Fixed Point Analysis of Single Cell IEEE 802.11e WLANs: Uniqueness, Multistability and Throughput Differentiation

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ABSTRACT

We consider the vector fixed point equations arising out of the analysis of the saturation throughput of a single cell IEEE 802.11e wireless local area network with nodes that have different back-off parameters, including different Arbitration InterFrame Space (AIFS) values. We consider balanced and unbalanced solutions of the fixed point equations arising in homogeneous and nonhomogeneous networks. We are concerned, in particular, with (i) whether the fixed point is balanced within a class, and (ii) whether the fixed point is unique. Our simulations show that when multiple unbalanced fixed points exist in a homogeneous system then the time behaviour of the system demonstrates severe short term unfairness (or *multistability*). Implications for the use of the fixed point formulation for performance analysis are also discussed. We provide a condition for the fixed point solution to be balanced within a class, and also a condition for uniqueness. We then provide an extension of our general fixed point analysis to capture AIFS based differentiation; again a condition for uniqueness is established. An asymptotic analysis of the fixed point is provided for the case in which packets are never abandoned, and the number of nodes goes to ∞. Finally the fixed point equations are used to obtain insights into the throughput differentiation provided by different initial back-offs, persistence factors, and AIFS, for finite number of nodes, and for differentiation parameter values similar to those in the standard.

Categories and Subject Descriptors

C.2.5 [**Computer-Communication Networks**]: Local and Wide-Area Networks—*Access schemes*; I.6.4 [**Simulation and Modeling**]: Model Validation and Analysis

General Terms

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1. INTRODUCTION

A new component of the IEEE 802.11e medium access control (MAC) is an enhanced DCF (EDCF), which provides differentiated channel access to packets by allowing different backoff parameters (see [2]). Several traffic classes are supported, the classes being distinguished by different back-off parameters. Thus, whereas in the legacy DCF all nodes have a single back-off "state machine" all with the same back-off parameters (we say that the nodes are *homogeneous*), in EDCF the nodes can have multiple back-off state machines with different parameters, and hence are permitted to be *nonhomogeneous*.

This paper is concerned with the saturation throughput analysis of single cell networks with nonhomogeneous nodes. We limit our study to the case in which each node has one EDCF queue; the further generalisation to multiple traffic classes per node can also be done using the same techniques but is not reported here for lack of space. Thus in the nonhomogeneous case our analysis is applicable to a single cell ad hoc network of IEEE 802.11e nodes (single cell meaning that all nodes are within control channel range of each other), with each node offering traffic of a single class. We consider an ideal channel (without capture, fading or frame error) and assume that packets are lost only due to collision of simultaneous transmissions.

Much work has been reported on the performance evaluation of EDCF to support differentiated service. Most of the analytical work reported has been based on a decoupling approximation proposed initially by Bianchi ([3]). While keeping the basic decoupling approximation, in [1] Kumar et al. presented a significant simplification and generalisation of the analysis of the IEEE 802.11 back-off mechanism. This analysis led to a certain one dimensional fixed point equation for the collision probability experienced by the nodes in a homogeneous system (i.e., one in which all the nodes have the same back-off parameters). In this paper we consider *multidimensional fixed point equations* for a homogeneous system of nodes, and also for a nonhomogeneous system of nodes. The nonhomogeneity could arise due to different initial back-offs, or different back-off multipliers, or different amounts of time that nodes wait after a transmission before restarting their back-off counters (i.e., the AIFS (Arbitration InterFrame Space) mechanism of IEEE 802.11e). We consider *balanced* solutions of the resulting multidimensional fixed point equations (i.e., solutions in which all the coordinates are equal), and also *unbalanced* fixed points.

The main contributions of this paper are the following:

1. We provide examples of homogeneous systems in which, even though a unique balanced fixed point exists, there can be multiple unbalanced fixed points, thus suggesting *multistability*. We demonstrate by simulation that, in such cases, significant short term unfairness can be observed and the unique balanced fixed point fails to capture the system performance.

2. Next, in the case where the backoff increases multiplicatively (as in IEEE 802.11), we establish a simple sufficient condition for the uniqueness of the solution of the multidimensional fixed point equation in the homogeneous and the nonhomogeneous cases.

3. We perform an analytical study of the differentiation provided by each of the three mechanisms that we model. We then provide an asymptotic analysis of the service differentiation (as the number of nodes become large), and also some approximate results for a finite number of nodes.

A survey of the literature: There has been much research activity on modeling the performance of IEEE 802.11 and IEEE 802.11e medium access standards. The general approach has been to extend the decoupling approximation introduced by Bianchi ([3]). Without modeling the AIFS mechanism, the extension is straightforward. Only the initial back-off, and the back-off multiplier (*persistence factor*) are modeled. In [4], [5] and [6], such a scheme is studied by extending Bianchi's Markov model per traffic category. In this paper, in Section 3, we will provide a generalisation and simplification of this approach. We will provide examples where nonunique fixed points can exist, the consequences of such nonuniqueness, and also conditions that guarantee uniqueness.

AIFS technique is a further enhancement in IEEE 802.11e that provides a sort of priority to nodes that have smaller values of AIFS. After any successful transmission, whereas high priority nodes (with AIFS = DIFS) wait only for DIFS (DCF Interframe Space) to resume counting down their back-off counters, low priority nodes (with AIFS > DIFS) defer the initiation of countdown for an additional AIFS−DIFS slots. Thus a high priority node decrements its back-off counter faster than a low priority node and also has fewer collisions.

Among the approaches that have been proposed for modeling the AIFS mechanism (for example, [7], [8], [9], [10], [11] and [12]) the ones in [11] and [12] come much closer to capturing the service differentiation provided by the AIFS feature. In [11] the authors propose a Markov model to capture both the different back-off window expansion approach and AIFS. AIFS is modelled by expanding the state-space of the Markov chain to include the number of slots elapsed since the previous transmission attempt on the channel. In [12] the authors observe that the system exists in states in which only nodes of certain traffic classes can attempt. The approach is to model the evolution of these states as a Markov chain. The transition probabilities of this Markov chain are obtained from the assumed, decoupled attempt probabilities. This approach yields a fixed point formulation. This is the approach we will discuss in Section 6.

Relation of the existing literature to our work: We note that the analyses in [11] and [12] are based on Bianchi's approach to modeling the residual back-off by a Markov chain. In this paper, we have extended the simplification reported in [1] (which was for a homogeneous system of nodes) to nonhomogeneous nodes with different backoff parameters and also AIFS based priority schemes. Thus, in our work, we have provided a simplified and integrated model to capture all the essential service differentiation mechanisms of IEEE 802.11e.

In the previous literature, it is assumed that the collision rate experienced by a node of each traffic category is constant over time. There appears to have been no attempt to study the phenomenon of short term unfairness in the fixed point framework. Also, all the existing work assumes that the collision probabilities of all the nodes of a given traffic category are the same. Thus there appears to have been no earlier work on studying the possibility of unbalanced solutions of the fixed point equations. In addition, the possibility of nonuniqueness of the solution of the fixed point equations arising in the analyses seems to have been missed in the earlier literature. In our work, we study the fixed point equations for IEEE 802.11e networks and take into account all these possibilities.

Outline of the paper: In Section 2 we review the generalised back-off model that was first presented in [1]. In Section 3 we develop the multidimensional fixed point equation for the homogeneous and nonhomogeneous cases, and obtain the necessary and sufficient conditions satisfied by the solutions to the fixed point equations. We provide examples in Section 4 to show that even in the homogeneous case there can exist multiple unbalanced fixed points and show the consequence of this. In Section 5.1, we analyse the fixed point equations for a homogeneous system of nodes and obtain a condition for the existence of only one fixed point. In Sections 5.2 and 6, we extend the analysis to nonhomogeneous system of nodes, with different back-off parameters. In Section 7 we provide an analytical study of the service differentiation provided by the various mechanisms. Section 8 concludes the paper and discusses future work. *The proofs of all lemmas and theorems, if not in the paper, are provided in [18].*

2. THE GENERALISED BACK-OFF MODEL

There are n nodes, indexed by $i, 1 \leq i \leq n$, each with one EDCF queue. We adopt the notation in [1] whose authors consider a generalisation of the back-off behaviour of the nodes, and define the following back-off parameters (for node i)

$K_i :=$ At the $(K_i + 1)$th attempt either the packet being attempted by node i succeeds or is discarded

$b_{i,k} :=$ The *mean* back-off (in slots) at the kth attempt for a packet being attempted by node i, $0 \leq k \leq K_i$

Definition 2.1. *A system of n nodes is said to be* ***homogeneous****, if the back-off parameters of the nodes K_i, $b_{i,k}, 0 \leq k \leq K_i$ are the same for all $i, 1 \leq i \leq n$. A system of nodes is called* ***nonhomogeneous*** *if the back-off parameters of the nodes are not identical.* □

Remark: IEEE 802.11e permits different backoff parameters to differentiate channel access obtained by the nodes in an attempt to provide QoS. The above definitions capture the possibility of having different CW_{min} and CW_{max} values, different exponential back-off multiplier values and even different number of permitted attempts. For ease of discussion and understanding, we will postpone the topic of AIFS until Section 6. Hence in the discussions up to Section 5.2, all the nodes wait only for a DIFS after a busy channel. □

It has been shown in [1] that under the decoupling assumption, introduced by Bianchi in [3], the attempt probability of node i (conditioned on being in back-off) for given collision probability γ_i is given by,

$$G_i(\gamma_i) := \frac{1+\gamma_i+\cdots+\gamma_i^{K_i}}{b_{i,0}+\gamma_i b_{i,1}+\cdots+\gamma_i^{K_i} b_{i,K_i}} \tag{1}$$

Remark: When the system is homogeneous then we will drop the subscript i from $G_i(\cdot)$, and write the function simply as $G(\cdot)$.

3. THE FIXED POINT EQUATION

It is important to note that in the present discussion all rates are conditioned on being in the back-off periods; i.e., we have eliminated all durations other than those in which nodes are counting down their back-off counters (see [1]). This suffices to obtain the collision probabilities. Now consider a nonhomogeneous system of n nodes. Let $\boldsymbol{\gamma}$ be the vector of collision probabilities of each node. With the slotted model for the back-off process and the decoupling assumption, the natural mapping of the attempt probabilities of other nodes to the collision probability of a node is given by

$$\gamma_i = \Gamma_i(\beta_1, \beta_2, \ldots, \beta_n) = 1 - \prod_{j=1, j\neq i}^{n} (1-\beta_j)$$

where $\beta_j = G_j(\gamma_j)$. We can now expect that the equilibrium behaviour of the system will be characterised by the solutions of the following system of equations. For $1 \le i \le n$,

$$\gamma_i = \Gamma_i(G_1(\gamma_1), \cdots, G_n(\gamma_n))$$

We write these n equations compactly in the form of the following multidimensional fixed point equation.

$$\boldsymbol{\gamma} = \mathbf{\Gamma}(\mathbf{G}(\boldsymbol{\gamma})) \tag{2}$$

Since $\mathbf{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$ is a composition of continuous functions it is continuous. We thus have a continuous mapping from $[0,1]^n$ to $[0,1]^n$. Hence by Brouwer's fixed point theorem there exists a fixed point in $[0,1]^n$ for the equation $\boldsymbol{\gamma} = \mathbf{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$.

Consider the i^{th} component of the fixed point equation, i.e.,

$$\gamma_i = 1 - \prod_{1\le j\le n, j\neq i} (1 - G_j(\gamma_j))$$

or equivalently,

$$(1-\gamma_i) = \prod_{1\le j\le n, j\neq i} (1 - G_j(\gamma_j))$$

Multiplying both sides by $(1 - G_i(\gamma_i))$, we get,

$$(1-\gamma_i)(1-G_i(\gamma_i)) = \prod_{1\le j\le n} (1 - G_j(\gamma_j))$$

Thus a *necessary and sufficient condition* for a vector of collision probabilities $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_n)$ to be a fixed point solution is that, for all $1 \le i \le n$,

$$(1-\gamma_i)(1-G_i(\gamma_i)) = \prod_{j=1}^{n} (1 - G_j(\gamma_j)) \tag{3}$$

where the right-hand side is seen to be independent of i.

Define $F_i(\gamma) := (1-\gamma)(1-G_i(\gamma))$. From Equation 3 we see that if $\boldsymbol{\gamma}$ is a solution of Equation 2, then for all $i, j, 1 \le i, j \le n$,

$$F_i(\gamma_i) = F_j(\gamma_j) \tag{4}$$

Notice that this is only a *necessary condition.* For example, in a homogeneous system of nodes, the vector $\boldsymbol{\gamma}$ such that $\gamma_i = \gamma$ for all $1 \le i \le n$, satisfies Equation 4 for any $0 \le \gamma \le 1$, but not all such points are solutions of the fixed point Equation 2.

DEFINITION 3.1. *We say that a fixed point $\boldsymbol{\gamma}$ (i.e., a solution of $\boldsymbol{\gamma} = \mathbf{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$) is a **balanced** fixed point if $\gamma_i = \gamma_j$ for all $1 \le i, j \le n$; otherwise, $\boldsymbol{\gamma}$ is said to be an **unbalanced fixed point**.* □

REMARKS 3.1.

1. It is clear that if there exists an unbalanced fixed point for a homogeneous system, then every permutation is also a fixed point and hence, in such cases, we do not have a unique fixed point.

2. In the homogeneous case, by symmetry, the average collision probability at each node must be the same for every node. If the collision probabilities correspond to a fixed point (see 3, next), then this fixed point will be of the form $(\gamma, \gamma, \cdots, \gamma)$ where γ solves $\gamma = \Gamma(G(\gamma))$ (since $\Gamma_i(\cdot) = \Gamma(\cdot)$ and $G_i(\cdot) = G(\cdot)$ for all $1 \le i \le n$). Such a fixed point of $\gamma = \Gamma(G(\gamma))$ is guaranteed by Brouwer's Fixed Point. The uniqueness of such a balanced fixed point was studied in [1]. We reproduce this result in Theorem 5.1.

3. There is, however, the possibility that even in the homogeneous case, there is an unbalanced solution of $\boldsymbol{\gamma} = \mathbf{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$. By simulation examples we observe in Section 4 that when there exist unbalanced fixed points, the balanced fixed point of the system does not characterise the average performance, even if there exists only one balanced fixed point. In Section 5.1, we provide a condition for IEEE 802.11 type nodes with geometric backoff under which there is a unique balanced fixed point and no unbalanced fixed point for a homogeneous system of nodes. In such cases, it is now well established, that the unique balanced fixed point accurately predicts the saturation throughput of the system.

4. For the homogeneous case the back-off process can be exactly modeled by a positive recurrent Markov chain (see [1]). Hence the attempt and collision processes will be ergodic and, by symmetry, the nodes will have equal attempt and collision probabilities. In such a situation the existence of multiple unbalanced fixed points will suggest short term unfairness or short term multistability. We will observe this phenomenon in Section 4.

5. Consider a system of homogeneous nodes having unbalanced solutions for the fixed point equation $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$ (i.e., there exists i, j such that $\gamma_i \neq \gamma_j$), then from Equation 4, we see that $F(\gamma_i) = F(\gamma_j)$, or the function F is many-to-one. Hence for a homogeneous system of nodes, if the function F is one-to-one then there cannot exist unbalanced fixed points. In Section 5.2 we use this observation to obtain a sufficient condition for the uniqueness of the fixed point in the nonhomogeneous case.

4. NONUNIQUE FIXED POINTS AND MULTISTABILITY: SIMULATION EXAMPLES

4.1 Example 1

Consider a homogeneous system (let us call it System-I) with $n = 10$ nodes. The function $G(\cdot)$ of the nodes is given by,

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \gamma^3 + \dots}{1 + \gamma + \gamma^2 + \gamma^3 + 64(\gamma^4 + \gamma^5 + \dots)}$$

The system corresponds to the case where $K = \infty$, $b_0 = b_1 = b_2 = b_3 = 1$ and $b_4 = b_5 = b_6 = \dots = 64$. From the form of function G, we can see that a node which is currently at backoff stage 0 is more likely to remain at that stage as it *takes* 4 *successive collisions* to make the attempt rate of the node < 1. Likewise, a node that is in the larger back-off stages $b_4 = b_5 = \dots = 64$, will retry continuously with mean inter-attempt slots of 64 until it succeeds. Observe that only one node can be at backoff stage 0 at any time. This leads to the apparent multistability of the system.

Figure 1 plots $G(\gamma)$, the corresponding $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and shows the balanced fixed point of the system for $n = 10$ nodes. The balanced fixed point of the system shown in the figure is obtained using the fixed point equation $\gamma = 1 - (1 - G(\gamma))^9$. Observe that the function $F(\cdot)$ is not one-to-one (the function $F(\cdot)$ not being one-to-one does not necessarily imply that there exist multiple fixed point solutions; see Remarks 3.1, 5).

Figure 2 shows the existence of unbalanced fixed points for System-I. These fixed points are obtained as follows. Assume that we are interested in fixed points such that $\gamma_1 \neq \gamma_2 = \dots = \gamma_n$. Given $\gamma_2 = \dots = \gamma_n$, the attempt probability of the nodes is given by $G(\gamma_2)$. Hence, the collision probability of node 1 is given by $\gamma_1 = 1 - (1 - G(\gamma_2))^{n-1}$. The attempt probability of node 1 would then be $G(\gamma_1)$. Using the decoupling assumption, the collision probability of any of the $n-1$ nodes would then be, $1 - (1 - G(\gamma_2))^{n-2}(1 - G(\gamma_1)) = \gamma_2$. Thus we obtain a fixed point equation for γ_2 (and hence for all the other $\gamma_j, 3 \le j \le n$). In Figure 2 we plot $1 - (1 - G(\gamma))^8(1 - G(1 - (1 - G(\gamma))^9))$ (plotted as line marked with dots), the intersection of which with the "y=x" line shows the solutions for $\gamma_2 = \dots = \gamma_n$. In the same way, by eliminating γ_2 from the multidimensional system of equations, we can obtain a fixed point equation for γ_1. This function is also plotted in Figure 2 (using pluses and lines) and the intersetion of this curve with the "y=x" line shows the solutions for γ_1. We see that there are three solutions in each case. The smallest values of γ_1 (approx. 0.14) pairs up with the largest value of $\gamma_2 = \dots = \gamma_n$ (approx. 0.97). Notice that the balanced fixed point of the system is also a fixed point in the plot (compare with Figure 1). Then there is one remaining unbalanced fixed point whose values can be read off the plot. We note that there could exist many other unbalanced fixed points for this system of equations, as we have considered only a particular variety of fixed points that have the property that $\gamma_1 \neq \gamma_2 = \dots = \gamma_n$.

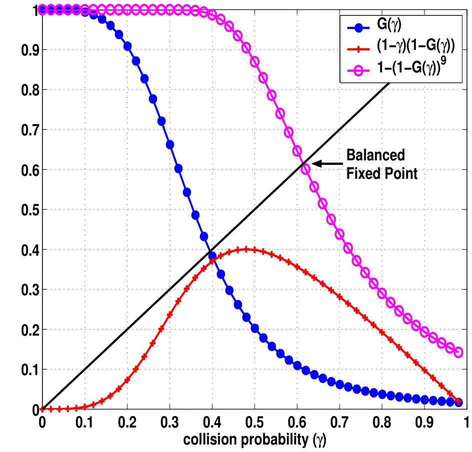

Figure 1: Example System-I: The balanced fixed point. Plots of $G(\gamma)$, $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and $1-(1-G(\gamma))^9$ vs. the collision probability γ; we also show the "y=x" line.

In order to examine the consequences of multiple unbalanced fixed points we simulated the back-off process with the back-off parameters of System-I. The following remarks summarise our simulation approach in this paper.

REMARKS 4.1 (ON THE SIMULATION APPROACH USED).

1. All the simulation results reported in this paper are based on simulations of the coupled multidimensional back-off processes of the various nodes. We are not simulating the actual Wireless LAN system (as is done in an *ns2* simulation). The main aim of the simulations is to understand the backoff behaviour of the nodes with respect to the different backoff parameters. From the point of view of performance analysis, it may also be noted that once the back-off behaviour is correctly modelled the channel activity can easily be added analytically, and thus throughput results can be obtained (see [3] and [1]). Note that a good match between analysis that uses a decoupled Markov model of the back-off process and *ns2* simulations has already been reported in earlier work (see the literature survey in Section 1).

2. Thus our simulation is programmed as follows. The system evolves over back-off slots. All the nodes are assumed to be in perfect slot synchronisation. The actual coupled evolution of the backoff process is modeled. The backoff distribution is uniform and the residual backoff time is the state for each node. At every slot, depending on the state of the back-off process, there are three possibilities: the slot is idle, there is a successful transmssion, or there is a collision. This causes further evolution of the back-off process.

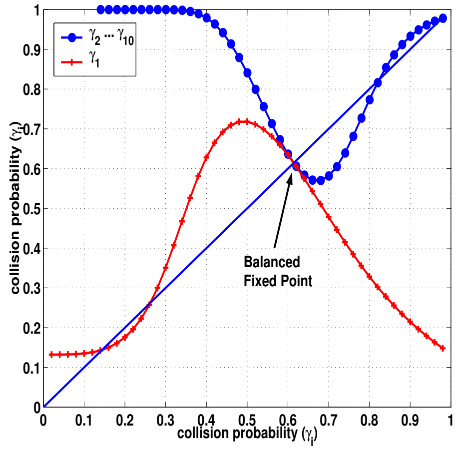

Figure 2: Example System-I: Demonstration of unbalanced fixed points. Plots of $1-(1-G(\gamma))^8(1-G(1-(1-G(\gamma))^9))$ (the curve drawn with dots and lines) and the function for the unbalanced fixed point equation for γ_1 (see text).

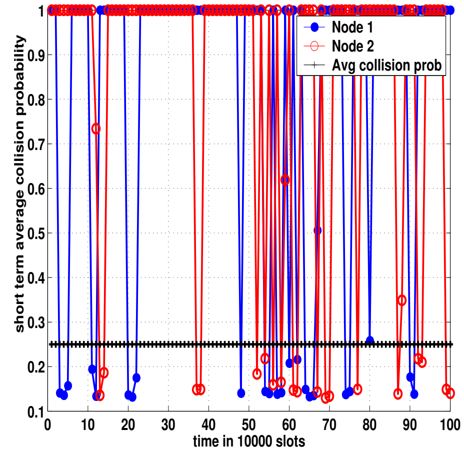

Figure 3: Example System-I: Snap-shot of short term average collision probability of 2 of the 10 nodes. Also plotted is the average collision probability of the nodes (averaged *over all slots and nodes*). The 95% confidence interval for the average collision probability lies within 0.7% of the mean value.

3. In Figures 3 and 5, for the purpose of reporting the short term unfairness results, the entire duration of simulation is divided into k frames, where the size of each frame is 10,000 slots. The short-term average of the collision probability of each node $j, 1 \leq j \leq n$, is calculated as $\frac{C_j(i)}{A_j(i)}$ where $C_j(i)$ and $A_j(i)$ correspond to the number of collisions and attempts in frame $i, 1 \leq i \leq k$, for node j. The long-term average is similarly calculated as $\frac{1}{n}\sum_{j=1}^{n}\frac{\sum_{i=1}^{k}C_j(i)}{\sum_{i=1}^{k}A_j(i)}$ where n is the number of nodes. Notice that the long-term average collision rate is a batch biased average of the short-term collision rates. Hence, when looking at the graphs, it will be incorrect to visually average the short-term collision rate plots in an attempt to obtain the long-term average collision rate. This is because when a node is shown to have a low collision probability, it is the one that is attempting every slot (while the other nodes attempt with a mean gap of 64 slots), and hence it sees a low probability of collision. In this case $A_j(\cdot)$ is large and $C_j(\cdot) \ll A_j(\cdot)$. On the other hand, when a node is shown to have a high collision probability it is attempting at an average rate of $\frac{1}{64}$ and almost all its attempts collide with the node that is then attempting in every slot. In this case $A_j(\cdot)$ is small and $C_j(\cdot) \approx A_j(\cdot)$. Thus, in obtaining the linear average, it is essential to account for the large variation in $A_j(\cdot)$ between the two cases. □

In Figure 3 we plot a (simulation) snap shot of the short term average collision probability of 2 of the 10 nodes of System-I and the average collision probability of the nodes (The average is calculated over all frames and all nodes. Since the nodes are identical, the average collision probability is the same for all the nodes). Observe that the short term average has a huge variance around the long term average. It is evident that over 1000's of slots one node or the other monopolises the channel (and the remaining nodes see a collision probability of 1 during those slots). This could be described as short term multistability. A look into the fairness index (see Figure 6) plotted as a function of the frame size used to calculate throughput suggests that System-I exhibits significant unfairness in service even over reasonably large time intervals.

Implication for the use of the balanced fixed point: Notice also that the average collision rate shown in Figure 3 is about 0.25, whereas the balanced fixed point shown in Figure 1 shows a collision probability of about 0.62. *Hence we see that in this case, where there are multiple fixed points, the balanced fixed point does not capture the actual system performance.*

4.2 Example 2

Let us now consider yet another homogeneous example (let us call it System-II) with $n = 20$ nodes. The function $G(\cdot)$ of the nodes is given by,

$$G(\gamma) = \frac{1+\gamma+\gamma^2+\cdots+\gamma^7}{1+3\gamma+9\gamma^2+27\gamma^3+\cdots+2187\gamma^7}$$

The system corresponds to the case where $K = 7$, $b_0 = 1$, $p = 3$ and $b_k = p^k b_0$ for all $0 \leq k \leq K$. We notice that in this example the way the back-off expands is similar to the way it expands in the IEEE 802.11 standard, except that the initial back-off is very small (1 slot), and the multiplier is 3, rather than 2. We observe that, similar to Example System-I, this system also has multiple (unbalanced) fixed points and exhibits short-term unfairness in service (A detailed comment on System-II is provided in [18]).

Discussion of Examples 1 and 2: From the simulation examples, we can make the following inferences

1. When there are multiple unbalanced fixed points in a homogeneous system then the system can display short term multistability, which manifests itself as significant short term unfairness in channel access.

2. When there are multiple unbalanced fixed points in a homogeneous system then the collision probability

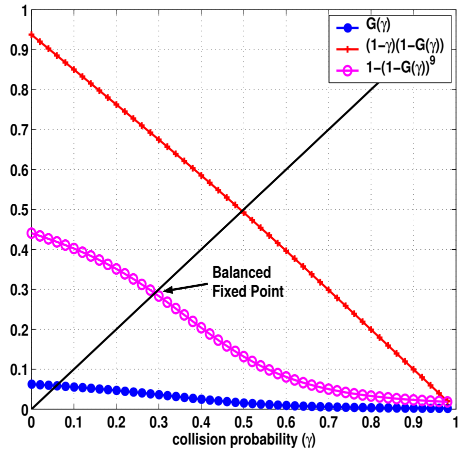

Figure 4: Example System-III: Plots of $G(\gamma)$, $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and $1-(1-G(\gamma))^9$ vs. the collision probability γ; the line "y=x" is also shown.

obtained from the balanced fixed point may be a poor approximation to the long term average collision probability.

3. Similar conclusions can be drawn for nonhomogeneous systems when the system of fixed point equations have multiple solutions.

It appears that the existence of multiple-fixed points is a consequence of the form of the $G(\cdot)$ function in the above examples, where $G(\cdot)$ is similar to a switching curve; see, for example, Figure 1 where there is a very high attempt probability at low collision probabilities and a very low attempt probability at high collision probabilities.

4.3 Example 3

Consider a homogeneous system in which backoff increases multiplicatively as in IEEE 802.11 DCF (let us call it System-III), with $n = 10$ nodes. The function $G(\cdot)$ is given by,

$$G(\gamma) = \frac{1+\gamma+\gamma^2+\cdots+\gamma^7}{16+32\gamma+64\gamma^2+\cdots+2048\gamma^7}$$

The system corresponds to the case where $K = 7$, $p = 2$ and $b_0 = 16$ and $b_k = p^k b_0$ for all $0 \le k \le K$. Figure 4 plots $G(\cdot)$, the corresponding $F(\gamma) = (1-\gamma)(1-G(\gamma))$ and the unique balanced fixed point of the system (Notice that F is one-to-one and uniqueness of the fixed point will be proved in Section 5.1) The balanced fixed point of the system is obtained using the fixed point equation $\gamma = 1-(1-G(\gamma))^9$. The balanced fixed point yields a collision probability of approximately 0.29.

Figure 5 plots a snap shot of the short term average collision probability (from simulation) of 2 of the 10 nodes and the average collision probability of the nodes of the Example System-III. Notice that the short term average collision rate is close to the average collision rate (the vertical scale in this figure is much finer than in the corresponding figures for System-I and System-II, see [18]). Also, the average collision rate matches well with the balanced fixed point solution obtained in Figure 4.

Remark: Thus we see that in a situation in which there is a unique fixed point not only is there lack of short term

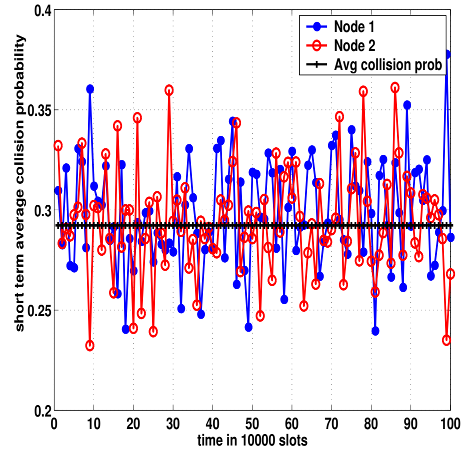

Figure 5: Example System-III: Snap-shot of short term average collision probability of 2 of the 10 nodes. Also plotted is the average collision probabilty obtained by the nodes. The 95% confidence interval of the average collision rate lies within 0.2% of the mean value.

multistability, but also the fixed point solution yields a good approximation to the long run average behaviour.

4.4 Short Term Fairness in Examples 1, 2, 3

Figure 6 plots the throughput fairness index $\frac{1}{n}\frac{(\sum_{i=1}^{n}\tau_i)^2}{\sum_{i=1}^{n}\tau_i^2}$ (where τ_i is the average throughput of node i over the measurement frame, see [15]) against the frame size used to measure throughput. The fairness index is obtained for each frame and is averaged over the duration of the simulation. Also plotted in the figure is the 95% confidence interval. We note that values of this index will lie in the interval $[0, 1]$, and smaller values of the index correspond to greater unfairness between the nodes. The performance of all the three example systems are compared. Notice that Example System-III (similar to IEEE 802.11 DCF) has the best fairness properties. The system achieves fairness of 0.9 over 1000's of slots (or packets). However, for Example System-I and II, similar performance is achieved only over 100,000 and 1,000,000 slots (or packets). The unfairness of Example Systems-I and II can be attributed to their apparent multi-stability.

In the subsequent sections we establish conditions for the uniqueness of the solutions to the multidimensional fixed point equation.

5. ANALYSIS OF THE FIXED POINT

5.1 The Homogeneous Case

The following two results are adopted from [1].

LEMMA 5.1. *$G(\gamma)$ is nonincreasing in γ if $b_k, k \ge 0$, is a nondecreasing sequence. In that case, unless $b_k = b_0$ for all k, $G(\gamma)$ is strictly decreasing in γ.* □

THEOREM 5.1. *For a homogeneous system of nodes, $\Gamma(G(\gamma)) : [0,1] \to [0,1]$, has a unique fixed point if $b_k, k \ge 0$, is a nondecreasing sequence.* □

Remark: The fixed point $(\gamma, \gamma, \cdots, \gamma)$ is the unique balanced fixed point for $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$. From Equation 4, we

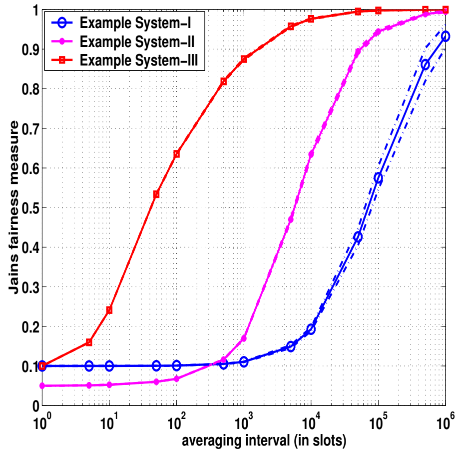

Figure 6: Jain's throughput fairness index is plotted against the number of slots used to measure throughput. The dotted lines mark the 95% confidence interval.

see that a *necessary* condition for the existence of unbalanced fixed points in a homogeneous system of nodes is that the function $F(\gamma) = (1-\gamma)(1-G(\gamma))$ needs to be many-to-one. In other words, if the function $(1-\gamma)(1-G(\gamma))$ is one-to-one and if $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ is a solution of the system $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$, then $\gamma_i = \gamma_j$ for all i, j.

Consider the multiplicatively increasing back-off model for which $G(\cdot)$ is given by,

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \cdots + \gamma^K}{b_0(1 + p\gamma + p^2\gamma^2 + \cdots + p^K\gamma^K)} \tag{5}$$

Clearly, $G(\gamma)$ is a continuously differentiable function and so is $F(\gamma) = (1-\gamma)(1-G(\gamma))$. The following simple lemma is a consequence of the mean value theorem.

LEMMA 5.2. *$F(\gamma)$ is one-to-one* if $F^{'}(\gamma) \neq 0$ *for all* $0 \leq \gamma \leq 1$. □

REMARKS 5.1.

When $F(\cdot)$ is one-to-one, the following hold
(i) $F(\gamma) = 0$ iff $\gamma = 1$,
(ii) $F(0) > 0$, since $G(\gamma) \leq 1$ for all $0 \leq \gamma \leq 1$, and
(iii) $F(\gamma)$ is a decreasing function of γ. □
Now the derivative of F is

$$F^{'}(\gamma) = -1 + G(\gamma) - G^{'}(\gamma)(1-\gamma)$$

LEMMA 5.3. *If $K \geq 1, p \geq 2$ and $G(\cdot)$ is as in Equation 5, then $|G^{'}(\gamma)| \leq \frac{2p}{b_0}$ for all $0 \leq \gamma \leq 1$.* □

Clearly, $G(\gamma) \leq \frac{1}{b_0}$ and $G^{'}(\gamma) \leq 0$ and $(1-\gamma) \geq 0$ for all $0 \leq \gamma \leq 1$. Substituting into the expression for $F^{'}(\gamma)$, we get,

$$F^{'}(\gamma) \leq -1 + \frac{1+2p}{b_0}$$

The following result is then immediate.

THEOREM 5.2. *For a function $G(\cdot)$ defined as in Equation 5 if $K \geq 1, p \geq 2$ and $b_0 > 2p+1$, then the system $\boldsymbol{\gamma} = \boldsymbol{\Gamma}(\mathbf{G}(\boldsymbol{\gamma}))$ has a unique fixed point which is balanced.* □

Remark: It can be shown that if Lemma 5.3 holds for $G(\cdot)$ as in Equation 5 it also holds for any case in which $b_k = p^k b_0$ for $0 \leq k \leq m \leq K$ and $b_k = p^m b_0$ for $m < k \leq K$. The latter is the situation in the IEEE 802.11 standard (with $b_0 = 16, p = 2, K = 7, m = 5$). Hence a homogeneous IEEE 802.11 WLAN has a unique fixed point which is also balanced. In general, if the function $G(\cdot)$ is arbitrary (as in Equation 1) but monotone decreasing, there exists a unique balanced fixed point for the system as long as the function $(1-\gamma)(1-G(\gamma))$ is one-to-one.

5.2 The Nonhomogeneous Case

In this section, we will extend our results to systems with nonhomogeneous nodes. AIFS will be introduced in Section 6. Nonhomogeneity is introduced by varying b_0, p and K of the nodes.

Consider a nonhomogeneous system of n nodes, with $G_i(\cdot)$ a monotonically decreasing function and the function $(1-\gamma)(1-G_i(\gamma))$ being one-to-one for all i. Let there be two fixed point solutions $\boldsymbol{\gamma}_1 = (\gamma_{1,1}, \gamma_{1,2}, \ldots, \gamma_{1,n})$ and $\boldsymbol{\gamma}_2 = (\gamma_{2,1}, \gamma_{2,2}, \ldots, \gamma_{2,n})$ for the above system. From the necessary condition (Equation 4) we require that, for all i, and for some $J_1 > 0$ and $J_2 > 0$,

$$(1-\gamma_{1,i})(1-G_i(\gamma_{1,i})) = J_1$$
$$(1-\gamma_{2,i})(1-G_i(\gamma_{2,i})) = J_2$$

Since $(1-\gamma)(1-G_i(\gamma))$ is one-to-one, we require $J_1 \neq J_2$. Without loss of generality, assume $J_1 < J_2$. Hence, $\gamma_{1,i} > \gamma_{2,i}$ for all i. Using Equation 3 we have,

$$\begin{aligned}\gamma_{2,i} &= 1 - \prod_{j\neq i}(1 - G_j(\gamma_{2,j})) \\ &\geq 1 - \prod_{j\neq i}(1 - G_j(\gamma_{1,j})) \\ &= \gamma_{1,i}\end{aligned}$$

a contradiction. Hence, we require $J_1 = J_2$ or there exists a unique fixed point.

Notice that the arguments above immediately imply the following result.

THEOREM 5.3. *If $G_i(\gamma)$ is a decreasing function of γ for all i and $(1-\gamma)(1-G_i(\gamma))$ is a monotone function in $[0,1]$, then the system of equations $\beta_i = G_i(\gamma_i)$ and $\gamma_i = \Gamma_i(\beta_1, \ldots, \beta_i, \ldots, \beta_n)$ has a unique fixed point.* □

Where nodes use exponentially increasing back-off, the next result then follows.

THEOREM 5.4. *For a system of nodes with $G_i(\cdot)$ as in Equation 5 that satisfy $K_i \geq 1$, $p_i \geq 2$ and $b_{0_i} > 2p_i + 1$, there a exists a unique fixed point for the system of equations $\gamma_i = 1 - \prod_{j\neq i}(1 - G_j(\gamma_j))$ for $1 \leq i \leq n$.*

Remark: The above result has relevance in the context of the IEEE 802.11e standard where the proposal is to use differences in back-off parameters to differentiate the throughputs obtained by the various nodes. While Theorem 5.4 only states a sufficient condition, it does point to a caution in choosing the back-off parameters of the nodes.

6. ANALYSIS OF THE AIFS MECHANISM

Our approach for obtaining the fixed point equations is the same as the one developed in [12]. However, we develop the analysis in the more general framework introduced in [1]. We show that under the condition that $F(\cdot)$ is one-to-one there exists a unique fixed point for this problem as well. The analysis is presented here for the two priority class case, but can be extended to any number of classes.

Let us begin by recalling the basic idea of AIFS based service differentiation (see [13]). In legacy DCF, a node decrements its back-off counter and attempts to transmit only after it senses an idle medium for more than a DCF interframe space (DIFS). However, in EDCF (Enhanced Distributed Coordination Function) based on the access category of a node (and its AIFS value), a node attempts to transmit only after it senses the medium idle for more than its AIFS. Higher priority nodes have smaller values of AIFS (though not less than DIFS), and hence obtain a lower average collision probability, since these nodes can decrement their back-off counters, and even transmit, in slots in which lower priority nodes (waiting to complete their AIFSs) cannot. Thus, *nodes of higher priority (lower AIFS) not only tend to transmit more often but also have fewer collisions compared to nodes of lower priority (larger AIFS).*

6.1 The Fixed Point Equations

Let us consider two classes of nodes of two different priorities. The priority for a class is supported by using AIFS as well as b_0, p and K. All the nodes of a particular priority have the same values for all these parameters. There are $n^{(1)}$ nodes of Class 1 and $n^{(0)}$ nodes of Class 0. Class 1 corresponds to a higher priority of service. The AIFS for Class 1 is DIFS, and for Class 0 the AIFS is DIFS$+l$ slots. Thus, after every transmission activity in the channel, only Class 1 nodes attempt to transmit in the first l slots following an idle DIFS, while Class 0 nodes wait to complete their AIFS. Also, if there is any transmission activity (by Class 1 nodes) during those l slots, then again the Class 0 nodes wait for another l slots following an idle DIFS, and so on.

As in [3] and [1], we need to model only the evolution of the back-off process of a node (i.e., the back-off slots after removing any channel activity such as transmissions or collisions) to obtain the collision probabilities. For convenience, let us call the slots in which only Class 1 nodes can attempt as *Excess AIFS* slots, which will correspond to the subscript EA in the notation. In the *remaining* slots (corresponding to the subscript R in the notation) nodes of either class can attempt. Let us view such groups of slots, where different sets of nodes contend for the channel, as different *contention periods*. Let us define

$\beta_i^{(1)}$:= the attempt probability of a Class 1 node for all $i, 1 \leq i \leq n^{(1)}$, in the slots in which a Class 1 node can attempt (i.e., all the slots)

$\beta_i^{(0)}$:= the attempt probability of a Class 0 node for all $i, 1 \leq i \leq n^{(0)}$, in the contention periods during which Class 0 nodes can attempt (i.e., slots that are not Excess AIFS slots)

Note that in making these definitions we are modeling the attempt probabilities for Class 1 as being constant over all slots, i.e., the Excess AIFS slots and the remaining slots. This simplification is just an extension of the basic decoupling approximation, and has been shown to yield results that match well with simulations (see [12]).

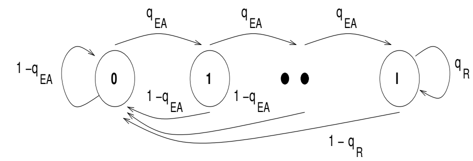

Figure 7: AIFS differentiation mechanism: Markov model for remaining number of AIFS slots.

Now the collision probabilities experienced by nodes will depend on the contention period that the system is in. The approach is to model the evolution over contention periods as a Markov Chain over the states $(0, 1, 2, \cdots, l)$, where the state s, $0 \leq s \leq (l-1)$, denotes that an amount of time equal to DIFS $+s$ slots has elapsed since the end of the previous channel activity. These states correspond to the Excess AIFS period in which only Class 1 nodes can attempt. In the remaining slots, where the state is $s = l$, all nodes can attempt.

In order to obtain the transition probabilities for this Markov chain we need the probability that a slot is idle. Using the decoupling assumption, the idle probability in any slot during the Excess AIFS period is obtained as,

$$q_{EA} = \prod_{i=1}^{n^{(1)}} (1 - \beta_i^{(1)}) \tag{6}$$

Similarly, the idle probability in any of the remaining slots is obtained as,

$$q_R = \prod_{i=1}^{n^{(1)}} (1 - \beta_i^{(1)}) \prod_{j=1}^{n^{(0)}} (1 - \beta_j^{(0)}) \tag{7}$$

The transition structure of the Markov chain is shown in Figure 7. As compared to [12], we have used a simplification that the maximum contention window is much larger than l. If this were not the case then some nodes would certainly attempt before reaching l. In practice, l is small (e.g., 1 slot or 5 slots; see [2]) compared to the maximum contention window.

Let $\pi(EA)$ be the stationary probability of the system being in the Excess AIFS period; i.e., this is the probability that the above Markov chain is in states 0, or 1, or $\cdots$, or $(l-1)$. In addition, let $\pi(R)$ be the steady state probability of the system being in the any of the remaining slots, i.e., state l of the Markov chain. Solving the balance equations for the steady state probabilities, we obtain,

$$\begin{aligned} \pi(EA) &= \frac{1 + q_{EA} + q_{EA}^2 + \cdots + q_{EA}^{l-1}}{1 + q_{EA} + q_{EA}^2 + \cdots + q_{EA}^{l-1} + \frac{q_{EA}^l}{1-q_R}} \\ \pi(R) &= \frac{\frac{q_{EA}^l}{1-q_R}}{1 + q_{EA} + q_{EA}^2 + \cdots + q_{EA}^{l-1} + \frac{q_{EA}^l}{1-q_R}} \end{aligned} \tag{8}$$

Average collision probability of a node is then obtained by averaging the collision probability experienced by a node over the different contention periods. Average collision probability for Class 1 nodes is given by, for all i, $1 \le i \le n^{(1)}$,

$$\gamma_i^{(1)} = \pi(EA)(1 - \prod_{j=1, j\neq i}^{n^{(1)}} (1-\beta_j^{(1)})) + \pi(R)(1 - (\prod_{j=1, j\neq i}^{n^{(1)}} (1-\beta_j^{(1)}) \prod_{j=1}^{n^{(0)}} (1-\beta_j^{(0)}))) \quad (9)$$

Similarly, the average collision probability of a Class 0 node is given by, for all i, $1 \le i \le n^{(0)}$,

$$\gamma_i^{(0)} = 1 - (\prod_{j=1}^{n^{(1)}} (1-\beta_j^{(1)}) \prod_{j=1, j\neq i}^{n^{(0)}} (1-\beta_j^{(0)})) \quad (10)$$

Our analysis in the remaining section now generalises the analysis of [12] and also establishes uniqueness of the fixed point and the property that the fixed point is balanced over nodes in the same class. Define $G^{(1)}(\cdot)$ and $G^{(0)}(\cdot)$ as in Equation 1 (except that the superscripts here denote the class dependent back-off parameters, with nodes within a class having the same parameters). Then the average collision probability obtained from the previous equations can be used to obtain the attempt rates by using the relations

$$\beta_i^{(1)} = G^{(1)}(\gamma_i^{(1)}), \text{ and } \beta_j^{(0)} = G^{(0)}(\gamma_j^{(0)}) \quad (11)$$

for all $1 \le i \le n^{(1)}$, $1 \le j \le n^{(0)}$. We obtain fixed point equations for the collision probabilities by substituting the attempt probabilities from Equation 11 into Equations 9 and 10 (and also into Equations 6 and 7). We have a continuous mapping from $[0,1]^{n^{(1)}+n^{(0)}}$ to $[0,1]^{n^{(1)}+n^{(0)}}$. It follows from Brouwer's fixed point theorem that there exists a fixed point.

6.2 Uniqueness of the Fixed Point

LEMMA 6.1. *If $F^{(\cdot)}$ is one-to-one, then collision probabilities of all the nodes of the same class are identical; i.e., the fixed points are balanced within each class.* □

THEOREM 6.1. *The set of Equations 9, 10 and 11 (together with 8, 6 and 7), representing the fixed point for the AIFS model, has a unique solution if the corresponding functions $F^{(1)}(\cdot)$ and $F^{(0)}(\cdot)$ are one-to-one.* □

Remark: It follows from the earlier results in this paper (see, for example, Theorem 5.2) that if $G^{(0)}(\cdot)$ and $G^{(1)}(\cdot)$ are of the form in Equation 5, and if $K^{(i)} \ge 1, p^{(i)} \ge 2$, and $b_0^{(i)} \ge 2p^{(i)} + 1$, for $i = 0, 1$, then the fixed point will be unique.

7. THROUGHPUT DIFFERENTIATION: AN ANALYTICAL STUDY

It should be noted that all the results in this section are for the fixed point solution. Hence, when we use the term "collision probability" and "attempt rate" it is only in so far as a good match between the fixed point analysis and simulation has already been reported in earlier literature (see Section 1).

We will consider two alternatives for K, the maximum retransmission attempts allowed for a packet, namely $K = \infty$ and K finite. In this section, for the finite K case, the form of the function $G(\gamma)$, for all γ, $0 \le \gamma \le 1$ is,

$$G(\gamma) = \frac{1+\gamma+\gamma^2+\cdots+\gamma^K}{b_0(1+p\gamma+p^2\gamma^2+\cdots+p^K\gamma^K)} \quad (12)$$

It is clear that for finite K the attempt rate of a node is lower bounded, and hence as the number of nodes increases to infinity the collision probability of any node goes to 1. Hence, for this case, we will obtain insights regarding performance differentiation only for a finitely large number of nodes. For the infinite K case, however, we will study (as in [1]) the asymptotics of performance differentiation as the number of nodes tends to ∞. In the $K = \infty$ case, the function $G(\gamma)$ simplifies to,

$$G_\infty(\gamma) = \begin{cases} \frac{(1-\gamma p)}{b_0(1-\gamma)} & 0 \le \gamma < \frac{1}{p} \\ 0 & \gamma \ge \frac{1}{p} \end{cases} \quad (13)$$

In the nonhomogeneous case we will write $G_\infty^{(1)}(\gamma)$ and $G_\infty^{(0)}(\gamma)$. For the homogeneous case with $K = \infty$, the (balanced fixed point) asymptotic analysis as $n \to \infty$ was performed in [1].

Consider a set of nodes, divided into two classes, Class 1 and Class 0, with Class 1 corresponding to a higher priority of service. *For simplicity, we assume that $n^{(1)}$ and $n^{(0)}$, the number of nodes of Class 1 and Class 0 respectively, are related as, $n^{(1)} = \alpha n, n^{(0)} = (1-\alpha)n$ for some n and $\alpha, 0 < \alpha < 1$.* Let $\gamma^{(1)}(K,n)$ and $\beta^{(1)}(K,n)$ be the fixed point solutions for the collision probability and attempt rate of a Class 1 node for a given K and total number of nodes n. Similarly, let $\gamma^{(0)}(K,n)$ and $\beta^{(0)}(K,n)$ be the corresponding values for a Class 0 node.

We will study three cases:

Case 1: $b_0^{(1)} < b_0^{(0)}$, $p^{(1)} = p^{(0)} = p$, $AIFS^{(1)} = AIFS^{(0)} = DIFS$

Case 2: $b_0^{(1)} = b_0^{(0)} = b_0$, $p^{(1)} < p^{(0)}$, $AIFS^{(1)} = AIFS^{(0)} = DIFS$

Case 3: $b_0^{(1)} = b_0^{(0)} = b_0$, $p^{(1)} = p^{(0)} = p$, $AIFS^{(1)} < AIFS^{(0)}$

Note that in the analysis in earlier sections, we used the Binomial model for the number of attempts in a slot. With $n \to \infty$, in this section, we will use the Poisson batch model for the number of attempts in a slot (as in [1]).

7.1 Case 1: Differentiation by b_0

7.1.1 $K = \infty$, *Asymptotic Analysis as* $n \to \infty$

With the random number of attempts of each class in a back-off slot being modeled as Poisson distributed, the collision probabilities $\gamma^{(\cdot)}(\infty,n)$ and the attempt rates $\beta^{(\cdot)}(\infty,n)$ are related by

$$\begin{aligned} \gamma^{(1)}(\infty,n) &= 1 - e^{-((n^{(1)}-1)\beta^{(1)}(\infty,n)+n^{(0)}\beta^{(0)}(\infty,n))} \\ \gamma^{(0)}(\infty,n) &= 1 - e^{-(n^{(1)}\beta^{(1)}(\infty,n)+(n^{(0)}-1)\beta^{(0)}(\infty,n))} \end{aligned} \quad (14)$$

Substituting $\beta^{(\cdot)}(\infty,n) = G_\infty^{(\cdot)}(\gamma^{(\cdot)}(\infty,n))$ in the above equations gives the desired fixed point equations governing the system. Trivially, we see that,

$$(1-\gamma^{(1)}(\infty,n))e^{-\beta^{(1)}(\infty,n)} = (1-\gamma^{(0)}(\infty,n))e^{-\beta^{(0)}(\infty,n)} \quad (15)$$

LEMMA 7.1. *For $i \in \{0,1\}$, $F_\infty^{(i)}(\gamma) := (1-\gamma)e^{-G_\infty^{(i)}(\gamma)}$ is one-to-one for all $\gamma, 0 \le \gamma \le 1$ if $b_0^i \ge 2p+1$.* $\square$

THEOREM 7.1. *In Case 1, with $K = \infty$, when $F_\infty^{(i)}$ is one-to-one for $i \in \{0,1\}$,*

1. $\gamma^{(1)}(\infty, n) < \gamma^{(0)}(\infty, n)$ *for all* n
2. $\lim_{n\to\infty} \gamma^{(1)}(\infty, n) \uparrow \frac{1}{p}, \lim_{n\to\infty} \gamma^{(0)}(\infty, n) \uparrow \frac{1}{p}$
3. $\lim_{n\to\infty}(n^{(1)}\beta^{(1)}(\infty,n)+n^{(0)}\beta^{(0)}(\infty,n)) \uparrow \ln(\frac{p}{p-1})$ $\square$

THEOREM 7.2. *In Case 1, with $K = \infty$, the ratio of the throughputs of Class 1 and Class 2 converges to $\frac{b_0^{(0)}-p}{b_0^{(1)}-p}$ as $n \to \infty$.* $\square$

Remark: Thus, for example, if $b_0^{(1)} = 16$, $b_0^{(0)} = 32$, and $p = 2$ then the ratio of the Class 1 to Class 0 node throughput will be approximately 30/14 for large n.

7.1.2 Finite K, Approximate Analysis for Large n

With finite K, as the number of nodes increases, the collision probability of either class increases to 1 (since the attempt rate is lower bounded) and $G^{(\cdot)}$ is small (since it decreases like $\frac{1}{b_0 p^{K+1}}$, see Equation 12). Then the difference between the collision probabilities (we drop the arguments K and n in the following)

$$\begin{aligned}\gamma^{(1)} - \gamma^{(0)} &= (G^{(0)}(\gamma^{(0)}) - G^{(1)}(\gamma^{(1)})) \\ &\quad (1-G^{(0)}(\gamma^{(0)}))^{(n^{(0)}-1)}(1-G^{(1)}(\gamma^{(1)}))^{(n^{(1)}-1)}\end{aligned}$$

also becomes insignificant. Hence, we can assume that $\gamma^{(1)} \approx \gamma^{(0)}$. For equal packet length transmission, the ratio of the throughputs of a Class 1 node to a Class 0 node corresponds to the ratio of their success probabilities, hence the throughput ratio is given by,

$$\begin{aligned}&\frac{G^{(1)}(\gamma^{(1)})(1-G^{(1)}(\gamma^{(1)}))^{n^{(1)}-1}(1-G^{(0)}(\gamma^{(0)}))^{n^{(0)}}}{G^{(0)}(\gamma^{(0)})(1-G^{(1)}(\gamma^{(1)}))^{n^{(1)}}(1-G^{(0)}(\gamma^{(0)}))^{n^{(0)}-1}} \\ &\qquad = \frac{\frac{G^{(1)}(\gamma^{(1)})}{(1-G^{(1)}(\gamma^{(1)}))}}{\frac{G^{(0)}(\gamma^{(0)})}{(1-G^{(0)}(\gamma^{(0)}))}} \qquad (16)\end{aligned}$$

Using $\gamma^{(1)} \approx \gamma^{(0)}$, writing this as γ, and using the fact that $G^{(\cdot)}(\gamma) \approx 0$ for large n, we have,

$$\approx \frac{\frac{G^{(1)}(\gamma)}{(1-G^{(1)}(\gamma))}}{\frac{G^{(0)}(\gamma)}{(1-G^{(0)}(\gamma))}} \approx \frac{G^{(1)}(\gamma)}{G^{(0)}(\gamma)} = \frac{b_0^{(0)}}{b_0^{(1)}}$$

It follows that when service differentiation is provided by the back-off window, for a large number of nodes, the throughput ratio roughly corresponds to $\frac{b_0^{(0)}}{b_0^{(1)}}$, which, for large values of $b_0^{(0)}$ and $b_0^{(1)}$ is almost that same as that obtained for the asymptotic analysis with $K = \infty$ in Theorem 7.2

Remark: For finite K case, this observation (throughput ratio is approximately equal to $\frac{b_0^{(0)}}{b_0^{(1)}}$) is well known. This result has been shown analytically (using similar approximations) and also has been observed in simulations (see [6], [11] and [14]). It has been observed in [1] that for a given number of nodes, n, there will exist a $K(n)$ such that the system performance will not vary much for all $K > K(n)$. Hence, an asymptotic analysis would suffice for such cases. Moreover, we have obtained this result in a much more general setting, using the function $G(\cdot)$.

7.2 Case 2: Differentiation by p

It may be noted that in the current version of IEEE 802.11e standard this mechanism no longer exists [2].

7.2.1 $K = \infty$, Asymptotic Analysis as $n \to \infty$

The fixed point equation governing the collision probability and the attempt rate is the same as Equation 14. The following theorem summarizes the main results for Case 2.

THEOREM 7.3. *In Case 2, with $K = \infty$, when $F_\infty^{(i)}$ is one-to-one for $i \in \{0,1\}$, the following hold:*

1. $\gamma^{(1)}(\infty, n) < \gamma^{(0)}(\infty, n)$ *for all* n
2. $\lim_{n\to\infty} \gamma^{(1)}(\infty, n) \uparrow \frac{1}{p^{(1)}}, \lim_{n\to\infty} \gamma^{(0)}(\infty, n) \uparrow \frac{1}{p^{(1)}}$
3. $\lim_{n\to\infty} n^{(1)}\beta^{(1)}(\infty,n) \uparrow \ln(\frac{p^{(1)}}{p^{(1)}-1})$
4. $\lim_{n\to\infty} n^{(0)}\beta^{(0)}(\infty,n) = 0$ $\square$

Remark: Thus we see that, with $K = \infty$ and a large number of nodes, unlike initial back-off based differentiation, the persistence factor based differentiation completely suppresses the class with the larger value of p. $\square$

7.2.2 Finite K, Approximate Analysis for Large n

For finite K, with the approximation $\gamma^{(1)} \approx \gamma^{(0)}$ and the fact that $G^{(\cdot)}(\gamma^{(\cdot)}) \approx 0$, the throughput ratio approximates to $\frac{(1+p^{(0)}\gamma+p^{(0)2}\gamma^2+\cdots+p^{(0)K}\gamma^K)}{(1+p^{(1)}\gamma+p^{(1)2}\gamma^2+\cdots+p^{(1)K}\gamma^K)}$ (see Equation 16). Hence, as the collision probability of the system increases with load, the ratio of the throughputs of Class 1 to Class 0 also increases (depending on $p^{(1)}$, $p^{(0)}$ and the value of K). We note that as $n \to \infty$, the throughput ratio for the finite K case is finite, unlike the asymptotic case ($K = \infty$). However, the ratio tends to infinity when we consider $K \to \infty$.

7.3 Case 3: Differentiation by AIFS

7.3.1 $K = \infty$, Asymptotic Analysis for $n \to \infty$

In this case service differentiation is provided only by AIFS and we let $G_\infty^{(1)} = G_\infty^{(0)} = G_\infty$ (i.e., the back-off parameters b_0 and p are the same). With the assumption that the number of attempts in each slot is Poisson distributed, the fixed point equations for the AIFS model are (see Equations 9 and 10)

$$\begin{aligned}\gamma^{(1)}(\infty,n) &= \pi(EA)(1-e^{-(n^{(1)}-1)\beta^{(1)}(\infty,n)}) + \\ &\quad \pi(R)(1-e^{-(n^{(1)}-1)\beta^{(1)}(\infty,n)-n^{(0)}\beta^{(0)}(\infty,n)}) \\ \gamma^{(0)}(\infty,n) &= (1-e^{-n^{(1)}\beta^{(1)}(\infty,n)-(n^{(0)}-1)\beta^{(0)}(\infty,n)})\end{aligned}$$

THEOREM 7.4. *In Case 3, with $K = \infty$, when $F_\infty^{(i)}$ is one-to-one for $i \in \{0,1\}$,*

1. $\gamma^{(1)}(\infty, n) < \gamma^{(0)}(\infty, n)$ *for all* n
2. $\lim_{n\to\infty} \gamma^{(1)}(\infty, n) \uparrow \frac{1}{p}, \lim_{n\to\infty} \gamma^{(0)}(\infty, n) \uparrow \frac{1}{p}$

3. $\lim_{n\to\infty} n^{(1)}\beta^{(1)}(\infty,n) \uparrow \ln(\frac{p}{p-1})$

4. $\lim_{n\to\infty} n^{(0)}\beta^{(0)}(\infty,n) = 0$ □

Remark: Again we see that using AIFS for differentiation, when $K=\infty$ and large n, completely suppresses the class with the larger value of AIFS. Observe that Parts 3 and 4 of Theorem 7.4 imply that the individual node attempt ratio $\frac{\beta^{(1)}(\infty,n)}{\beta^{(0)}(\infty,n)}$ goes to ∞ as $n\to\infty$. Some insight into this result will be obtained from the analysis in the following section.

7.3.2 Finite K, Approximate Analysis

Lemma 7.2. *In Case 3 for finite K, with $l=1$, if the fixed point collision probabilities are $\gamma^{(1)}$ and $\gamma^{(0)}$, then the ratio of the throughputs of Class 1 to Class 0 is given by*

$$\frac{\frac{G^{(1)}(\gamma^{(1)})}{(1-G^{(1)}(\gamma^{(1)}))}}{\frac{G^{(0)}(\gamma^{(0)})}{(1-G^{(0)}(\gamma^{(0)}))}}\frac{1}{q_R} \qquad \square$$

Using this result and approximating $(1-G^{(i)}(\gamma^{(i)}))\approx 1$ as before, the ratio of throughput equals

$$\frac{\frac{G^{(1)}(\gamma^{(1)})}{(1-G^{(1)}(\gamma^{(1)}))}}{\frac{G^{(0)}(\gamma^{(0)})}{(1-G^{(0)}(\gamma^{(0)}))}}\frac{1}{q_R} \approx \frac{G^{(1)}(\gamma^{(1)})}{G^{(0)}(\gamma^{(0)})}\frac{1}{q_R} \tag{17}$$

For general l, we can expect a factor like $\frac{1}{q_R^l}$ in the previous expression. For low loads, when q_R is not close to 0, the dominating term in the previous expression is $\frac{G^{(1)}(\gamma^{(1)})}{G^{(0)}(\gamma^{(0)})}$. At high loads, both the terms contribute to throughput differentiation depending on the values of $n^{(1)}$ and $n^{(0)}$.

7.4 Numerical Study and Discussion

In Figure 8 we plot throughput ratios obtained from a simulation of the coupled back-off processes of two classes of nodes (the simulation approach is explained in Remarks 4.1). We note that this is the throughput ratio if the packet sizes of the two classes are equal. If the packet sizes are unequal then we only need to multiply the throughput ratio plotted here by the ratio of the packet lengths of the two classes. The following remarks help in interpreting the results in Figure 8.

Remarks 7.1.

1. For finite K the attempt rates are bounded below, and the term $\frac{G^{(1)}(\gamma^{(1)})}{G^{(0)}(\gamma^{(0)})}$ is bounded, but as $(n^{(1)}+n^{(0)})\to\infty$ the idle probability $q_R\to 0$ ensuring (see Equation 17) that the individual node throughput ratio goes to ∞ for finite K as well (similar to the asymptotic results in Theorem 7.4). In addition, when $n^{(1)}$ increases, $\pi(EA)$ increases to 1. Hence, the lower priority nodes (with larger AIFS) rarely get a chance to attempt and the throughput ratio goes to infinity; this is demonstrated by the simulation results in Figure 8, plots with $+$ and $\star$. When $n^{(1)}$ is kept constant and $n^{(0)}$ is increased (which is more typical), the collision probability of Class 0 nodes increases to 1 and their success probability tends to 0. However, the collision probability of Class 1 nodes remains much less than 1 depending on the value of $n^{(1)}$ and hence again the throughput ratio tends to ∞ (see Figure 8, plots with $\circ$). Figure 8 also shows the throughput ratio when only b_0 is used for differentiation (plots with $\bullet$); notice that, as shown earlier, the throughput ratio is just the reciprocal of the ratios of the initial back-off durations, and does not change with n.

2. For Case 3, in general, $\gamma^{(1)}$ and $\gamma^{(0)}$ are different, unlike in Cases 1 and 2. This is captured by the first term in the expression $\frac{G^{(1)}(\gamma^{(1)})}{G^{(0)}(\gamma^{(0)})}\frac{1}{q_R}$.

3. Notice that the above results for AIFS hold even when the functions $G^{(1)}$ and $G^{(0)}$ are not identical (see Figure 8, plot with $\star$). A comparison between the plots with $+$ and $\star$ in Figure 8 shows the effect of using both b_0 and AIFS for throughput differentiation. The b_0 based differentiation causes the entire curve to shift up (in favour of the higher priority class), and AIFS still causes the ratio to increase with increasing n.

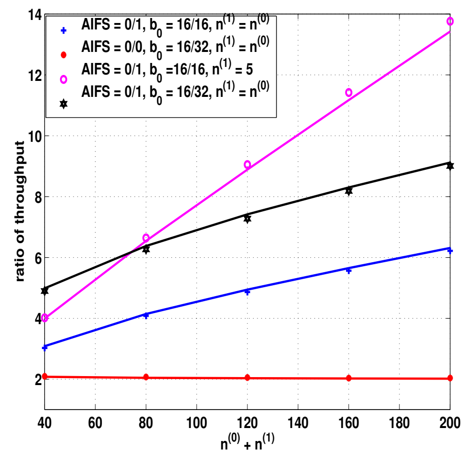

Figure 8: Ratio of the throughput of a Class 1 (higher priority) node to the throughput of a Class 0 node (lower priority). Analysis results (solid lines) and simulation results (symbols). Four cases are considered: $+$: differentiation only by AIFS with equal number of nodes, $n^{(1)}=n^{(0)}$; $\star$: differentiation by AIFS and by b_0 with equal number of nodes, $n^{(1)}=n^{(0)}$; $\bullet$: differentiation only by b_0 with equal number of nodes, $n^{(1)}=n^{(0)}$; $\circ$: differentiation only by AIFS with, $5=n^{(1)}\ll n^{(0)}$. In all cases $p=2$ and $K=7$ for either class. For the simulation results, the 95% confidence interval lies within 1% of the average value.

8. SUMMARY

In this paper we have studied a multidimensional fixed point equation arising from a model of the back-off process of the EDCF access mechanism in IEEE 802.11 and 11e Wireless LANs. Our first concern was the consequences of the nonuniqueness of the fixed point solution and conditions for uniqueness. We demonstrated via examples of homogeneous systems that even when the balanced fixed point is unique, the existence of unbalanced fixed points coexists with the observation of severe short term unfairness in simulations.

Further, in such examples the balanced fixed point solution does not capture the long run average behaviour of the system. With these observations in mind, we concluded that it is desirable to have systems in which there is a unique fixed point, even for a nonhomogeneous system.

We have provided simple sufficient conditions on the node back-off parameters that guarantee that a unique fixed point exists. We have shown that the default IEEE 802.11 parameters satisfy these sufficient conditions. The IEEE 802.11e standard motivated us to consider the nonhomogeneous case, and in this case our results suggest certain *safe* ranges of parameters that guarantee the uniqueness of the fixed point while providing service differentiation.

Using the fixed point analysis, we were also able to obtain insights into how the different back-off parameters provide throughput differentiaton between the nodes in a nonhomogeneous system. We observed that using initial back-off window, in general, a fixed throughput ratio can be achieved. On the other hand, using p and $AIFS$ the service can be significantly biased towards the high priority class, with the differentiation increasing in favour of the high priority class as the load in the system increases.

This paper concerns with the saturation throughput analysis of an IEEE 802.11e single cell WLAN without fading and capture. Also we only consider one EDCF queue per node in this work. In our recent work, we have studied multiple EDCF queues per node (see [18]). We have also developed a general framework to analyse single cell systems with capture in [16]. Extending to multi-cell scenario, in [17], performance analysis of IEEE 802.11 networks comprising intefering co-channel cells was studied using the fixed point approach.

Future work on the topic of this paper can include an analytical linkage between a coupled Markov model of the back-off process and the fixed point analysis. The fixed point approach is simply a heuristic that is found to work well in some cases. Our work in this paper suggests where it might not work and where it might work. An analytical study of this is an important future research problem. On the topic of QoS differentiation using the back-off parameters it will be important to map actual required system performance to various combinations of the back-off parameters.

9. ACKNOWLEDGMENTS

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