Saturation Throughput Analysis of a System of Interfering IEEE 802.11 WLANs *

Manoj K. Panda †   Anurag Kumar ‡   S H Srinivasan §

Abstract

IEEE 802.11 wireless local area networks that span large buildings or campuses must comprise multiple cells, several of which must necessarily be cochannel cells. It can be shown that in such multicell networks, typically, the radio ranges of cochannel cells overlap. The concern in this paper is to study the saturation throughput performance of two cochannel cells with ***critical overlap****, i.e., their interference ranges completely overlap but no node in either cell can decode any transmissions from the other cell. We identify that the difference between the two MAC parameters, EIFS and DIFS is a key issue and incorporate this difference as a parameter into an analytical model. The model yields a fixed point equation that yields an approximation to the saturation throughputs of the two cells. The results from the analysis are validated against ns-2 simulations. We find that with EIFS>DIFS there is substantial temporal unfairness in the channel access between the two cells, but, because of fewer collisions, the critical overlap configuration has higher per cell throughput than if the cells' decoding ranges overlapped.*

1 Introduction

In the recent past, much effort has been directed toward gaining an analytical understanding of the IEEE 802.11 standard. Most of the published analyses of IEEE 802.11 based WLANs are for the case of a *single cell* network. A single cell is a system of wireless devices or *nodes* located within such a distance of each other that any node n_i in the system can decode the *control* transmissions from any other node n_j in the system provided that all other nodes $n_k, k \neq j$, are silent. In a single cell, there is at most one successful transmission at any point of time.

In this paper, we are concerned with the analysis of wireless networks that are based on the IEEE 802.11 standard and span larger geographical areas, e.g., large buildings. Such networks must necessarily comprise multiple cells. Cell layout, cell coverage and channel allocation are some of the problems that have to be addressed together. In IEEE 802.11 networks, these problems are complicated and performance analysis of such scenarios should involve the consideration of a system of interfering cells.

Our work is motivated by these issues, and our aim is to provide an analytical study of the saturation throughput performance of a multicell wireless network, in which the radio ranges of the cells overlap. The main contributions of this paper are the following:

(a) Based on the measurements and cell layouts reported in existing literature, we predict the possible interdependencies that may exist between the adjacent cochannel cells in multicell wireless networks. We propose four important parameters to quantify the nature of overlap between the adjacent cochannel cells. We focus on a specific type of cell placement, which we call the *critical placement*, and argue that critical placement can actually arise in practice.

(b) We then provide a saturation throughput analysis of a pair of critically placed cells with arbitrary number of nodes in each cell. Our analysis is based on an extension of the fixed point approach provided in [7].

(c) Finally, we provide simulation results that corroborate our analytical approach, demonstrate the unfairness that may exist in multicell networks because of the proximity of adjacent cochannel cells, and show that in certain multicell networks, where hidden terminals do not exist, fairness can be significantly improved by a simple modification made to the EIFS value.

The remaining of this paper is organized as follows. In Section 2, we briefly discuss the work reported in the literature related to multicell networks. In Section 3, we discuss some of the features of the IEEE 802.11 standard that are important in the context of our work. In Section 4, we discuss various possible cell placements at length. In Section 5, we provide a saturation throughput analysis of a pair of critically placed cells. In Section 6, we provide results obtained from ns-2 simulations and compare them with the

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†Applied Research Group, Satyam Computer Services Ltd., SID Block, IISc Campus, Bangalore - 560012.

‡ECE Department, Indian Institute of Science, Bangalore, INDIA

§Applied Research Group, Satyam Computer Services Ltd., SID Block, IISc Campus, Bangalore - 560012.

results obtained from analysis. We conclude the paper in Section 7.

2 Literature Survey

The pioneering work on the performance of single cell networks was done by Bianchi [3]. The analysis has been simplified and generalized by Kumar et al. [7]. In this paper, we have extended the approach in [7] to a pair of critically placed cells.

Wireless networks having "ring" and "mesh" topologies have been analytically modeled for *multihop* operation by Gupta and Kumar [10]. In performing the analysis, the authors have considered a simplified model by ignoring the mechanism of doubling of the maximum back-off window on collisions. Our emphasis in this paper is on the *muticell* case where each transfer is over a single hop but the cells are interfering.

Analysis of a so called "three pair" system has been reported in [4]. In [4], the adjacent pairs are critically placed (see Section 4.4 for the definition of critical placement). The authors have shown that severe unfairness would result for this specific placement of nodes because of the large difference between the values of EIFS and DIFS as defined in the IEEE 802.11 standard. However, the scalability of their model with respect to the number of nodes is very poor because of the fact that they have considered the remaining back-off count of each transmitting node individually to describe the system state. In this paper, we propose a scalable cell level analysis of the system.

The effects of EIFS on network performance for various scenarios have been examined in detail by Gupta et al. [9]. The authors have proposed a useful technique that uses different values of EIFS in different circumstances to enhance the fairness. In this paper, we propose a similar simple modification to the default value of EIFS, which results in significant enhancement of fairness in scenarios where hidden terminals are not present.

3 The IEEE 802.11 DCF Standard

In this section, we briefly summarize the key features of the IEEE 802.11 standard which are relevant to our purpose. A complete specification can be found in [1]. In IEEE 802.11 based wireless systems, traffic originates and terminates at stations (STAs) or access points (APs). Data transfer is possible by a two-way handshake of DATA-ACK exchange called the *Basic Access* mechanism, or a four-way handshake of RTS-CTS-DATA-ACK exchange called the *RTS/CTS* mechanism.

A transmitting STA infers a collision if either a CTS frame is not received correctly within the $CTSTimeout$ or an ACK frame is not received correctly within the $ACKTimeout$. After each unsuccessful attempt (either due to collision or transmission errors), a retransmission attempt is scheduled up to a specified number of times called the *retry count*. The retry count is different for RTS and DATA transmissions and depends upon the physical (PHY) layer being used. When the number of unsuccessful attempts exceeds the retry count, the packet is dropped and transmission of the next head of line (HOL) packet is scheduled.

After an erroneous frame is received (either due to collisions, transmission errors or insufficient power), a STA must defer channel access at least for a duration called Extended Inter Frame Space (EIFS). The EIFS interval begins when the PHY indicates a medium IDLE condition at the end of the transmission of the erroneous frame. The value of EIFS is defined in the IEEE 802.11 standard as

$$EIFS := SIFS + T_{ACK} + DIFS$$

where T_{ACK} is the time required for the transmission of an ACK frame. Similarly, T_{RTS}, T_{CTS} and T_{DATA} are defined as the time required for the transmission of an RTS, a CTS and a DATA frame, respectively.

4 Radio Ranges and Cell Overlap

In this section, for illustration purposes, we assume that radio ranges are *hard* boundaries. In practice, of course, due to the randomness in the propagation phenomena, a statement like "A is within some range of B" can only be interpreted in a probabilistic sense.

4.1 The Radio Ranges

There exist three different ranges in IEEE 802.11 based wireless networks:

Decoding Range (R_d) represents the range within which a packet is successfully received (i.e., it can be decoded and the contents can be extracted) provided that there is no interference from other radios. R_d depends mainly on the transmitter power level, the rate of transmission and the nature of the intervening wireless medium.

Carrier Sensing Range (R_{cs}) is the range within which a transmitter triggers a channel busy condition at idle receivers. R_{cs} depends mainly on the transmitter power level and the sensitivity of the receiver.

Interference Range (R_i) is the range within which stations in receive mode will be interfered with. R_i depends upon the relative distance of the receiving STA from the transmitting STA and from the potential interferers.

For a given distance between a potential interferer and a receiver, the probability of collision due to interference decreases as the distance of the transmitter from the receiver decreases. This happens because of the *capture effect*. Hence, R_i is closely related to the capture capabilities of the receiver. It is possible that multiple potential interferers of a given receiver R_x might not cause collisions when they transmit alone but when two or more of them transmit

together, cause collision of the ongoing packet reception at R_x. For the sake of simplicity of analysis, however, we ignore the effect of capture, which has two implications:
(i) Any node within the R_{cs} of a receiver can interfere with its ongoing receptions. Thus $R_i = R_{cs}$ and we have to consider only two ranges, viz., R_d and R_i.
(ii) The results that we obtain without capture will be pessimistic because the collision probability would be less if the capture effect were to be incorporated.

4.2 Effect of Data Rate

Measurements on real testbeds have been carried out both for 802.11 as well as 802.11b [2]. It has been found that the carrier sensing range R_{cs} depends on the transmitter power and the receiver sensitivity only and does not depend upon the rate of transmission. For 802.11b networks it was found that for all data rates, the physical carrier sensing range for outdoor environments is about 250m [2].

For 802.11, the decoding range at the only allowed data rate of 2Mbps is about 40m in an outdoor environment. For 802.11b, there could be a huge difference between the *decoding range corresponding to data rate*, R_{dd} and the *decoding range corresponding to control rate*, R_{dc}. This is because, 802.11b allows three different data rates, viz., 2Mbps, 5.5Mbps, 11 Mbps, whereas the control rate has a fixed value of 2Mbps. From measurement, the R_{dd} corresponding to the data rates of 2Mbps, 5.5Mbps, 11 Mbps are found to be 90m, 70m, 30m, respectively [2]. These values are for an outdoor environment.

4.3 Quantifying the Type and Extent of Overlap

A hexagonal arrangement of cells and an assignment of frequencies to them in a 3-reuse pattern is shown in Figure 2(a) [8]. For designing campus wide networks for indoor environments, linear (Figure 2(b)) and rectangular (Figure 2(c)) arrangements have also been considered [5]. Whether a linear or a rectangular arrangement is to be used depends upon the length to width ratio of the area to be covered. For both linear and rectangular arrangements, the area being covered has been shown by a rectangle $LMNO$.

It has been assumed that channel 1, channel 6 and channel 11 of IEEE 802.11 specification are used. Cells that are assigned with channel 1 have all been shaded to emphasize the relative placement of adjacent cochannel cells. In all these arrangements, R denotes the radius of a cell and D denotes the distance between the centers of the adjacent cochannel cells.

The extent of overlap between the coverage areas of adjacent non-cochannel cells in the linear arrangement is decided by the width LM of the rectangle $LMNO$. It can be noted that some of the cochannel cells touch each other in the rectangular arrangement. This is necessary to avoid coverage gaps even though cochannel interference increases. In fact, there are three possible values of D for the rectangular arrangement, viz., $2R$, $\sqrt{10}R$ and $3\sqrt{2}R$. Henceforth, for simplicity, we consider only the linear and the hexagonal arrangements.

In wireless LANs, peer-to-peer communication can be provided in one of two ways. In some wireless LANs it is possible for a STA to communicate directly with another STA. In others, even though they are both within the communication range of each other, can communicate only by having their transmissions relayed by an AP. Depending on whether or not direct communication between STAs is allowed, the radius R of the cell should satisfy the following conditions: (a) If direct communication is allowed: $R \leq \frac{R_{dd}}{2}$ (b) If direct communication is not allowed: $R \leq R_{dd}$.

Hence, depending on (i) the data rate (ii) whether direct communication is allowed , and (iii) the type of placement of cells (viz., linear or rectangular or hexagonal) various possible scenarios may arise. These possibilities have been summarized in Table 1 for an IEEE 802.11b network with three non-overlapping channels for the linear and hexagonal arrangements [1]. To quantify the type and extent of overlap between adjacent cochannel cells, we define four parameters, the values of which have been summarized in the last four columns of Table 1. Their meaning and significance are discussed in the following.

(a) The *interference separation ratio* is defined as:

$f_{i,separation} := \frac{R_i}{D-2R}$

If $f_{i,separation} < 1$ (case 1a and 1b in Table 1), then the minimum separation between the adjacent cochannel cells is greater than the interference range R_i. *This means that adjacent cochannel cells are totally independent. Hence, the performance of each cell can be predicted by single cell analysis* (see [3] and [7]).

(b) The *interference overlap ratio* is defined as:

$f_{i,overlap} := \frac{R_i}{D+2R}$

If $f_{i,overlap} \geq 1$ (case 1c, 1c', 2c, 2a', 2b' and 2c' in Table 1), then the maximum distance between any two nodes in the adjacent cochannel cells is less than the interference range R_i. It can be noted that *there are no hidden terminals in this case* such that more than one attempt from nodes belonging to either of the cells always result in collision and all the nodes in the system defer for EIFS. Let us compare this case with the case $f_{i,overlap} \leq 1$ and $f_{i,separation} \geq 1$, which corresponds to the situation where only a fraction of a cell's area lies within the interference range of the adjacent cochannel cell. In the latter case, hidden terminals are present and multiple attempts made by nodes in different cells do not necessarily result in collision.

(c) The *control separation ratio* is defined as:

[1]For simplicity of calculations, we assume $D = 6R$ for the linear arrangement, which implies zero overlap between the adjacent non-cochannel cells.

Figure 1. Various arrangement of cells: (a) Hexagonal (b) Linear (c) Rectangular

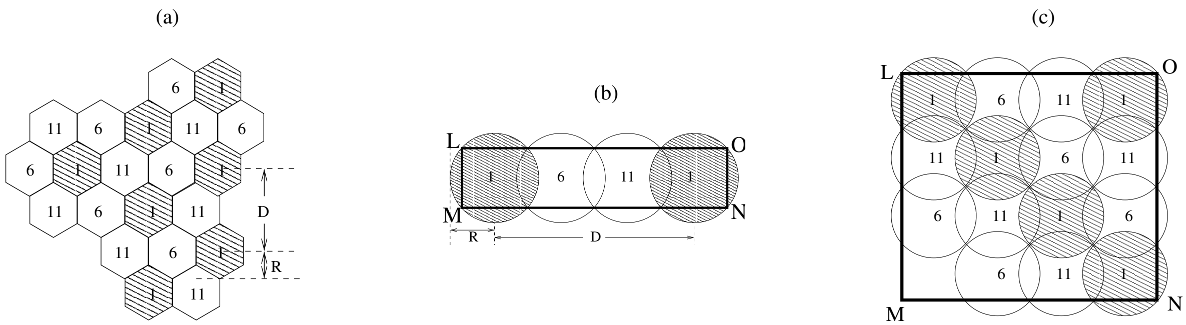


Table 1. Overlapping of 802.11b based cells with various arrangements and at different data rates: (The interference range R_i is assumed to be equal to the carrier sensing range $R_{cs} = 250m$.)

Case	$R \leq \frac{R_{dd}}{2}$?	Arrangement	Data Rate in Mbps	R in m	D in m	$f_{i,separation} = \frac{R_i}{D-2R}$	$f_{i,overlap} = \frac{R_i}{D+2R}$	$f_{dc,separation} = \frac{R_{dc}}{D-2R}$	$f_{dc,overlap} = \frac{R_{dc}}{D+2R}$
1a	No	Linear	2	90	540	0.6944	0.3472	0.2500	0.1250
1b	No	Linear	5.5	70	420	0.8928	0.4464	0.3214	0.1607
1c	No	Linear	11	30	180	2.0833	1.0416	0.7500	0.3750
1a’	Yes	Linear	2	45	270	1.3888	0.6944	0.5000	0.2500
1b’	Yes	Linear	5.5	35	210	1.7857	0.8928	0.6428	0.3214
1c’	Yes	Linear	11	15	90	4.1666	2.0833	1.500	0.7500
2a	No	Hexagonal	2	90	270	2.7777	0.5555	1.000	0.2000
2b	No	Hexagonal	5.5	70	210	3.5714	0.7143	1.2857	0.2571
2c	No	Hexagonal	11	30	90	8.3333	1.6666	3.0000	0.6000
2a’	Yes	Hexagonal	2	45	135	5.5555	1.1111	2.000	0.4000
2b’	Yes	Hexagonal	5.5	35	105	7.1428	1.4285	2.5714	0.5143
2c’	Yes	Hexagonal	11	15	45	16.6666	3.3333	6.0000	1.2

$$f_{dc,separation} := \frac{R_{dc}}{D-2R}$$

If $f_{dc,separation} < 1$ (case 1a, 1b, 1c, 1a’ and 1b’ in Table 1), then the minimum separation between the adjacent cochannel cells is greater than the decoding range at control rate, R_{dc}. This means that nodes cannot decode the control transmissions from the adjacent cochannel cell. Since, $R_{dd} \leq R_{dc}$, the data transmissions cannot be decoded either. But, because of the fact that $R_{dc} < R_i$, nodes may detect the transmission power from the adjacent cochannel cell. The consequence of this is that *both successful transmissions and collisions in a cell are interpreted as collisions by the nodes in the other cell.*

(d) The *control overlap ratio* is defined as:

$$f_{dc,overlap} := \frac{R_{dc}}{D+2R}$$

If $f_{dc,overlap} \geq 1$ (case 2c’ Table 1), then the maximum distance between any two nodes in adjacent cochannel cells is less than the decoding range at control rate R_{dc}. *This means that a pair of adjacent cochannel cells would behave as if they form a single big cell.* This is because, the RTS, CTS and ACK transmissions from one cell can be decoded by all the nodes in the other cell such that their NAVs are properly set. The transmission of the DATA frame may or may not be decoded depending on the rate of data transmission. This is because, $R_{dd} \leq R_{dc}$ and $f_{dc,overlap} \geq 1$ does not always guarantee that the maximum distance between any two nodes in adjacent cochannel cells is less than the decoding range at data rate R_{dd}. The nodes in the other cell defer for EIFS after the transmission of the DATA frame is finished in case DATA transmissions cannot be decoded. However, transmission of the ACK frame following the DATA frame at control rate resynchronizes the nodes in the other cell with the nodes in the transmitting cell.

4.4 Critical Placement

We say that a pair of adjacent cochannel cells is *critically placed* when the following conditions hold.

(a) $2R \leq R_{dc}$: This means that a node can decode the control transmission from any other node in the same cell.

(b) $f_{dc,separation} < 1$: This means that a node cannot decode the transmission from a node in the other cell.

(c) $f_{i,overlap} \geq 1$: This means that any node can interfere with the ongoing reception of any other node in the system.

In Section 5, we provide a saturation throughput analysis of a pair of critically placed cells. The general case is when (i) the cells overlap only partially (ii) the minimum separation between them is less than R_{dc}. Further, ideally, the entire system of cells that can affect each other should be

considered together rather than considering them in pairs. However, we have considered this problem because of the following reasons:
(a) If the receivers had no capture capability, the problem of critically placed cells could actually have occurred in real world because of the large carrier sensing range and small decoding range. For example, case 1c in Table 1 is a case of critical placement. Further, case 1b' almost qualifies to be called a case of critical placement.
(b) As explained further in Section 6.3, capture can reduce the unfairness associated with critical placement only to a very small extent. Therefore, our assumption that capture being absent provides a worst case analysis and is very close to the actual average case.
(c) We obtain new insights into the working of multi-cell wireless networks by solving the problem of critically placed cells. Therefore, solving the problem of critically placed cells is an important next step toward the solution of the general multicell problem.

5 Analysis of Critical Placement

The two critically placed cells in our problem will be called Cell 0 and Cell 1, respectively. We assume that there are several STAs (or nodes) in each of the cells. Each STA (or AP) has an infinite backlog of UDP packets to be transmitted to one or more STAs (or APs) in the same cell. When any STA transmits its data packets, it does so at a rate C bits per second, irrespective of the destination STA. In this symmetric and saturated situation, our objective is to obtain the average rate, in bits per second, at which a node can transmit its backlogged data.

Since the entire system is within a circle of diameter R_i, hidden terminals are not present. We assume that there is no fading and that there is no interference from unknown external interferes, e.g., home appliances operating on the ISM band. Therefore, collisions in the system can occur only when the back-off counters of two or more nodes simultaneously reach zero. This means that only the RTS frames can collide if RTS/CTS mechanism is used and success of an RTS frame guarantees the success of the CTS-DATA-ACK exchange that follows [2].

5.1 Evolution of Back-off and Channel Activity

Figure 2 shows the back-off evolution of the system for 3 nodes in Cell 0 and 2 nodes in Cell 1 starting from some point of time when nodes in both cells are found decrementing their back-off counters in synchrony. Such times are called *back-off times*. There are times when the nodes in Cell 0 find the medium idle and decrement their back-off counters but the nodes in Cell 1 infer the medium busy so that they freeze their back-off counters and defer channel access. Such times are back-off time for Cell 0 and *excess deferral time* for Cell 1. As explained further, these times occur after a successful transmission finishes in Cell 0. Similarly, excess deferral times occur in Cell 0 after a successful transmission finishes in Cell 1.

Time is measured in slots during back-off times. We call these *system slots*. Nodes are allowed to attempt only at the beginning of the system slots. If none of the nodes attempt at the beginning of a system slot, an idle slot elapses in the system and all the nodes decrement their back-off counters by one. If there is a single attempt resulting in a successful transmission or there are multiple attempts resulting in collision, all the nodes sense the medium to be busy and freeze their back-off counters. Therefore, as explained in the following, the evolution of channel activity after attempts, as seen by any node in the system, is deterministic.

Suppose that multiple attempts, may be from the nodes in the same cell or that from different cells, are made in the system. Then, all the nodes in the system infer a collision. The nodes that attempted, wait for a time $CTSTimeout$ after transmitting their RTS frames. At the end of $CTSTimeout$, they defer for an additional $DIFS$, and then resume their back-off counters. Nodes that did not attempt, sense the medium busy until the transmission of collided RTS frames is finished and then defer channel access for $EIFS$ because they receive corrupted frames (see Section 3). It turns out that $CTSTimeout + DIFS$ is equal to $EIFS$. Hence, after a collision finishes, all the nodes in the system resume their back-off counters simultaneously.

The effect of collisions is shown in Figure 2. Node $N_{0,0}$ and node $N_{0,1}$ attempt simultaneously which results in a collision such that all nodes in the system freeze their back-off counters during the *collision time* T_c. At the end of the collision time T_c, all the nodes in the system, including nodes $N_{0,0}$ and $N_{0,1}$, resume their back-off counters simultaneously. The collision time T_c is defined as:

$$T_c := T_{RTS} + EIFS$$

Let's now assume that there is a single attempt made by node $N_{0,2}$ and all other nodes in the system keep quiet Figure 2. All the nodes in Cell 0 can decode the RTS (and possibly DATA) transmissions from node $N_{0,2}$ and the CTS and ACK transmissions from node $N_{0,2}$'s intended recipient, which is also in Cell 0. Hence, the nodes in Cell 0 freeze their back-off counters until the entire exchange of RTS-CTS-DATA-ACK frames is finished and resume their back-off counters after deferring for an additional duration of $DIFS$. Since $f_{i,overlap} \geq 1$ for critical placement, transmissions from Cell 0 nodes can be detected by Cell 1 nodes. Hence, the nodes in Cell 1 also freeze their back-off counters until the entire exchange of RTS-CTS-DATA-ACK frames is finished in Cell 0. However, transmissions from Cell 0 nodes cannot be decoded by Cell 1 nodes because $f_{dc,separation} < 1$ for critical placement. Therefore, the

[2] We explain our analysis for the RTS/CTS mechanism. The arguments for the basic access mechanism are very similar.

nodes in Cell 1 have to defer for an additional duration of $EIFS$ before they can resume their back-off counters.

We define the time required for the transmission of the entire exchange of RTS-CTS-DATA-ACK frames followed by an idle $DIFS$ period as *success time* T_s, or

$$T_s := T_{RTS} + SIFS + T_{CTS} + SIFS + T_{DATA} + SIFS + T_{ACK} + DIFS$$

In summary, after a successful transmission finishes, nodes in the same cell defer channel access for a time T_s and then resume their back-off counters whereas nodes in the other cell defer for a time $T_s + l$, where l is the excess deferral time given by $l := EIFS - DIFS$.

5.2 Channel Slots

The beginning of system slots are of particular importance because attempts in the system can be made only at these instants. Further, between any two consecutive channel activity periods (i.e., success or collision), at least one idle system slot elapses in the medium such that all the nodes decrement their back-off counters by one [3]. In effect, therefore, an activity period can be clubbed together with the idle slot that follows to form a *channel slot*. Channel slots corresponding to Figure 2 are shown in Figure 3.

It can be noted that attempts in the system can occur only at the channel slot boundaries. These instants have been indicated by arrows in Figure 3. If none of the nodes attempt at the beginning of a given system slot, an idle system slot elapses in the system and we say that an *idle channel slot*, which is equal to a system slot, has elapsed. If there is a collision in the system, all the nodes in the system infer the medium busy for a duration T_c. Hence, we define the *collision channel slot* to be of duration $T_c + 1$. If there is a single attempt resulting in a success, nodes in the same cell freeze their back-off counters for a success time T_s. Hence, we define the *success channel slot* to be equal to $T_s + 1$. Nodes in the other cell also freeze their back-off counters for a time T_s due to the interference power that they hear. Hence, an *interference channel slot* is also defined to be equal to $T_s + 1$. Every interference channel slot is followed by *excess deferral channel slots*, each of duration equal to an idle channel slot. A maximum of l such excess deferral channel slots can occur consecutively.

5.3 The Embedded Markov Process

Since the contention mechanism evolves over the channel slots, we embed the renewal process at channel slot boundaries, which we index by $k \geq 1$. We represent the states of the system by a pair of quantities $\mathbf{S}(k) = (S_0(k), S_1(k)), k \geq 1$, where k is the discrete time index.

[3] This is because a node attempting at the beginning of the very first system slot following an activity period would mean that its back-off counter had already reached zero, which, in turn, means that it should have attempted at the beginning of the previous slot.

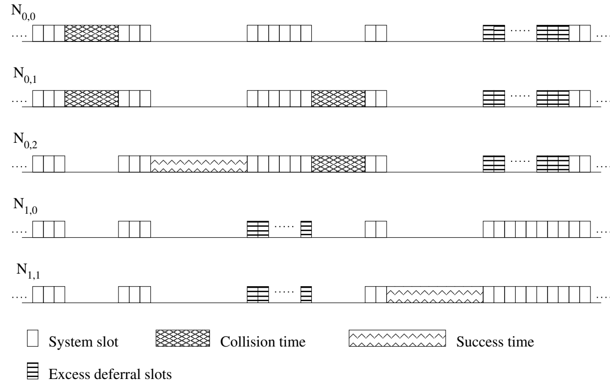


Figure 2. The evolution of back-off periods and channel activity for a pair of critically placed cells: The jth node in Cell i is denoted as $N_{i,j}$. Also shown are the back-offs, the successful transmissions, collisions and excess deferral slots.

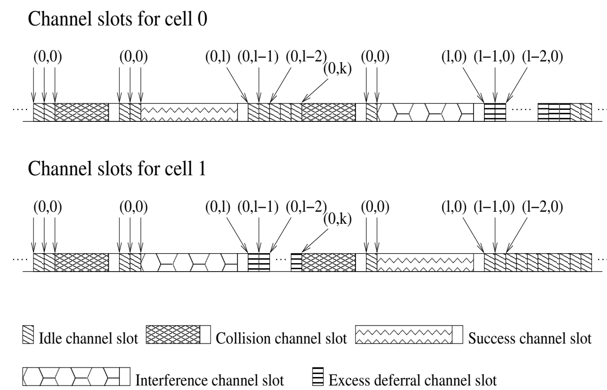


Figure 3. Evolution of channel slots for a pair of critically placed cells: The state of the system changes at the the beginning of channel slots. These instants have been indicated by arrows. The embedded process evolves as a Discrete Time Markov Chain (DTMC).

We call $\mathbf{S}(k)$ the *state vector*. The elements $S_0(k)$ and $S_1(k)$ of the state vector correspond to Cell 0 and Cell 1, respectively. We say that a cell attempted when at least one node belonging to the cell attempts. As explained further, at a given point of time, either both or only one, or none of the cells can attempt, depending upon which cell attempted last and what the outcome of that attempt was (viz, a success or a collision).

At a given point of time, the cell that can attempt would be taken as the reference. The element of the state vector corresponding to the reference cell shall assume the value 0 signifying the fact that it is the reference. The element of the state vector corresponding to the other (non-reference) cell contains the remaining number of idle slots that must elapse

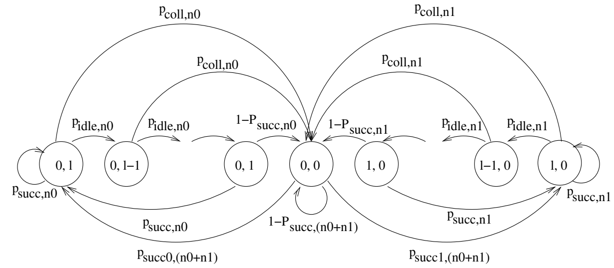


Figure 4. The transition structure of the ($S_0(k), S_1(k)$) DTMC: state $(0, 0)$ is the only possible state in which both cells can attempt simultaneously. The left (right) branch corresponds to the states in which only Cell 0 (Cell 1) can attempt.

in the medium after which it can attempt. For example, a state $(0, m)$ implies that (i) Cell 0 is the reference (ii) it is the only cell that can attempt currently and (iii) Cell 1 can attempt if the medium remains idle for m consecutive slots. State $(0, 0)$ is a special case, when both cells can attempt and either of the cells can be taken as the reference.

It can be noted that the state of the system changes only at channel slot boundaries. The evolution of state of the system depends on (i) the current state that it is in at a given channel slot boundary and (ii) the event that occurs at the given channel slot boundary. It does not depend on the history of events. Hence, we model the system as a Discrete Time Markov Chain (DTMC) embedded at the channel slot boundaries as shown in Figure 4. The transition probabilities shown in Figure 4 have been defined in Section 5.4.

Let us consider Figure 3, where the channel slot boundaries have been indicated by arrows. It is shown that, initially, the system is in the state $(0, 0)$ because, all the nodes in the system are decrementing their back-off counters in synchrony and there can be attempts from any of the cells. If none of the nodes attempt, the system spends one idle channel slot in state $(0, 0)$. The state of the system does not change after this idle channel slot, and, again, at the beginning of the next slot there can be an attempt from any of the cells. Similarly, if there is a collision in the system in state $(0, 0)$, the system spends one collision channel slot in state $(0, 0)$. But, the state of the system at the end of this collision channel slot does not change because all the nodes in the system resume their back-off counters simultaneously (see Figure 2 and Figure 3). This is also shown in Figure 4, where the the system remains in the state $(0, 0)$ until a success occurs.

When a single attempt is made from Cell 0 at the state $(0, 0)$, the system spends one success channel slot in the state $(0, 0)$. At the end of this success channel slot, Cell 0 nodes resume their back-off counters and can potentially attempt but Cell 1 nodes are l consecutive idle slots away from the time when they can attempt. Hence, after a success in Cell 0, the state of the system changes to $(0, l)$. Similarly, after a success in Cell 1, the state of the system changes to $(l, 0)$.

Suppose that the system is in state $(0, l)$ such that only Cell 0 nodes can attempt. If none of the nodes in Cell 0 attempt at the beginning of the following system slot, the system spends one idle channel slot in the state $(0, l)$ and the state of the system changes to $(0, l-1)$ at the end of the idle channel slot. This is because, the nodes in Cell 1 are now $l - 1$ consecutive idle slots away from the time when they can attempt. Similarly, starting at the state $(0, l)$, if none of the nodes in Cell 0 attempt during the next $m, 1 \leq m \leq l$ system slots, the system spends one idle channel slot in each of the states $(0, k), l \leq k \leq l - m + 1$ and the state of the system changes to $(0, l - m)$.

If there is a success at any of the states $(0, m), 1 \leq m \leq l$, the system spends one success channel slot in the state $(0, m)$ and the state of the system changes to $(0, l)$ at the end of the success channel slot. If, however, there is a collision, the system is thrown back to state $(0, 0)$ after it spends one collision time in the state $(0, m)$. This is shown in Figure 3 (also see Figure 2), where Cell 0 nodes are decrementing their back-off counters but Cell 1 nodes defer channel access during their excess deferral time. Node $N_{0,1}$ and node $N_{0,2}$ both attempt when the Cell 1 nodes are k slots away from the time when they can also attempt. The excess deferral time of Cell 1 nodes is terminated prematurely. A collision occurs because of multiple attempts and the system assumes the state $(0, 0)$ at the end of the collision channel slot.

5.4 The Transition Probabilities

We define the following quantities pertaining to each cell. In each of the following, the cell index i is either equal to 0 or 1, i.e., we have $i \in \{0, 1\}$.

$n_i :=$ The number of STAs in Cell i

$\beta_i :=$ The attempt probability (per channel slot) of an [4] STA in Cell i during times when nodes in Cell i can attempt

$\gamma_i :=$ The collision probability as seen by a node in Cell i (conditioned on an attempt being made by the node)

The transition probabilities shown in Figure 4 are defined as follows. Given that the n_i nodes of Cell i only can attempt, P_{idle,n_i}, P_{coll,n_i} and P_{succ,n_i} are defined as the probability that the system remains idle, a collision occurs in the system and a success occurs in the system, respectively. Similarly, given that all the $(n_0 + n_1)$ nodes in the

[4] Note that β_i is the attempt probability (per channel slot) of nodes in Cell i (when nodes in Cell i can attempt), irrespective of nodes in the other cell can simultaneously attempt or not. This is a simplification and can be viewed as an extension of the basic decoupling approximation introduced in [3].

system can attempt, $P_{succ_i,(n_0+n_1)}$ and $P_{succ,(n_0+n_1)}$ are defined as the probability that a success occurs in Cell i and that a success occurs in any of the cells, respectively. Using the above definitions, the transition probabilities can be expressed in terms of the attempt rates as:

$$\begin{aligned}
P_{idle,n_i} &= (1-\beta_i)^{n_i} \\
P_{succ,n_i} &= n_i\beta_i(1-\beta_i)^{n_i-1} \\
P_{coll,n_i} &= 1 - P_{idle,n_i} - P_{succ,n_i} \\
P_{succ_0,(n_0+n_1)} &= n_0\beta_0(1-\beta_0)^{n_0-1}(1-\beta_1)^{n_1} \\
P_{succ_1,(n_0+n_1)} &= n_1\beta_1(1-\beta_1)^{n_1-1}(1-\beta_0)^{n_0} \\
P_{succ,(n_0+n_1)} &= P_{succ_0,(n_0+n_1)} + P_{succ_1,(n_0+n_1)} \quad (1)
\end{aligned}$$

5.5 Collision Probability and Attempt Probability from Fixed-Point Analysis

The fixed-point analysis that we develop in this paper is based on the approach taken in [7]. In [7] the back-off parameters of a node are defined as:

$K :=$ The maximum number of retransmissions allowed, i.e., at the $(K+1)$th attempt either the packet being attempted succeeds or is discarded

$b_k :=$ The mean back-off (in slots) at the kth attempt, $0 \le k \le K$

It was shown in [7] that, in the single cell case, under the decoupling assumption introduced by Bianchi in [3] and when the back-off parameters are identical for all the nodes, the attempt rate β of a node at a channel slot boundary, for a given collision probability γ is given by:

$$G(\gamma) := \frac{1+\gamma\cdots+\gamma^K}{b_0+\gamma b_1\cdots+\gamma^k b_k+\cdots+\gamma^K b_K} \quad (2)$$

For a pair of critically placed cells we continue to assert that the decoupling assumptions still remains valid. But the attempt rates for the nodes in the two cells could be different owing to the different number of nodes in the two cells. So, the attempt rate β_i, when nodes of Cell i can attempt, is given by:

$$G_i(\gamma_i) := \frac{1+\gamma_i\cdots+\gamma_i^K}{b_0+\gamma_i b_1\cdots+\gamma_i^k b_k+\cdots+\gamma_i^K b_K} \quad (3)$$

In a single cell, all nodes can attempt simultaneously because they are in *synchrony*. For a pair of critically placed cells, the nodes in the same cell are still in synchrony whereas that in different cells are not. To quantify the asynchrony between the cells we define the quantity a_i (corresponding to Cell i) as the probability that nodes in the other cell can also attempt given that nodes in Cell i can attempt. The quantities a_0 and a_1 can be expressed in terms of the steady state probabilities as:

$$\begin{aligned}
a_0 &= \frac{\pi(0,0)}{\pi(0,0)+\sum_{j=1}^{l}\pi(0,j)} \\
a_1 &= \frac{\pi(0,0)}{\pi(0,0)+\sum_{j=1}^{l}\pi(j,0)} \quad (4)
\end{aligned}$$

where $\pi(0,j)$ and $\pi(j,0)$, $0 \le j \le l$, denote the stationary probabilities of the finite irreducible DTMC shown in Figure 4. By symmetry, $a_0 = a_1$ when both the cells contain equal number of nodes. Also, since the transition probabilities are functions of the attempt probabilities β_i's alone, so are the quantities a_0 and a_1.

The collision probability γ_i as seen by nodes in Cell i can be obtained by conditioning upon the state probabilities. Nodes in both cells can attempt simultaneously iff the system is in state (0,0). Otherwise, nodes in only one of the cells can attempt. Thus, the collision probability γ_0 and γ_1 can be expressed in terms of a_0 and a_1 as (see [7]):

$$\begin{aligned}
\Gamma_0(\beta_0,\beta_1) &= (1-a_0)[1-(1-\beta_0)^{n_0-1}] \\
&\quad +a_0[1-(1-\beta_0)^{n_0-1}(1-\beta_1)^{n_1}] \\
\Gamma_1(\beta_0,\beta_1) &= (1-a_1)[1-(1-\beta_1)^{n_1-1}] \\
&\quad +a_1[1-(1-\beta_1)^{n_1-1}(1-\beta_0)^{n_0}] \quad (5)
\end{aligned}$$

where we recall the decoupling assumption that nodes in Cell i attempt at a rate β_i irrespective of whether or not nodes in the other cell can simultaneously attempt.

The equilibrium behavior of the system is, thus, characterized by the solution of the following set of fixed point equations.

$$\begin{aligned}
\gamma_0 &= \Gamma_0(G_0(\gamma_0), G_1(\gamma_1)) \\
\gamma_1 &= \Gamma_1(G_0(\gamma_0), G_1(\gamma_1)) \quad (6)
\end{aligned}$$

5.6 Aggregate Saturation Throughput

We define the following to calculate the aggregate throughput of a cell in bits per second.

$T_o :=$ The fixed overhead (in second) associated with the successful transmission of a DATA frame

$T_c :=$ The fixed overhead (in second) associated with an RTS collision

$L :=$ Length of payload (in bits) in the DATA frame

$C :=$ Rate of data transmission (in bits per second)

$\sigma :=$ Length of a system slot (in second)

As we have seen, the process $\mathbf{S}(k)$ is a Markov chain embedded at the beginnings of successive channel slots. The channel slot k, is of random length $T(k)$ (i.e., an idle slot, or a collision slot, or a success slot), and with a probability depending on the state $\mathbf{S}(k)$ there is a success, resulting in the transmission of L bits. Let us denote the number of bits successfully transmitted in channel slot k as a reward $R(k)$. We thus have a Markov renewal reward process $(\mathbf{S}(k), T(K), R(k)), k \ge 1$. Then the aggregate throughput of Cell 0 is given by

$$\Theta_0 = \frac{\mathbf{E}_\pi R}{\mathbf{E}_\pi T} \quad (8)$$

where $\mathbf{E}_\pi(\cdot)$ denotes expectation with respect to the stationary distribution π of the Markov chain $\mathbf{S}(k)$. The resulting expression is displayed in Equation 7. A similar equation

$$\Theta_0 = \frac{\pi(0,0)P_{succ_0,(n_0+n_1)}L + \sum_{j=1}^{l}\pi(0,j)P_{succ,n_0}L}{\left(\sigma + \sum_{j=1}^{l}\pi(0,j)\left[P_{succ,n_0}(\frac{L}{C}+T_o) + P_{coll,n_0}T_c\right] + \sum_{j=1}^{l}\pi(j,0)\left[P_{succ,n_1}(\frac{L}{C}+T_o) + P_{coll,n_1}T_c\right] + \pi(0,0)\left[P_{succ,(n_0+n_1)}(\frac{L}{C}+T_o) + (1 - P_{idle,n_0}P_{idle,n_1} - P_{succ,(n_0+n_1)})T_c\right]\right)} \quad (7)$$

can also be written for the aggregate throughput of Cell 1. Assuming that the aggregate throughput of a cell is shared fairly among the nodes in the cell, the throughput per node in Cell 0, is given by $\theta_0 := \frac{\Theta_0}{n_0}$.

6 Results and Discussion

We conducted extensive simulations using ns-2 [11]. ns-2 implements the Direct Sequence Spread Spectrum (DSSS) PHY layer specification of the IEEE 802.11 standard. We suppressed the capture effect in these simulations. The function "***fsolve()***" of OCTAVE was used for solving the fixed-point equations [5]. The following default values of ns-2 implementation have been used for solving the fixed-point equations: $K = 7$ [6]; $CW_{min} = 32$; $b_k = \frac{2^k CW_{min} - 1}{2}$ for $0 \leq k \leq 5$ and is equal to $CW_{max} = 2^5 CW_{min}$ for $k = 6, 7$; $l = 16$ slots, where each slot duration is $\sigma = 20\mu s$; $L = 1000$ bytes; $C = 2$ Mbps; $T_o = 5616\mu s = 280.8$ slots and $T_c = 402\mu s = 20.1$ slots.

6.1 Comparison of Values Obtained from Fixed-Point Analysis and ns-2 Simulations

The values of collision probability γ (attempt probability β) obtained from fixed-point analysis have been compared with the values obtained from ns-2 simulations in Figure 6(a) (Figure 6(b)). These plots correspond to the cases where both cells contain equal number of nodes. The collision probability, γ, and throughput per node, θ, for a pair of critically placed cells each having n nodes have been compared with the corresponding values for a single cell having $2n$ nodes in Table 2. Results pertaining to the cases where the cells contain unequal number of nodes have been summarized in Table 3 [7]. The following observations are made:

1.) It can be seen from Figure 6(a), Figure 6(b) and Table 3 that the fixed-point analysis is capable of predicting the collision probability and attempt probability reasonably accurately. It was found that the difference between the values predicted by the fixed-point analysis and that obtained from ns-2 simulations is about 6-8%.

2.) It can be noted that $\gamma = 0.2955$ for a single cell having 10 nodes and $\gamma = 0.3222$ for a pair of critically placed cells each having 10 nodes (see the first and second rows of Table 2). Similarly, the collision probabilities given in Table 3 for the 10 node cell is found to be higher than the collision probability for a single cell having 10 nodes in all the cases. Therefore, it is easy to see that the collision probability in multicell environments is greater than the corresponding singlecell values due to the presence of proximal cochannel cells.

3.) We see from Table 2 that *The collision probability γ for a pair of critically placed cells each having n nodes is significantly less than the collision probability for a single cell having $2n$ nodes.* This is an important consequence of critical placement and is true, in spite of the fact that in both the cases, there are $2n$ nodes within a circle of diameter R_i such that the channel is time shared among the $2n$ nodes. The following inequality can be inferred from the tables.

$$\gamma_{singlecell,n} \leq \gamma_{twocell-critical,n} \leq \gamma_{singlecell,2n} \quad (9)$$

That such is the case can also be seen from Equation 5.

4.) It can be noted from Table 2 that throughput per node θ for the case of a pair of critically placed cells each having n nodes is marginally higher than the throughput per node for a single cell having $2n$ nodes. The enhancement of throughput per node for critical placement becomes more significant as the number of nodes per cell decreases. The following inequality is supported by the numerical results.

$$\theta_{singlecell,2n} \leq \theta_{twocell-critical,n} \leq \theta_{singlecell,n} \quad (10)$$

6.2 Fairness

We use the fairness index given in [6]. When n entities share a resource, suppose that the ith entity obtains a throughput Θ_i. The fairness index defined in [6], writing $\mathbf{\Theta} = [\Theta_1, \Theta_2, \cdots\cdots, \Theta_n]$, can be expressed as:

$$F(\mathbf{\Theta}) = \frac{(\frac{1}{n}\sum_{i=1}^{n}\Theta_i)^2}{\frac{1}{n}\sum_{i=1}^{n}\Theta_i^2} \quad (11)$$

Note that $F(\mathbf{\Theta}) \leq 1$ with equality iff $\Theta_1 = \Theta_2 = \cdots\cdots = \Theta_n$. In our current context, there are two entities (i.e., two cells) sharing the resource (wireless spectrum) such that we have $\mathbf{\Theta} = [\Theta_0, \Theta_1]$, where Θ_0 and Θ_1 are the aggregate throughputs of Cell 0 and Cell 1, respectively. Since, $\Theta_0, \Theta_1 \geq 0$, the most unfair situation occurs when $\Theta_0 = 0, \Theta_1 \neq 0$ or $\Theta_0 \neq 0, \Theta_1 = 0$ such that $F(\mathbf{\Theta}) = 0.5$.

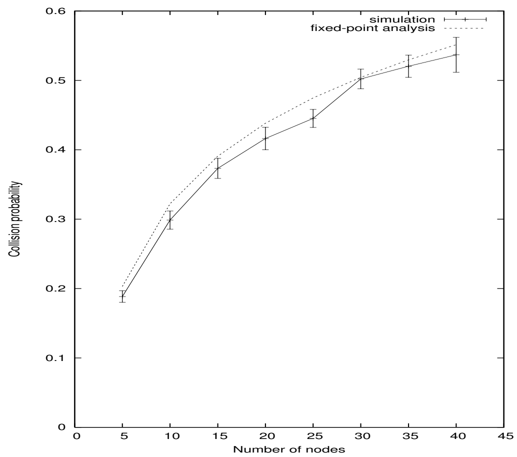
[5] OCTAVE is available for free with many Linux distributions and possesses MATLAB like features.

[6] This is because the retry count for RTS collisions is equal to 7 for the DSSS PHY layer. We analyze for the RTS/CTS mechanism and since $f_{i,overlap} \geq 1$ for critical placement, only RTS collisions can occur.

[7] For lack of space, we do not provide the 99% confidence limits for these cases. However, the trend is very similar to the cases where both cells contain equal number of nodes.

Figure 5. Analysis and ns-2 simulation results for (a) Collision probability γ and (b) Attempt rate β for a pair of critically placed cells each having n nodes. The mean and the 99% confidence limits have been shown for various values of n.

(a)



(b)

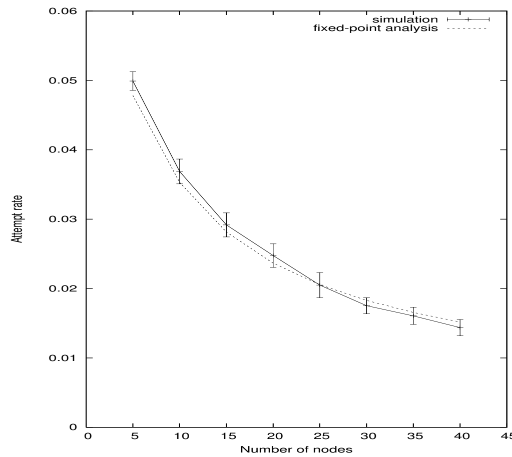


Table 2. Collision probability γ and Throughput per node θ from both analysis and simulation for a single cell having $2n$ nodes and that of a pair of critically placed cells each having n nodes are compared. Also shown are the 99% confidence limits.

$2n$	$\gamma_{ana,single}$	$\gamma_{sim,single}$	$\gamma_{ana,crit}$	$\gamma_{sim,crit}$	$\theta_{ana,single}$	$\theta_{sim,single}$	$\theta_{ana,crit}$	$\theta_{sim,crit}$
10	0.2955	0.2760±0.0152	0.2031	0.1885±0.0083	81.881	52.927±10.500	81.949	61.280±9.666
20	0.4039	0.3858±0.0286	0.3222	0.2988±0.0130	40.801	33.527±6.164	40.900	38.325±4.055
30	0.4651	0.4440±0.0127	0.3908	0.3732±0.0144	27.123	23.550±3.014	27.208	26.668±3.616
40	0.5081	0.4929±0.0164	0.4383	0.4263±0.0162	20.212	19.689±3.064	20.366	20.995±2.745

The most fair situation occurs when $\Theta_0 = \Theta_1 \neq 0$ such that $F(\mathbf{\Theta}) = 1$. Also note that a larger value of F_Θ implies more fairness. Figure 7(a) and Figure 7(b) compare the fairness of systems having 10 and 20 nodes, respectively. The following observations are made:

5.) Over a window of less than or equal to 10ms, in all cases, only one or the other cell obtains access. However, over a window of between 20ms and 1 sec the access for a pair of critically placed cells can be very unfair as compared to the case in which the two cells have completely overlapping decoding ranges.

6.) By comparing Figure 7(a) and Figure 7(b), it can be seen that the fairness for critical placement is better for the case when each cell contains 10 nodes than the case when each cell contains 5 nodes. In general, it was found that fairness for critical placement improves with the increase in the number of nodes in the system.

7.) Fairness of critically placed cells can be significantly improved and can be made as good as that of a single cell by making EIFS = DIFS.

6.3 Effect of capture

When the system is in state $(0, 0)$, depending on the relative magnitudes of R and D and for certain specific locations of transmitter-receiver pairs, it is theoretically possible that a transmission in each of the cells may be "captured" such that a success occurs in both the cells. To study this possibility, we simulated case 1a', 1b' and 1c' of Table 1 by enabling the capture effect. These cases were chosen because they are closer to the case of critical placement (case 1c) than all other cases. It was found that the collision probability decreases by $\sim 1-2\%$ for all these cases. However, events of simultaneous success in both cells were not observed. In the real world, where the effect of capture is further limited by the fact that interference powers from multiple transmissions add up to cause collisions, it would be rare that all of the frames in the RTS-CTS-DATA-ACK exchange can be captured.

7 Conclusions

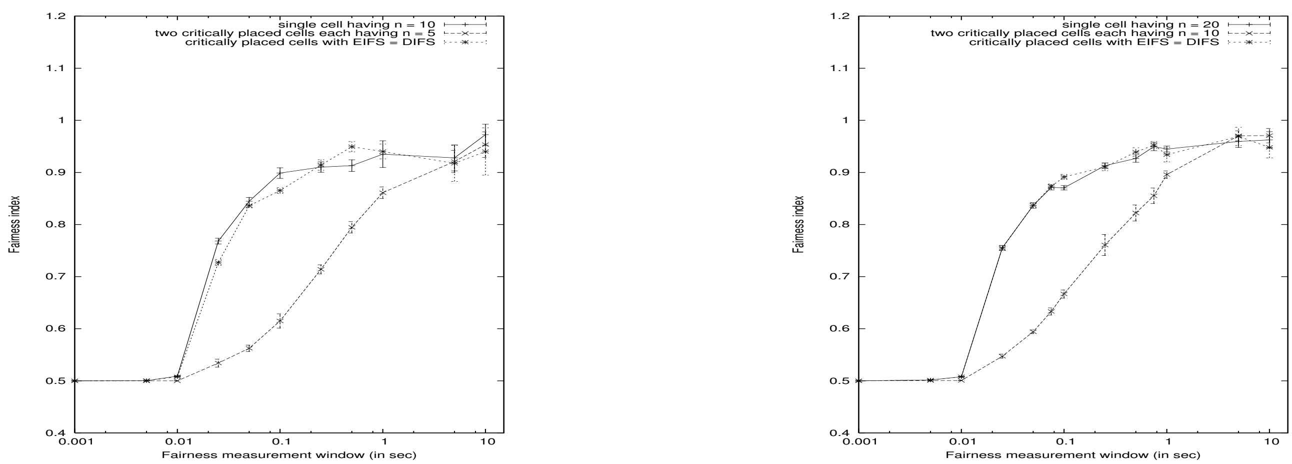
In this paper we have provided a performance analysis of an IEEE 802.11 network comprising two critically placed cochannel cells, i.e., the cells have non-overlapping decoding ranges but their interference ranges overlap. As a consequence of EIFS being larger than DIFS, we find that the system can exist in states where either one cell or the other accesses the channel, but not both. We model the evolution

Table 3. Collision probability γ, Attempt rate β and Throughput per node θ from both analysis and simulation for a pair of critically placed cells having unequal number of nodes.

n_0	n_1	$\gamma_{0,ana}$	$\gamma_{0,sim}$	$\gamma_{1,ana}$	$\gamma_{1,sim}$	$\beta_{0,ana}$	$\beta_{0,sim}$	$\beta_{1,ana}$	$\beta_{1,sim}$	$\theta_{0,ana}$ in kbps	$\theta_{0,sim}$ in kbps	$\theta_{1,ana}$ in kbps	$\theta_{1,sim}$ in kbps
10	5	0.3129	0.2912	0.2140	0.1994	0.0363	0.0384	0.0467	0.0488	42.583	40.945	78.580	61.484
10	15	0.3285	0.2903	0.3849	0.3402	0.0346	0.0359	0.0287	0.0290	40.986	38.439	27.151	24.197
10	20	0.3335	0.3197	0.4283	0.4191	0.0341	0.0346	0.0246	0.0248	40.985	36.885	20.324	19.962
10	25	0.3377	0.4174	0.4615	0.4431	0.0336	0.0353	0.0216	0.0223	40.914	37.378	16.259	17.053
10	30	0.3414	0.3252	0.4883	0.4932	0.0332	0.0345	0.0183	0.0216	40.808	39.618	13.562	12.570

Figure 6. Simulation results showing the comparison of fairness index for (i) a single cell having 2n nodes (ii) a pair of critically placed cells each having n nodes and (iii) the same pair of critically placed cells with EIFS made equal to DIFS: (a) n = 5 (b) n = 10.

(a) (b)



of this channel access behavior via a discrete time Markov chain embedded at channel slot boundaries, and obtain a fixed point equation for the collision probabilities of the two cells. The analysis yields cell throughputs that are found to compare well with ns-2 simulations. Important insights obtained are that (i) the cell throughputs are better in the situation analyzed because of less contention when only one cell accesses the channel, (ii) but there can be significant short term unfairness (iii) in absence of hidden nodes, we can make EIFS equal to DIFS and improve the fairness in multicell networks.

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