Competition for Content Spread over Multiple Social Networks

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Abstract—We consider the competition between two competing content creators who can reach out to their potential consumers via two different online social networks. The *efficiency* of a network for information spread is characterized by two simple properties: the level of activity within the network and the popularity of the network among the population of consumers. We assume that the contents under our consideration are exclusive in nature, i.e., each consumer is interested in receiving only one of the competing contents. Each content creator optimizes the total budget spent across the two social networks. We study the non-cooperative game and characterize the best response functions for the content creators. From the best response functions we observe that there exists a *hysteresis*-like behavior when it comes to resource allocation across multiple networks, i.e., as a player responds to increasing budget of the competitor, there is an interval of the opponent's budget between when the player saturates the resources in the *better* network and when he begins to invest in the *worse* network. A similar behavior is also observed when the player begins reducing resources, responding to a much higher budget of the competitor. We also observe that the larger the difference between the networks' efficiency levels, the larger the interval. We then numerically evaluate the Nash equilibria from the best response functions and conclude with discussions on possible future scope of this work.

I. Introduction

Social media provide a more direct interface, as against traditional media, for content creators to reach out to their potential consumers. With increasing activity in Online Social Networks (OSN) like Facebook and Twitter, and with a greater fraction of the global population being present on these networks, content creators have begun investing in online social media advertisements. The proliferation of mobile Internet and smartphones have ensured that the consumers stay online in one or more of these social networks most of the time, hence permitting timely delivery of content as opposed to other traditional media (such as print media, television, etc.)

There has been considerable interest in the research community to study influence spread in the context of viral marketing, and develop algorithms for choosing the most influential set of initial nodes in the social network [1], [2] under various influence spread models (such as Linear Threshold, Independent Cascade, etc.). Since modern day social networks have a large user base (for instance, Facebook has 1billion+ users), there have also been efforts to use fluid limits [3] to approximate the influence spread on such networks [4]. These yield differential equations capturing the average dynamics of such processes, much similar to the ones traditionally used in epidemiology [5]. Recent efforts have also focused on epidemic games [6], [7], [8] to study competition between content creators on a social network.

One aspect that is largely unexplored in the literature of viral marketing, is characterizing competition across multiple social networks. We observe that there are several online social networks, such as Google+, Facebook, Twitter with considerable user-base derived from a common pool of consumers (i.e., the set of all Internet users). Each consumer spends varying amounts of time in each of these social networks, depending on the network's popularity and usefulness. Thus, the content creators need to simultaneously manage campaigns across multiple social networks. Recent literature has studied the aspect of migration across such social networks [9], but modeling competition across several such social networks has not received sufficient attention.

In this work, we characterize the competition between two content creators for a common user base, via two social networks. Each social network is characterized by its level of activity and its popularity among the consumers. There is a maximum number of resources that each content creator can use in each of the networks, and the aim is to optimize the net budget for competition. Our main contributions are as follows.

- Introduce two simple parameters to capture the information spread efficiency in different social networks
- We study the non-cooperative game between the two content creators and characterize the best response.
- From the best response function we observe that, for increasing values of the competitor's net allocation, the content creator begins by allocating his budget to the *better* social network. Interestingly, we observe that after exhausting the resources in the better network, there is an intermediate interval of competitor's allocation, for which the content creator refrains from investing in the second social network. A similar behavior is observed when the content creator starts reducing his budget in response to a much higher budget of the competitor.
- We obtain the Nash equilibria from the intersections of the best response functions. We observe that the equilibrium budget for competition increases with
  - decrease in the player's own cost per unit budget
  - increase in the opponent's cost per unit budget

The paper is organized as follows: In Section II, we describe the system model for the competition framework and the social network, and justify the assumptions involved using real-world instances. In Section III we derive the utility functions from the system dynamics and formulate the non-cooperative game in terms of the net budget used by each of the content creators. In Section IV we obtain the best response functions for each of the users, and thus characterize the Nash equilibria, as the
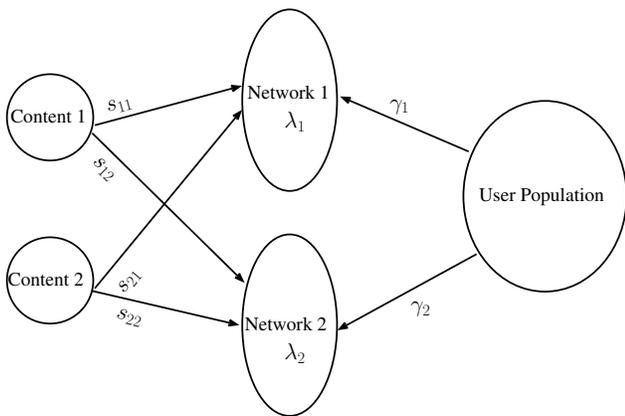
Fig. 1. Interaction between various components of the system. Sources recruit promotional accounts (s_{ij}) in each of the social networks, and users visit each network with a probability γ_j. Once inside the network, they receive updates from the promotional accounts at the rate λ_j

intersection of the best response functions. We also provide interesting remarks on the structure of the best response function and the hysteresis behavior. We also discuss a variant of the cost structure and discuss its implications. Section V provides numerical study of the best response functions and the Nash equilibrium, and provides interesting insights into the solution. Finally we conclude by providing possible future directions for this work.

II. SYSTEM MODEL

In this section, we will introduce the system model and justify the associated assumptions by considering examples from real world networks. We begin with the description of the competition framework and then describe the social network.

A. Competition Framework

We restrict our attention to publish-subscribe based networks, i.e., networks in which there are sources which generate content and users follow the sources. Consider M content sources trying to spread their content to a consumer population through K different such networks. We primarily deal with digital content, say news articles, popular song videos or podcasts. We also assume that the content sources specialize in the same genre of content, and hence there is a competition among them to attract a larger fraction of the common user base. We also assume that the contents are exclusive in nature, i.e., a particular consumer would adopt the first content he comes across, and is no longer interested in others. This is true in cases such as news podcasts (CNN, BBC,etc.) covering the same sports event, media streaming sites (Youtube, Vimeo, etc.) providing the same music video, etc. Figure 1 shows the various components (content sources, social network and the user population) in the system. We look at the scenario where the users could be present in one of several such networks at any given point of time. These networks vary in their popularity in the user population, and also in the level of interaction within the network. Thus each network j is characterized by a visit probability per user, γ_j, and a meeting

rate within the network, λ_j. Note that $\sum_{j \in K} \gamma_j \leq 1$. The system parameters γ_j and λ_j can be interpreted as follows: in each time slot, every user chooses which network to be present in, according to the probability γ_j, and once in Network j, he/she visits the sources' pages at the rate λ_j.

Though there are several online social networks, each of them have evolved to provide a unique service, and hence it is common for users to maintain accounts simultaneously on multiple social networks. For instance, Gmail is primarily intended for personal and professional communications among a small group of individuals. Facebook is used to stay in touch with our social circle (acquaintances and friends) and broadcast contents to all (or a majority) of friends. Twitter is primarily used as a content-sharing network, where users follow (unidirectional) content sources for latest updates. γ_j represents how users divide their attention across these various social networks. This will highly depend on individual preferences, but for simplicity we have assumed this to be constant for the user population under consideration.

λ_j is dependent on the network, since different networks may operate on different time scales and have varying mechanisms by which users access the content. For instance, it has been empirically observed [10], that the dynamics of news cycles on Facebook and Twitter occur at different rates, with Twitter being more up to date. This may be due to the different reasons for which the networks have evolved, as hinted earlier. Consider a single user who has subscribed to a mailing list on Gmail, has *liked* a page on Facebook (thus receives updates) and also follows a particular account on Twitter. Receiving half a dozen updates about a content from a single account over the period of a day may be perceived as normal on Twitter, while on Facebook it might be annoying and on Gmail subscriptions it might border on spamming. Thus content sources are restricted to using a λ_j as dictated by the timescale at which the network operates.

Each content source creates aliases in each of the networks, to spread his particular content. This can be done by creating accounts/pages on these networks to promote their content, and recruit personnel to manage and update these accounts/pages. We denote by s_{ij}, the fraction of promoters for content i in network j. The net budget spent by a given content source i is then $\sum_j s_{ij}$. We will also assume that there is a cost per unit budget for each source and is given by ϕ_i. We also assume there is a maximum limit to the number of promotion accounts that can be created on each network. s_{ij}'s are hence normalized with respect to the user population, i.e., $s_{ij} \leq 1$. The aim of the content provider would then be to optimize the net budget to maximize his revenue. The exact utility to be optimized is provided in Section III.

For this work, we will restrict our attention to two content sources and social networks (M=2, K=2), though the methodology and the results in this work can be generalized for the (M, K) case.

B. Social Network

Consider the social network as shown in Figure 2. We have considered Network 1 for example. s_{i1} denotes the promotion
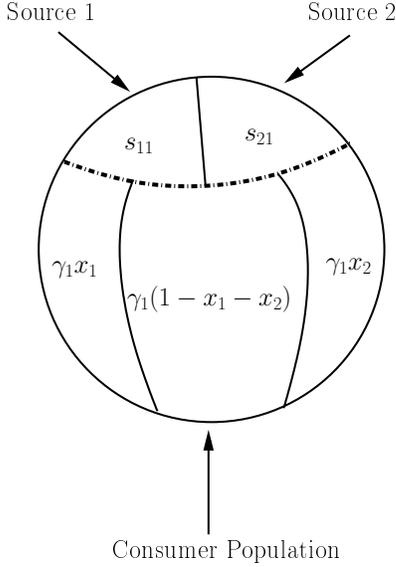
Fig. 2. The different subsets of users within a given social network (Social Network 1 is considered as an example). s_{i1} represents the promotion accounts related to source i and x_i denotes the fraction of total user population that has consumed content i. γ_1 is the probability with which a user is present in Network 1.

accounts promoting content from source i. The total set of users in the population is normalized to 1 and the fraction of users currently present in Network 1 is given by γ_1. Among these, $\gamma_1 x_1$ fraction of users have already consumed content of source 1 and $\gamma_1 x_2$ fraction of users have consumed content of source 2. Hence the competition between the sources is for the fraction of users who haven't consumed either of the content and are currently present in Network 1, i.e., $\gamma_1(1 - x_1 - x_2)$. These users receive content updates from each of the sources (in s_{11} or s_{21}) at rate λ_j and depending on which source reaches them first, they get converted to x_1 or x_2. We can use this to write down the system dynamics.

We will now write down the system of o.d.e.'s for the system evolution under fluid limit assumptions [3], [4] with the total user population normalized to 1. Let x_1, x_2 denote the fraction of users who have seen content 1 and 2 respectively ($x_1 + x_2 \leq 1$). The system evolution can then be given by,

$$\dot{x}_1 = (s_{11}\lambda_1\gamma_1 + s_{12}\lambda_2\gamma_2)(1 - x_1 - x_2) \quad (1)$$
$$\dot{x}_2 = (s_{21}\lambda_1\gamma_1 + s_{22}\lambda_2\gamma_2)(1 - x_1 - x_2) \quad (2)$$

In the above equations, the first term represents the fraction of nodes delivered the content in Network 1 (by s_{i1}), and the second term represents the fraction of nodes delivered the content in Network 2 (by s_{i2}). Let $a = \lambda_1\gamma_1$ and $b = \lambda_2\gamma_2$. The system evolution o.d.e.'s then become:

$$\dot{x}_1 = (as_{11} + bs_{12})(1 - x_1 - x_2) \quad (3)$$
$$\dot{x}_2 = (as_{21} + bs_{22})(1 - x_1 - x_2) \quad (4)$$

Let X_i represent the final fraction of users who have seen content i, i.e., $X_i := x_i(\infty)$ from equations (3) and (4). By

solving the above system we get,

$$X_i = \frac{as_{i1} + bs_{i2}}{as_{i1} + bs_{i2} + as_{j1} + bs_{j2}} \quad (5)$$

Note that since we are restricting ourselves to publish-subscribe based networks, we do not consider content sharing among users in this work. Modeling information spread in such system taking into account content sharing is a promising future direction. This will require a realistic modeling of the graph structure within each of these networks. This would also involve addressing issues such as, users sharing contents across the social networks (sharing an article read on Facebook to his followers on Twitter), and the distinction between user sharing and promotion from sources (users generally tend to share only once, since they are not interested in maximizing the visibility of any particular content).

III. GAME FORMULATION

We model the competition between the two content creators (players) as a non-cooperative game, with strategies (s_{11}, s_{12}) for player 1 and (s_{21}, s_{22}) for player 2. s_{ij}'s are normalized with the user population, hence $s_{ij} \leq 1$. The general utility function for player i is of the form,

$$U_i = X_i - \phi_i(s_{i1} + s_{i2})$$

where X_i represent the final fraction of users who have seen content i, as given in Equation 5 and ϕ_i is the cost per unit budget for content source i. We have made the cost depend only on the source, assuming that he does not pay differently, the personnel involved in spreading his content on different networks. Section IV-D has discussions on a variant of the cost structure, which depends both on the source and the corresponding social network.

We see that $\lambda_j\gamma_j$ serves as a measure of how efficient a network is for spreading information quickly to the common pool of consumers. This is intuitive, since λ_j indicates the rate at which you can generate content in Network j and γ_j indicates how often users are present in Network j. Without loss of generality, assume $a, b > 0$, with $a > b$ i.e. $\lambda_1\gamma_1 > \lambda_2\gamma_2$. Then if we fix the total budget, say $B_i = s_{i1} + s_{i2}$, then we can show that in the optimal allocation of the net budget (assuming a cost structure independent of the network) is, $s_{i1} = \min(B_1, 1)$ and $s_{i2} = \min((B_1 - 1)^+, 1)$. This implies that since Network 1 is *better* than Network 2 ($a > b$), given a total budget, it is optimal to invest in Network 1 until the maximum limit is reached, and then invest in Network 2. It is evident that, even in the case of a general K network system, the networks can be ordered in decreasing order of $\lambda_j\gamma_j$ and the budget can be allocated beginning with the *best* network and proceeding to the next network once the maximum limit is reached.

Define,

$$z_1 = a\min(B_1, 1) + b\min((B_1 - 1)^+, 1)$$

$$z_2 = a\min(B_2, 1) + b\min((B_2 - 1)^+, 1)$$

We can then rewrite the utilities as function of the budgets (B_1, B_2).

$$U_1 = \frac{z_1}{z_1 + z_2} - \phi_1 B_1$$

$$U_2 = \frac{z_2}{z_1 + z_2} - \phi_2 B_2$$

Hence we have,

$$U_i = \begin{cases} \frac{aB_i}{aB_i + z_j} - \phi_i B_i, & B_i \leq 1 \\ \frac{a - b + bB_i}{a - b + bB_i + z_j} - \phi_i B_i, & 1 \leq B_1 \leq 2 \end{cases} \quad (6)$$

Thus, by observing the optimal allocation across the social networks, we have reformulated the utility in terms of a single strategy for each user, i.e., the net budget B_i. We can now solve for the Nash equilibrium of the above non-cooperative game by studying the best response dynamics of (B_1, B_2).

Observe that when $B_2 = 0$, the strategy $B_1 = \epsilon > 0$ yields a utility of $U_1 = 1 - \phi_1 \epsilon$ for player 1 and $\forall \epsilon > 0$, $B_1 = \frac{\epsilon}{2}$ yields a higher utility. But, by the system definition, $B_1 = 0$ yields zero utility. Thus there is no best response for $B_2 = 0$, and the utility function U_1 is discontinuous at $B_1 = 0$. Hence we require $B_i \geq \delta > 0$, the minimum budget for each player. Also, since $s_{ij} \leq 1$, it is sufficient to restrict the strategy set of each user to $B_i \in [\delta, 2]$.

IV. BEST RESPONSE DYNAMICS

In this section, we first establish the existence of a Nash equilibrium and then proceed to characterize the best response function for each player. We can then obtain the Nash equilibrium as the intersection of the best response functions.

A. Existence of PSNE

Theorem 1 (Debreu, Glicksberg, Fan [11]): Consider a strategic form game $< \mathcal{I}; (S_i); (u_i) >$, where $\mathcal{I}$ is a finite set. Assume that the following conditions hold for each $i \in \mathcal{I}$.

- S_i is a non-empty, convex, and compact subset of a finite-dimensional Euclidean space.
- $u_i(s)$ is continuous in s.
- $u_i(s_i, s_{-i})$ is quasi-concave in s_i

Then the game $< \mathcal{I}; (S_i); (u_i) >$ has a pure strategy Nash equilibrium.

Since the set of strategies (the net budget), $B_i \in [\delta, 2]$ the first condition is satisfied. From the utility functions (6), the remaining two conditions can be verified, and hence the game has a pure strategy Nash equilibrium.

B. Characterizing the Best Response Function

Let $BR_i(B_j)$ be the best response of player i corresponding to the strategy B_j by player j. Let $U_i^*(B_j)$ be the corresponding maximum utility. We then have the following cases:

Case (i) $BR_i(B_j) = \delta$, $U_i^*(B_j) = \frac{a\delta}{a\delta + z_2} - \phi_1 \delta$

Case (ii): $0 < BR_i(B_j) < 1$

Setting $\frac{dU_i}{dB_i} = 0$ in (6) for $B_i \leq 1$, we get

$$BR_i(B_j) = \frac{1}{a}\left(\sqrt{\frac{z_2 a}{\phi_1}} - z_2 \right)$$

$$U_i^*(B_j) = \left(1 - \sqrt{\frac{z_2 \phi_1}{a}} \right)^2$$

Case (iii): $BR_i(B_j) = 1$, $U_i^*(B_j) = \frac{a}{a + z_2} - \phi_1$

Case (iv): $1 < BR_i(B_j) < 2$

Setting $\frac{dU_i}{dB_i} = 0$ in (6) for $1 \leq B_i \leq 2$, we get

$$BR_i(B_j) = \frac{1}{b}\left(\sqrt{\frac{z_2 b}{\phi_1}} - a + b - z_2 \right)$$

$$U_i^*(B_j) = \left(1 - \sqrt{\frac{z_2 \phi_1}{b}} \right)^2 + \phi_1 \frac{a - b}{b}$$

Case (v): $BR_i(B_j) = 2$, $U_i^*(B_j) = \frac{a+b}{a+b+z_2} - 2\phi_1$

We note that the best response can belong to any of the above cases, and hence by comparing the maximum utility $U_i^*(B_j)$ obtained in each of the cases, we can construct the best response function.

Define,

$$f_1 = \frac{1}{a}\left(\sqrt{\frac{z_2 a}{\phi_1}} - z_2 \right)$$

$$g_1 = \frac{1}{b}\left(\sqrt{\frac{z_2 b}{\phi_1}} - a + b - z_2 \right)$$

If $f_1 \geq g_1$ (which is true for sufficiently large ϕ_1), then we have the following structure for the best response function.

Let Θ_{f_1}, Θ'_{f_1} be the values of B_2 for which $f_1 = 1$ and similarly Θ_{g_1}, Θ'_{g_1} be the values of B_2 for which $g_1 = 1$. The best response function $BR_1(B_2)$ is of the following form

$$BR_1(B_2) = \begin{cases} \max(f_1, \delta), & 0 < B_2 < \Theta_{f_1} \\ 1, & \Theta_{f_1} < B_2 < \Theta_{g_1} \\ \min(g_1, 2), & \Theta_{g_1} < B_2 < \Theta'_{f_1} \\ 1, & \Theta'_{g_1} < B_2 < \Theta'_{f_1} \\ \max(f_1, \delta), & B_2 > \Theta'_{f_1} \end{cases}$$

The max and min terms feature, in order to ensure that the budget remains in the feasible set $[\delta, 2]$. We can similarly characterize $BR_2(B_1)$. The PSNE of the game can then be obtained as the intersection of the best response curves.

C. Hysteresis Behavior of Resource Allocation

Figure 3 shows the structure of the best response function. An interesting feature that arises out of this structure is that, in the intervals $\Theta_{f_1} < B_2 < \Theta_{g_1}$ and $\Theta'_{g_1} < B_2 < \Theta'_{f_1}$, it is optimal to use the budget $B_1 = 1$, i.e., exploit the resources in Network 1 to the maximum, but refrain from investing in Network 2. Especially as the opponent (player 2) increases his budget from δ to 2, player 1 increases his investments in Network 1. When Network 1 is saturated for Player 1, i.e., $s_{11} = 1$, he does not immediately begin investing in Network 2. Thus we observe there is an interval $\Theta_{f_1} < B_2 < \Theta_{g_1}$ when the optimal response remains at $B_1 = 1$. A similar behavior is observed as Player 2 begins approaching his maximum budget $B_2 = 2$. In such a scenario, it is no longer optimal to maintain $B_1 = 2$ and hence Player 1 starts lowering his budget (thus reducing his investment in Network 2). Once he relieves all his resources in Network 2, it is observed that Player 1 does not immediately begin reducing resource usage in Network 1. Instead there is an interval $\Theta'_{g_1} < B_2 < \Theta'_{f_1}$, where

Fig. 3.   Structure of the Best Response Function

Fig. 4.   Best response function for player 2 $BR_2(B_1)$ for various values of ϕ_2

Fig. 5.   Pure strategy Nash equilibrium obtained as the intersection of the best response curves. The blue curves represent the best responses of player 2 $BR_2(B_1)$ and the red curves represent the best responses of player 1 $BR_1(B_2)$.
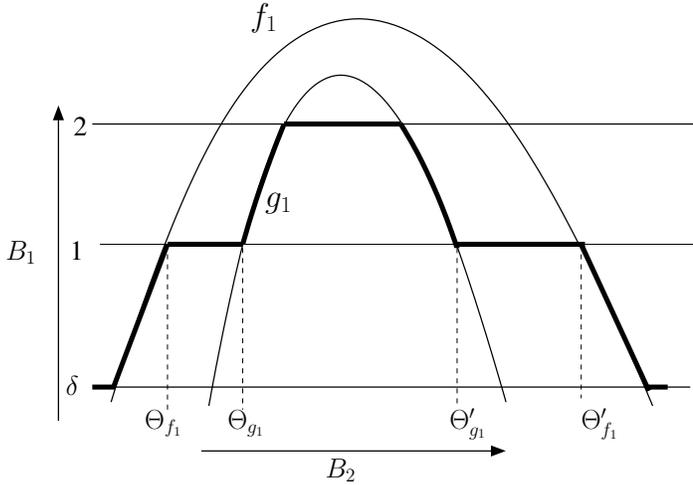
it is still optimal to operate his resources in Network 1 at full capacity. Such a behavior is similar to hysteresis curves generally observed in magnetism and thermodynamics. There are also several observed instances of hysteresis in economics [12], [13].

We can also infer that the intervals become larger, as the difference in the networks' efficiency ($\lambda_1\gamma_1 - \lambda_2\gamma_2$) increases. For instance, when the networks are equally efficient ($\lambda_1\gamma_1 = \lambda_2\gamma_2$), then the problem is similar to a Cournot duopoly model[11] with the inverse demand function $P(B_1, B_2) = \frac{1}{B_1+B_2}$ and a budget constraint. Such a demand function is termed as *iso-elastic*, since the quantity demanded is reciprocal to price, and thus reflects a case where the consumers spend a constant amount on the commodity, irrespective of the price [14]. Also, for instance, when $\lambda_2\gamma_2 = 0$, and $\lambda_1\gamma_1 > 0$, we can see that the players will never invest in Network 2 (implying a possibly infinite hysteresis gap).

D. Remarks on Cost Structure

We have assumed that a source i pays ϕ_i per unit budget. This assumption is valid, when the payment/cost incurred per promotional account is independent of the social network. This may not be the case, and in general, various social networks could charge differently for promotional accounts on their site. Twitter and Facebook have their own mechanisms for promoting content spread, and charge the users per account. With slight abuse of notation, let ϕ_j denote the cost per unit budget, in the social network j. In such a case, the total cost incurred by source i would be $\phi_1 s_{i1} + \phi_2 s_{i2}$. As a special case, when $\phi_j = C\lambda_j\gamma_j$, i.e. proportional to the network efficiency, then the utility functions will be of the form,

$$U_i = \frac{Q_i}{Q_i + Q_{i'}} - CQ_i \qquad (7)$$

where $Q_i = \lambda_1\gamma_1 s_{i1} + \lambda_2\gamma_2 s_{i2}$. Thus we again get a variation on the Cournot duopoly game[11] with inverse demand function $P(Q_1, Q_2) = \frac{1}{Q_1+Q_2}$ and a budget constraint.
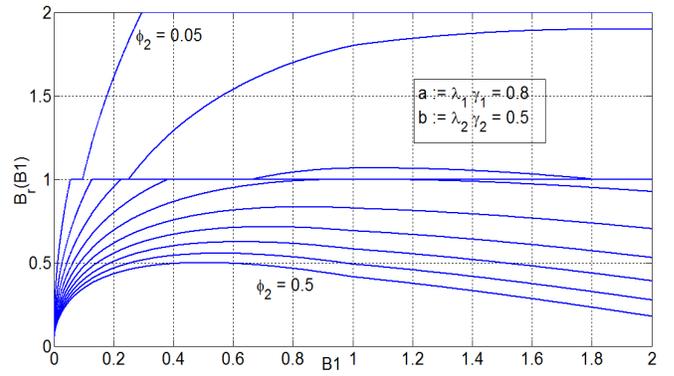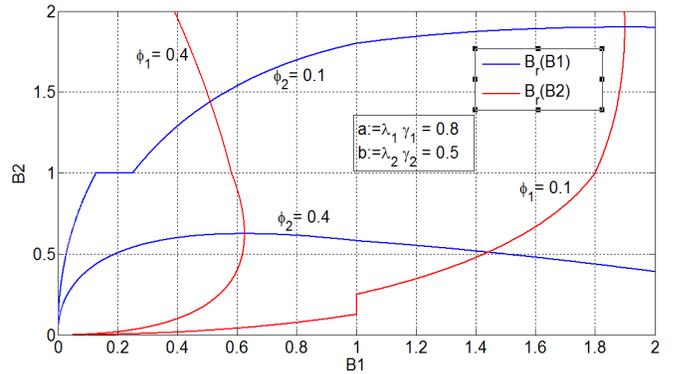
V. NUMERICAL RESULTS

In this section, we numerically study the best response function for various values of cost per unit budget ϕ_i and also compute the Nash equilibrium numerically.

A. Best response function

Figure 4 shows the best response function for player 2 $BR_2(B_1)$ for various values of player 2's cost per unit budget ϕ_2. We observe that for increasing values of ϕ_2, player 2 becomes more conservative, i.e., if $\phi_2' < \phi_2^o$, $BR_2'(B_1) \geq BR_2^o(B_1)$ for all values of B_1. We can also see that the simulations verify the hysteresis behavior observed in Section IV.

B. Nash equilibrium

Figure 5 shows the computation of Nash equilibrium obtained by intersection of the best response functions. We can see that, for fixed cost per unit budget of player 2 ($\phi_2 = 0.4$) the equilibrium budget of player 1 is greater for $\phi_1 = 0.1$ than when $\phi_1 = 0.4$. This implies that the equilibrium budget for competition increases with decrease in the player's own cost per unit budget (as observed in Section V-A). Also, we see that for fixed cost per unit budget of player 1 ($\phi_1 = 0.4$), the equilibrium budget of player 1 is greater for $\phi_2 = 0.4$ than when $\phi_2 = 0.1$. This implies that the equilibrium budget for

competition increases with increase in the opponent's cost per unit budget.

VI. CONCLUSION

In this work, we have looked at a simple model for studying competition between content creators for a common user base, via multiple social networks. We characterized the efficiency of each network for information spread by two simple properties, i.e., the level of activity (λ_j) and the network's popularity (γ_j). We formulated a non-cooperative game between the content creators and obtained the best response functions. We observed that there is a hysteresis-like behavior when content creators allocate resources between the networks. Finally we compute the Nash equilibrium numerically and discuss some interesting properties of the equilibrium.

This work can be taken forward in several directions. In the current formulation, the convergence of best response dynamics and the uniqueness, stability of the Nash equilibrium can be investigated. Puu [14] has an interesting characterization of chaotic dynamics that can arise in such a Cournot system with iso-elastic demand function, and it would interesting to study such a dynamics for a system with budget constraint and multiple markets (social networks). The model can also be improved in several ways. Firstly, the model can be made more realistic by including content sharing among users and a more general graph structure within the social network. We may also include variation among the users in their perceived popularity of a social network γ_j and their level of activity in that network. We can then generalize the work for multiple content creators and multiple social networks. Finally, it would be interesting to validate the observed hysteresis behavior using real-world data from online social networks.

VII. ACKNOWLEDGEMENTS

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