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Cooperative and Non-Cooperative Control in IEEE 802.11 WLANs

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Abstract. Numerous techniques for optimal performance of an IEEE 802.11 WLAN have been investigated. These techniques make use of either power control or PHY (physical layer) rate control or both to achieve maximum throughput levels for the network at minimum power consumption. However most of these techniques are non-cooperative by definition. Here, we analyse cooperative and non-cooperative rate and power control in an 802.11 WLAN that uses the Distributed Coordination Function (DCF). We formulate a payoff function comprising of the throughput and costs related to power consumption. The payoff function is optimized and closed form expressions for the optimal PHY rate are obtained. In the cooperative approach we seek to obtain the optimal rates under two different scenarios – *max-min fair* rate and *global multirate* allocation. In the non-cooperative approach we consider only *multirate* allocation. We consider optimization problems for both finite number of nodes n and for the limit $n \to \infty$ and obtain explicit expressions for the optimal PHY rate. Single node throughputs corresponding to the optimal PHY rates are numerically studied and it is observed that network performance in the cooperative scenario is superior to that in the non-cooperative scenario.
Keywords: IEEE 802.11, PHY rate, power control, WLAN, bandwidth sharing, GPS queue

1 Introduction

We analyse *cooperative* and *non-cooperative* power and rate control in an IEEE 802.11 WLAN environment, based on an explicit throughput expression [1] validated in [2] using ns2 simulations. We consider optimizing either the achieved aggregate network throughput (cooperative approach) or an individual node's achieved throughput (in a non-cooperative setup) by adaptively selecting one of the available PHY data rates. In the formulation of the optimization problems we further take into account a cost for power consumption. We formulate a payoff function W_n for n users which comprises a utility part representing the throughput and a cost part related to power consumption. In the cooperative case the global payoff comprising the total network throughput and total transmission power costs of all mobile nodes is maximized. In the non-cooperative game case, each player seeks to maximize its own payoff. The corresponding solution concept is then the Nash equilibrium. In the cooperative control analysis, we seek to maximize the payoff with two different approaches: (i) obtaining an optimal fair assignment of PHY rates, with a *max-min* flavor, to all nodes irrespective of their channel conditions (of course, this means that a channel with bad conditions will have to use larger power); (ii) *global multirate* approach, we allow each node to use a different PHY rate and seek to obtain the optimal rate for each node. In this case, the optimal PHY rate used by each node will depend on its channel conditions. We also present a queueing model that allows us to study the dynamic behavior aspects and expected transfer time and steady state probabilities for data transfers. Our main contribution is in obtaining explicit expressions (or set of equations that can be solved numerically in the case of $n \to \infty$) for the optimal PHY rate. These expressions are then used to calculate explicit throughput values. Our discussion takes into account both ad-hoc and infrastructure networks.

Related Work and Motivation: Application of power control in WLAN systems to minimize the required power in the *transmit mode* and adaptive selection of PHY rates has been studied by many researchers. In [9] and [10], the authors have proposed rate adaptation algorithms–Auto Rate Fallback (ARF) and Receiver based Auto Rate (RBAR). These are non-cooperative algorithms that use only PHY rate control to achieve maximum throughput levels without considering any potential benefits that can be achieved by combining power control. Some other schemes have been proposed in [12] and [13] which incorporate only power control without considering the idea of an optimal PHY rate selection. The MiSer algorithm in [11] which is based on the 802.11a/h standards, is probably one of the few algorithms that combines the idea of PHY rate and power control. MiSer is also a non-cooperative attempt to obtain optimality by using combined rate and power control. Most control schemes in previous work either consider only rate control or only power control to maximize the application throughput. Some other schemes like MiSer use both rate and power control to maximize the energy efficiency. However, all these schemes are valid only for a non-cooperative environment. That is, they attempt to optimize an individual node's performance in terms of throughput or power consumption, as mentioned before. But optimizing an individual node's performance may cause the overall network performance to suffer. Interestingly, Tan et al. in [4] have shown that in a non-cooperative scenario under DCF, a "rational" node may achieve a higher throughput by using a lower transmission rate than by using a higher transmission rate, but at the expense of a reduced overall network throughput. We will show later in Section 6 that a part of this result by Tan et al. can also be derived from our analysis. Our contributions with respect to [4] are (i) we have explicit formulas for the equilibrium throughputs, where as in [4] the throughputs are obtained numerically (ii) the formula used in [4] for the throughput (as a function of the parameters choice) depends on the frame success rate for which there is no analytical expression in [4], whereas we have an explicit expression for the frame success rate.

2 Model and background

Our analysis is based on the results obtained in [1]. Let there be n active nodes in a *single cell* IEEE 802.11 WLAN contending to transmit data. Each node uses the Distributed Coordination Function (DCF) protocol with an RTS/CTS frame exchange before any data-ack frame exchange and each node has an equal probability of the channel being allocated to it. It is assumed that there is no limit on the transmit power of any node and that every node has infinitely many packets backlogged in its transmission buffer. In other words, the transmission buffer of each node is *saturated*: there are always packets to transmit when a node gets a chance to do so. It is assumed that all nodes use the same *back-off* parameters. Let β denote the long run average attempt rate per node per *slot* ($0 < \beta < 1$) in *back-off time* [1] (conditions for the existence of a unique such β are given in [1]). We assume the *decoupling approximation* [3] which says that from the point of view of a given node, the number of attempts by the other nodes in successive slots are i.i.d. binomial random variables with parameters $(n-1)$ and β. It is assumed that the decoupling approximation and the ensuing fixed point analysis in [1] yield an accurate estimate of the attempt rate. Let the MAC frame size of node i be L_i bits and let the PHY rate used by this node be denoted by C_i bits per slot. Let T_o be defined as the transmission overhead in slots related to a frame transmission, which comprises of the SIFS/DIFS, etc and let T_c be defined as the fixed overhead for an RTS collision in slots. Then it follows from [1] that the throughput of node i is given by

$$\theta(i,n) = \frac{\beta(1-\beta)^{n-1}L_i}{1 + n\beta(1-\beta)^{n-1}\left(T_o - T_c + \frac{1}{n}\sum_{i=1}^{n}\frac{L_i}{C_i}\right) + \left(1-(1-\beta)^n\right)T_c} \tag{1}$$

where $\beta = \beta(n)$ for the case when $K \to \infty$ is given by [1]

$$\beta = \frac{\eta p - LambertW(\eta(p-1)e^{\eta p})}{\eta b_0}$$

with $\eta = \frac{n-1}{b_0}$. Note that β does not depend on L_i or C_i, and all nodes achieve the same single node throughput even if they use different rates. As is the case in IEEE 802.11, for all nodes that use an

RTS/CTS frame exchange before the data-ack frame transmission, we assume throughout our discussion that $T_o \geq T_c$. In our analysis in the following sections, we will consider optimization problems for both finite n and for the limit $n \to \infty$. For handling the latter case, we identify here the asymptotic aggregate throughput as $n \to \infty$ (this derivation can be found in [1] for the special symmetric case where all L_i's and C_i's are equal). An appealing feature of the asymptotic case is the *explicit* expression for β.
Asymptotic throughput: In our discussion we use asymptotic throughput in the following two contexts:
(i) In the max-min fair (MMF) case where we assign the same PHY rate to all mobile nodes, we consider all nodes to be symmetric, i.e., they all use the same PHY rate C (they still may have different channel conditions). In this case, if first $K \to \infty$ [1] and then $n \to \infty$, the global throughput is given by Sec. VII.C in [1] as:

$$\tau(C) = \frac{L\left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + \left(\frac{L}{C}\right)\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{2}$$

where p is the exponential back-off multiplier, i.e., if b_k is the mean back-off duration (in slots) at the kth attempt for a frame then $b_k = p^k b_0$. According to the IEEE 802.11 specifications $p = 2$.
(ii) In the case where we consider global multirate PHY rate assignment to all nodes, i.e., each node uses one of the c distinct available values of the parameters (C_i, L_i) with $(C_i, L_i) \in \{(C_1, L_1), ..., (C_c, L_c)\}$, we derive here the corresponding asymptotic throughput. Assume that there are m_i nodes using parameters (C_i, L_i). Denote by $\alpha_i(n) = m_i/n$ the fraction of the nodes using (C_i, L_i) among all nodes in the cell. Then the throughput of all nodes using (C_i, L_i) is given by

$$\theta(\alpha_i(n)) = \frac{m_i \beta e^{-n\beta} L_i}{1 + n\beta e^{-n\beta}\left(T_o - T_c + \sum_{i=1}^{c} \frac{\alpha_i(n) L_i}{C_i}\right) + \left(1 - e^{-n\beta}\right) T_c} \tag{3}$$

where we use the Binomial to Poisson approximated version of the throughput expression for the asymptotic case mentioned in Section VII.C of [1]. It is assumed that $\alpha_i(n)$ converges to a limit α_i which is a probability measure. Note that the attempt rate $\beta = \beta(n)$ and the collision probability γ as defined in [1] are not functions of L_i nor C_i. Now, first taking $K \to \infty$ [1] and then taking the limit $n \to \infty$, it can be observed that $\lim_{n\to\infty} n\beta(n) \uparrow \ln\left(\frac{p}{p-1}\right)$ (see Theorem VII.2 in [1]). Combining this result with Equation (3) we get as $n \to \infty$ the following expression for the aggregate throughput of all nodes using (C_i, L_i):

$$\tau(\alpha_i) = \frac{\alpha_i L_i \left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + \sum_{i=1}^{c}\left(\frac{\alpha_i L_i}{C_i}\right)\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{4}$$

Denote $E_\alpha[L/C] = \sum_{i=1}^{c} \frac{\alpha_i L_i}{C_i}$ and $E_\alpha[L] = \sum_{i=1}^{c} \alpha_i L_i$. Then it follows from Equation (4) that the asymptotic global throughput is given by

$$\tau(\alpha) = \frac{E_\alpha[L]\left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + E_\alpha[L/C]\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{5}$$

3 Defining the payoff function

In an efficiently working WLAN, the goal of the mobile nodes is to achieve maximum throughput levels with minimized power consumption costs. In a cooperative scenario, the nodes should cooperate to achieve maximum overall network throughput at minimum combined power consumption. If each node uses the highest available PHY rate, which is say common for all nodes, it may not be the best strategy to achieve the most efficient overall network performance. The reason being that under the given channel conditions, a node may be unnecessarily consuming more power by transmitting at the highest available rate if transmitting at a lower PHY rate does not degrade the combined network throughput. Based on this thought and the fact that under DCF, each node has an equal probability of gaining access to the channel we define a long-term payoff function W_n for n active nodes in the WLAN as

$$W_n := \sum_{i=1}^{n} (\theta(i,n) - \zeta_i Q_i(C_i)) \tag{6}$$

where $\theta(i,n)$ is the throughput of node i as defined in Equation (1). $Q_i(C_i)$ is a cost related to the power consumption of node i and is a function of the PHY rate C_i and ζ_i is a weight that gives relative importance for node i to the cost versus the throughput. Note that, maximizing this payoff function leads to maximizing the throughput and minimizing the costs related to power consumption. Experiments conducted by Gruteser et al. in [5] with IEEE 802.11 equipment reveal that under given channel conditions and a low transmission power range, the power consumed by a mobile node can be approximated as being linearly proportional to the PHY rate used. Consequently, $Q_i(C_i)$ can be considered as a linear cost function of the form: $Q_i^{lin}(C_i) = a_i C_i$, where a_i is a random variable that may depend on the path attenuation under given channel conditions. Next, motivated by the Shannon's theorem and assuming an AWGN channel that uses complex symbols , the transmission rate of a node is of the form $C(\pi) = W \log_2 \left(1 + \frac{\pi}{z}\right)$, where W is the passband spectrum in $Hertz$. π is the transmission power of the node and $z = WN_o/h$, where N_o is the one-sided *power spectral density* of the channel noise and h is a random variable that characterizes the signal attenuation. z is therefore a random variable that may depend on channel fading and shadowing. The previous equation can be rewritten as: $\pi(C) = z(e^{\psi C} - 1)$, where $\psi = \frac{ln2}{W}$. It has also been seen in the results of the experiments in [5] that the power consumed by mobile nodes is piecewise linearly proportional to the transmission power. Therefore, an exponential cost $Q_i^{exp}(C_i)$ can be assumed, which is of the form: $Q_i^{exp}(C_i) = z_i(e^{\psi C_i} - 1)$. From the definitions of a_i and z_i in the foregoing discussion it is evident that their values may vary from one mobile node to another. We denote the expected values of a_i and z_i by $E[a_i]$ and $E[z_i]$.

4 Cooperative approach

In the cooperative approach to PHY rate and power control, we shall consider two different scenarios. In the first *max-min fair* scenario, we assign each node the same PHY rate C and MAC frame size L at all channel states. This will of course require an appropriate power control so that in bad channel conditions the transmitted power is larger. We seek to obtain the optimal PHY rate that will maximize the overall payoff of the network. As discussed before, an optimal PHY rate may not be the highest available PHY rate. In the second *global multirate* scenario, we allow each node to use a different PHY rate C_i depending on its channel conditions and we seek to obtain the globally optimal PHY rates for all nodes. In both the scenarios, it is assumed that all nodes use the same MAC frame size L. We will pursue analysis for finite n number of nodes and also consider the situation when $n \to \infty$ for both cases. We shall consider the set of possible values of C or C_i, as the case may be, to lie in an interval of the form $\underline{\mathcal{C}} := [C_l, C_u]$. In 802.11a, this interval could be $[6, 54]$.

4.1 Obtaining the max-min fair PHY rates

A max-min assignment of resources to users is a fairness concept characterized by the property that no user i can be assigned more resources unless we decrease the assignment to another user j who already has the same amount or a lesser amount of resources than user i. This is an efficient assignment in the Pareto sense. In our case it is the PHY rates that are assigned according to the max-min approach leading to an identical assignment to all users. Note that the actual throughputs are already the same for all users even if the PHY rates are different.

Finite number of nodes: We seek to maximize the payoff function defined in Section 3 while assigning the same PHY rate C to each node irrespective of the channel conditions. Consider the following problem:

$$\text{Find } C^* \text{ that maximizes } W_n := \sum_{i=1}^{n} (\theta(i,n) - \zeta_i Q_i(C)) \tag{7}$$

W_n is concave with respect to C (see [6] for proof) and thus has a unique maximizer C^*. In particular, we have the linear and the exponential costs as: $Q_i^{lin}(C) = E[a_i]C, \quad Q_i^{exp}(C) = E[z_i]\Big(e^{\psi C} - 1\Big)$. Denote $u^{lin} = \sum_{i=1}^{n} \zeta_i a_i$ and $u^{exp} = \sum_{i=1}^{n} \zeta_i z_i$ and set

$$q_1 = n\beta(1-\beta)^{n-1}L, \quad q_2 = 1 + n\beta(1-\beta)^{n-1}(T_o - T_c) + (1-(1-\beta)^n)\,T_c \tag{8}$$

$$\text{Then} \quad W_n^{lin}(C) = \frac{q_1}{q_2 + q_1/C} - E[u^{lin}]C \qquad W_n^{exp}(C) = \frac{q_1}{q_2 + q_1/C} - E[u^{exp}]\Big(e^{\psi C} - 1\Big) \tag{9}$$

By differentiating the payoff w.r.t. C and equating the derivative to zero, we get the following results:

(i) In the linear case, the unique positive solution of $\frac{dW_n^{lin}(C)}{dC} = 0$ is given by

$$C^* = \frac{q_1}{q_2}\left(\frac{1}{\sqrt{E[u^{lin}]}} - 1\right) \tag{10}$$

provided that $0 < E[u^{lin}] < 1$. If $E[u^{lin}] \geq 1$ then there is no positive solution.

(ii) In the exponential case the unique positive solution of $\frac{dW_n^{exp}(C)}{dC} = 0$ is given by

$$C^* = \frac{2}{\psi} LambertW\left(\frac{1}{2}\frac{q_1}{q_2}\sqrt{\frac{\psi}{E[u^{exp}]}}\exp\left(\frac{1}{2}\frac{q_1}{q_2}\psi\right)\right) - \frac{q_1}{q_2} \tag{11}$$

See [6] for the definition of *LambertW* function. In either the linear or the exponential case, if C^* lies within $\underline{\mathcal{C}}$ then it is the unique globally optimal rate assignment solution for problem (7). If not, then the optimal solution is obtained on one of the two boundary points of $\underline{\mathcal{C}}$. We defer the discussion on the numerical computations of C^* to Section 6.

The asymptotic case: We present here the asymptotic behaviour for large number of nodes. The optimization is based on the expression for the asymptotic throughput given by Equation (2). Here we assume that a_i, z_i and ζ_i have the same distribution for all mobiles. Consider the following problem:

$$\text{Find } C^* \text{ that maximizes } W(C) := \tau(C) - \zeta Q(C) \tag{12}$$

where $Q(C) = E[a]C$ for the linear cost and $Q(C) = E[z]\Big(e^{\psi C} - 1\Big)$ for the exponential one. $W(C)$ turns out to be concave in C (see [6] for proof) and therefore it has a unique maximizer. Writing $W(C)$ for the linear and exponential case as

$$W^{lin}(C) = \frac{q_1}{q_2 + q_1/C} - E[a]C \quad \text{and} \quad W^{exp}(C) = \frac{q_1}{q_2 + q_1/C} - E[z]\Big(e^{\psi C} - 1\Big)$$

$$\text{where} \quad q_1 = L\left(1 - \frac{1}{p}\right), \qquad q_2 = \frac{1 + T_c/p}{\ln\left(\frac{p}{p-1}\right)} + \left(1 - \frac{1}{p}\right)(T_o - T_c). \tag{13}$$

Then the optimal C is obtained by differentiating $W^{lin}(C)$ and $W^{exp}(C)$ and equating them to zero, which gives the following unique positive solution for the linear and exponential cases, respectively:

$$C^*_{lin} = \frac{q_1}{q_2}\left(\frac{1}{\sqrt{\zeta E[a]}} - 1\right), \quad C^*_{exp} = \frac{2}{\psi} LambertW\left(\frac{1}{2}\frac{q_1}{q_2}\sqrt{\frac{\psi}{\zeta E[z]}}\exp\left(\frac{1}{2}\frac{q_1}{q_2}\psi\right)\right) - \frac{q_1}{q_2}.$$

If C^* lies within $\underline{\mathcal{C}}$ then it is the unique globally optimal rate assignment solution for problem (12). If not then the optimal solution is obtained on one of the two boundary points of $\underline{\mathcal{C}}$. Also note that C^* here has the same form as in the finite n case but with different q_1 and q_2.

4.2 The dynamic case

So far we have considered a fixed number of nodes in the system. In this section we consider a dynamic setting. Let nodes arrive in a WLAN system according to an independent Poisson process with rate λ. The nth node is assumed to have a service requirement σ_n where σ_n are i.i.d. generally distributed.

Max-min fair case: We assume that the assignment of physical rates follows the max-min fairness approach, so that the physical rate of each mobile node is given by $C(n)$, where n is the number of mobiles in the system; n can be referred to as the system state. Then the throughput of each mobile when the system is in state n is given by

$$\theta(n) = \frac{\beta(1-\beta)^{n-1}L}{1 + n\beta(1-\beta)^{n-1}\left(T_o - T_c + \frac{L}{C}\right) + \Big(1-(1-\beta)^n\Big)T_c} \tag{14}$$

Since all mobiles use an identical MAC frame size L in the max-min fairness approach and hence theoretically achieve the same throughput, the whole network can be viewed as an $M/G/1/\infty$ queue where the service discipline is a generalized processor sharing (GPS) [8]. If we denote $\rho := \lambda E[\sigma_0]$ and define $\phi(n) = 1/\prod_{i=1}^{n}\theta(i)$ then applying the general theory of [8] for GPS queues, we obtain the following expressions for the steady state probabilities and sojourn times:

Theorem 1. *Assume that $\sum_{i=1}^{\infty}\rho^i\phi(i)/i! < \infty$. Then the steady state probabilities exist. They, as well as the expected sojourn time of a mobile are given by*

$$Pr(N=n) = \frac{\rho^n\phi(n)}{n!\sum_{i=0}^{\infty}\frac{\rho^i}{i!}\phi(i)}, \qquad E[T] = E[\sigma]\frac{\sum_{j=0}^{\infty}\frac{\rho^j}{j!}\phi(j+1)}{\sum_{j=0}^{\infty}\frac{\rho^j}{j!}\phi(j)}$$

Remark 1. Using [8] one can in fact (i) obtain explicit expressions for the QoS of more complex arrival process, and in particular for on/off sources having general thinking times, (ii) obtain explicit expressions for the case when there is a limit (enforced by a call admission control) on the number of active mobiles.

Proportional fairness: We next consider the case in which the MAC frame size L_i is not identical for all nodes but is taken to be proportional to the PHY rate C_i, so that $\Omega := L_i/C_i$ does not depend on i. Then even if the MAC frame size L_i is not the same for all nodes (unlike in all other cases in the paper), we still get the throughput of each node in terms of *packets per second* to be identical and is given by:

$$\overline{\theta}(n) := \frac{\theta(n)}{L_i} = \frac{\beta(1-\beta)^{n-1}}{1 + n\beta(1-\beta)^{n-1}\left(T_o - T_c + \Omega\right) + \Big(1-(1-\beta)^n\Big)T_c} \tag{15}$$

Now, (i) Theorem 1 still holds with $\overline{\theta}(n)$ replacing $\theta(n)$ in the definition of $\phi(n)$, and (ii) the expected sojourn time of mobile i is given by:

$$E[T_i] = \frac{E[\sigma]}{L_i}\frac{\sum_{j=0}^{\infty}\frac{\rho^j}{j!}\phi(j+1)}{\sum_{j=0}^{\infty}\frac{\rho^j}{j!}\phi(j)}.$$

4.3 Global multirate (channel dependent) optimization

In this section we consider global optimization in which we allow each node to use a different PHY rate C_i and we seek to obtain the best choice of C_i, $i = 1..n$. We assume that all values of $E[a_i]$, $E[z_i]$ and ζ_i are known. By allowing C_i to differ from one node to another we expect to achieve higher efficiency.
Finite number of nodes, channel-dependent case: Consider the following problem:

$$\text{Find } \mathbf{C}^* = (C_1^*, ..., C_n^*) \text{ that maximizes } W_n := \sum_{i=1}^{n}(\theta(i,n) - \zeta_i Q_i(C_i)) \tag{16}$$

where $\theta(i,n)$ is defined by Equation (1). Then we have,

$$W_n^{lin} = \frac{q_1}{q_2 + \frac{q_1}{n}\sum_{i=1}^{n}(\frac{1}{C_i})} - \sum_{i=1}^{n}\zeta_i E[a_i]C_i \text{ and } W_n^{exp} = \frac{q_1}{q_2 + \frac{q_1}{n}\sum_{i=1}^{n}(\frac{1}{C_i})} - \sum_{i=1}^{n}\zeta_i E[z_i](e^{\psi C_i} - 1) \tag{17}$$

where q_1 and q_2 are defined in (8). Define

$$q_2^i = q_2 + \frac{q_1}{n}\sum_{j=1, j\neq i}^{n}\frac{1}{C_j}, \qquad \hat{C} = \left(\sum_{i=1}^{n}\frac{1}{C_i}\right)^{-1} \qquad \text{and} \qquad H(\hat{C}) = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 n}.$$

With these definitions and by differentiating W_n^{lin} we get

$$\frac{\partial W_n^{lin}}{\partial C_i} = \frac{nq_1^2}{(nq_2^i C_i + q_1)^2} - \zeta_i E[a_i] = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 nC_i^2} - \zeta_i E[a_i] \tag{18}$$

and similarly by differentiating W_n^{exp} we get

$$\frac{\partial W_n^{exp}}{\partial C_i} = \frac{nq_1^2}{(nq_2^i C_i + q_1)^2} - \psi\zeta_i E[z_i]e^{\psi C_i} = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 nC_i^2} - \psi\zeta_i E[z_i]e^{\psi C_i} \tag{19}$$

Now by equating the derivatives in Equations (18) and (19) to zero, we obtain:

(i) In the linear case, we get from Equation (18)

$$C_i = \sqrt{\frac{H(\hat{C})}{\zeta_i E[a_i]}} \text{ and also, } \hat{C} = \left(\sum_{i=1}^{n}\sqrt{\frac{\zeta_i E[a_i]}{H(\hat{C})}}\right)^{-1} = \frac{\sqrt{H(\hat{C})}}{Y}, \quad \text{where } Y = \sum_{i=1}^{n}\sqrt{\zeta_i E[a_i]} \tag{20}$$

which implies that the solution $\hat{C}^*$ is given by $\hat{C}^* = \frac{1}{n}\frac{q_1}{q_2}\left(\frac{\sqrt{n}}{Y} - 1\right)$. Substituting the solution of this equation in (20) gives the C_i^*'s.

(ii) In the exponential case, we get from Equation (19)

$$C_i = \frac{2}{\psi} LambertW\left(\frac{1}{2}\sqrt{\frac{\psi H(\hat{C})}{\zeta_i E[z_i]}}\right) \tag{21}$$

Therefore, using the definition of $\hat{C}$, $\hat{C}^*$ is the solution of $\hat{C} = \frac{2}{\psi \sum_{i=1}^{n}\left[LambertW\left(\frac{1}{2}\sqrt{\frac{\psi H(\hat{C})}{\zeta_i E[z_i]}}\right)\right]^{-1}}$

which yields the C_i^*'s through (21).

The above solutions are globally optimal provided they are within the range $\underline{\mathcal{C}}$. We defer the discussion on the numerical computations of C_i^*'s to Section 6.

Large number of nodes: To model the case of a large number of users we shall use a fluid approximation in which there are (non-countably) infinite number of users. We introduce R population classes of mobiles. The parameter z in the exponential cost function will be the same for all mobiles of the same type $r, r = 1, 2, ..., R$ so that mobiles belonging to a given class r have the same channel conditions. We shall thus use the notation $z^{(r)}$ to indicate this dependence. We shall use similarly the notation $a^{(r)}$ for the coefficient appearing in the linear cost. In short, mobiles with the same value of $(a^{(r)}, \zeta^{(r)})$ (in the linear case) or $(z^{(r)}, \zeta^{(r)})$ (in the exponential case) are said to belong to the same class of mobiles having identical channel conditions. We define for each r the vector $\mathbf{x}^{(r)} = (x_1^{(r)}, ..., x_c^{(r)})$ to be the amount of r-type mobiles that use each of the rates $C_1, ..., C_c$. Define $\mathbf{x} = (\mathbf{x}^{(1)}, ..., \mathbf{x}^{(R)})$ to be a multistrategy for all mobiles. With some abuse of notation, let $x_i := \sum_{r=1}^{R} x_i^{(r)}$ denote the global amount of mobiles that use the rate C_i under $\mathbf{x}$. Denote $\overline{\nu}$ to be the total amount of users. Then $\overline{\nu} = \sum_{i=1}^{c} x_i$. Define $\alpha_i(\mathbf{x}) = x_i/\overline{\nu}$. It follows from Equation (5) that

$$\tau(\alpha(\mathbf{x})) = \frac{E_{\alpha(\mathbf{x})}[1]q_1}{q_2 + q_1 E_{\alpha(\mathbf{x})}[1/C]} = \frac{\overline{\nu}q_1}{\overline{\nu}q_2 + q_1\sum_{i=1}^{c} x_i C_i^{-1}}$$

where q_1 and q_2 are given by Equation (13). To simplify, we shall denote $\tau(\mathbf{x}) = \tau(\alpha(\mathbf{x}))$. Define $Q_i^{(r)}(x_i^{(r)}) = a^{(r)}C_i$ for the linear cost and $Q_i^{(r)}(x_i^{(r)}) = z^{(r)}\left(e^{\psi C_i} - 1\right)$ for the exponential cost. Then our problem of maximizing the payoff function turns out to be a non-linear optimization problem defined by:

$$\max_{\mathbf{x}} W(\mathbf{x}) \quad \text{where } W(\mathbf{x}) := \tau(\mathbf{x}) - \sum_{r=1}^{R}\zeta^{(r)}\sum_{i=1}^{c} x_i^{(r)} Q_i^{(r)}(x_i^{(r)})$$

$$= \frac{\overline{\nu}q_1}{\overline{\nu}q_2 + q_1\sum_{i=1}^{c}\left(\sum_{r=1}^{R} x_i^{(r)}\right)C_i^{-1}} - \sum_{r=1}^{R}\zeta^{(r)}\sum_{i=1}^{c} x_i^{(r)} Q_i^{(r)}(x_i^{(r)})$$

$$\text{subject to} \qquad \sum_{i=1}^{c} x_i^{(r)} = g_r, \forall r, \quad x_i^{(r)} \geq 0, \forall i, r$$

where g_r is the predefined constraint on the number of mobiles in class r. Describing the solution for this problem is outside the scope of this paper. However, $W(\mathbf{x})$ turns out to be concave and the feasible set is compact. We conclude that there exists a unique solution.

5 Non-cooperative game

In this section we analyse the non-cooperative behaviour of mobile nodes. We shall model this situation using non-cooperative game theory and obtain the equilibrium. Here we will consider only the case when there are finite number of nodes n (see [6] for the asymptotic case). In a non-cooperative setting, each node uses the same MAC frame size L and is allowed to use a different PHY rate as in the global multirate allocation in the cooperative approach. But here the objective of each node is to maximize its individual payoff function which can be denoted by $\Omega_i(C_i)$ and defined as

$$\Omega_i(C_i) = \theta(i,n) - \zeta_i Q_i(C_i) \tag{22}$$

For every i, Ω_i is concave w.r.t. C_i and continuous w.r.t. $C_j, j \neq i$. It then follows from Rosen [7] that a Nash equilibrium exists. In particular, we shall be interested in the linear $Q_i^{lin}(C_i) = E[a_i]C_i$ and the exponential $Q_i^{exp}(C_i) = E[z_i]\left(e^{\psi C_i} - 1\right)$ cases. We have

$$\frac{\partial \Omega_i^{lin}}{\partial C_i} = \frac{H(\hat{C})}{nC_i^2} - \zeta_i E[a_i], \qquad \frac{\partial \Omega_i^{exp}}{\partial C_i} = \frac{H(\hat{C})}{nC_i^2} - \psi\zeta_i E[z_i]e^{\psi C_i} \tag{23}$$

These are the same equations as we had in Section 4.3 except an extra factor of n in the denominator. Equating them to zero:

(i) For the linear case: $\hat{C}^* = \frac{1}{n}\frac{q_1}{q_2}\left(\frac{1}{Y} - 1\right)$, Y is the same as defined in (20) and C_i^*'s are obtained from

$$C_i = \sqrt{\frac{H(\hat{C})}{n\zeta_i E[a_i]}}. \tag{24}$$

(ii) Similarly for the exponential case, we obtain $\hat{C}^*$ as a solution of

$$\hat{C} = \frac{2/\psi}{\sum_{i=1}^{n}\left[LambertW\left(\frac{1}{2}\sqrt{\frac{\psi H(\hat{C})}{n\zeta_i E[z_i]}}\right)\right]^{-1}} \text{ which gives } C_i^* = \frac{2}{\psi} LambertW\left(\frac{1}{2}\sqrt{\frac{\psi H(\hat{C}^*)}{n\zeta_i E[z_i]}}\right)$$

We defer the discussion on the numerical computations of C_i^*'s to Section 6.

6 Numerical studies

In this section, we numerically examine the closed form expressions for the optimal transmission rates, single node throughputs and overall payoffs. We compute these quantities as a function of number of nodes n (see[6] for variation with frame size L). The following set of parameters have been used to study the optimal transmission rates and the corresponding single node throughputs and overall payoffs. In the linear cost $E[a_i]$ is set to vary uniformly from $a_i^{min} = 0.5 * 10^{-3}$ to $a_i^{max} = 1 * 10^{-3}$ $watts$ per $bits/slot$ for each mobile i. In the exponential cost $E[z_i]$ is set to vary uniformly from $z_i^{min} = WN_o/h_i^{min}$ to $z_i^{max} = WN_o/h_i^{max}$ where value of W (passband spectrum) is taken as 20 MHz for an 802.11a system, N_o (one-sided power spectral density) is taken as $5.52 * 10^{-21}$ $watts/Hz$ and for the Rayleigh fading case $h_i^{min} = 10^{-11}$ and $h_i^{max} = 10^{-8}$. The back-off multiplier $p = 2$ and $b_0 = 16$ slots in $b_k = p^k b_0$. The data frame transmission overhead $T_o = 52$ slots, the RTS collision overhead $T_c = 17$ slots and the MAC frame size $L = 12000 bits (1500 bytes)$. The slot size is taken as $20\mu s$ and $K = 10$ [1]. For simplification the parameter ζ_i is taken to be the same for all nodes $i = 1..n$. The values are displayed below each plot. The plots obtained from the numerical computations are presented at the end of the paper.

Comparison of cooperative and non-cooperative solutions: In the PHY rate plots (Figure 1- 3 & 7- 9), we observe that the optimal PHY rate of each node decreases with increasing number of nodes. It can be seen that in cooperative global multirate and non-cooperative multirate allocations, for a given number of nodes, each node is assigned a different rate depending on the parameters $E[a_i]$ and $E[z_i]$ for channel conditions. When we have a *linear* cost associated with the power consumption, then for $n = 2$, the single node throughput in the cooperative global multirate case (Figure 5) is around 11% higher than in the non-cooperative multirate case (Figure 6). In fact with increasing n the single node throughput percentage gain in the cooperative global multirate scenario over the non-cooperative multirate scenario goes from 11% for $n = 2$ to up to more than 200% for $n = 10$. When the cost associated with power consumption is *exponential* the single node throughput percentage gain in the cooperative allocation (Figure 11) over the non-cooperative allocation (Figure 12) varies from around 12% for $n = 2$ to up to 100% for $n = 10$. We also observe that the cooperative max-min fair scheme (Figure 4,10) performs almost equally well as the cooperative global multirate scheme (Figure 5,11). These observations clearly illustrate that cooperative rate allocation strategy results in higher single node throughputs and hence higher total network throughput as against non-cooperative strategy. Our analysis thus confirms the results obtained by Tan et al. in [4]. Indeed the DCF protocol under non-cooperative setting is not efficient.

7 Conclusion and future work

In this paper, we have analysed cooperative and non-cooperative rate and power control in an IEEE 802.11 WLAN, by optimizing a payoff function that comprises of the throughput and costs related to power consumption. It is observed through numerical studies that cooperative control is more efficient than non-cooperative control. With a linear cost approximation, the single node throughput in the cooperative approach is observed to be 11% to 200% more than in the non-cooperative game approach. The improvement varies from 12% to 100% in the exponential cost approximation case. Thus a first glimpse of cooperative and non-cooperative control in an 802.11 WLAN by our analysis shows that the currently used mandatory DCF protocol in 802.11 does not perform with the highest efficiency in a non-cooperative setting. Our future work will include designing an efficient cooperative rate and power control algorithm based on the analysis illustrated in this paper.

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Variation with n and linear cost

Cooperative Approach

Max-Min fair

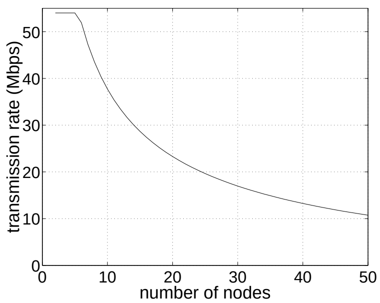

Fig. 1. Using Eqn. 10, $\zeta_i = 6$

Global multirate

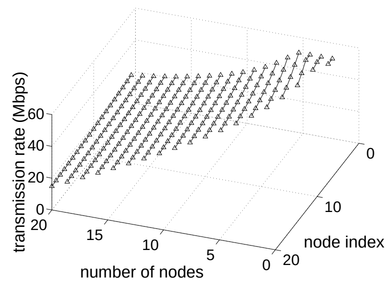

Fig. 2. Using Eqn. 20, $\zeta_i = 9$

Non-cooperative game

Multirate

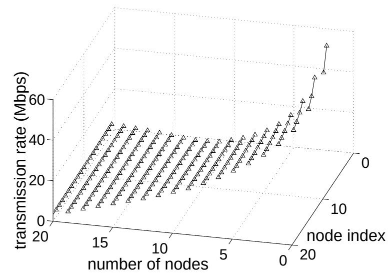

Fig. 3. Using Eqn. 24, $\zeta_i = 9$

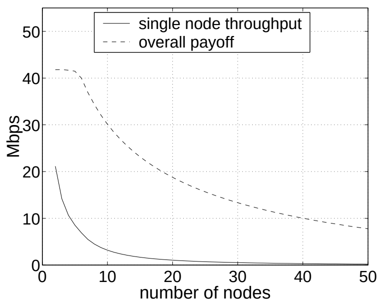

Fig. 4. Using Eqn. 9, $\zeta_i = 6$

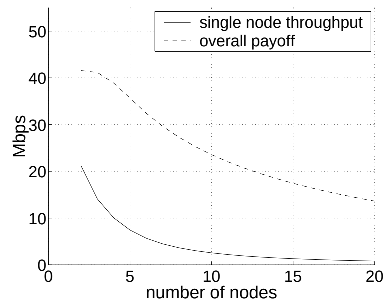

Fig. 5. Using Eqn. 17, $\zeta_i = 9$

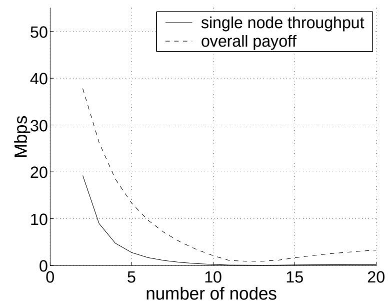

Fig. 6. Using Eqn. 22, $\zeta_i = 9$

Variation with n and exponential cost

Cooperative Approach

Max-Min fair

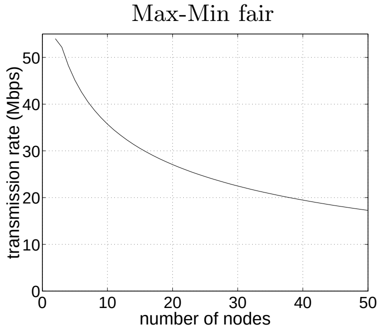

Fig. 7. Using Eqn. 11, $\zeta_i = e^5$

Global multirate

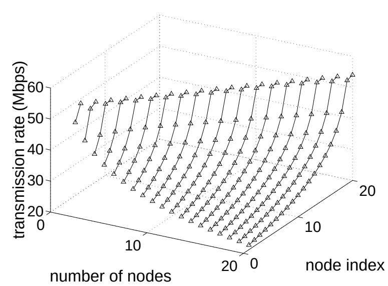

Fig. 8. Using Eqn. 21, $\zeta_i = e^5$

Non-cooperative game

Multirate

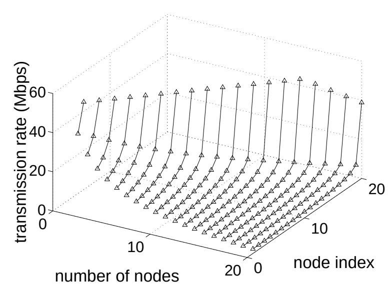

Fig. 9. Using Eqn. 5, $\zeta_i = e^5$

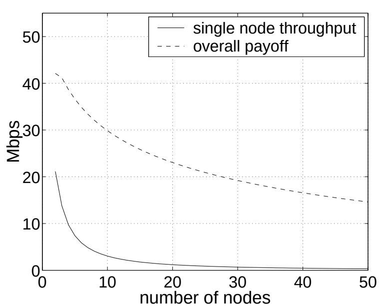

Fig. 10. Using Eqn. 9, $\zeta_i = e^5$

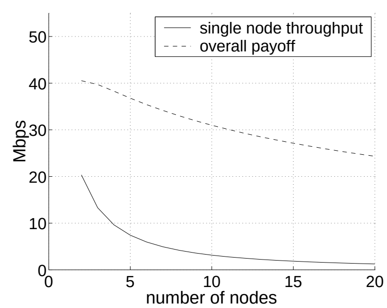

Fig. 11. Using Eqn. 17, $\zeta_i = e^5$

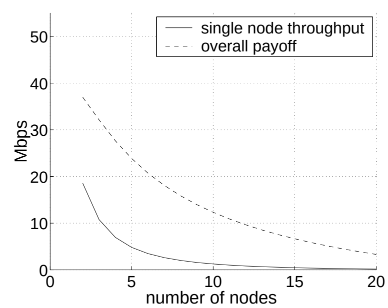

Fig. 12. Using Eqn. 22, $\zeta_i = e^5$