MU-MIMO NOMA with Linear Precoding Techniques in Indoor Downlink VLC Systems

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Abstract—In this paper, we consider non-orthogonal multiple access (NOMA) transmission with linear precoding techniques in downlink multiuser indoor multiple-input multiple-output (MIMO) visible light communication (VLC) systems. In particular, we propose a MIMO-NOMA transmission scheme in which the users having highly correlated channels are grouped into a single cluster and a common precoder is designed for all the users within this cluster. For the proposed scheme, we present an algorithm for grouping the users into clusters based on the correlation among their channel gains. Our performance results show that the proposed MIMO-NOMA transmission scheme achieves significantly higher spectral efficiency as compared to the conventional multiuser MIMO precoding schemes and can serve more number of users simultaneously.

Index Terms — Visible light communication, MIMO VLC systems, NOMA, clustering.

I. INTRODUCTION

Visible light communication is increasingly gaining recognition as a promising technology for wireless communication in indoor and vehicular environments [1]. It is attractive as the light emitting diodes (LEDs) which serve as transmitters can be used for lighting purposes along with the transmission of data through intensity modulation. The intensity modulated signals can be detected by photo diodes (PDs) at the receiver using direct detection. The achieved data rates in VLC systems are limited by the narrow modulation bandwidth of the LEDs (due to slow rise/fall times). Multiple-input multiple-output (MIMO) techniques [2]-[4] can be employed to increase the spectral efficiency in VLC systems. In [2], it is shown that spatial multiplexing (SMP) improves the spectral efficiency when the channel correlation is less. Also, it is shown that at low SNRs, spatial modulation (SM) achieves high spectral efficiency. Multiuser interference (MUI) cancellation is one of the topics that has been considered in the literature when it comes to multiuser MIMO VLC systems [5],[6]. To eliminate MUI at each user terminal, zero forcing (ZF) precoding technique in VLC systems has been investigated in [5] and block diagonalization (BD) precoding technique has been investigated in [6]. The performance of these precoding techniques in terms of spectral efficiency are affected when the channel gain vectors at different receivers are highly correlated.

Non-orthogonal multiple access (NOMA), a multiple access approach being considered in 5G to enhance system capacity and user experience [7], is getting increased attention recently for use in multiuser VLC systems [8]-[12]. In [13],[14], NOMA is applied in a multiuser MIMO VLC system. In these works, the authors have showed that the achievable sum-rate can be significantly improved by using NOMA with power allocation technique called normalized gain difference power allocation (NGDPA). In [15], the authors have analyzed the bit error rate performance of OQAM-OFDM based MIMO-NOMA over VLC as a function of ratio of power allocated to different users. We note that the papers on multiuser MIMO VLC networks reported in the literature so far have not considered using MIMO-NOMA transmission to alleviate the performance degradation of multiuser precoding schemes due to high correlation between different users’ channel gain vectors. This forms the key focus in this paper. Specifically, we propose a multiuser MIMO-NOMA transmission scheme, where the users having high correlation among their channel gain vectors are grouped into a single cluster. Multiaccess within a cluster is provided using NOMA. ZF or BD precoding is used in combination with NOMA to reduce the interference across the clusters. The proposed scheme can also be used when the number of receiver PDs is more than the number of LEDs. For the proposed system, we present an algorithm that groups the users into clusters based on the correlation among their channel vectors. The spectral efficiency performance of the proposed scheme is evaluated in an indoor MIMO-VLC setting and compared with those of conventional precoding schemes. The proposed scheme is shown to achieve better spectral efficiency performance.

II. SYSTEM MODEL

Consider an indoor multiuser MIMO VLC system consisting of $N_t$ transmit Luminaires (LEDs) and $K$ receivers, where each receiver is provided with $N_r$ PDs. The schematic showing various parameters which determine the line-of-sight (LOS) path gain between an LED and a PD in a VLC link is shown in Fig. 1. The LOS optical channel gain from a transmitter luminaire to a receiver PD is given by [16]

$$ h = \frac{A}{d^2} L_r(\phi) F(\theta) C(\theta) \cos \theta, $$

(1)

where $A$ is the area of the PD, $\theta$ is the angle of incidence at the receiver with respect to its normal, $\phi$ is the angle of emergence with respect to the normal at the transmitter, $d$ is the distance between the transmitter luminaire and the receiver PD, $L_r(\phi)$ is the function that represents the Lambertian radiation pattern of the luminaire, $F(\theta)$ is the gain of optical filter, and $C(\theta)$ is the gain of optical concentrator. The Lambertian radiation pattern of the luminaire is given by

$$ L_r(\phi) = \frac{m + 1}{2\pi} \cos^m \phi, m = -\ln 2 \ln \cos \frac{\Phi}{2}, $$

(2)
where \( m \) represents the order of Lambertian emission and \( \Phi_\frac{1}{2} \) is the half-power semi-angle of the transmitter luminaire. The optical concentrator gain is expressed as

\[
C(\theta) = \begin{cases} \frac{n^2}{\sin^2 \Theta}, & 0 \leq \theta \leq \Theta \\ 0, & \text{otherwise} \end{cases},
\]

where \( n \) denotes the refractive index and \( \Theta \) is the field-of-view (FOV) of the PD.

**A. Precoding in multiuser MIMO systems**

Let \( \mathbf{x} \in \mathbb{R}^{N_t \times 1} \) be the transmit vector, where the \( q \)th element in \( \mathbf{x} \) is transmitted by the \( q \)th LED. Let \( \mathbf{H}_i \in \mathbb{R}^{N_r \times N_t} \) denote the channel gain matrix at the \( i \)th user. The received vector \( \mathbf{y}_i \) of size \( N_r \times 1 \) at the \( i \)th user is given by

\[
\mathbf{y}_i = a\mathbf{H}_i\mathbf{x} + \mathbf{n}_i + \mathbf{b}_a, \quad i \in \mathcal{K} = \{1, 2, 3, \ldots, K\},
\]

where \( \mathbf{n}_i \in \mathbb{R}^{N_r \times 1} \) is zero mean AWGN vector at the \( i \)th user, and \( \mathbf{b}_a \in \mathbb{R}^{N_r \times 1} \) is DC-bias added to the transmit vector \( \mathbf{x} \) to make transmit signal vector real and non-negative, \( a \) is the PD responsivity, and \( \mathbf{n}_i \) is the zero mean AWGN vector at the \( i \)th user. The variance of noise vector for all the users is \( \sigma^2 \), i.e., \( \mathbb{E}[|\mathbf{n}_i|^2] = \sigma^2, \forall i \in \mathcal{K} = \{1, 2, 3, \ldots, K\} \). The average transmit SNR per LED is given by

\[
\Gamma = \frac{\mathbb{E}[|\mathbf{x}|^2]}{N_r \sigma^2}.
\]

Precoding at the transmitter can be used to mitigate inter-user interference in multiuser MIMO systems. Some of the well known precoding techniques for multiuser MIMO systems are ZF, BD, and DPC [17]. While DPC is an optimal precoding technique, it has higher complexity compared to linear precoding techniques like ZF and BD techniques. Also, the performance of DPC is worse compared to ZF and BD with imperfect channel state information (CSIT) [17]. Here, we consider ZF and BD precoding techniques for the scheme that we propose. The transmit vector \( \mathbf{x} \) with linear precoding given by

\[
\mathbf{x} = \mathbf{W}\mathbf{u},
\]

where \( \mathbf{W} = [\mathbf{W}^{(1)} \  \mathbf{W}^{(2)} \  \cdots \  \mathbf{W}^{(K)}] \) is precoder matrix, \( \mathbf{W}^{(i)} \) is precoder matrix corresponding to the \( i \)th user, \( \mathbf{u} = [\mathbf{u}_1^T \  \mathbf{u}_2^T \  \cdots \  \mathbf{u}_K^T]^T \) is unprecoded symbol vector, and \( \mathbf{u}_i \) is symbol vector corresponding to the \( i \)th user. Now, the received vector \( \mathbf{y}_i \), at the \( i \)th user is given by

\[
\mathbf{y}_i = a\mathbf{H}_i\mathbf{W}^{(i)}\mathbf{u}_i + \sum_{j \neq i} a\mathbf{H}_i\mathbf{W}^{(j)}\mathbf{u}_j + a\mathbf{H}_i\mathbf{b}_a + \mathbf{n}_i, \forall i \in \mathcal{K}.
\]

1) **ZF precoder**: In this linear precoding technique, the precoder matrix \( \mathbf{W}^{(i)}_{\text{ZF}} \in \mathbb{R}^{N_r \times N_t} \) for the \( i \)th user is obtained from the null space of the matrix \( \mathbf{H}_i \in \mathbb{R}^{(K N_r - N_r) \times N_t} \), where

\[
\mathbf{H}_i = [\mathbf{H}_1^T \  \mathbf{H}_2^T \  \cdots \  \mathbf{H}_{i-1}^T \  \mathbf{H}_{i+1}^T \  \cdots \  \mathbf{H}_K^T]^T.
\]

Using singular value decomposition (SVD), the precoder matrix \( \mathbf{W}^{(i)}_{\text{ZF}} \) can be obtained as

\[
\mathbf{W}^{(i)}_{\text{ZF}} = \mathbf{H}_i \mathbf{U}_i \mathbf{S}_i^{-1} \mathbf{V}_i^H.
\]

2) **BD precoder**: The ZF precoder matrix \( \mathbf{W}^{(i)}_{\text{ZF}} \in \mathbb{R}^{N_r \times K N_r} \) is obtained using \( \mathbf{H} = [\mathbf{H}_1^T \  \mathbf{H}_2^T \  \cdots \  \mathbf{H}_K^T]^T \in \mathbb{R}^{K N_r \times N_t} \) as

\[
\mathbf{W}^{(i)}_{\text{BD}} = \mathbf{H}_i \mathbf{U}_i \mathbf{S}_i^{-1} \mathbf{V}_i^H \mathbf{R}_i^{-1},
\]

where \( \mathbf{W}^{(i)}_{\text{BD}} \) can be represented as the concatenation of precoder matrices of the users as

\[
\mathbf{W}^{(i)}_{\text{ZF}} = [\mathbf{W}^{(1)}_{\text{ZF}} \  \mathbf{W}^{(2)}_{\text{ZF}} \  \cdots \  \mathbf{W}^{(K)}_{\text{ZF}}].
\]

Note that precoder matrix \( \mathbf{W}^{(i)}_{\text{ZF}} \) will be full rank only when \( K N_r \leq N_t \). Hence, ZF precoder is also valid only when \( K N_r \leq N_t \). Note that inter-user interference term in (7) is 0 for both ZF and BD precoding.

**III. PROPOSED MULTIUSER MIMO-NOMA TRANSMISSION SCHEME**

When the channel gains of any two receiver PDs are highly correlated, the ZF and BD precoding do not perform well. So, we propose a NOMA transmission scheme for multiuser MIMO VLC systems, where we use combination of NOMA and precoding. The users are divided into different clusters (groups) based on correlation among their channel gain vectors. The clustering is done in such a way that the channel gain vectors of the users within any given cluster have high correlation. Multiple access across the clusters is provided using precoding and multiple access within any given cluster is provided using NOMA.

We first discuss the user clustering algorithm in Sec. III-A. Then, for the given clustering of users, we discuss the transmitter and receiver framework of the proposed scheme in Sec. III-B. For the rest of this paper, it is assumed that each of the users is provided with one receiver PD (i.e., \( N_r = 1 \)).

**A. Clustering algorithm**

The pseudo-code of the proposed clustering algorithm is given in Algorithm 1. The inputs to the algorithm are number of transmit LEDs (\( N_t \)), number of receiver PDs (\( K \)), and channel gain vectors of the receivers (\( \mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_K \)), where \( \mathbf{h}_i \) is...
\(N_t \times 1\) vector containing channel gains from \(N_t\) LEDs to receiver PD of the \(i\)th user. Minimum number of clusters that is required \((S_0)\) and correlation threshold \((\rho)\) are additional inputs for the case \(N_t \geq K\). The steps followed to cluster the users in the proposed algorithm can be split into two parts.

In the first part (steps 3-16), the number of clusters is initially chosen to be \(S\), and \(S\) out of \(K\) users are chosen and assigned as representative users for each of these \(S\) clusters, where \(S = S_0\) if \(N_t \geq K\) and \(S = N_t\) otherwise. The representative users are chosen in such a way that the correlations between their channel gains are less.

The steps followed in the second part (steps 17-28) are listed below:

**Step 1:** A random user is picked from the remaining \(K - S\) users.

**Step 2:** The normalized cross correlations between the channel gains of the chosen user and the representative users of the clusters are computed and the representative user with which the normalized cross correlation of channel gains is maximum is determined.

**Step 3:** The procedure followed in this step is different for the cases \(N_t \geq K\) and \(N_t < K\). The procedure followed in these cases are discussed below.

*Case 1 \((N_t \geq K)\):* If the normalized cross correlation of the channel gains of the chosen user and the representative user corresponding to maximum normalized cross correlation is greater than \(\rho\), then the chosen user is assigned to the cluster in which the representative user corresponding to maximum normalized cross correlation is assigned. Otherwise, the chosen user is assigned to a new cluster and the chosen user becomes the representative user of the newly formed cluster.

*Case 2 \((N_t < K)\):* The chosen user is assigned to the cluster in which the representative user corresponding to maximum normalized cross correlation is assigned.

**Step 4:** Remaining unassigned users are picked one by one randomly and they are assigned to one of the already formed clusters or a new cluster based on 2) and 3).

Note that, at the completion of the clustering algorithm, in Case 1, the number of clusters can be more than \(S_0\), whereas in Case 2 number of clusters is fixed to \(N_t\). Also note that, in Case 1, if normalized cross correlation of all the possible pairs of receivers’ channel gain vectors is less than \(\rho\) then every cluster will have only one user and there will be no NOMA transmission.

**B. Transmitter and receiver framework**

In the proposed scheme, the ZF or BD precoders are obtained based on equivalent channel gain vectors of the clusters, where the equivalent channel gain vector corresponding to \(j\)th cluster \(C_j\) is given by

\[
\hat{h}_j = \frac{\sum_{k \in C_j} h_k}{\|\sum_{k \in C_j} h_k\|}, \quad j \in \mathbb{R},
\]

(12)

**Algorithm 1 User clustering algorithm**

1. **Inputs:** \(N_t, K, h_1, h_2, \ldots, h_K\)
2. **Additional inputs if \((N_t \geq K)\):** \(S_0, \rho\)
3. For all \(i, i' \in K = \{1, 2, 3, \ldots, K\}\) and \(i \neq i'\) compute \(\rho_{i,i'} = (h_i^T h_{i'})/((\|h_i\| \times \|h_{i'}\|))\)
4. \((k,k') = \arg\min_{i,j \in K, i \neq j} \rho_{i,j}\)
5. \(R = \{k,k'\}\)
6. if \((N_t \geq K)\)
7. \(S = S_0\)
8. else (i.e., \(N_t < K\))
9. \(S = N_t\)
10. end
11. while \(|R| < S\)
12. For all \(n \in \mathbb{R}\) and \(m \in K \setminus R\) compute \(\zeta_{m,n} = (h_n^T h_m)/((\|h_n\| \times \|h_m\|))\)
13. \(l = \arg\max_{m \in \mathbb{R} \setminus R} \|\zeta_{m,n}\|\), where \(\zeta_m = [\zeta_{m,1}, \zeta_{m,2}, \ldots, \zeta_{m,|R|}]^T\)
14. \(R = R \cup \{l\}\)
15. end (end of ‘while’ loop)
16. \(C_j = \{j\}, \forall j \in R\) (assign user \(j \in R\) to cluster \(C_j\))
17. \(R' = R\)
18. while \(|R \setminus R'| > 0\)
19. Choose a random user \(\hat{j}\) from the set \(K \setminus R'\)
20. \(p = \max_{j \in R'} \rho_{j,\hat{j}}\)
21. if \((N_t < K)\) or \(\rho_{p,\hat{j}} \geq \rho\)
22. \(C_p = C_p \cup \{\hat{j}\}, R' = R' \cup \{\hat{j}\}\)
23. else
24. \(R = R \cup \{\hat{j}\}, R' = R' \cup \{\hat{j}\}\)
25. \(C_j = \{\hat{j}\}\), (new cluster formed )
26. end (end of ‘if-else’ conditional statement)
27. end (end of ‘while’ loop)
28. **Output:** Clusters \(C_j\)’s, \(j \in R\)

where \(h_k\) is the channel gain vector of the \(k\)th user in cluster \(C_j\). Now, the transmit vector is given by

\[
x = W \left[ \sum_{k \in C_1} \sqrt{p_{k_1}} u_{k_1} \sum_{k_2 \in C_2} \sqrt{p_{k_2}} u_{k_2} \cdots \sum_{k_S \in C_S} \sqrt{p_{k_S}} u_{k_S} \right]^T,
\]

(13)

where \(u_{k_j}\) is the modulation symbol of the \(k_j\)th user of cluster \(C_j\), \(p_{k_j}\) is the power allocated to the \(k_j\)th user of cluster \(C_j\), and \(W\) is either ZF or BD precoder obtained using equivalent channel gain vectors \(h_1, h_2, \ldots, h_S\). For a given cluster \(C_j\), the transmit power \(p_{k,j}\)s are obtained using GRPA [11]. That is, in a given cluster, the powers are allocated based on the norms of the channel gains. The user with highest norm of the channel gain will be allocated least power and the user with lowest norm of the channel gain will be allocated highest power.
TABLE I: Simulation parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>User height</td>
<td>( d_r )</td>
<td>0.85 m</td>
</tr>
<tr>
<td>Transmitter LED height</td>
<td>( d_t )</td>
<td>3 m</td>
</tr>
<tr>
<td>Room dimension</td>
<td></td>
<td>6 m×4 m×5 m</td>
</tr>
<tr>
<td>Half power semi-angle</td>
<td>( \Phi_0 )</td>
<td>60°</td>
</tr>
<tr>
<td>PD field of view</td>
<td>( \Theta )</td>
<td>80°</td>
</tr>
<tr>
<td>Area of PD</td>
<td>A</td>
<td>10^2 m^2</td>
</tr>
<tr>
<td>Responsivity of PD</td>
<td>( n_o )</td>
<td>0.3 A/W</td>
</tr>
<tr>
<td>Gain of optical filter</td>
<td>( G(\theta) )</td>
<td>1</td>
</tr>
<tr>
<td>Refractive index</td>
<td>( n_r )</td>
<td>1.5</td>
</tr>
</tbody>
</table>

TABLE II: Luminaires locations.

<table>
<thead>
<tr>
<th>Luminaires number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-axis (m)</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>2</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>Y-axis (m)</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>Z-axis (m)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The received signal at the \( k_j \)th user of the cluster \( C_j \) after compensating for the DC-bias term is given by

\[
y_{k_j} = a\alpha_{kj} \sum_{k_j' \in C_j} \sqrt{p_{k_j'} u_{k_j'}^T} + I_{k_j} + n_{k_j}, \quad \forall k_j \in C_j, \ j \in S,
\]

where \( \alpha_{kj} = h_{kj}^T w_j \), \( n_{k_j} \) is AWGN at \( k_j \)th receiver of cluster \( C_j \), \( w_j \) is \( j \)th column of precoder matrix \( W \), and \( I_{k_j} \) represents the interference from the users of other clusters, which is given by

\[
I_{k_j} = a h_{kj}^T \sum_{j' \neq j} \sum_{k_j' \in C_{j'}} \sqrt{p_{k_j'} u_{k_j'}.} \quad (15)
\]

Each user’s receiver performs successive interference cancellation (SIC) decoding ignoring the interference from the users of other clusters. In any given cluster, the users are decoded in ascending order of norm of their channel gain vectors. Although the users within the clusters have high channel correlation, the norm of their channel gains may be different. Since the effectiveness of the SIC decoding is mainly determined by norm of the channel gains, grouping highly correlated users in one cluster may not affect the effectiveness of SIC decoding. With SIC decoding, the received SINR for the \( k_j \)th user of cluster \( C_j \), assuming \( p_1 \geq p_2 \geq \cdots \geq p_{K_j} \) is given by

\[
\gamma_{k_j} = \frac{a^2(\alpha_{kj})^2 p_{k_j}}{a^2 \left( \sum_{r=k_j}^{K_j} \frac{p_r + \xi \alpha_{kr}^2 \sum_{r=k_j+1}^{K_j-1} p_r + I_{kr}}{\sum_{k_j}^{K_j}} \right) + \sigma^2}, \quad (16)
\]

where \( \xi \) denotes residual factor which accounts for interference due to incorrectly decoded symbols corresponding to users \( 1, 2, \cdots, k_j - 1 \) [19] and \( \sigma^2 \) is AWGN variance. The spectral efficiency (in b/s/Hz) of the \( k_j \)th user of cluster \( C_j \) is given by

\[
\eta_{k_j} = \frac{1}{2} \log_2 (1 + \gamma_{k_j}). \quad (17)
\]

IV. RESULTS AND DISCUSSIONS

In this section, we present the spectral efficiency performance of the proposed multiuser MIMO-NOMA scheme in comparison with that of conventional multiuser MIMO precoding scheme (with out NOMA). The parameters considered for the simulation are given in Table I. The locations of the luminaires are specified in Table II. The spectral efficiency performance plots in Figs. 2, 3, 4, 5 and 6 are obtained by averaging over large number of user locations that are generated using uniform random distribution.

In Fig. 2, we compare the spectral efficiency of the proposed scheme with \( S_0 = 1 \), \( S_0 = 2 \), \( S_0 = 3 \), \( S_0 = 4 \), and \( S_0 = 5 \) for different values of correlation threshold \( \rho \), where the transmit SNR is taken as 130 dB and the number of users is \( K = 6 \). As \( N_t = 6 \) and \( K = 6 \), the considered example belongs to the case \( N_t \geq K \). It can be observed from Fig. 2 that the performance of the proposed scheme using ZF is best for \( S_0 = 4 \) and \( \rho = 0.85 \), and the proposed scheme using BD is best for \( S_0 = 4 \) and \( \rho = 0.9 \). It can also be observed that the proposed scheme using ZF precoding technique for \( S_0 = 4 \) performs best among all the cases for \( \rho \leq 0.85 \). With these values of \( S_0 \) and \( \rho \), proper balance between precoding and NOMA is achieved.

Next, in Fig. 3, we compare the spectral efficiency of the proposed scheme with multiuser MIMO precoding as a function of
transmit SNR for \( K = 6 \) users and \( S_0 = 4 \), where \( \rho = 0.85 \). It can be observed that the proposed scheme shows better spectral efficiency than conventional multiuser MIMO precoding using both ZF and BD techniques at relatively low average transmit SNR per LED values (i.e., the proposed scheme performs better than multiuser MIMO precoding using ZF precoding technique for average transmit SNR per LED less than or equal to 152 dB and using BD precoding technique for average transmit SNR less than or equal to 135 dB). The gain in the spectral efficiency for the proposed scheme as compared to multiuser MIMO using ZF precoding at transmit SNR = 130 dB is about 4.3 b/s/Hz, and gain in the spectral efficiency for the proposed scheme using BD precoding technique is about 0.6 b/s/Hz. The interference from users of other clusters is more at high SNR values. So, the performance of the proposed scheme is worse than conventional precoding in which there is no interference from the other users. However, at low average transmit SNR per LED values, the interference from users of other clusters is negligible and the proposed scheme performs better compared to conventional precoding schemes by using NOMA to provide multiple access across highly correlated users.

Next, in Fig. 4, we present the spectral efficiency of MIMO-NOMA with the proposed scheme and conventional multiuser MIMO precoding for \( K = 12 \) and \( N_t = 6 \). As \( N_t = 6 \) and \( K = 12 \), the considered example belongs to the case \( N_t < K \). Note that the conventional multiuser MIMO precoding techniques can only serve at most \( N_t \) users in a given time slot, whereas the proposed scheme can serve all the \( K \) users simultaneously. So, under this case for conventional multiuser MIMO precoding, the transmitter needs \( \lceil K/N_t \rceil \) time slots to serve all \( K \) users. Therefore, for this case, we compare performance of the proposed scheme with conventional precoding scheme where at each time slot \( N_t \) users are chosen randomly (assuming uniform distribution) without replacement from \( K \) users. It can be observed from Fig. 4 that the proposed scheme outperforms conventional multiuser MIMO with ZF precoding as well as BD precoding schemes. For example, the gain in the spectral efficiency for the proposed scheme as compared to conventional multiuser MIMO with ZF precoding is about 3.5 b/s/Hz and with BD precoding is about 3 b/s/Hz for an average transmit SNR per LED of 130 dB.

Note that the effective channel matrix (matrix containing effective channel vectors of clusters as rows, i.e., \( [\mathbf{h}_1 \mathbf{h}_2 \cdots \mathbf{h}_S]^T \)) for the proposed scheme for the case \( N_t \geq K \) is a fat matrix and the effective channel matrix for the proposed scheme for the case \( N_t < K \) is a square matrix. So, ZF precoder will be pseudo-inverse of effective channel matrix and BD precoder with suitable scaling of columns will also be pseudo-inverse (as the each user has one PD) of effective channel matrix for the case \( N_t \geq K \), and ZF precoder will be inverse of effective channel matrix and BD precoder with suitable scaling of columns will be inverse of effective channel matrix for the case \( N_t < K \). As pseudo-inverse is not unique and inverse is unique and since the columns of ZF matrix are normalized to ensure uniform power from all the LEDs, the ZF and BD precoders will be same for the case \( N_t < K \) and
they need not be same for the case $N_i \geq K$. Therefore, for the case $N_i \geq K$, the performance of proposed scheme using ZF precoding is different from the performance of proposed scheme using BD (see Fig. 3), whereas for the case $N_i < K$, the performance of proposed scheme using ZF is same as the performance of proposed scheme using BD (see Fig. 4).

Next, in Fig. 5, we compare the spectral efficiency of the proposed scheme using ZF with that of random clustering schemes using ZF, gain based clustering schemes using ZF, and NOMA without clustering for the case $N_i \geq K$ ($K = 6$ and $N_i = 6$). In random clustering schemes, users are randomly grouped in such a way that there are at least $S_i$ clusters. In the gain based clustering schemes, first, the number of clusters is fixed to $S$. Then the users are sorted in the decreasing order in terms of norm of their channel gain vectors. After sorting, the $i$th, $(S+i)$th, $(2S+i)$th…$\left([K/S]-1\right)S+i$th users are grouped in the $i$th cluster, where $i = 1, 2, 3, \ldots, S$. We can observe that MIMO-NOMA with proposed clustering performs better than MIMO-NOMA using random clustering with $S_0 = 1, 2, 3, 4, 5$ for average transmit SNR less than 150 dB. Also, MIMO-NOMA using proposed clustering performs better than MIMO-NOMA without clustering for average transmit SNR greater than 110 dB and it significantly performs better than MIMO-NOMA using gain based clustering with $S = 2$ and 3.

Finally, in Fig. 6, we compare the spectral efficiency of the proposed scheme using ZF with that of random clustering scheme using ZF, gain based clustering schemes using ZF and NOMA without clustering for the case $N_i < K$ ($K = 12$ and $N_i = 6$). Note that, for this case in random clustering scheme, the number of clusters is fixed to be 6 ($S = 6$) and the gain based scheme is same as that explained for the case $N_i \geq K$. We can observe that the MIMO-NOMA using proposed clustering performs better than MIMO-NOMA using random clustering with $S_0 = 6$ for average transmit SNR less than 145 dB. Also, MIMO-NOMA with proposed clustering performs better than MIMO-NOMA without clustering for average transmit SNR greater than 113 dB and it performs significantly better than MIMO-NOMA using gain based clustering with $S = 2, 3,$ and 6.

V. CONCLUSIONS

We proposed a multiuser MIMO-NOMA transmission scheme in which the users having high correlation among their channel gain vectors are grouped into a single cluster and the users present in same cluster are multiplexed using NOMA technique. For the proposed scheme, we presented a clustering algorithm that groups the users into clusters based on correlation among their channel gains. For the case $N_i \geq K$, we compared the performance of the proposed scheme with that of the conventional multiuser MIMO precoding scheme. For the case $N_i < K$, we compared the performance of the proposed scheme with that of the conventional multiuser MIMO precoding scheme where $K$ users are served in $\left[ K/N_i \right]$ time slots by time sharing. Our results showed that the proposed scheme performs better than the multiuser MIMO precoding schemes like ZF and BD in terms of spectral efficiency, especially at relatively low average transmit SNR values. We also compared the performance of MIMO-NOMA with proposed clustering algorithm with that of MIMO-NOMA with random clustering and gain based clustering algorithms. Results show that MIMO-NOMA with proposed clustering algorithm performs significantly better than MIMO-NOMA with gain based clustering and it performs better than MIMO-NOMA with random clustering at relatively low SNR values. Investigating the effect of limited dynamic range and non-linearity of the LEDs on the performance of the proposed precoded MIMO-NOMA scheme and analyzing the bit error rate performance of the proposed scheme are some of the interesting topics for future research.

REFERENCES