hop direct transmission between the source-destination pair), versus using low power and shorter hop lengths (multihop communication using intermediate nodes), with the latter necessitating more packets to be transported in the network. The objective of the present paper is to study optimal network operation, in terms of the hop length and optimal power control (for a fading channel), when the network (described above) is used in a multihop mode. Our objective is to maximize a certain measure of network transport capacity (measured in bit-meters per second; see Section 4), subject to a network power constraint. A network power constraint determines, to a first order, the lifetime of the network.

Situations and considerations such as those that we study could arise in a dense ad hoc wireless sensor network. Ad hoc wireless sensor networks are now being studied as possible replacements for wired measurement networks in large factories. For example, a distillation column in a chemical plant could be equipped with pressure and temperature sensors and valve actuators. The sensors monitor the system and communicate the pressure and temperature values to a central controller which in turn actuates the valves to operate the column at the desired operating point. Direct communication between the sensors and actuators is also a possibility. Such installations could involve hundreds of devices, organized into a single cell ad hoc wireless network because of the physical proximity of the nodes. There would be many flows within the network and there would be multihopping. We wish to address the

- *V. Ramaiyan is with the Department of Electrical Engineering, Indian Institute of Technology Madras, Chennai 600 036, Tamil Nadu, India. E-mail: rvenkat@ee.iitm.ac.in.*
- *A. Kumar is with the Department of Electrical Communication Engineering, Indian Institute of Science, Bengaluru 560 012, Karnataka, India. E-mail: anurag@ece.iisc.ernet.in.*
- *E. Altman is with INRIA, Route des Lucioles, B.P.93, Sophia-Antipolis Cedex 06902, France. E-mail: Eitan.Altman@sophia.inria.fr.*

Manuscript received 2 Aug. 2009; revised 29 June 2011; accepted 25 Aug. 2011; published online 27 Sept. 2011.
For information on obtaining reprints of this article, please send e-mail to: tmc@computer.org, and reference IEEECS Log Number TMC-2009-08-0316.
Digital Object Identifier no. 10.1109/TMC.2011.204.

question of optimal organization of such an ad hoc network so as to maximize its transport capacity subject to a power constraint. The power constraint relates to the network lifetime and would depend on the application. In a factory situation, it is possible that power could be supplied to the devices; hence, large power would be available. In certain emergencies, "transient" sensor networks could be deployed for situation management; we use the term "transient" as these networks are supposed to exist for only several minutes or hours, and the devices could be disposable. Such networks need to have large throughputs, but, being transient networks, the power constraint could again be loose. On the other hand, sensor networks deployed for monitoring some phenomenon in a remote area would have to work with very small amounts of power, while sacrificing transport capacity. Our formulation aims at providing insights into optimal network operation in a range of such scenarios.

1.1 Preview of Contributions

We motivate the definition of the transport capacity of the network as the product of the aggregate throughput (in bits per second) and the hop distance (in meters). For random spatiotemporal fading, we seek the power control and the hop distance that jointly maximize the transport capacity, subject to a network average power constraint. For a fixed data transmission time strategy (discussed in Section 3.2), we show that the optimal power allocation function has a water-pouring form (Section 5.1). At the optimal operating point (hop distance and power control), the network throughput (Θ_{opt}, in bits per second) is shown to be a fixed quantity, depending only on the contention mechanism and fading model, but independent of the network power constraint (Section 5.2). Further, we show that the optimal transport capacity is of the form $d_{opt}(\bar{P}_t) \times \Theta_{opt}$, with d_{opt} scaling as $\bar{P}_t^{\frac{1}{\eta}}$, where $\bar{P}_t$ is the available time average transmit power, and η is the power law path loss exponent (Theorem 5.2). Finally, we provide a condition on the fading density that leads to a simple characterization of the optimal hop distance (Section 5.3). This paper is an extended version of our earlier paper [20] and includes proofs and numerical work that were not a part of [20].

1.2 Outline of the Paper

In Section 2, we discuss the single cell assumption and related literature. In Section 3, we describe the system model and in Section 4, we motivate the objective. We study the transport capacity of a single cell multihop wireless network, operating in the fixed transmission time mode, in Section 5. Numerical results are discussed in Section 6 and we conclude the paper in Section 7.

2 Motivation and Related Literature

In this context (an ad hoc wireless network), the seminal paper by Gupta and Kumar [2] would suggest that each node should communicate with neighbors as close as possible while maintaining network connectivity. This maximizes network transport capacity (in bit-meters per second), while minimizing network average power. However, for small extent dense wireless networks, it has been observed by Dousse and Thiran in [5], that if, unlike [2], a practical model of bounded received power for finite transmitter power is used, then the increasing interference with an increasing density of simultaneous transmitters is not consistent with a minimum SINR requirement at each receiver. A dense wireless network becomes interference limited and hence infinite spatial reuse becomes inefficient (see also [11]). El Gamal and Mammen, in [12], have shown that, if the transceiver energy and communication overheads at each hop are factored in, then the operating regime studied in [2] is neither energy efficient nor delay optimal. Fewer hops between the transmitter and receiver (and hence, less spatial reuse) reduce the overhead energy consumption and lead to a better throughput-delay tradeoff [12], [13]. Also, in [13], it is shown that single cell operation is optimal for large network power scenarios (single hop TDM is the optimal strategy).

While optimal operation of a dense network might suggest using some spatial reuse, coordinating the optimal number of simultaneous transmissions (in a distributed fashion), in a constrained area, is difficult and the associated time, energy, and synchronization overheads have to be accounted for. In view of the above discussion, in this paper, we have chosen to study the performance of a dense ad hoc wireless network operating as a single cell, i.e., we assume that the medium access control (MAC) is such that only one transmitter-receiver pair communicates at any time in the network (there is no spatial reuse).

We also assume that data packets can be communicated between source-destination pairs by multihop relaying (without spatial reuse in the network). Multihopping is known to be a useful strategy to minimize energy consumption (or maximize throughput) even without spatial reuse. For example, IEEE 802.16j [23] uses relays in WiMAX networks to enhance throughput and extend coverage within a cell (even without spatial reuse). Also, works that concern with bits per Joule capacity of the network [7], [15], [22] allow multihopping while simultaneously avoiding any interference in the network. The scheduler that maximizes the data transferred per Joule schedules the different end-to-end flows separately (avoiding interference and loss of bit rate). Further, multihopping strategy is used to minimize the energy consumed in transferring the data. Thus, we note that, no spatial reuse and multihopping are good design strategies especially for low network power scenarios and energy efficient applications. Hence, in this work, we study and characterize the throughput performance of a dense wireless multihop network operating as a single cell without spatial reuse. The objective is to determine the optimal operation of the network in terms of the hop distance and the channel state dependent transmit power.

Related literature. In [2], Gupta and Kumar study the transport capacity of an extended ad hoc wireless network without multipath fading. They show that the optimal strategy to maximize transport capacity is to maximize spatial reuse and multihop packets between source-destination pairs. The transport capacity of a wireless fading channel for an extended wireless network is studied in [9]. It is shown in [9] that spatial reuse and multihopping is optimal even with fading. In [12], El Gamal and Mammen derive the transport capacity of ad hoc wireless networks with communication overheads. They study the tradeoff between the data transmission power and the communication overhead power and derive the optimal spatial reuse for the network. We study a dense wireless network, whose optimal spatial reuse is finite, irrespective of the number of nodes and network power.

The asymptotic transport capacity of a dense wireless network is studied in [5] and [11]. The optimal transport capacity is derived for increasing number of nodes and network power. The transport capacity of a dense wireless sensor network is studied in [10]. The general approach in [5], [11], and [10] is to consider a random deployment of nodes and characterize their average performance. Our approach to modeling dense wireless networks is similar to [18], [14], [17] where the network is assumed to be made up of a continuum of nodes. In [14], the authors study the load balancing problem in a dense wireless multihop network by formulating it as a minmax problem. In [18], the authors study the optimality of single path routes between source-destination pairs in massively dense wireless multihop networks. Optimal routing principles for dense wireless networks are proposed based on geometrical optics in [17]. We do not focus on optimal routing paths but instead study optimizing hop distance for a given path between the source-destination pairs.

There is considerable literature on cross-layer strategies aimed at optimizing network performance for finite sized networks (see [6], [3], [4], [8], [19]). Given node placements and channel parameters, the network objective (throughput maximization, energy minimization, etc.) is modeled as an optimization problem and the solution provides the optimal routing, scheduling and power allocation strategies. For example, in [6], the authors study the joint scheduling, routing, and power control to maximize throughput of a source-destination pair in a multihop wireless network. A joint routing and power allocation policy is provided in [3] which stabilizes the system and provides bounded delay guarantees whenever the input rates are within the capacity region. In [4], Cruz et al., provide an integrated routing, link scheduling, and power allocation policy that minimizes the total average power consumption to support minimum average rate requirements per link. In our work, we do not model individual nodes; we assume a continuum of nodes and aim at a characterization of the network performance in terms of the network power and channel parameters.

3 The Network Model

There is a dense collection of immobile nodes that use multiaccess multihop radio communication with single user decoding and packet forwarding to transport packets between various source-destination pairs

- All nodes use the same contention mechanism with the same parameters (e.g., all nodes use IEEE 802.11 DCF with the same back-off parameters).
- We assume that nodes send control packets (such as RTS/CTS in IEEE 802.11) with a constant power (i.e., power control is not used for the control packets) during contention, and these control packets are decodable by *every* node in the network. As in IEEE 802.11, this can be done by using a low rate, robust modulation scheme and by restricting the diameter of the network. This is the "single cell" assumption, also used in [16], and implies that there can be only one successful ongoing transmission at any time.
- During the control packet exchange, each transmitter learns about the channel "gain" to its intended receiver, and decides upon the power level that is used to transmit its data packet. For example, in IEEE 802.11, the channel gain to the intended receiver could be estimated during the RTS/CTS control packet exchange. Such channel information can then be used by the transmitter to do power control. In our paper, we assume that such channel estimation and power control is possible on a transmission-by-transmission basis.
- In this work, we model only an average power constraint and not a peak power constraint.
- Saturation assumption: We assume that the traffic is homogeneous in the network and all the nodes have data to send at all times; these could be locally generated packets or transit packets. In [14], the authors study the problem of load balancing in dense multihop wireless networks with arbitrary traffic requirements. In our work, we do not restrict to straight line paths, and permit such a load balancing routing strategy as in [14], which then ensures that the load and the channel access pattern are identical for all the nodes.

Data packets are sent between source-destination pairs by multihop relaying. Based on the dense network and traffic homogeneity assumption, we further make the following assumption:

- The nodes self-organize so that all hops are of length d, i.e., a one hop transmission always traverses a distance of d meters. This hop distance, d, will be one of our optimization variables.

For a random node deployment, the hop distance that maximizes the system throughput need not be the same for every node and every flow. However, the approximation should hold good for a homogeneous network with large number of nodes. Further, it will be practically infeasible to optimize every hop in a dense setup with hundreds of nodes.

3.1 Channel Model: Path Loss, Fading and Transmission Rate

The channel gain between a transmitter-receiver pair for a hop is assumed to be a function of the hop length (d) and the multipath fading "gain" (h). The path loss for a hop distance d is given by

$$\frac{1}{(\frac{d}{d_0})^\eta} = \frac{d_0^\eta}{d^\eta},$$

where η is the path loss exponent, chosen depending on the propagation characteristics of the environment (see, e.g., [25]) and d_0 is the far field reference distance. This variation of path loss with d holds for $d > d_0$; we will assume that this inequality holds (i.e., $d > d_0$), and will justify this assumption in the course of the analysis (see Theorem 5.2).

We assume a flat and slow fading channel with additive white Gaussian noise of power σ^2. We assume that for each transmitter-receiver pair, the channel gain due to multipath fading may change from transmission to transmission, but remains constant over any packet transmission duration. Since successive transmissions can take place between randomly selected pairs of nodes (as per the outcome of the distributed contention mechanism), we are actually modeling a spatiotemporal fading process. We assume that this fading process is stationary in space and time with some given marginal distribution H. Let the cumulative distribution of H be $A(h)$ (with a p.d.f. $a(h)$), which by our assumption of spatiotemporal stationarity of fading is the same for all transmitter-receiver pairs and for all transmissions. We assume that the channel coherence time, τ_c, applicable to all the links in the network, upper bounds every data transmission duration in the network. Further, we assume that H and τ_c are independent of the hop distance d.

When a node transmits to another node at a distance d (in the transmitting antenna's far field), using transmitter power P, with channel power gain due to fading, h, then we assume that the transmission rate given by Shannon's formula is achieved over the transmission burst; i.e., the transmission rate is given by

$$C = W \log\left(1 + \frac{hP\alpha}{\sigma^2 d^\eta}\right),$$

where W is the signal bandwidth and α is a constant accounting for any fixed power gains between the transmitter and the receiver (e.g., α includes d_0^η). Note that in writing this expression, we have accounted only for noise, but not cochannel interference, because of our assumption that the network operates without spatial reuse; at any time exactly one transmission is active in the network.

3.2 Fixed Transmission Time Strategy

We consider a fixed transmission time scheme, where all data transmissions are of equal duration, T ($<\tau_c$) secs, independent of the bit rate achieved over the wireless link. This implies that the amount of data that a transmitter sends during a transmission opportunity is proportional to the achieved physical link rate. Upon a successful control packet exchange, the channel (between the transmitter, that "won" the contention, and its intended receiver) is reserved for a duration of T seconds independent of the channel state h. This is akin to the "TXOP" (transmission opportunity) mechanism in the IEEE 802.11 standard. Thus, when the power allocated during the channel state h is $P(h)$, $C(h)T$ bits are sent across the channel, where $C(h) = W\log(1 + \frac{P(h)h\alpha}{\sigma^2 d^\eta})$. When $P(h) = 0$, we assume that the channel is left idle for the next T seconds. The transmitter does not relinquish the channel immediately, and the channel reserved for the transmitter-receiver pair (for example, by the RTS/CTS signalling) is left empty for the duration of T seconds.

The optimality of a fixed transmission time scheme, for throughput, as compared to a fixed packet length scheme, can be formally established (see [27]), we only provide an intuition here. When using fixed packet lengths, a transmitter may be forced to send the entire packet even if the channel is poor, thus taking longer time and more power. On the other hand, in a fixed transmission time scheme, we send more data when the channel is good and limit our inefficiency when the channel is poor.

4 Multihop Transport Capacity

Let d denote the common hop length and $\{P(h)\}$ a power allocation policy, with $P(h)$ denoting the transmit power used when the channel state is h. We take a simple model for the random access channel contention process. The channel goes through successive contention periods. Each period can be either an idle slot, or a collision period, or a successful transmission with probabilities p_i, p_c, and p_s, respectively. Let T_i, T_c, and T_o be the average time overheads associated with an idle slot, collision slot, and data transmission, respectively. For example, in IEEE 802.11, with the RTS/CTS mechanism being used, a collision takes a fixed time independent of the data transmission rate. Under the node saturation assumption, the aggregate bit rate carried by the system, $\Theta_T(\{P(h)\}, d)$, for the hop distance d and power allocation $\{P(h)\}$, is given by (see [16])

$$\Theta_T(\{P(h)\}, d) := \frac{p_s(\int_0^\infty L(h)\mathrm{d}A(h))}{p_iT_i + p_cT_c + p_s(T_o + T)}, \quad (1)$$

where $L(h) := C(h)T$, where, as shown earlier, $C(h) = W\log(1 + \frac{hP(h)\alpha}{\sigma^2 d^\eta})$. The denominator of (1) is the average time duration of a contention period and the numerator is the average data transmitted in a contention period (a function of the power control $\{P(h)\}$ and the hop distance d). We note that p_i, p_s, p_c, T_i, T_o, and T_c depend only on the parameters of the distributed contention mechanism (MAC protocol) and the channel, and not on any of the decision variables that we consider. See, e.g., [16] and the references therein for an approach for obtaining p_i, p_s, and p_c in the context of the IEEE 802.11 CSMA/CA medium access mechanism.

With $\Theta_T(\{P(h)\}, d)$ defined as in (1), we consider $\Theta_T(\{P(h)\}, d) \times d$ as our measure of transport capacity of the network. This measure can be motivated in several ways. $\Theta_T(\{P(h)\}, d)$ is the rate at which bits are transmitted by the network nodes. When transmitted successfully, each bit traverses a distance d. Hence, $\Theta_T(\{P(h)\}, d) \times d$ is the rate of spatial progress of the flow of bits in the network (in bit-meters per second). Viewed alternatively, it is the weighted average of the end-to-end flow throughput with respect to the distance traversed. Suppose that a flow i covers a distance D_i with $\frac{D_i}{d}$ hops (assumed to be an integer for this argument). Let $\beta_i\Theta_T(\{P(h)\}, d)$ be the fraction of throughput of the network that belongs to flow i. Then,

$$\frac{\beta_i\Theta_T(\{P(h)\}, d)}{\frac{D_i}{d}}$$

is the end-to-end throughput for flow i and

$$\frac{\beta_i\Theta_T(\{P(h)\}, d)}{\frac{D_i}{d}} \times D_i = \beta_i\Theta_T(\{P(h)\}, d) \times d$$

is the end-to-end throughput for flow i in bit-meters per second. Summing over all the flows, we have

$\Theta_T(\{P(h)\}, d) \times d$, the aggregate end-to-end flow throughput in bit-meters per second.

With the above motivation, our aim in this paper is to maximize the quantity $\Theta_T(\{P(h)\}, d) \times d$ over the hop distance d and over the power control $\{P(h)\}$, subject to a *network average power constraint*, $\bar{P}$. We use a network power constraint that accounts for the energy used in data transmission as well as the energy overheads associated with communication. The network average power, $\mathcal{P}(\{P(h)\})$, is given by

$$\mathcal{P}(\{P(h)\}) := \frac{p_i E_i + p_c E_c + p_s(E_o + T\int_0^\infty P(h)\,\mathrm{d}A(h))}{p_i T_i + p_c T_c + p_s(T_o + T)}. \quad (2)$$

E_i, E_c, and E_o correspond to the energy overheads associated with an idle period, collision, and successful transmission. Thus, E_i denotes the total energy expended in the network over an idle slot, E_c denotes the total average energy expended by the colliding nodes, as well as the idle energy of the idle nodes, and E_o denotes the average energy expended in the successful contention negotiation between the successful transmitter-receiver pair, the receive energy at the receiver (in the radio and in the packet processor), and the idle energy expended by all the other nodes over the time $T_o + T$. We assume that E_i, E_c, and E_o depend only on the contention mechanism and not on the decision variables d and $\{P(h)\}$.

5 Optimizing the Transport Capacity

For a given $\{P(h)\}$ and d, and the corresponding throughput $\Theta_T(\{P(h)\}, d)$, the transport capacity in bit-meters per second, which we will denote by $\psi(\{P(h)\}, d)$, is given by

$$\psi(\{P(h)\}, d) := \Theta_T(\{P(h)\}, d) \times d.$$

Maximizing $\psi(\cdot,\cdot)$ involves optimizing over d, as well as $\{P(h)\}$. However, we observe that, it would not be possible to vary d with fading, as routes cannot vary at the fading time scale. Hence, we propose to optimize first over $\{P(h)\}$ for a given d, and then optimize over d, i.e., we seek to solve the following problem:

$$\max_d \max_{\{\{P(h)\}:\mathcal{P}(\{P(h)\})\le\bar{P}\}} \psi(\{P(h)\}, d). \quad (3)$$

For a given d and power allocation $\{P(h)\}$, define the time average transmission power, $\bar{P}_t(\{P(h)\}, d)$, and the time average overhead power, $\bar{P}_o$, as

$$\bar{P}_t(\{P(h)\}, d) := \frac{p_s(\int_0^\infty P(h)\,\mathrm{d}A(h))T}{p_i T_i + p_c T_c + p_s(T_o + T)},$$
$$\bar{P}_o := \frac{p_i E_i + p_c E_c + p_s E_o}{p_i T_i + p_c T_c + p_s(T_o + T)}.$$

Observe that $\bar{P}_o$ does not depend on $\{P(h)\}$ and d. Now, the network power constraint can be viewed as

$$\bar{P}_t(\{P(h)\}, d) \le \bar{P} - \bar{P}_o,$$

where the right-hand side is independent of $\{P(h)\}$ or d. Define $\bar{P}_t := \bar{P} - \bar{P}_o$, the *time average* data transmission power constraint for the network.

5.1 Optimization over $\{P(h)\}$ for a Fixed d

Consider the optimization problem (from (3))

$$\max_{\{\{P(h)\}:\mathcal{P}(\{P(h)\})\le\bar{P}\}} \psi(\{P(h)\}, d). \quad (4)$$

The denominators of $\Theta_T(\cdot,\cdot)$ in (1) and of $\mathcal{P}$ in (2) are independent of d and the power control $\{P(h)\}$. Thus, with d fixed, the optimization problem simplifies to maximizing $\int_0^\infty L(h)\,\mathrm{d}A(h)$ or,

$$\int_0^\infty \log\left(1 + \frac{P(h)h\alpha}{\sigma^2 d^\eta}\right)\mathrm{d}A(h)$$

subject to the power constraint,

$$\int_0^\infty P(h)\,\mathrm{d}A(h) \le \bar{P}_t',$$

where $\bar{P}_t'$ is given by

$$\bar{P}_t' := \frac{(p_i T_i + p_c T_c + p_s(T_o + T))}{p_s T}\bar{P}_t.$$

$\bar{P}_t'$ is the average transmit power constraint averaged only over the transmission periods (successful contention slots).

This is a well-known problem whose optimal solution has the water-pouring form (see [1] and [26, Chapter 6]).[1] The optimal power allocation function $\{P(h)\}$ is given by

$$P(h) = \left(\frac{1}{\lambda} - \frac{d^\eta\sigma^2}{h\alpha}\right)^+,$$

where λ is obtained from the power constraint equation

$$\int_{\frac{\lambda\sigma^2 d^\eta}{\alpha}}^\infty a(h)P(h)dh = \bar{P}_t'.$$

The optimal power allocation is a nonrandomized policy, where a node transmits with power $P(h)$ every time the channel is in state h (whenever $P(h) > 0$), or leaves the channel idle for h such that $P(h) = 0$.

5.2 Optimization over d

By defining $\xi(h) := \frac{P(h)}{d^\eta}$, the problem of maximizing the throughput over power controls, for a fixed d, can be rewritten as

$$\max \int_0^\infty \log\left(1 + \frac{\alpha h}{\sigma^2}\xi(h)\right)a(h)dh$$

subject to

$$\int_0^\infty \xi(h)a(h)dh \le \frac{\bar{P}_t'}{d^\eta}.$$

Observe that $\bar{P}_t'$ and d influence the optimization problem only as $\frac{\bar{P}_t'}{d^\eta}$. Denoting by $\Gamma(\frac{\bar{P}_t'}{d^\eta})$ the optimal value of this problem, the problem of optimization over the hop-length, d, now becomes

1. The "water pouring" terminology comes from the following viewpoint. Plot $P(h)$ versus $\frac{d^\eta\sigma^2}{h\alpha}$ with h varying. On the x-scale, a small x value corresponds to a better channel. Consider the triangular region between the y-axis and the y = x line. Pour "water" in this region until it fills up to the level $\frac{1}{\lambda}$. Any h at which there is "water" accumulated is assigned the power between the level $\frac{1}{\lambda}$ and the y = x line. Any h at which there is no "water" (these will be poor channels) is assigned no power.

$$\max_{d} d \times \Gamma\left(\frac{\bar{P}_t{}'}{d^{\eta}}\right). \quad (5)$$

Theorem 5.1. *In the problem defined by (5), the objective $d \times \Gamma(\frac{\bar{P}_t{}'}{d^{\eta}})$, when viewed as a function of d, is continuously differentiable. Further, when the channel fading random variable, H, has a finite mean ($\mathbf{E}(H) < \infty$), then*

1. *$\lim_{d\to 0} d \times \Gamma(\frac{\bar{P}_t{}'}{d^{\eta}}) = 0$ and*
2. *if in addition, $\eta \geq 2$, $\frac{1}{h^2} a(\frac{1}{h})$ is continuously differentiable and $\mathbf{P}(H > h) = O(\frac{1}{h^2})$ for large h, then, $\lim_{d\to\infty} d \times \Gamma(\frac{\bar{P}_t{}'}{d^{\eta}}) = 0$,*

Proof. The proofs of continuous differentiability of $d \times \Gamma(\frac{\bar{P}_t{}'}{d^{\eta}})$, and of assertions 1 and 2 are provided in Appendix B, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2011.204. □

Remarks 5.1.

1. Under the conditions proposed in Theorem 5.1, it follows that $d \times \Gamma(\frac{\bar{P}_t{}'}{d^{\eta}})$ is bounded over $d \in [0, \infty)$ and achieves its maximum in $d \in (0, \infty)$.
2. When the objective function (5) is unbounded (can happen if the conditions are violated), the optimal solution occurs at $d = \infty$ (follows from the continuity results).
3. We note that, in practice, $\eta \geq 2$.

Let d_0 be the far field reference distance (discussed in Section 3.1).

Theorem 5.2. *The following hold for the problem in (5):*

1. *Without the constraint $d > d_0$, the optimum hop distance d_{opt} scales as $(\bar{P}_t{}')^{\frac{1}{\eta}}$.*
2. *There is a value $\bar{P}'_{t_{\min}}$ such that, for $\bar{P}_t{}' > \bar{P}'_{t_{\min}}$, $d_{opt} > d_0$, and the optimal solution obeys the scaling shown in 1.*
3. *For $\bar{P}_t{}' > \bar{P}'_{t_{\min}}$, the optimum power control $\{P(h)\}$ is of the water-pouring form and scales as $\bar{P}_t{}'$.*
4. *For $\bar{P}_t{}' > \bar{P}'_{t_{\min}}$, the optimal transport capacity scales as $(\bar{P}_t{}')^{\frac{1}{\eta}}$.*

Proof.

1. Let d_{opt} be optimal for $\bar{P}_t{}' > 0$. We claim that, for $x > 0$, $x^{\frac{1}{\eta}} d_{opt}$ is optimal for the power constraint $x\bar{P}_t{}'$. For suppose this was not so, it would mean that there exists $d > 0$ such that
$$x^{\frac{1}{\eta}} d_{opt} \Gamma\left(\frac{x\bar{P}_t{}'}{(x^{\frac{1}{\eta}} d_{opt})^{\eta}}\right) < d\Gamma\left(\frac{x\bar{P}_t{}'}{d^{\eta}}\right)$$
or, equivalently,
$$d_{opt} \Gamma\left(\frac{\bar{P}_t{}'}{d_{opt}^{\eta}}\right) < x^{-\frac{1}{\eta}} d\Gamma\left(\frac{\bar{P}_t{}'}{(x^{-\frac{1}{\eta}} d)^{\eta}}\right),$$
which contradicts the hypothesis that d_{opt} is optimal for $\bar{P}_t{}'$.
2. With the path loss model $\frac{P}{d^{\eta}}$, we see that for $d < d_0$, the received power is scaled more than the transmitted power P, due to the factor $\frac{1}{d^{\eta}}$, and an d_0^{η} factor in α, i.e., the model overestimates the received power and the transport capacity. Hence, the achievable transport capacity for $d < d_0$ is definitely less than $d \times \Gamma(\frac{\bar{P}_t{}'}{d^{\eta}})$. The result now follows from the scaling result in 1.
3. It follows from 1 that, if $\bar{P}_t{}'$ scales by a factor x, then the optimum d scales by $x^{\frac{1}{\eta}}$, so that, at the optimum, $\frac{\bar{P}_t{}'}{d^{\eta}}$ is unchanged. Hence, the optimal $\{\xi(h)\}$ is unchanged, which means that $\{P(h)\}$ must scale by x. The water-pouring form is evident.
4. Again, by 1 and 2, if $\bar{P}_t{}'$ scales by a factor x, then the optimum d scales by $x^{\frac{1}{\eta}}$, so that, at the optimum, $\frac{\bar{P}_t{}'}{d^{\eta}}$ is unchanged. Thus, $\Gamma(\frac{\bar{P}_t{}'}{d^{\eta}})$ is unchanged, and the optimal transport capacity scales as the optimum d, i.e., by the factor $x^{\frac{1}{\eta}}$. □

Remarks 5.2. The above theorem yields the following observations for the fixed transmission time model:

1. As an illustration, with $\eta = 3$, in order to double the transport capacity, we need to use 2^3 times the $\bar{P}_t{}'$. This would result in a considerable reduction in network lifetime, assuming the same battery energy.
2. We observe that as the power constraint $\bar{P}_t{}'$ scales, the optimal bit rate carried in the network, $\Gamma(\frac{\bar{P}_t{}'}{d^{\eta}})$, stays constant, but the optimal transport capacity increases since the optimal hop length increases. Further, because of the way the optimal power control and the optimal hop length scale together, the nodes transmit at the same *physical bit rate* in each fading state; see the proof of Theorem 5.2, part 3).

5.3 Characterization of the Optimal d

By the results in Theorem 5.1, we can conclude that the optimal solution of the maximization in (5) lies in the set of points for which the derivative of $d \times \Gamma(\frac{\bar{P}_t{}'}{d^{\eta}})$ is zero. For a fixed $\bar{P}_t{}'$, define $\pi(d) := \frac{\bar{P}_t{}'}{d^{\eta}}$. Differentiating $d \times \Gamma(\pi(d))$, we obtain (see Appendix A, available in the online supplemental material),

$$\frac{\partial}{\partial d}(d\,\Gamma(\pi(d))) = \Gamma(\pi(d)) - \eta\pi(d)\lambda(\pi(d)),$$

where $\lambda(\pi(d))$ is the Lagrange multiplier for the optimization problem that yields $\Gamma(\pi(d))$. Since d appears only via $\pi(d)$, we can view the right-hand side as a function only of π. We are interested in the zeros of the above expression. Clearly, $\pi = 0$ is a solution. The solution $\pi = 0$ corresponds to the case $d = \infty$. However, we are interested only in solutions of d in $(0, \infty)$, and hence, we seek positive solutions of π of

$$\Gamma(\pi) - \eta\pi\lambda(\pi) = 0. \quad (6)$$

Remarks 5.3. In Appendix A, available in the online supplemental material, we consider a continuously distributed fading random variable H with p.d.f. $a(h)$. The analysis can be done for a discrete valued fading distribution as well, and we provide this analysis in Appendix C, available in the online supplemental material. The following example then illustrates that,

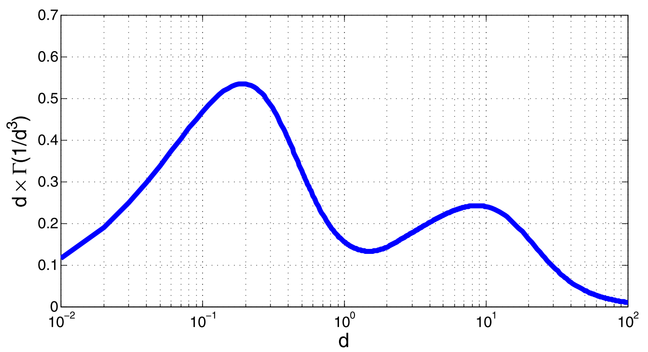


Fig. 1. Plot of $d \times \Gamma(\frac{1}{d^3})$ (linear scale) versus d (log scale) for a channel with two fading states h_1, h_2. The fading gains are $h_1 = 100$ and $h_2 = 0.1$, with probabilities $a_{h_1} = 0.01 = 1 - a_{h_2}$. The function has three nontrivial stationary points.

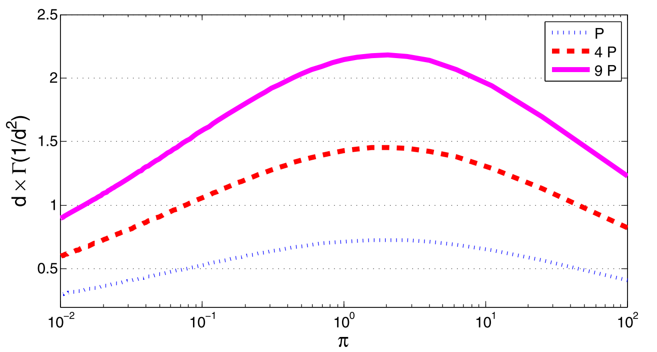


Fig. 2. Plot of $d \times \Gamma(\frac{\bar{P}_t'}{d^\eta})$ (linear scale) versus π $(= \frac{\bar{P}_t'}{d^\eta})$ (log scale) for a fading channel (with exponential distribution). We consider three power levels ($\bar{P}_t'$, $4\bar{P}_t'$, and $9\bar{P}_t'$) and $\eta = 2$. The function has a unique optimum $\pi_{opt} (\pi_{opt} \approx 2)$ for all the three cases.

in general, the function $\Gamma(\pi) - \eta\pi\lambda(\pi) = 0$ can have multiple solutions. Consider a fading distribution that takes two values: $h_1 = 100$ and $h_2 = 0.1$, with probabilities $a_{h_1} = 0.01 = 1 - a_{h_2}$. Fig. 1 plots $d \times \Gamma(\frac{1}{d^3})$ for the system with $\eta = 3$. Notice that there are three stationary points other than the trivial solution $d = \infty$ (which is not shown in the figure). The maximizing solution here is the first stationary point (the stationary point close to 0). If, on the other hand, $a_{h_1} = 0.1 = 1 - a_{h_2}$, we will again have three stationary points, but the optimal solution now will be the third stationary point (see Section 6).

More generally, and still pursuing the discrete case, let $\mathcal{H}$ denote the set of fading states when the fading random variable is discrete with a finite number of values; $|\mathcal{H}|$ denotes the cardinality of $\mathcal{H}$.

Theorem 5.3. *There are at most* $2|\mathcal{H}| - 1$ *stationary points of* $d\,\Gamma(\pi(d))$ *in* $0 < d < \infty$.

Proof. See Appendix C, available in the online supplemental material, for the related analysis and the proof of this theorem. □

We conclude from the above discussion that it is difficult to characterize the optimal solution when there are multiple stationary points. Hence, we seek conditions for a unique positive stationary point, which must then be the maximizing solution. In Appendix A, available in the online supplemental material, we have shown that the equation characterizing the stationary points, $\Gamma(\pi) - \eta\pi\lambda(\pi) = 0$, can be rewritten as

$$\int_0^1 (\log(y) - \eta(y-1)) \frac{\lambda}{y^2} f\left(\frac{\lambda}{y}\right) dy = 0 \qquad (7)$$

for $f(x) := a(\frac{\sigma^2 x}{\alpha})\frac{\sigma^2}{\alpha}$, the density of the random variable $X := \frac{\alpha H}{\sigma^2}$. Notice that π does not appear in this expression. The solution directly yields the Lagrange multiplier of the throughput maximization problem for the optimal value of hop length. The following theorem guarantees the existence of at most one solution of (7).

Theorem 5.4. *If for any* $\lambda_1 > \lambda_2 > 0$, $f(\frac{\lambda_2}{y})/f(\frac{\lambda_1}{y})$ *is a strictly monotone decreasing function of* y*, then the objective function* $d \times \Gamma(\frac{\bar{P}_t'}{d^\eta})$ *has at most one stationary point* $d_{opt}, 0 < d_{opt} < \infty$.

Proof. The proof follows from the discussion in Appendix A, available in the online supplemental material, and Lemma A.1. □

Corollary 5.1. *If* H *has an exponential distribution and* $\eta \geq 2$, *then the objective in the optimization problem of (5) has a unique stationary point* $d_{opt} \in (0, \infty)$, *which achieves the maximum.*

Proof. $a(h)$ is of the form $\mu e^{-\mu h}$. From Theorem 5.1, we see that $\lim_{d\to 0} d \times \Gamma(\frac{\bar{P}_t'}{d^\eta}) = 0$ and $\lim_{d\to\infty} d \times \Gamma(\frac{\bar{P}_t'}{d^\eta}) = 0$. The monotonicity hypothesis in Theorem 5.4 holds for $a(h)$. □

Remarks 5.4.

1. Hence, for $\eta \geq 2$, for the Rayleigh fading model, there exists a unique stationary point which corresponds to the optimal operating point.
2. For $\bar{P}_t' > \bar{P}_{t\,\min}'$, and for the conditions in Theorem 5.1 and 5.4, let π_{opt} denote the unique stationary point of (6). Then, define $\Gamma(\pi_{opt}) = \Theta_{opt}$. It follows from Theorem 5.2 that the optimal transport capacity takes the form $(\frac{\bar{P}_t'}{\pi_{opt}})^{\frac{1}{\eta}}\Theta_{opt}$, where Θ_{opt} depends on $a(h)$ and the MAC parameters but not on $\bar{P}$ (or $\bar{P}_t$).
3. Fig. 2 numerically illustrates our results for the Rayleigh fading distribution and $\eta = 2$. Scaling $\bar{P}_t'$ by four scales the transport capacity from 0.72 to 1.44, i.e., by $4^{\frac{1}{\eta}} = \sqrt{4}$ and similarly for scaling $\bar{P}_t'$ by 9.

The uniqueness result (Theorem 5.4) guarantees that a distributed implementation of the problem (optimization over the hop distance), if it converges, shall converge to the unique stationary point, which is the optimal solution.

6 Numerical Results and Simulations

In Section 6.1, we discuss the consequences of multiple stationary points and the importance of there being a unique stationary point (in d) for $d \times \Gamma(\frac{\bar{P}_t'}{d^\eta})$. In Section 6.2, we describe a technique for carrying out the hop distance optimization by utilizing a distance discretization technique studied in [21]. In Section 6.3, we compare the performance of our optimization strategy with well-known policies.

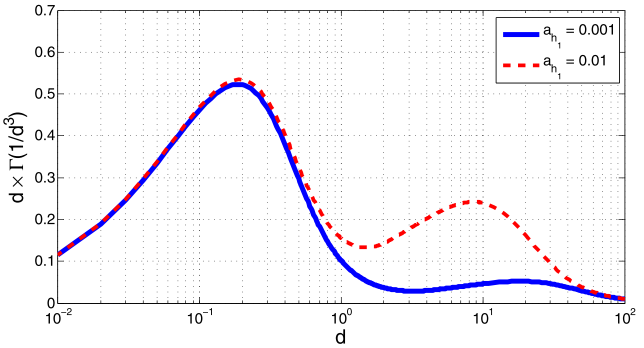


Fig. 3. Plots of $d \times \Gamma(\frac{1}{d^\beta})$ (linear scale) versus d (log scale) for a channel with two fading states h_1, h_2. The fading gains are $h_1 = 100$ and $h_2 = 0.1$. The plots correspond to two probability distributions, $a_{h_1} = 0.001$ and $a_{h_1} = 0.01$.

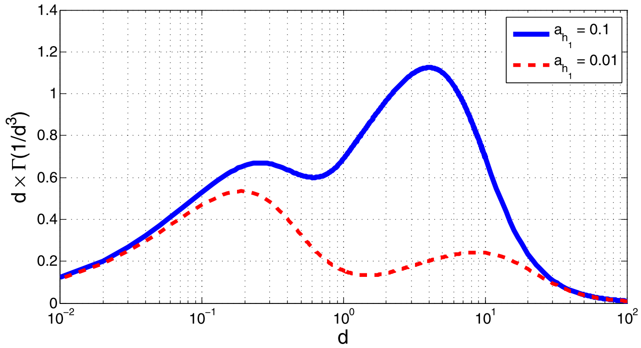


Fig. 4. Plots of $d \times \Gamma(\frac{1}{d^\beta})$ (linear scale) versus d (log scale) for a channel with two fading states h_1, h_2. The fading gains are $h_1 = 100$ and $h_2 = 0.1$. The plots correspond to two probability distributions, $a_{h_1} = 0.1$ and $a_{h_1} = 0.01$.

6.1 Multiple Stationary Points

In Section 5.3, we noted that $d \times \Gamma(\frac{\bar{P}_t'}{d^\eta})$ can have multiple stationary points. For the discrete fading distribution, $h_1 = 100, h_2 = 0.1$ with $a_{h_1} = 0.01 = 1 - a_{h_2}$, $d \times \Gamma(\frac{1}{d^\beta})$ has three nontrivial stationary points (plotted in Fig. 1). In this case, the first stationary point (the stationary point close to 0) is the optimum and the penalty of choosing the third stationary point as the operating point would be approximately 2. Fig. 3 plots $d \times \Gamma(\frac{1}{d^\beta})$ for the same h_1 and h_2 with $a_{h_1} = 0.001 = 1 - a_{h_2}$. Here again, the first stationary point is the optimum; however, the penalty of choosing the third stationary point would now be 10. In Fig. 4, we plot $d \times \Gamma(\frac{1}{d^\beta})$ again for $h_1 = 100, h_2 = 0.1$ now with $a_{h_1} = 0.1 = 1 - a_{h_2}$. Observe that the third stationary point is now the optimum (for $a_{h_1} = 0.1$) unlike the previous cases in Fig. 3 (for $a_{h_1} = 0.01$ and $a_{h_1} = 0.001$). Figs. 3 and 4 illustrate the usefulness of Theorem 5.4 and the importance of the uniqueness of the stationary point, as the penalty of choosing an arbitrary stationary point as the operating point can be large.

6.2 A Distance Discretization Algorithm for d_{opt}

In [21], Acharya et al. report on an approach for obtaining an approximation to the optimum hop distance, d_{opt}, in the same network setting as in our paper. In [21], the authors first provide a distributed technique for constructing the critical geometric graph[2] (CGG) on the node locations. They propose DISCRIT, a distributed and asynchronous algorithm for obtaining an approximation of the critical geometric graph on the node locations. The algorithm does not require the knowledge of node locations or internode distances, nor does it require received signal strength measurements. Instead, the algorithm makes use of successful Hello packet receipt counts (obtained during a neighbor discovery process) as edge weights, along with a simple distributed min-max computation algorithm. Extensive simulation results are used to demonstrate the efficacy of DISCRIT in obtaining an approximation of the CGG. The CGG on the node locations is then used to approximate the distances between nodes using the approximation that the hop distance between nodes is proportional to the euclidean distance between them. The CGG over the node locations can be expected to provide the most accurate approximation of the euclidean distance. Then, a distributed distance vector algorithm is employed to determine shortest paths between nodes. As an application, in [21], DISCRIT is used to obtain the optimal transport capacity (in our sense) for a simple no-fading scenario. Fixing a $k \geq 1$, the network is operated so that each node considers nodes k hops away from it as radio neighbors. The transport capacity is measured for each $k \geq 1$, and the network is then set to operate at the value of k that maximizes the transport capacity. Simulation results show that the resulting network provides a transport capacity that is an excellent approximation to the case when optimization is done over exact internode distances. In this section and in Section 6.3, we report our simulation results using this distance discretization technique from [21].

Simulation setup. Here, we provide a simulation study of this technique in the presence of Rayleigh fading; we also study the effect of the power constraint and the node density. We consider N nodes deployed uniformly and independently in an area 50 metres by 50 metres. The critical geometric graph on the deployment is then used to identify h hop neighbors for each node, for each $h = 1, 2, \ldots$. Fixing an hop length h, we calculate the average transport capacity when all nodes forward packets to neighbors h hops away. We assume a Rayleigh fading channel with average power gain $\mathbf{E}[H]$ and the path loss between the transmitter and the receiver is assumed to be a function of the actual euclidean distance between the nodes (with the path loss exponent η). Assuming saturated queues, the average transport capacity is measured for each hop length h. The transport capacity is maximized by choosing the common hop length h that maximizes the average transport capacity.

For each h, the average distance between neighbors is computed by taking the average of the distance between all pairs of nodes that are separated by h hops on the CGG. Fig. 5 shows the results from this simulation. We plot the average transport capacity versus the average hop distance, for two different values of $N = 100, 1{,}000$ and for two values of the time average transmit power constraint $\bar{P}_t' = 40$ dB, 30 dB (relative to the noise power); $\mathbf{E}[H] = 1$ and $\eta = 4$. Also, plotted in the figure is

2. Given a set of points, the geometric graph of radius r on these nodes is the graph obtained by placing an undirected edge between any pair of nodes separated by an euclidean distance of at most r. The critical geometric graph on the set of nodes is the *connected* graph with minimum radius.

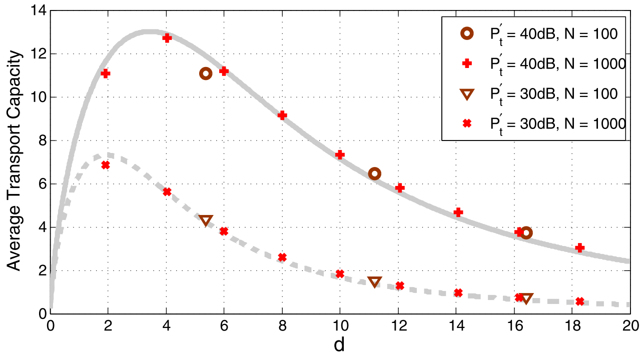


Fig. 5. Average transport capacity versus average hop distance using the distance discretization technique. The continuous curves are a plot of $d \times \Gamma(\frac{\bar{P}'_t}{d^\eta})$ versus d. The discrete points superimposed on the continuous curves correspond to the transport capacity for $h = 1, 2, \ldots$ hops, from left to right, for each $\bar{P}'_t$ and N. We consider $N = 100, 1{,}000$ nodes, in an area of 50 metres $\times$ 50 metres, $\bar{P}'_t = 30$ dB, 40 dB and $\eta = 4$.

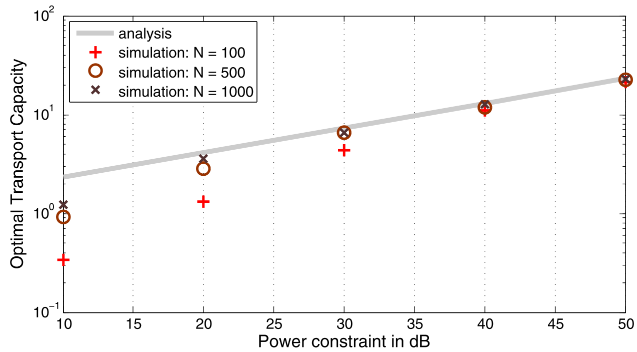


Fig. 6. Optimal transport capacity from simulation (performance of hop-count-based optimization) for different network densities N as a function of the transmit power constraint $\bar{P}'_t$. The continuous curve is the optimal transport capacity from the analysis $(\max_d d \times \Gamma(\frac{\bar{P}'_t}{d^\eta}))$. We consider three different network densities, $N = 100, 500, 1{,}000$ and $\eta = 4$.

$d \times \Gamma(\frac{\bar{P}'_t}{d^\eta})$. Observe that the distance discretization technique provides an approximation to d_{opt} from the analysis. For $N = 1{,}000$ and $\bar{P}'_t = 40$ dB (30 dB), the optimum value of h is 2 (1), and the optimal transport capacity from simulation is approximately 12.8 (6.8) in comparison with 13 (7.2) from the analysis. For $N = 100$, the best that can be done is for each node to send to its one hop neighbor in the CGG, and the plots show the performance penalty as compared to optimum performance, due to the discrete node locations. As expected from Theorem 5.4, since we have taken Rayleigh fading, there is a unique stationary point. As the power constraint decreases, the value of d_{opt} decreases, as does the optimum transport capacity.

Fig. 6 plots the optimal transport capacity from simulation using the distance discretization technique (in comparison with the analysis) for different values of $N = 100, 500, 1{,}000$ and for increasing time average transmit power constraint $\bar{P}'_t$. We consider $\mathbf{E}[H] = 1$ and $\eta = 4$. Observe that, for large values of network power, the simulation (with $N = 100,\ 500, 1{,}000$) better approximates the analysis. For small values of network power, d_{opt} is small, and larger node density is required for the distance discretization to track d_{opt}. From Fig. 6, we also note that the match between the simulation and the analysis is better with higher network density, for every network power constraint.

We conclude that the distance discretization technique described above is a useful approach for obtaining an approximation to the optimum hop distance, d_{opt}, in a practical setup. Further, we note that it is sufficient to optimize transport capacity over the hop distance h, rather than the exact distances.

6.3 Performance Evaluation

In this section, we compare the performance of our optimization strategy, i.e., to optimize both the hop distance and power, with simple policies. We report, through extensive simulations, scenarios where it is useful to optimize both the hop distance and power.

We compare our optimization strategy, OHOP (Optimal Hop distance and Optimal transmit Power for a fading channel), with

1. SHOP (Single Hop direct transmission between source-destination pairs with Optimal transmit Power for a fading channel),
2. OHCP (Optimal Hop distance with Constant transmit Power in each hop), and
3. SHCP (Single Hop direct transmission between source-destination pairs with Constant transmit Power).

In all our simulations, we use the distance discretization technique and hop-count-based optimization to optimize over the hop distance (i.e., for OHOP and OHCP).

6.3.1 SHOP: Single Hop Transmission with Power Control

We report the performance of a simple single hop direct transmission strategy with power control in comparison with optimal multihopping and power control. We assume a uniform and independent deployment of source and destination nodes. The single hop strategy assumes that the source-destination pairs are chosen randomly and that they communicate directly with each other. We assume a Rayleigh fading channel with mean $\mathbf{E}[H]$ and the path loss is assumed to be a function of the actual euclidean distance between the source-destination pair (with path loss exponent η). We maximize the transport capacity by optimally allocating the transmit power subject to the transmit power constraint $\bar{P}'_t$.

Fig. 7 plots the ratio of the maximum transport capacity with OHOP and the maximum transport capacity achieved using the single hop strategy SHOP. We consider two different network densities $N = 100, 1{,}000$ and two different values of $\eta = 3, 4$. Clearly, OHOP performs better than SHOP as we optimize over hop distance as well. Observe that the performance ratio approaches unity as the time average transmit power increases. As the network power increases, d_{opt} (equivalently, the optimal hop length) increases and, hence, for large network powers, direct transmission becomes optimal. However, for low values of the available average power, we observe that it is better to optimize over both the hop distance and power. Also, observe that the performance of OHOP (with respect to SHOP) improves as the network density increases. As

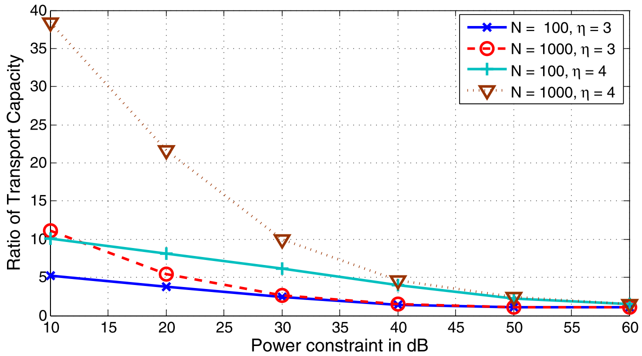


Fig. 7. Ratio of optimal transport capacity provided by OHOP and the optimal transport capacity provided by SHOP. We consider a Rayleigh fading channel, $N = 100, 1{,}000$ and $\eta = 3, 4$.

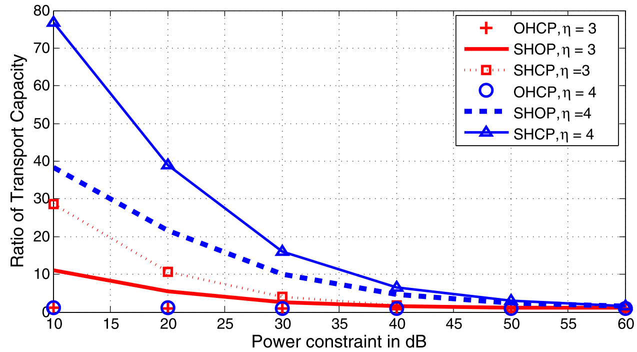


Fig. 9. Ratio of the optimal transport capacity provided by OHOP and 1) OHCP, 2) SHOP, and 3) SHCP. We consider $N = 1{,}000$ nodes, $\mathbf{E}[H] = 1$ and $\eta = 3, 4$.

observed in Section 6.2, large node densities helps the distance discretization technique in tracking d_{opt} accurately, thus improving the transport capacity. Further, in Fig. 7, we note that as η increases OHOP performs better. As η increases, the path loss between the nodes increases and optimizing hop distance becomes essential to maximize transport capacity.

6.3.2 *OHCP: Multihopping with Constant Transmit Power*

In Fig. 8, we report the performance of a multihopping strategy without power control (OHCP) in comparison with joint optimization of hop distance and power control (OHOP). In OHCP, the network does hop distance optimization but does not do power control. Hence, every transmission is assumed to use the same power $\bar{P}_t'$ for every channel fade. We assume that the network parameters are the same for either case.

Fig. 8 plots the ratio of the maximum transport capacity with OHOP and the maximum transport capacity achieved using OHCP. We consider four different network scenarios, including different network densities, η and Rayleigh fade power gain $\mathbf{E}[H]$. Observe that in all the four cases, the ratio of the transport capacity is at best two. Power control is helpful to counter the effect of fading. However, for different values of $\bar{P}_t'$, channel conditions η and average fades $\mathbf{E}[H]$, we observe that the ratio of the performance is close to 1. In Section 5.2, for OHOP, we observed that the

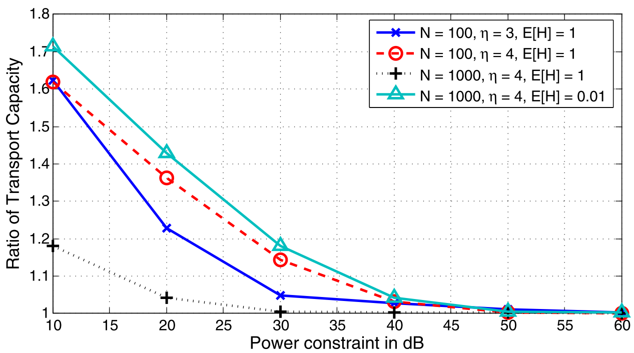


Fig. 8. Ratio of optimal transport capacity provided by OHOP and the optimal transport capacity provided by OHCP. The plots correspond to four different combinations of network densities N, η and channel average power gain $\mathbf{E}[H]$.

optimal transport capacity scales as $(\bar{P}_t')^{\frac{1}{\eta}}$ and the power control at d_{opt} was such that the bit rate at any channel fade is independent of the power constraint. Similarly, for OHCP, the optimal transport capacity,

$$\max_d d \times \mathbf{E}\left[\log\left(1 + \frac{\alpha \bar{P}_t' h}{d^\eta \sigma^2}\right)\right]$$

scales as $(\bar{P}_t')^{\frac{1}{\eta}}$ with the corresponding d_{opt} scaling as $(\bar{P}_t')^{\frac{1}{\eta}}$ keeping the bit rate at any channel fade independent of the power constraint (the proof is similar to the one in Theorem 5.1). Hence, the difference in performance (between OHOP and OHCP) is a function only of the optimal bit rate achieved at the optimal hop distance and is independent of the network power. From extensive simulations, we observe that this difference is small (in comparison with SHOP) for a variety of network scenarios.

6.3.3 *SHCP: Single Hop Transmission with Constant Power*

Fig. 9 compares SHOP and OHCP with our optimization strategy OHOP, along with a simple direct transmission, constant transmit power strategy SHCP. The single hop strategy SHCP assumes that the source-destination pairs are chosen randomly and that they communicate directly with each other. Further, in SHCP, every transmission is assumed to use the same transmit power $\bar{P}_t'$ for every channel fade. We plot the ratio of the optimal transport capacity provided by OHOP and the optimal transport capacity for the three policies OHCP/SHOP/SHCP for two different values of $\eta = 3, 4$. From Fig. 9, we infer that multihopping is essential for low network powers. For large network powers, single hop transmission is sufficient. Also, we observe that, for maximizing transport capacity, multihopping is more effective than power control.

7 Conclusion

In this paper, we have studied a problem of optimal power control and optimal forwarding distance (hop length) in a single cell, dense, ad hoc multihop wireless network. We formulated the problem as one of maximizing the transport capacity of the network subject to an average power constraint. We showed that, for a fixed transmission time scheme, there corresponds an intrinsic aggregate packet carrying capacity at which the network operates at the optimal operating point, independent of the average power constraint. We also obtained the scaling law relating the optimal hop distance to the power constraint, and hence relating the optimal transport capacity to the power constraint (see Theorem 5.2). Because of the way the power control and the optimal hop length scale, the optimal physical bit rate in each fading state is invariant with the power constraint. In Theorem 5.4, we provide a characterization of the optimal hop distance for cases in which the fading density satisfies a certain monotonicity condition. From extensive simulations, we observe that, for dense ad hoc networks, optimizing the hop distance is essential to maximize transport capacity, especially for low average network powers.

One motivation for our work is the optimal operation of sensor networks. If a sensor network is supplied with external power, or if the network is not required to have a long lifetime, then the value of the power constraint, $\bar{P}$, can be large, and a long hop distance will be used, yielding a large transport capacity. On the other hand, if the sensor network runs on batteries and needs to have a long lifetime then $\bar{P}$ would be small, yielding a small hop length. In either case, the optimal aggregate bit rate carried by the network would be the same.

Acknowledgments

This work was supported by the Indo-French Centre for the Promotion of Advanced Research (IFCPAR) under research contract Nos. 2900-IT-1 and 4000-IT-1. This is an extended version of a paper that appeared in IEEE WiOpt 2007.

References

[1] A.J. Goldsmith and P.P. Varaiya, "Capacity of Fading Channels with Channel Side Information," *IEEE Trans. Information Theory,* vol. 43, no. 6, pp. 1986-1992, Nov. 1997.

[2] P. Gupta and P.R. Kumar, "The Capacity of Wireless Networks," *IEEE Trans. Information Theory,* vol. 46, no. 2, pp. 388-404, Mar. 2000.

[3] M.J. Neely, E. Modiano, and C.E. Rohrs, "Dynamic Power Allocation and Routing for Time Varying Wireless Networks," *Proc. IEEE INFOCOM,* 2003.

[4] R.L. Cruz and A.V. Santhanam, "Optimal Routing, Link Scheduling and Power Control in Multihop Wireless Networks," *Proc. IEEE INFOCOM,* 2003.

[5] O. Dousse and P. Thiran, "Connectivity vs Capacity in Dense Ad Hoc Networks," *Proc. IEEE INFOCOM,* 2004.

[6] R. Bhatia and M. Kodialam, "On Power Efficient Communication over Multihop Wireless Networks: Joint Routing, Scheduling and Power Control," *Proc. IEEE INFOCOM,* 2004.

[7] Z. Wenrui, M. Ammar, and E. Zegura, "The Energy-Limited Capacity of Wireless Networks," *Proc. Ann. IEEE Comm. Soc. Conf. Sensor and Ad Hoc Comm. and Networks (SECON),* 2004.

[8] G. Kulkarni, V. Raghunathan, and M. Srivastava, "Joint End-to-End Scheduling, Power Control and Rate Control in Multihop Wireless Networks," *Proc. IEEE GlobeCom,* 2004.

[9] F. Xue, L. Xie, and P.R. Kumar, "The Transport Capacity of Wireless Networks over Fading Channels," *IEEE Trans. Information Theory,* vol. 51, no. 3, pp. 858-865, Mar. 2005.

[10] H. El Gamal, "On the Scaling Laws of Dense Wireless Sensor Networks: The Data Gathering Channel," *IEEE Trans. Information Theory,* vol. 51, no. 3, pp. 1229-1234, Mar. 2005.

[11] A. Behzad and I. Rubin, "High Transmission Power Increases the Capacity of Ad Hoc Wireless Networks," *IEEE Trans. Wireless Comm.,* vol. 5, no. 1, pp. 156-165, Jan. 2006.

[12] A. El Gamal and J. Mammen, "Optimal Hopping in Ad Hoc Wireless Networks," *Proc. IEEE INFOCOM,* 2006.

[13] L.L. Dai and V.W.S. Chan, "WLCp1-08: Throughput of Power-Limited Wireless Networks with Processing Energy Considerations," *Proc. IEEE GlobeCom,* 2006.

[14] E. Hyytia and J. Virtamo, "On Load Balancing in a Dense Wireless Multihop Network," *Proc. Second Conf. Next Generation Internet Design and Eng.,* 2006.

[15] V. Rodoplu and T.H. Meng, "Bits-per-Joule Capacity of Energy-Limited Wireless Networks," *IEEE Trans. Wireless Comm.,* vol. 6, no. 3, pp. 857-865, Mar. 2007.

[16] A. Kumar, E. Altman, D. Miorandi, and M. Goyal, "New Insights from a Fixed Point Analysis of Single Cell IEEE 802.11 Wireless LANs," *IEEE/ACM Trans. Networking,* vol. 15, no. 3, pp. 588-601, June 2007.

[17] R. Catanuto, S. Toumpis, and G. Morabito, "Opti{c,m}al: Optical/Optimal Routing in Massively Dense Wireless Networks," *Proc. IEEE INFOCOM,* 2007.

[18] E. Hyytia and J. Virtamo, "On Optimality of Single-Path Routes in Massively Dense Wireless Multi-Hop Networks," *Proc. 10th ACM Symp. Modeling, Analysis, and Simulation of Wireless and Mobile Systems (MSWiM),* 2007.

[19] C. Shuguang, R. Madan, A.J. Goldsmith, and S. Lall, "Cross-Layer Energy and Delay Optimization in Small-Scale Sensor Networks," *IEEE Trans. Wireless Comm.,* vol. 6, no. 10, pp. 3688-3699, Oct. 2007.

[20] V. Ramaiyan, A. Kumar, and E. Altman, "Jointly Optimal Power Control and Routing for a Single Cell, Dense, Ad Hoc Wireless Network," *Proc. IEEE Int'l Symp. Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks and Workshops (WiOpt),* 2007.

[21] S. Acharya et al., "Distributed Construction of the Critical Geometric Graph in Dense Wireless Sensor Networks," http://arxiv.org/abs/1011.3482v1, 2010.

[22] H. Zhang and J.C. Hou, "On the Asymptotic Minimum Transporting Energy and Its Implication on the Wireless Network Capacity," *IEEE/ACM Trans. Networking,* vol. 16, no. 5, pp. 1175-1187, Oct. 2008.

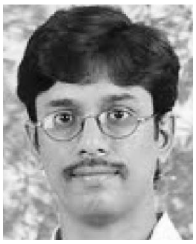
[23] *IEEE Std 802.16j-2009, IEEE Standard for Local and Metropolitan Area Networks, Air Interface for Broadband Wireless Access Systems, Amendment 1: Multihop Relay Specification,* IEEE, 2009.

[24] D.G. Luenberger and Y. Yinyu, *Linear and Nonlinear Programming,* third ed. Springer, 2008.

[25] T.S. Rappaport, *Wireless Communications: Principles and Practice.* Prentice Hall, 2002.

[26] A. Kumar, D. Manjunath, and J. Kuri, *Wireless Networking.* Morgan Kaufmann, Mar. 2008.

[27] V. Ramaiyan, A. Kumar, and E. Altman, "Optimal Routing and Power Control for a Single Cell, Dense, Ad Hoc Wireless Network," http://arxiv.org/abs/0907.3616, 2012.



Venkatesh Ramaiyan received the BE degree from the College of Engineering, Guindy at Chennai, and the master's and PhD degrees in communications from the Indian Institute of Science, Bengaluru. He is currently an assistant professor in the Department of Electrical Engineering, Indian Institute of Technology, Madras. His research interests include performance modeling, analysis, control and optimization of telecommunication systems. He is a member of the IEEE.

Anurag Kumar received the BTech degree from the Indian Institute of Technology at Kanpur and the PhD degree from Cornell University, both in electrical engineering. He was then with Bell Laboratories, Holmdel, New Jersey, for more than six years. Since 1988, he has been with the Indian Institute of Science (IISc), Bengaluru, in the Department of Electrical Communication Engineering, where he is now a professor. He is currently also the chair of the Electrical Sciences Division at IISc. From 1988 to 2003, he was the coordinator at IISc for the Education and Research Network Project (ERNET), India's first wide-area packet switching network. His area of research is communication networking, specifically, modeling, analysis, control, and optimization problems arising in communication networks and distributed systems. Recently, his research has focused primarily on wireless networking. He is a fellow of the IEEE, the Indian National Science Academy (INSA), the Indian Academy of Science (IASc), and the Indian National Academy of Engineering (INAE). He is a coauthor of the postgraduate textbooks *Communication Networking: An Analytical Approach* and *Wireless Networking*, both by Kumar, Manjunath, and Kuri, published by Elsevier.

Eitan Altman received the BSc degree in electrical engineering in 1984, the BA degree in physics in 1984, and the PhD degree in electrical engineering in 1990, all from the Technion-Israel Institute, Haifa. In 1990, he further received the BMus degree in music composition in Tel-Aviv university. Since 1990, he has been a researcher at INRIA (National Research Institute in Computer Science and Control) in Sophia-Antipolis, France. He has been on the editorial boards of several scientific journals: *Wireless Networks (WINET)*, *Computer Networks (COMNET)*, *Computer Communications (Comcom)*, *Journal of Discrete Event Dynamic Systems (JDEDS)*, *SIAM Journal of Control and Optimisation (SICON)*, *Stochastic Models*, and *Journal of Economy Dynamic and Control (JEDC)*. He received the best paper award from Networking 2006, Globecom 2007, and IFIP Wireless Days 2009 conferences, and is a coauthor of two papers that received the best student paper award (at QoFis 2000 and at Networking 2002). His areas of interest include networking, stochastic control, and game theory. He is a fellow of the IEEE. More informaion can be found at http://www-sop.inria.fr/members/Eitan.Altman.