DELAY-DOPPLER DOMAIN CHANNEL ESTIMATION FOR DZT-BASED OTFS SYSTEMS

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***Abstract*—Traditionally, orthogonal time frequency space (OTFS) modulation has been realized using a two-step approach where the information symbols mounted in the delay-Doppler (DD) domain are first converted to time-frequency (TF) domain and then to time domain for transmission. Recently, a low-complexity approach to OTFS has been proposed using discrete Zak transform (DZT), wherein information symbols in the DD domain are directly converted to time domain for transmission. In this paper, we consider the problem of DD domain channel estimation for DZT-based OTFS systems in fractional DD channels. Towards this, we propose a low-complexity iterative algorithm which carries out estimation of the DD domain channel parameters on a path-by-path basis. For each path, the algorithm first obtains an estimate of the integer part of delay and Doppler, and then estimates the fractional part where, instead of performing a brute-force search, the algorithm iteratively increases the DD resolution till a stopping criterion is reached. Our simulation results show that the proposed algorithm achieves good normalized mean square error performance and that the bit error performance achieved using the proposed estimation algorithm is close to that with perfect channel knowledge.**

***Index Terms*—OTFS modulation, delay-Doppler domain, discrete Zak transform, channel estimation, fractional delay-Doppler.**

I. Introduction

As we progress from 5G to 6G and beyond, it has become more necessary than ever to support reliable communication in channels with high Doppler spreads that manifest due to high mobility and increased carrier frequency of operation. It has been shown that orthogonal time frequency space (OTFS) modulation performs well in high-Doppler channels [1]-[5]. In OTFS modulation, data symbols are multiplexed in delay-Doppler (DD) domain instead of conventional frequency domain or time domain. The DD multiplexed data symbols are transformed to time domain and transmitted. In most of the exisiting literature on OTFS, this transformation is carried out in two steps, namely, DD domain to time frequency (TF) domain conversion using inverse symplectic finite Fourier transform (ISSFT) and TF domain to time domain conversion using Heisenberg transform [1]-[5]. Corresponding inverse transforms are carried out at the receiver to bring the received time domain signal to DD domain for detection.

An alternate approach to realize OTFS is through Zak transform approach which transforms a DD domain signal to time domain in one step [6],[7]. In the recent works reported in [8],[9], Zak based OTFS has been shown to achieve better performance compared to OTFS with two-step transformation in channels with large Doppler spreads. They also explain why Zak based OTFS waveform is naturally suited for large doubly-spread channels and why it achieves better performance compared to TDM, FDM, and OTFS with two-step transformation. Inspired by the digital counterpart of OFDM which has been widely adopted in 4G/5G systems, discrete implementation of OTFS can be realised using discrete Zak transform (DZT) [10],[11]. Also, DZT of a sequence can be viewed as discrete Fourier transform of the sub-sampled sequence [10], and hence DZT based OTFS implementation is computationally efficient. The input-output relation for DZT based OTFS (DZT-OTFS) is derived in [11]. A low-complexity maximal ratio combining detector for DZT-OTFS is presented in [12]. However, perfect channel knowledge has been assumed. In the above context, estimation of the DD domain channel in DZT-OTFS systems is of interest, and this forms the main focus of this paper.

In this paper, we consider the problem of DD domain channel estimation for DZT-OTFS systems in fractional DD channels. Specifically, we propose a low-complexity iterative algorithm for this purpose. The proposed algorithm carries out estimation of the DD domain channel parameters on a path-by-path basis, wherein all the parameters of a path, namely, channel coefficient, delay, and Doppler, are estimated before moving to the estimation of the next path. For each path, the algorithm first obtains an estimate of the integer part of delay and Doppler, and then estimates the fractional part. In the fractional estimation part, instead of performing a brute-force search, the algorithm iteratively increases the DD resolution till a stopping criterion is reached. This leads to low complexity of the algorithm without compromising much on the performance. Our simulation results show that the proposed algorithm achieves good normalized mean square error (NMSE) performance and that the bit error rate (BER) performance achieved using the proposed estimation algorithm is close to that with perfect channel knowledge.

II. DZT-OTFS system model

Let $\mathbf{Z}_x \in \mathbb{A}^{M\times N}$ be the DD domain frame of information symbols to be transmitted, where M and N are the number of delay and Doppler bins, respectively, and $\mathbb{A}$ is the modulation alphabet. The MN symbols are mounted on the DD grid in locations given by $(\frac{mT}{M}, \frac{n\Delta f}{N})$, $m = 0,\cdots,M-1$, $n = 0,\cdots,N-1$, where $\Delta f T = 1$, $M\Delta f = B$, and B is the bandwidth available for communication. $\mathbf{Z}_x$ is converted to time domain (TD) using inverse DZT before transmission. The TD signal vector $\mathbf{x} \in \mathbb{C}^{MN\times 1}$ is obtained from $\mathbf{Z}_x$ using IDZT as $\mathbf{x} = \text{vec}(\mathbf{Z}_x\mathbf{F}_N^H)$, where $\mathbf{F}_N$ is the N-point unitary DFT matrix. Cyclic prefix (CP) of length L_{CP} is added to $\mathbf{x}$ to obtain the vector $\mathbf{s}$, i.e.,

$$\mathbf{s}[u] = \begin{cases} \mathbf{x}[(u)_{MN}], & -L_{CP} \leq u \leq MN-1 \\ 0, & \text{otherwise.} \end{cases}$$

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s is converted to a continuous time signal as

$$s(t) = \sum_{u=-L_{CP}}^{MN-1} \mathbf{s}[u] g(t - uT_s), \tag{1}$$

where $g(t)$ is the transmit pulse and $T_s = 1/B$. $s(t)$ is transmitted through a time-varying channel with impulse response $h(\tau, \nu) = \sum_{i=1}^{I} \alpha_i \delta(\tau - \tau_i)\delta(\nu - \nu_i)$, where I is the number of paths, and α_i, τ_i, and ν_i are the channel coefficient, delay, and Doppler of the ith path, respectively. The received signal, $r(t)$, at the receiver is

$$r(t) = \sum_{i=1}^{I} \alpha_i s(t - \tau_i) e^{j2\pi\nu_i t} + w(t), \tag{2}$$

where $w(t)$ is the additive noise. The above signal is passed through the matched filter, whose output is

$$y(t) = \int_{-\infty}^{\infty} r(\tau) g^*(\tau - t) d\tau. \tag{3}$$

Using (1) and (2) in (3), we get

$$\begin{aligned} y(t) \;=\; & \sum_{i=1}^{I} \alpha_i \sum_{u=-L_{CP}}^{MN-1} \mathbf{s}[u] \\ & \int_{-\infty}^{\infty} g(\tau - uT_s - \tau_i) g^*(\tau - t) e^{j2\pi\nu_i \tau} d\tau + \tilde{w}(t), \end{aligned} \tag{4}$$

where $\tilde{w(t)}$ is the match filtered noise. Assuming that the maximum Doppler, $\max_i \{\nu_i\}$, is much lesser than the bandwidth of the pulse, and denoting $f(t) = \int g(\tau) g^*(\tau - t) d\tau$, $y(t)$ can be approximated as [11]

$$y(t) \approx \sum_{i=1}^{I} \alpha_i e^{j2\pi\tau_i\nu_i} \sum_{u=-L_{CP}}^{MN-1} \mathbf{s}[u] e^{j2\pi\nu_i u T_s} f(t - uT_s - \tau_i). \tag{5}$$

For the considered $g(t)$, $f(t)$ can be approximately bounded to finite duration in time [11]. $y(t)$ is sampled at rate $1/T_s$ to obtain the discrete signal vector

$$y[v] = \sum_{i=1}^{I} \alpha_i e^{j2\pi\tau_i\nu_i} \sum_{u=-L_{CP}}^{MN-1} \mathbf{s}[u] e^{j2\pi u\nu_i T_s} f_i[v - u], \tag{6}$$

where $f_i(u) = f(uT_s - \tau_i)$ and is assumed to have a finite support satisfying the condition that the range of the support is much less than MN. Removing the CP, (6) can be approximated as

$$y[v] \approx \sum_{i=1}^{I} \alpha_i e^{j2\pi \frac{l_i k_i}{MN}} \sum_{u=0}^{MN-1} \mathbf{s}[u] e^{j2\pi u \frac{k_i}{MN}} \tilde{f}_i[v - u], \tag{7}$$

where $\tilde{f}_i[u]$ is the periodic version of $f_i[u]$ with the period MN, $k_i = \nu_i MNT_s \in \mathbb{R}$, and $l_i = \frac{\tau_i}{T_s} \in \mathbb{R}^+$. (7) can be simplified as

$$\mathbf{y} = \sum_{i=1}^{I} \alpha_i e^{j2\pi \frac{l_i k_i}{MN}} [(\mathbf{x} \cdot \mathbf{v}_i) \circledast \tilde{\mathbf{f}}_i] + \tilde{\mathbf{w}}, \tag{8}$$

where $\mathbf{v}_i[u] = e^{j2\pi u \frac{k_i}{MN}}$, $\mathbf{x} \cdot \mathbf{v}_i$ denotes the element wise product of $\mathbf{x}$ and $\mathbf{v}_i$, $\circledast$ is the circular convolution operator, and $\tilde{\mathbf{w}}$ is the additive white Gaussian noise. $\mathbf{y}$ is transformed to DD domain using DZT to obtain $\mathbf{Z}_y$ as

$$\mathbf{Z}_y[m, n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mathbf{y}[m + kM] e^{\frac{-j2\pi nk}{N}}, \tag{9}$$

where $m = 0, \cdots, M-1$ and $n = 0, \cdots, N-1$. Using the properties of DZT, (9) can be written as

$$\mathbf{Z}_y = \sum_{i=1}^{I} \alpha_i e^{j2\pi\tau_i\nu_i} \mathbf{Z}_{y_i} + \mathbf{w}, \tag{10}$$

where

$$\mathbf{Z}_{y_i}[m, n] = \sum_{l=0}^{M-1} \left(\sum_{k=0}^{N-1} \mathbf{Z}_x[l, k] \mathbf{Z}_{v_i}[l, n-k] \right) \mathbf{Z}_{\tilde{f}_i}[m - l, n], \tag{11}$$

and $\mathbf{Z}_{v_i}$ and $\mathbf{Z}_{\tilde{f}_i}$ are Zak transforms of $\mathbf{v}_i$ and $\tilde{\mathbf{f}}_i$, respectively.

A. Vectorization of input-output relation

Let $\mathbf{z}_y, \mathbf{z}_{y_i}, \mathbf{z}_x$ denote the vectorized forms of $\mathbf{Z}_y, \mathbf{Z}_{y_i}, \mathbf{Z}_x$, respectively, i.e., $(nM + m)$th element in the vector is the $[m, n]$th entry in the corresponding matrix. The vectorized form of input-output relation between $\mathbf{z}_{y_i}$ and $\mathbf{z}_x$ is derived as follows.

Let $\mathbf{A} \in \mathbb{C}^{M \times N}$ and $\mathbf{B} \in \mathbb{C}^{2M-1 \times N}$ be two matrices with entries $\mathbf{A}[m, n] = \mathbf{Z}_{v_i}[m, n]$ and $\mathbf{B}[m, n] = \mathbf{Z}_{\tilde{f}_i}[m - (M-1), n]$, $m = 0, \cdots, M-1$, $n = 0, \cdots, N-1$. Also, let $\mathbf{R}_N \in \mathbb{C}^{N \times N}$ be a reversal matrix and $\mathbf{P}_N$ be a basic circulant permutation matrix of size N [13]. Define a matrix $\mathbf{H}_q^{(i)'} \in \mathbb{C}^{M \times N}$ as

$$\mathbf{H}_q^{(i)'}[m, n] = \begin{cases} \mathbf{A}[m, n], & \text{if } m = [q]_M \\ 0, & \text{otherwise,} \end{cases} \tag{12}$$

for $q = 0, 1, \cdots, MN-1$. Here $[\cdot]_M$ denotes the modulo-M operation. Let $\mathbf{H}_1^{(i)} \in \mathbb{C}^{MN \times MN}$ be a matrix whose qth row is filled with $\text{vec}(\mathbf{H}_q^{(i)'} \mathbf{R}_N \mathbf{P}_N^{\lfloor \frac{q}{M} \rfloor + 1})$, where $\lfloor \cdot \rfloor$ denotes the floor operator. Define $\mathbf{H}_q^{(i)''} \in \mathbb{C}^{M \times N}$ as

$$\mathbf{H}_q^{(i)''}[m, n] = \begin{cases} \mathbf{B}[m + [q]_M, n], & \text{if } n = \lfloor \frac{q}{M} \rfloor \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Also, define $\mathbf{H}_2^{(i)} \in \mathbb{C}^{MN \times MN}$ whose qth row is filled with $\text{vec}(\mathbf{R}_M \mathbf{H}_q^{(i)''})$. Finally, (11) and (10) can be vectorized as

$$\mathbf{z}_{y_i} = \mathbf{H}_2^{(i)} \mathbf{H}_1^{(i)} \mathbf{z}_x \tag{14}$$

and

$$\mathbf{z}_y = \sum_{i=1}^{I} \alpha_i e^{j2\pi \frac{l_i k_i}{MN}} \mathbf{z}_{y_i}, \tag{15}$$

respectively. Here, the matrix $\mathbf{H}_1^{(i)}$ effectively carries out element-wise multiplication with $\mathbf{v}_i$ and $\mathbf{H}_2^{(i)}$ carries out the circular convolution with $\tilde{\mathbf{f}}_i$ in (8).

III. Proposed DD channel estimation algorithm

To estimate the DD domain channel at the receiver, a known pilot frame is transmitted. We consider a pilot frame consisting of a pilot symbol at the center and zeros elsewhere, i.e.,

$$\mathbf{Z}_x[m,n] = \begin{cases} \sqrt{MNE_p}, & \text{if } m=\frac{M}{2}, n=\frac{N}{2} \\ 0, & \text{otherwise}, \end{cases} \tag{16}$$

where E_p is the average energy of each bin of the frame. The received pilot signal vector, $\mathbf{z}_y$, is used to estimate the channel, which is then used for detection of data symbols, at the receiver. The proposed channel estimation algorithm is described below. (14) and (15) can be alternatively written as

$$\mathbf{z}_y = \sum_{i=1}^{I} \mathbf{g}_i \alpha_i + \mathbf{w} = \mathbf{G}\boldsymbol{\alpha} + \mathbf{w}, \tag{17}$$

where $\mathbf{g}_i = e^{j2\pi\frac{l_i k_i}{MN}} \mathbf{H}_2^{(i)} \mathbf{H}_1^{(i)} \mathbf{z}_x \in \mathbb{C}^{MN\times 1}$, $\mathbf{G} = [\mathbf{g}_1(l_1,k_1), \mathbf{g}_2(l_2,k_2), \cdots, \mathbf{g}_I(l_I,k_I)] \in \mathbb{C}^{MN\times I}$, and $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_I]^T \in \mathbb{C}^{I\times 1}$. The maximum likelihood (ML) estimate of the tuple $(\mathbf{l},\mathbf{k},\boldsymbol{\alpha})$ is given by

$$[\hat{\mathbf{l}}, \hat{\mathbf{k}}, \hat{\boldsymbol{\alpha}}] = \underset{\mathbf{l},\mathbf{k},\boldsymbol{\alpha}}{\text{argmin}} \|\mathbf{z}_y - \mathbf{G}(\mathbf{l},\mathbf{k})\boldsymbol{\alpha}\|_2^2, \tag{18}$$

where $\|\cdot\|_2$ denotes 2-norm. For a given $\mathbf{l},\mathbf{k}$, the ML estimate of $\boldsymbol{\alpha}$ is obtained by

$$\tilde{\boldsymbol{\alpha}}(\mathbf{l},\mathbf{k}) = \left[\mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{G}(\mathbf{l},\mathbf{k})\right]^{-1} \mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{z}_y. \tag{19}$$

Substituting (19) in (18) and simplifying, we obtain

$$\left(\hat{\mathbf{l}}, \hat{\mathbf{k}}\right) = \underset{\mathbf{l},\mathbf{k}}{\text{argmax}} \Big[\mathbf{z}_y^H \mathbf{G}(\mathbf{l},\mathbf{k}) \left(\mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{G}(\mathbf{l},\mathbf{k})\right)^{-1} \mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{z}_y\Big]. \tag{20}$$

Using (19) in (20),

$$\left(\hat{\mathbf{l}}, \hat{\mathbf{k}}\right) = \underset{\mathbf{l},\mathbf{k}}{\text{argmax}} \left[\mathbf{z}_y^H \mathbf{G}(\mathbf{l},\mathbf{k})\tilde{\boldsymbol{\alpha}}(\mathbf{l},\mathbf{k})\right]. \tag{21}$$

Based on the estimates obtained in (20), we obtain the estimate of $\boldsymbol{\alpha}$ as

$$\hat{\boldsymbol{\alpha}}(\hat{\mathbf{l}},\hat{\mathbf{k}}) = \left[\mathbf{G}^H(\hat{\mathbf{l}},\hat{\mathbf{k}})\mathbf{G}(\hat{\mathbf{l}},\hat{\mathbf{k}})\right]^{-1} \mathbf{G}^H(\hat{\mathbf{l}},\hat{\mathbf{k}})\mathbf{z}_y. \tag{22}$$

We note that the product $\mathbf{G}(\mathbf{l},\mathbf{k})\tilde{\boldsymbol{\alpha}}(\mathbf{l},\mathbf{k})$ in (20) is the reconstructed signal with possible combinations of $\mathbf{l}$ and $\mathbf{k}$ in the search area, and (20) can be viewed as the correlation of the received signal with the reconstructed signal. To solve (20), we propose an algorithm which estimates the channel parameters in a path-by-path fashion, i.e., before estimation of the parameters of the pth path, the parameters of all the paths till the pth have been estimated. In other words, the channel parameters of the first, second, and so on till the $(p-1)$th path have been estimated before the estimation of the pth path can begin.

Figure 1 shows the flow chart of the proposed algorithm to estimate the channel parameters. For the pth path, for estimating the l_p and k_p of the path, the algorithm first estimates the integer part of l_p and k_p, called *coarse estimate* (denoted by $\sim$ over the estimates), and then the fractional part of l_p and k_p is estimated, called *fine estimate* (denoted by $\wedge$ over the estimate). The algorithm begins by initializing $\mathbf{G}(\mathbf{l},\mathbf{k}) = \left[\mathbf{g}_1(l_1,k_1), \mathbf{g}_2(l_2,k_2), \cdots, \mathbf{g}_{P_{\max}}(l_{P_{\max}}, k_{P_{\max}})\right] = \mathbf{0}_{MN\times P_{\max}}$, where $P_{\max}$ is the maximum number of paths the algorithm estimates before termination.

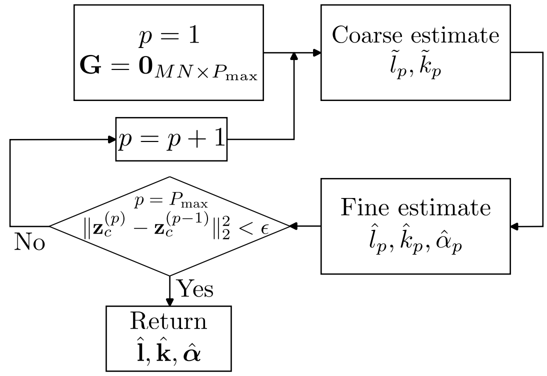

Fig. 1: Flow chart of the proposed algorithm to estimate DD channel parameters.

A. Coarse Estimation

For the pth path, the proposed algorithm first estimates the integer parts of l_p and k_p as described below. A search area is defined as $\mathcal{J} = \mathcal{L}\otimes\mathcal{K}$, where $\mathcal{L} = \{0,1,\cdots,\lceil l_{\max}\rceil\}, \mathcal{K} = \{-\lceil k_{\max}\rceil, \cdots, 0, \cdots, \lceil k_{\max}\rceil\}$, $l_{\max} = \max_i \{l_i\}$, $k_{\max} = \max_i \{k_i\}$, and $\otimes$ denotes the cartesian product. A cost function, $\boldsymbol{\Phi}_p(l_p,k_p)$, is maximized over the search area to obtain the integer estimates, as

$$(\tilde{l_p}, \tilde{k_p}) = \underset{(l_p,k_p)\in\mathcal{J}}{\text{argmax}}\, \boldsymbol{\Phi}_p(l_p,k_p), \tag{23}$$

$\boldsymbol{\Phi}_p(l_p,k_p) = \mathbf{z}_y^H \mathbf{G}_p(\mathbf{l},\mathbf{k})(\mathbf{G}_p^H(\mathbf{l},\mathbf{k})\mathbf{G}_p(\mathbf{l},\mathbf{k}))^{-1}\mathbf{G}_p^H(\mathbf{l},\mathbf{k})\mathbf{z}_y$, and $\mathbf{G}_p(\mathbf{l},\mathbf{k}) = [\mathbf{g}_1(\hat{l_1},\hat{k_1}), \cdots \bar{\mathbf{g}_p}(l_p,k_p), \mathbf{0}, \cdots, \mathbf{0}]$. Here, $\hat{l}_i$ and $\hat{k}_i$ denote the fine estimate of delay and Doppler, respectively, for $i = 1, 2, \cdots, p-1$. $\bar{\mathbf{g}_p}(l_p,k_p) = \frac{\mathbf{g}_p(l_p,k_p)}{\|\mathbf{g}_p(l_p,k_p)\|}$ is the normalization operation for each $(l_p,k_p)\in\mathcal{J}$ to aid the correlation computation (see (20)).

B. Fine Estimation

Once the coarse estimates of the channel parameters for the pth path have been obtained, the fine estimation of the channel parameters for this path are carried out as described below. The fine estimation of the channel parameters involves estimation of the fractional part of the channel parameters to the desired resolution. Figure 2 shows the flow chart of the iterative fine estimation algorithm. The algorithm begins by initializing $n=1$ and $l_p^{(0)} = \tilde{l_p}$ and $k_p^{(0)} = \tilde{k_p}$. For the nth iteration, the search area, $\mathcal{I}(n)$, is defined as

$$\mathcal{I}(n) = \left\{ \left\{ l_p^{(n-1)} - \frac{5}{10^n}, l_p^{(n-1)} - \frac{4}{10^n}, \cdots, l_p^{(n-1)} + \frac{5}{10^n} \right\} \otimes \left\{ k_p^{(n-1)} - \frac{5}{10^n}, k_p^{(n-1)} - \frac{4}{10^n}, \cdots, k_p^{(n-1)} + \frac{5}{10^n} \right\} \right\}. \tag{24}$$

Similar to the coarse estimate, the same cost function is maximized over $\mathcal{I}(n)$, given by

$$(\hat{l}_p^{(n)}, \hat{k}_p^{(n)}) = \underset{(l_p,k_p)\in\mathcal{I}(n)}{\text{argmax}}\, \boldsymbol{\Phi}_p(l_p,k_p). \tag{25}$$

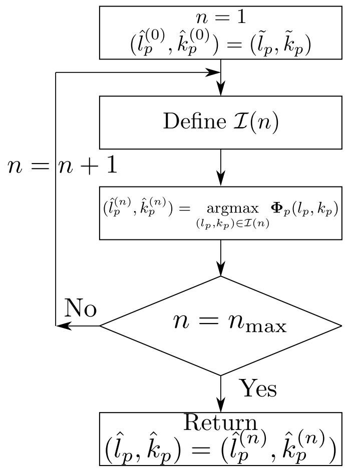

Fig. 2: Flow chart of fine estimation algorithm.

Following this, the value of n is incremented by 1. This iterative procedure is stopped when a pre-defined value for n is achieved, i.e., $n = n_{\max}$ (the estimate for delay and Doppler have been estimated to the $n_{\max}$th decimal place).

Stopping Criterion: The algorithm stops once $P_{\max}$ paths have been estimated, i.e., $p = P_{\max}$, or $\|\mathbf{z}_c^{(p)} - \mathbf{z}_c^{(p-1)}\|_2^2 < \epsilon$, where $\mathbf{z}_c^{(p)} = \mathbf{G}(\hat{\mathbf{l}}, \hat{\mathbf{k}})\hat{\alpha}(\hat{\mathbf{l}}, \hat{\mathbf{k}})$.

Remark: We note that the value of $P_{\max}$ determines the number of paths estimated. If $P_{\max} < I$, the number of estimated paths is less than the number of actual paths. Therefore, for our simulations, we consider $P_{\max} \gg I$.

IV. Results and Discussions

This section presents the performance of the proposed channel etimation algorithm. Two DZT-OTFS systems with $(M = N = 16)$ and $(M = 64, N = 32)$ are considered. Square-root raised cosine pulse with roll-off factor 0.5 is used as the transmit pulse. Two parameter sets are considered for the simulation: For the first set, $\Delta f = 3.75$ kHz, $I = 4$ with uniform power delay profile (PDP), delays are uniformly distributed in $(0, \tau_{\max}]$, $\tau_{\max} = 0.133$ ms, and $\nu_{\max} = 937$ Hz. The second set, a more practical scenario, considers Vehicular A (VehA) PDP [14] with $\Delta f = 156.25$ kHz, and $\nu_{\max} = 1700$ Hz. For both the cases, Dopplers are generated using Jakes' Doppler spectrum, $\nu_i = \nu_{\max}\cos(\theta_i)$, where θ_i is uniformly distributed in $(0, 2\pi]$, and carrier frequency $f_c = 4$ GHz.

Further, the following algorithm parameters are chosen: $P_{\max} = 15$, $n_{\max} = 2$, and $\epsilon = 20\sigma^2$, where σ^2 is the variance of noise. Also, in all the BER simulations, the pilot SNR is same as data SNR.

Brute-force search vs proposed search comparison: Figure 3 shows the NMSE performance comparison between the proposed low complexity search and the brute-force search in fine estimation of the delay and Doppler values. DZT-OTFS with $M = N = 16$ is considered. A channel with single path is assumed for this comparison. It is seen that the NMSE of the proposed low-complexity search matches with that of the brute force search. However, the brute-force search has a high complexity as detailed below. For a resolution of $10^{-n_{\max}}$, the brute-force search requires $10^{2n_{\max}}$ cost computations while the proposed low-complexity search requires only $n_{\max}10^2$ cost computations. For example, for a resolution of 0.01, brute-force search requires 10^4 computations, while the proposed search requires only 200 computations, making the proposed search computationally efficient.

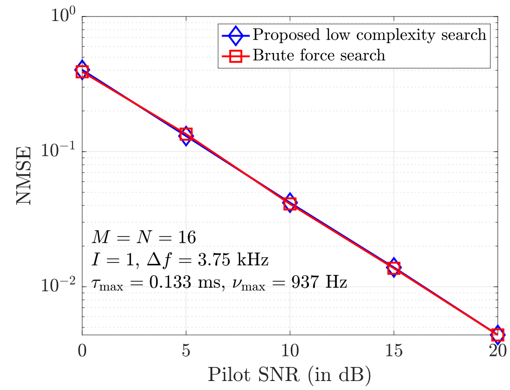

Fig. 3: NMSE performance comparison between brute-force search and proposed low-complexity search in fine estimation.

NMSE and BER performance: Figure 4 shows the NMSE performance of the proposed algorithm as a function of pilot SNR for DZT-OTFS systems with $M = N = 16$ and $M = 64, N = 32$. As expected, it is observed that the NMSE decreases with increase in pilot SNR. For the same set of parameters, the NMSE performance of the system with $M = 64, N = 32$ is observed to be better than that of the $M = N = 16$ system. This is because, as the values of M and N increase, the resolution of DD grid increases, and hence a better channel estimation accuracy is obtained. Also, it is observed that the NMSE performance with uniform PDP performs better than the VehA PDP. Figure 5 shows the corresponding BER performance of the two systems using BPSK modulation and minimum mean square error (MMSE) detection. The BER performance with perfect CSI is also added for comparison. It is seen that the BER performance using the proposed channel estimation algorithm is very close to the corresponding perfect CSI performance, demonstrating the effectiveness of the proposed estimation algorithm.

NMSE performance with and without an approximation: Recall from (20) that the cost function is given by $\mathbf{z}_y^H\mathbf{G}(\mathbf{l},\mathbf{k})\left(\mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{G}(\mathbf{l},\mathbf{k})\right)^{-1}\mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{z}_y$. The product $\mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{G}(\mathbf{l},\mathbf{k})$ can be approximated by a scaled identity matrix, i.e., $\mathbf{G}^H(\mathbf{l},\mathbf{k})\mathbf{G}(\mathbf{l},\mathbf{k}) \approx MN\mathbf{I}$. This results in further reduction of the number of operations required to compute the cost function. However, this comes at the cost of performance. Figure 6 shows the NMSE performance of the proposed algorithm with and without the above approximation. Uniform PDP is considered for the simulation. It is seen that the NMSE performance with approximation is relatively inferior compared that without approximation, which is expected. However, the achieved NMSE with approximation is adequately small so that the BER performance is not compromised much. This can be observed in Fig. 7 which implies that the approximation

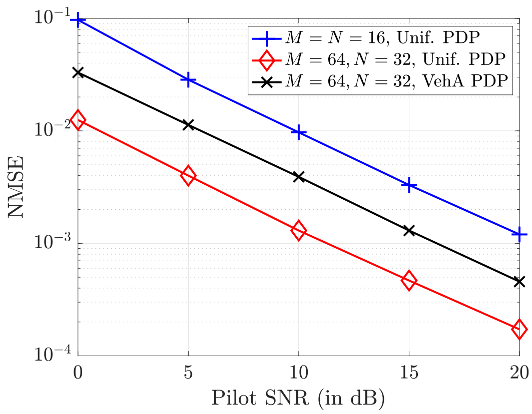

Fig. 4: NMSE performance of the proposed estimation algorithm for DZT-OTFS systems with $M = N = 16$ and $M = 64,\ N = 32$.

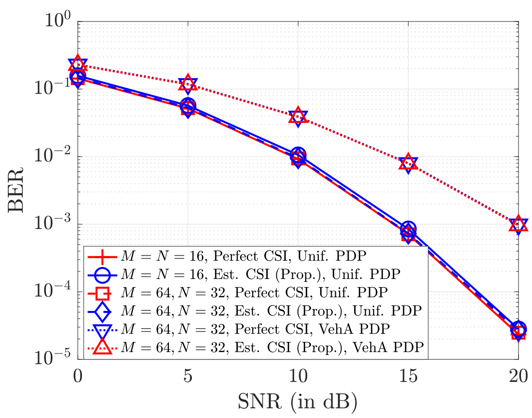

Fig. 5: BER performance achieved using the proposed estimation algorithm for DZT-OTFS systems with $M = N = 16$ and $M = 64,\ N = 32$.

can be used to reduce complexity without compromising much on performance.

V. Conclusions

We proposed a low-complexity iterative algorithm that estimates the DD domain channel parameters in DZT based OTFS systems. The proposed algorithm estimates fractional delays and Dopplers path-by-path. The fine estimate involved a low-compelxity search instead of a brute-force search. Numerical results showed that the proposed algorithm achieves good NMSE performance and BER performance that is very close to perfect CSI performance.

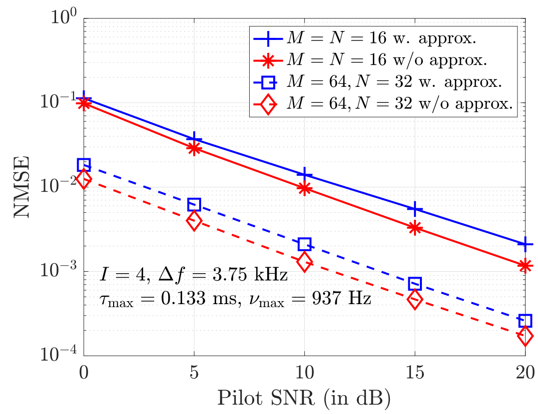

Fig. 6: NMSE performance of the proposed estimation algorithm with and without approximation.

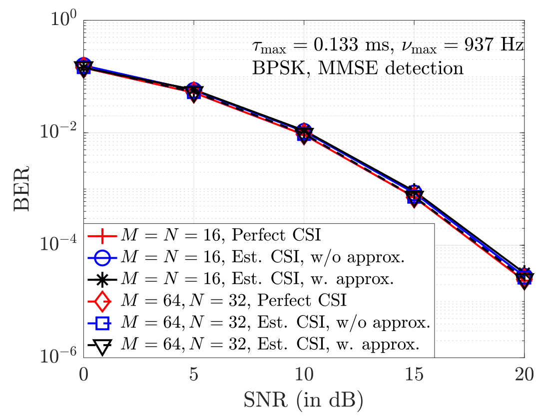

Fig. 7: BER performance achieved using the proposed estimation algorithm with and without approximation.

References

[1] R. Hadani et al., “Orthogonal time frequency space modulation,” *Proc. IEEE WCNC’2017*, pp. 1-6, Mar. 2017.

[2] Y. Hong, T. Thaj, and E. Viterbo, *Delay-Doppler Communications: Principles and Applications*, London UK: Elsevier, 2022.

[3] G. D. Surabhi, R. M. Augustine, and A. Chockalingam, “On the diversity of uncoded OTFS modulation in doubly-dispersive channels,” *IEEE Trans. Wireless Commun.*, vol. 18, no. 6, pp. 3049-3063, Jun. 2019.

[4] P. Raviteja, K. T. Phan, Y. Hong, and E. Viterbo, “Interference cancellation and iterative detection for orthogonal time frequency space modulation,” *IEEE Trans. Wireless Commun.*, vol. 17, no. 10, pp. 6501-6515, Oct. 2018.

[5] I. A. Khan and S. K. Mohammed, “Low complexity channel estimation for OTFS modulation with fractional delay and Doppler,” arXiv:2111.06009 [cs.IT], Nov. 2021.

[6] A. J. E. M. Janssen, “The Zak transform: a signal transform for sampled time-continuous signals,” Philips J. Res., 43, pp. 23-69, 1988.

[7] S. K. Mohammed, “Derivation of OTFS modulation From first principles,” *IEEE Trans. Veh. Tech.*, vol. 70, no. 8, pp. 7619-7636, Aug. 2021.

[8] S. K. Mohammed, R. Hadani, A. Chockalingam and R. Calderbank, “OTFS - a mathematical foundation for communication and radar sensing in the delay-Doppler domain,” *IEEE BITS the Information Theory Magazine*, vol. 2, no. 2, pp. 36-55, Nov. 2022.

[9] S. K. Mohammed, R. Hadani, A. Chockalingam, and R. Calderbank, “OTFS - predictability in the delay-Doppler domain and its value to communication and radar sensing,” available https://arxiv.org/abs/2302.08705.

[10] H. Bolcskei and F. Hlawatsch, “Discrete Zak transforms, polyphase transforms, and applications,” *IEEE Trans. Signal Processing*, vol. 45, no. 4, pp. 851-866, Apr. 1997.

[11] F. Lampel, A. Avarado, F. M. J. Willems, “On OTFS using the discrete Zak transform,” *IEEE ICC’2022 Workshops*, pp. 729-734, May 2022.

[12] T. Thaj, E. Viterbo, and Y. Hong, “General I/O relations and low-complexity universal MRC detection for all OTFS variants,” *IEEE Access*, vol. 10, pp. 96026-96037, 2022.

[13] R. Horn and C. Johnson, *Matrix Analysis*, Cambridge Univ. Press, 2013.

[14] ITU-R M.1225, “Guidelines for the evaluation of radio transmission technologies for IMT-2000,” International Telecommunication Union Radio communication, 1997.