Multiuser Detection in Large-Dimension Multicode MIMO-CDMA Systems with Higher-Order Modulation

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***Abstract*—In this paper, we are interested in high spectral efficiency multicode CDMA systems with large number of users employing single/multiple transmit antennas and higher-order modulation. In particular, we consider a local neighborhood search based multiuser detection algorithm which offers very good performance and complexity, suited for systems with large number of users employing M-QAM/M-PSK. We apply the algorithm on the chip matched filter output vector. We demonstrate near-single user (SU) performance of the algorithm in CDMA systems with large number of users using 4-QAM/16-QAM/64-QAM/8-PSK on AWGN, frequency-flat, and frequency-selective fading channels. We further show that the algorithm performs very well in multicode multiple-input multiple-output (MIMO) CDMA systems as well, outperforming other linear detectors and interference cancelers reported in the literature for such systems. The per-symbol complexity of the search algorithm is $O(K^2n_t^2n_c^2M)$, K: number of users, n_t: number of transmit antennas at each user, n_c: number of spreading codes multiplexed on each transmit antenna, M: modulation alphabet size, making the algorithm attractive for multiuser detection in large-dimension multicode MIMO-CDMA systems with M-QAM.**

Keywords – *Multicode MIMO, DS-CDMA, higher-order modulation, near-optimal performance, local neighborhood search.*

I. Introduction

MIMO techniques are attractive to achieve high-spectral efficiency wireless transmissions, and they have been widely adopted in current wireless standards including LTE-A and WiFi. Direct-sequence code division multiple access (CDMA) has been attractive in both military as well as non-military (2G/3G cellular) communication scenarios. It is of interest to increase the spectral efficiencies achieved in CDMA systems. This can be achieved through the use of i) higher-order modulation, ii) multicode operation, and iii) MIMO techniques. Multicode scheme divides a high rate data stream into multiple low rate substreams, and transmits these substreams in parallel using multiple spreading codes. For example, in CDMA based wireless communication standards, e.g., HSDPA and HSUPA [1]-[3], M-QAM modulated multicode operation has been introduced in the physical layer in order to provide enhanced data rates. In frequency-selective channels, multipath induced code-domain interference in such systems can seriously degrade performance. In addition to code-domain interference, spatial interference will result if MIMO transmission is adopted. Conventional matched filter (MF) detection results in poor performance in such cases. It has been shown that in order to achieve high spectral efficiencies using higher-order modulation in CDMA, multiuser detection [4] is essential [5].

Achieving optimal/near-optimal multiuser detection performance, when the dimensionality of the detection problem is large, is challenging. Particularly, the size of maximum-likelihood (ML) solution space in multicode MIMO-CDMA is $M^{Kn_tn_c}$ (M: modulation alphabet size, K: number of users, n_t: number of transmit antennas at each user, n_c: number of codes multiplexed on each transmit antenna), which makes ML detection impractical for large $Kn_tn_c \log_2 M$. In this paper, our focus is on achieving near-ML performance in large-dimension multicode MIMO-CDMA with higher-order modulation at low complexities.

Several papers on multiuser detection in CDMA with higher-order modulation (without or with multicode MIMO) mainly consider linear detectors including MF, decorrelating/zero-forcing (ZF), and minimum mean square error (MMSE) detectors. For example, in [5], the spectral efficiency analysis in large CDMA systems with higher-order modulation is for MF and MMSE detectors. Early performance analysis works on MMSE detection in CDMA with higher order modulation is due to Milstein *et al* [6]-[9]. In [10], van Houtum analyzed the BER performance of quasi-synchronous CDMA with M-QAM/M-PSK and Walsh-Hadamard codes in the presence of synchronization errors smaller than the chip time for MF detection. In [11], Rhee *et al* proposed a multicarrier 16-QAM CDMA transceiver system, which used a combination of decorrelator and RAKE receiver. In [12],[13], Yu *et al* developed approximations for the symbol/bit error rate of LMMSE receivers for M-QAM in Rayleigh faded CDMA channels. In [14], it has been shown that chip-level LMMSE equalizer with receiver diversity provides a significant performance enhancement in HSDPA system. Chip-level LMMSE has been extended to MIMO multicode CDMA in [15].

Interference canceling detectors with higher order modulation have also been widely studied [16],[17]. In [18], Huang *et al* proposed a group detection scheme for multicode MIMO. In [19], Meurer and Weber studied an iterative multiuser detection scheme, which is closely related to parallel interference cancellation (PIC), for TD-CDMA with higher-order modulation. Performance of multicode CDMA with successive interference cancellation (SIC) in Nakagami-m channel channels is presented in [20]. In [21], Xia and Wang derived analytical BER expressions for multicode CDMA with M-QAM using a PIC receiver. Recently, this work has been extended for bit-interleaved coded M-QAM (BIC-MQAM) [22]. A frequency-domain PIC scheme for multicode MIMO-CDMA is studied in [23]. In [24], Change and Lee presented a two stage interference cancelling scheme for detection in multicode MIMO. This paper also proposed power allocation strategies at the transmitter based on the feedback from receiver in order to balance signal-to-noise ratio in all substreams of data, and thereby improving the performance further. In [25], receivers that cancel code-domain and space-domain interferences in multicode MIMO in adhoc network settings are presented. In [26], Yang *et al* showed that with usage of LCZ and ZCZ sequences as spreading codes the BER performance of SIC schemes in multicode schemes can be improved significantly.

Higher-order QAM detection in CDMA using other techniques including semi-definite programming (SDP) have been studied [27]. However, most of the above mentioned detectors either scale well in complexity but perform poorly (e.g., MF, ZF, MMSE), or perform well but do not scale well in complexity (e.g., interference cancelers), making them inadequate for use in communication systems with *large dimensions*. Achieving high spectral efficiencies needs signaling to be done in large dimensions (e.g., in hundreds), i.e., large K, n_t, n_c, M in multicode MIMO-CDMA. Recently, algorithms rooted in artificial intelligence/machine learning have been shown to be attractive to achieve near-optimal performance in communication problems in large dimensions [28]-[32].

In this paper, we consider a low complexity algorithm based on local neighborhood search for multiuser detection in multicode MIMO-CDMA systems using M-QAM/M-PSK, and demonstrate its near-optimal performance in large (hundreds of) dimensions. The algorithm starts from an initial solution vector, which can be the output from any known low-complexity detector such as ZF, MMSE detectors. This initial solution is refined (in terms of ML cost) through an iterative low-complexity search in the neighborhood to get the final solution vector. We apply the algorithm on the chip-matched filter output vector in multicode MIMO-CDMA. Simulation results for 16-QAM/64-QAM/8-PSK show that the algorithm achieves near-single user BER performance in multiuser multicode MIMO CDMA. The per-symbol complexity of the algorithm is $O(K^2 n_t^2 n_c^2 M)$. A key advantage is that the algorithm offers both scalability to large dimensions as well as near-optimal performance in large dimensions.

II. System Model

We consider K-user DS-CDMA on the uplink. Without loss of generality, we consider synchronous CDMA where each user employs n_t transmit antennas and multiplexes n_c substreams of data in the code domain using n_c spreading codes on each transmit antenna with N chips per symbol. The base station (BS) receives the signal using n_r receive antennas. Figure 1 shows the system model considered.

Let $\mathbf{x}_k$ denote the kth user's transmit data symbol vector of size $n_t n_c \times 1$, which is given by

$$\mathbf{x}_k = \left[\mathbf{x}_{k,1}^T, \mathbf{x}_{k,2}^T, \cdots, \mathbf{x}_{k,n_t}^T\right]^T, \tag{1}$$

where $\mathbf{x}_{k,j}$ is the sub-vector of size $n_c \times 1$ on jth antenna of kth user, given by

$$\mathbf{x}_{k,j} = \left[x_{k,j,1}, x_{k,j,2}, \cdots, x_{k,j,n_c}\right]^T, \tag{2}$$

where $x_{k,j,i} \in \mathbb{A}$ is the complex data symbol mounted on ith code on jth antenna of kth user, and $\mathbb{A}$ is the modulation alphabet (e.g., M-QAM/M-PSK). Stacking the transmit vectors from all the K users, the overall $Kn_t n_c \times 1$ sized transmit vector is given by

$$\mathbf{x} = \left[\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_K^T\right]^T. \tag{3}$$

The spreading waveform on kth user's ith code, denoted by $c_{k,i}(t)$, is given by

$$c_{k,i}(t) = \sum_{n=0}^{N-1} c_{k,i}^n \phi(t - nT_c), \quad k = 1, \cdots, K, \tag{4}$$

where T_c is one chip duration, $c_{k,i}^n \in \{\pm 1\}$ is the nth chip of kth user's ith code, $\phi(t)$ is the chip waveform, which is assumed to be rectangular, i.e., unity for $0 \leq t \leq T_c$ and zero otherwise, $N = T_s/T_c$ is the processing gain, and T_s is one data symbol duration. There are n_c different spreading codes assigned to each user. So there are a total of Kn_c different spreading codes. Each user reuses its assigned codes on all its n_t transmit antennas. The ith code sequence of kth user, denoted by $\mathbf{c}_{k,i}$, is given by

$$\mathbf{c}_{k,i} = \left[c_{k,i}^0, c_{k,i}^1, \cdots, c_{k,i}^{N-1}\right]^T. \tag{5}$$

Putting all n_c codes on jth antenna of user k in a $N \times n_c$ matrix form, we write

$$\mathbf{C}_{k,j} = \left[\mathbf{c}_{k,1}, \mathbf{c}_{k,2}, \cdots, \mathbf{c}_{k,n_c}\right]. \tag{6}$$

Because of code reuse, same codes are used in all transmit antennas in a given user. We therefore write kth user's full code matrix of size $N \times n_t n_c$ as

$$\mathbf{C}_k = \left[\mathbf{C}_{k,j}, \mathbf{C}_{k,j}, \cdots n_t \text{ times}\right]. \tag{7}$$

Putting the code matrices of all users together, the overall system code matrix $\mathbf{C}$ of size $N \times Kn_t n_c$ is written as

$$\mathbf{C} = \left[\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_K\right]. \tag{8}$$

So, the overall input to the channel in N chip intervals is given by $\mathbf{Cx}$, where nth row denotes the transmit vector at the nth chip interval.

Consider frequency-selective fading channel model, where multiple resolvable paths exist between each user and the BS. Without loss of generality, let L denote the number of resolvable paths between any pair of transmit and receive antennas. Let $h_{k,p,j}^l \sim \mathcal{CN}(0, \Omega_l)$ denote the complex channel gain on the lth path between jth transmit antenna of kth user and the pth receive antenna at the BS. We assume that the channel coefficients remain constant over one symbol duration. Exponential power-delay profile with

$$\Omega_l = \mathbb{E}[|h_{k,p,j}^l|^2] = \Omega_0 e^{-\beta l}, \quad l = 0, \cdots, L-1, \tag{9}$$

is assumed, where β is the rate of exponential decay of average path power. It is also assumed that the delay spread is small compared to the symbol duration (i.e., $L << N$), so that there is inter-chip interference but no inter-symbol interference. The received sample at nth chip of pth receive antenna is

$$y_p(n) = \sum_{k=1}^{K} \sum_{l=0}^{L-1} \sum_{j=1}^{n_t} \sum_{i=1}^{n_c} h_{k,p,j}^l c_{k,i}^{n,l} x_{k,j,i} + w_p(n) \tag{10}$$

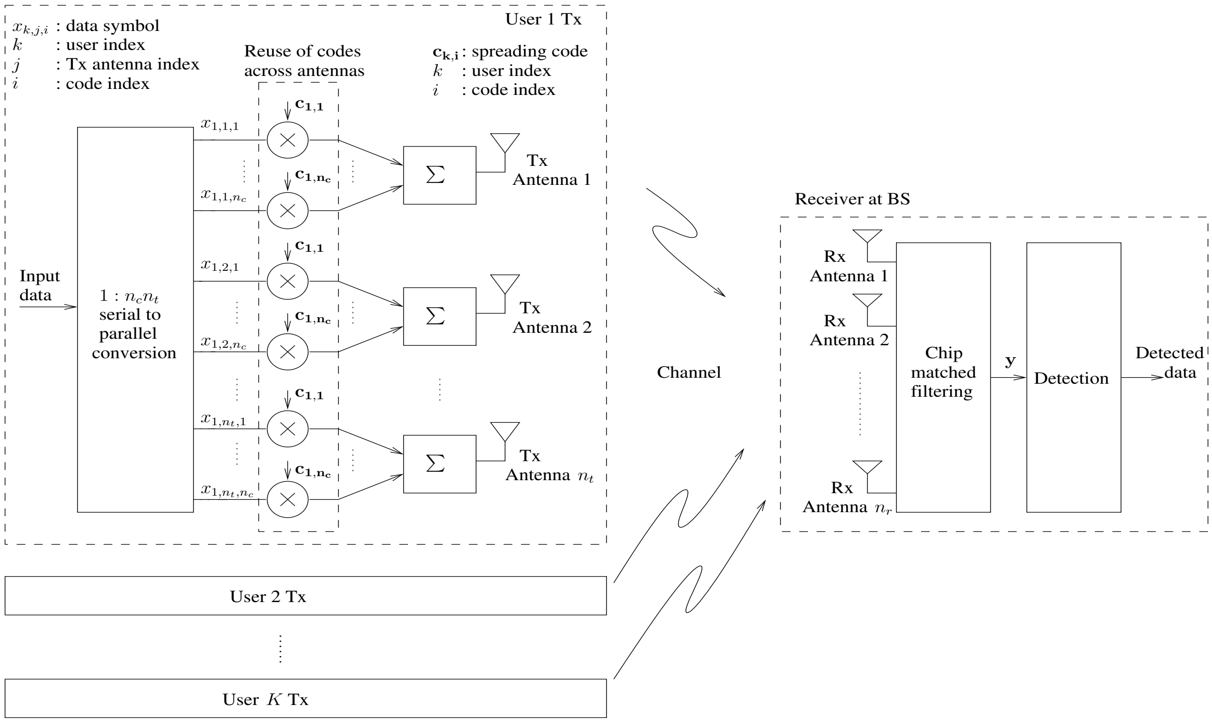

Fig. 1. Multicode MIMO-CDMA System Model.

where $c_{k,i}^{n,l}$ is the nth entry of the l chip delayed code sequence vector $\mathbf{c}_{k,i}^{l} = [0, 0, \cdots l \text{ times}, c_{k,i}^{0}, c_{k,i}^{1}, \cdots, c_{k,i}^{N-l-1}]^T$, and $w_p(n)$ is the complex Gaussian noise sample with zero mean and variance σ^2. Note that $\mathbf{c}_{k,i}^{0}$ in the above is the same as $\mathbf{c}_{k,i}$ defined in (5).

We want to develop a linear vector channel model comprising the received samples for receiver processing at the BS. Let us take the $N \times 1$ received vector at pth rx antenna $\mathbf{y}_p$ as

$$\mathbf{y}_p = [y_p(0), y_p(1), \cdots, y_p(N-1)]^T, \tag{11}$$

and the overall $n_r N \times 1$ received vector over all receive antennas as

$$\mathbf{y} = \left[\mathbf{y}_1^T, \mathbf{y}_2^T, \cdots, \mathbf{y}_{n_r}^T\right]^T. \tag{12}$$

We proceed to determine the equivalent code matrix ($\mathbf{S}$) and channel matrix ($\mathbf{H}$) at the receiver side, such that the linear vector channel equation with transmit vector $\mathbf{x}$ can be written in the form

$$\mathbf{y} = \mathbf{SHx} + \mathbf{w}, \tag{13}$$

where $\mathbf{w}$ is $n_r N \times 1$ noise vector, given by

$$\mathbf{w} = \left[\mathbf{w}_1^T, \mathbf{w}_2^T, \cdots, \mathbf{w}_{n_r}^T\right]^T, \tag{14}$$

where $\mathbf{w}_p$ is the noise vector at the pth rx antenna, given by

$$\mathbf{w}_p = [w_p(0), w_p(1), \cdots, w_p(N-1)]^T. \tag{15}$$

Let $\mathbf{h}_{k,p,j}$ denote the $L \times 1$ multipath channel coefficient vector corresponding to kth user's jth transmit and BS's pth receive antennas, which is given by

$$\mathbf{h}_{k,p,j} = \left[h_{k,p,j}^{0}, h_{k,p,j}^{1}, \cdots, h_{k,p,j}^{L-1}\right]. \tag{16}$$

Since the fades remain same for all the n_c codes for a given tx-rx antenna pair, we define a $n_c L \times n_c$ channel matrix

$$\mathbf{H}_{k,p,j} = diag\{\mathbf{h}_{k,p,j}, \mathbf{h}_{k,p,j}, \cdots n_c \text{ times}\}. \tag{17}$$

Putting together the above matrices for all n_t transmit antennas from user k to receive antenna p, define the $n_t n_c L \times n_t nc$ channel matrix

$$\mathbf{H}_{k,p} = diag\{\mathbf{H}_{k,p,1}, \mathbf{H}_{k,p,2}, \cdots, \mathbf{H}_{k,p,n_t}\}. \tag{18}$$

Likewise, the channel matrix of size $Kn_t n_c L \times Kn_t n_c$ at the pth rx antenna from all the K users can be written as

$$\mathbf{H}_p = diag\{\mathbf{H}_{1,p}, \mathbf{H}_{2,p}, \cdots, \mathbf{H}_{K,p}\}. \tag{19}$$

The overall channel matrix between all the users and the BS receiver is of size $n_r K n_t n_c L \times K n_t n_c$, and is given by

$$\mathbf{H} = \left[\mathbf{H}_1^T, \mathbf{H}_2^T, \cdots, \mathbf{H}_{n_r}^T\right]^T. \tag{20}$$

The receiver side code matrix $\mathbf{S}$ becomes different from transmit side code matrix $\mathbf{C}$ because of the fact that at the receiver side additional effect of multipath channel gains on the transmit symbols have to be considered. The data stream corresponding to ith code on jth transmit antenna of user k ($\mathbf{c}_{k,i}$) is received at the pth receive antenna through L resolvable paths. Combining the delayed code sequences corresponding to L paths, we write the $N \times L$ matrix

$$\mathbf{S}_{p,k,j,i} = \left[\mathbf{c}_{k,i}^{0}, \mathbf{c}_{k,i}^{1}, \cdots, \mathbf{c}_{k,i}^{L-1}\right]. \tag{21}$$

Putting together the code matrices corresponding to all n_c codes on jth tx antenna of user k, we write $N \times n_c L$ matrix

$$\mathbf{S}_{p,k,j} = \left[\mathbf{S}_{p,k,j,1}, \mathbf{S}_{p,k,j,2}, \cdots, \mathbf{S}_{p,k,j,n_c}\right]. \tag{22}$$

The $N \times n_t n_c L$ code matrix corresponding all transmit antennas of user k can be obtained by concatenating $\mathbf{S}_{p,k,j}$ n_t times as codes are reused across all tx antennas, i.e.,

$$\mathbf{S}_{p,k} = \big[\mathbf{S}_{p,k,j}, \mathbf{S}_{p,k,j}, \cdots n_t \text{ times}\big]. \tag{23}$$

The overall code matrix of size $N \times K n_t n_c L$ corresponding to pth rx antenna can be obtained by combining all $\mathbf{S}_{p,k}$'s as

$$\mathbf{S}_p = \big[\mathbf{S}_{p,1}, \mathbf{S}_{p,2}, \cdots, \mathbf{S}_{p,K}\big]. \tag{24}$$

The code matrices across the receive antennas remain same. Therefore, the overall receiver side code matrix $\mathbf{S}$ of size $n_r N \times n_r K n_t n_c L$ is given by

$$\mathbf{S} = diag\big\{\mathbf{S}_p, \mathbf{S}_p, \cdots n_r \text{ times}\big\}. \tag{25}$$

With the above definitions of channel and code matrices $\mathbf{H}$ and $\mathbf{S}$, we will carry out receiver processing on the linear vector channel model in (13).

III. Detection Algorithm

Consider the complex system model (13)

$$\mathbf{y} = \underbrace{\mathbf{SH}}_{\triangleq \mathbf{A}} \mathbf{x} + \mathbf{w}, \tag{26}$$

where $\mathbf{y} \in \mathbb{C}^{d_r}$, $d_r = n_r N$, $\mathbf{A} \in \mathbb{C}^{d_r \times d_t}$, $d_t = K n_t n_c$ $\mathbf{x} \in \mathbb{A}^{K n_t n_c}$ and $\mathbf{w} \in \mathbb{C}^{d_r}$. We convert the complex system model in (26) to a real system model, given by

$$\mathbf{y}_r = \mathbf{A}_r \mathbf{x}_r + \mathbf{w}_r, \tag{27}$$

where, defining $\mathbf{y} = \mathbf{y}_I + j\mathbf{y}_Q$, $\mathbf{A} = \mathbf{A}_I + j\mathbf{A}_Q$, $\mathbf{x} = \mathbf{x}_I + j\mathbf{x}_Q$, $\mathbf{w} = \mathbf{w}_I + j\mathbf{w}_Q$,

$$\mathbf{y}_r = [\mathbf{y}_I^T \;\; \mathbf{y}_Q^T]^T, \quad \mathbf{A}_r = \begin{pmatrix} \mathbf{A}_I & -\mathbf{A}_Q \\ \mathbf{A}_Q & \mathbf{A}_I \end{pmatrix},$$
$$\mathbf{x}_r = [\mathbf{x}_I^T \;\; \mathbf{x}_Q^T]^T, \quad \mathbf{w}_r = [\mathbf{w}_I^T \;\; \mathbf{w}_Q^T]^T, \tag{28}$$

so that $\mathbf{A}_r \in \mathbb{R}^{2d_r \times 2d_t}$, $\mathbf{y}_r \in \mathbb{R}^{2d_r \times 1}$, $\mathbf{x}_r \in \mathbb{S}^{2d_t \times 1}$, and $\mathbf{w}_r \in \mathbb{R}^{2d_r \times 1}$. With this real system model, when $\mathbf{x}$ takes values from square M-QAM alphabet, each entry of $\mathbf{x}_r$ can be thought of as an element from $\sqrt{M}$-PAM signal set $\mathbb{S}$ such that, $\mathbb{S} \equiv \{(2m-1-\sqrt{M}) : m = 1, 2, \cdots, \sqrt{M}\}$. Assuming channel state information to be available at the receiver, the optimal maximal likelihood (ML) solution vector, $\mathbf{x}_{r,ML}$, is

$$\begin{aligned} \mathbf{x}_{r,ML} &= \underset{\mathbf{x}_r \in \mathbb{S}^{2d_t}}{\arg\min} \; \|\mathbf{y}_r - \mathbf{A}_r \mathbf{x}_r\|^2 \\ &= \underset{\mathbf{x}_r \in \mathbb{S}^{2d_t}}{\arg\min} \; \underbrace{\mathbf{x}_r^T \mathbf{A}_r^T \mathbf{A}_r \mathbf{x}_r - 2\mathbf{y}_r^T \mathbf{A}_r \mathbf{x}_r}_{\psi(\mathbf{x}_r)}. \end{aligned} \tag{29}$$

The detection problem in (29) is exponentially complex in d_t. We consider a low-complexity algorithm, termed as likelihood ascent search (LAS) algorithm [28],[29],[30]. Details of the algorithm are developed below.

The algorithm attempts to reach the global minima of the objective function in (29) an iterative fashion, starting from an initial solution vector. This initial solution vector, $\mathbf{x}_r^{(0)}$, can be the output of a low-complexity linear detector, e.g., MF, ZF, MMSE detectors. Let $\mathbf{x}_r^{(t)}$ denote the solution vector after tth iteration of the algorithm. In each iteration, the solution vector updates one of its entry such that the cost function undergoes maximum reduction. The iterative procedure terminates when none of the possible updates reduces the cost function any further. The update rule at tth iteration, assuming the symbol update happens at the qth entry of the solution vector, can be written as

$$\mathbf{x}_r^{(t)} = \mathbf{x}_r^{(t-1)} + \theta_q^{(t)} \mathbf{e}_q, \tag{30}$$

where $\mathbf{e}_q$ is unit vector with its qth entry as one, and all other entries as zero, and $\theta_q^{(t)}$ is change in the value of qth entry of $\mathbf{x}_r^{(t-1)}$. The task at each iteration is to find out q and $\theta_q^{(t)}$ that minimizes the ML cost in that iteration. The change in cost function in the tth iteration is given by

$$\Delta_q^{(t)} \triangleq \psi\big(\mathbf{x}_r^{(t)}\big) - \psi\big(\mathbf{x}_r^{(t-1)}\big) = \theta_q^{(t)^2} \mathbf{G}_{(q,q)} - 2\theta_q^{(t)} z_q^{(t)}, \tag{31}$$

where $\mathbf{G}_{(q,q)}$ is the (q,q)th entry of $\mathbf{G} \triangleq \mathbf{A}_r^T \mathbf{A}_r$, $z_q^{(t)}$ is the qth entry of the $\mathbf{z}^{(t)} \triangleq \mathbf{A}_r^T(\mathbf{y}_r - \mathbf{A}_r \mathbf{x}_r^{(t-1)})$. If $\Delta_q^{(t)}$ is negative for some q, then reduction of the cost function at tth iteration is possible. Let us assume that qth entry of $\mathbf{x}_r^{(t-1)}$ has taken the value $s_q^{(t-1)} \in \mathbb{S}$. Then, $\theta_q^{(t)}$ can take values only from the set $\mathcal{S}_q^{(t)} \equiv \{(2m - 1 - \sqrt{M} - s_q^{(t-1)}) : m = 1, \cdots, \sqrt{M}\}$, so that the qth entry of $\mathbf{x}_r{}^{(t)}$ can also be an element of $\mathbb{S}$. We find out the cost function difference corresponding to each possible value of $\theta_q^{(t)} \in \mathcal{S}_q^{(t)}$ using (31) and determine the minimum among the negative cost difference values thus obtained. Let $\Delta_{q,min}^{(t)}$ be the minimum cost difference value for symbol update at the qth entry of $\mathbf{x}_r^{(t-1)}$. We compare the minimum cost difference value $\Delta_{q,min}^{(t)}$'s for $q = 1, 2, \cdots, 2d_t$, and determine the least among them. Let us call the least cost difference value thus obtained as $\Delta_{min}^{(t)}$, which is given by

$$\Delta_{min}^{(t)} = \min\{\Delta_{q,min}^{(t)} : q = 1, \cdots, 2d_t\}. \tag{32}$$

Let the values of q and $\theta_q^{(t)}$ corresponding to $\Delta_{min}^{(t)}$ be $q_{min}^{(t)}$ and $\theta_{min}^{(t)}$, respectively. The symbol update at ith iteration then becomes

$$\mathbf{x}_r^{(t)} \;=\; \mathbf{x}_r^{(t-1)} + \theta_{min}^{(t)} \mathbf{e}_{q_{min}^{(t)}}. \tag{33}$$

We also observe that $\mathbf{z}^{(t+1)}$ can be obtained from $\mathbf{z}^{(t)}$ as

$$\begin{aligned} \mathbf{z}^{(t+1)} &= \mathbf{A}_r^T(\mathbf{y}_r - \mathbf{A}_r \mathbf{x}_r^{(t)}) \\ &= \mathbf{A}_r^T\big(\mathbf{y}_r - \mathbf{A}_r \mathbf{x}_r^{(t-1)}\big) - \theta_{min}^{(t)} \mathbf{G}\, \mathbf{e}_{q_{min}^{(t)}} \\ &= \mathbf{z}^{(t)} - \theta_{min}^{(t)} \mathbf{G}\, \mathbf{e}_{q_{min}^{(t)}}. \end{aligned} \tag{34}$$

The algorithm terminates when $\Delta_q^{(t)} \geqslant 0$, $\forall q \in \{1, \cdots, 2d_t\}$ and for all possible values of $\theta_q^{(t)}$, i.e., there is no negative cost difference, and hence further reduction of cost of the objective function is not possible. Since the search is only based on local neighborhood (i.e., 1-symbol neighborhood) rather than exhaustive search, the final solution vector from the algorithm can be a local minima. However, the performance of this simple algorithm is found to get increasingly closer to optimal performance for increasing values of d_t.

A. Computational Complexity

The algorithm is comprised of the following main components: i) computation of $\mathbf{G}$, ii) computation of initial solution vector $\mathbf{x}_r^{(0)}$, iii) computation of $\mathbf{z}^{(1)}$, and iv) search operations. The complexity of calculating $\mathbf{G}$ is $O(d_t^2 d_r)$. Computation of initial vector using ZF/MMSE involves taking inverse of a matrix of size $d_t \times d_t$, whose complexity is $O(d_t^3)$. The computation of $\mathbf{z}^{(1)}$ comprises of two components: multiplication of $\mathbf{G}$ and $\mathbf{x}_r^{(0)}$, and subtraction of $\mathbf{A}_r^T \mathbf{y}_r$ from $\mathbf{G}\mathbf{x}_r^{(0)}$. The complexity involved in this is $O(d_t^2)$. From simulations, the average complexity involved in one search iteration and average number of iterations required are found to be clearly sub-cubic in d_t and linear in M. So the overall complexity order is dominated by the cubic term in the MMSE matrix inversion. Combining all the above complexities, the total complexity is $O(d_t^3 M)$. Since d_t symbols are detected, the average per-symbol complexity of the algorithm is $O(d_t^2 M)$, i.e., $O(K^2 n_t^2 n_c^2 M)$.

IV. Simulation Results and Discussions

We evaluated the BER performance of LAS detection algorithm through simulations in uplink DS-CDMA with 4-QAM, 16-QAM, 64-QAM and 8 PSK, as a function of average SNR under different channel conditions (AWGN, frequency-flat fading, and frequency-selective fading) for different values of n_c, n_t, and n_r. Simulations are carried out for systems with large dimensions, i.e., large Kn_tn_c, to illustrate the scalability of the algorithm to large dimensions as well as achievability of near-single user performance in large dimensions. We also present the comparison of BER performance of MMSE-LAS algorithm with those of few other algorithms available in the literature for both single user as well as multiuser multicode MIMO. The average SNR for kth user is P_k/σ^2, where $P_k = n_t E_s n_c N$ is the total transmit power of kth user in one symbol duration, and E_s is average symbol energy. The output of MMSE detector is used as the initial solution vector for LAS algorithm in all simulations; consequently, the algorithm is referred to as 'MMSE-LAS' in all the BER performance figures.

Performance on AWGN channel: In Fig. 2, we plot the BER performance of MMSE-LAS detector for DS-CDMA with 4-QAM, 16-QAM and 64-QAM on AWGN channels with $n_c = n_t = n_r = 1$ (i.e., system with single transmit antenna per user, single code per transmit antenna, and single receive antenna at the BS). The number of users and processing gain are kept at $K = 128$ and $N = 256$, respectively. The performance of MMSE detector as well as the single user (SU) performance are also plotted for comparison. Note that obtaining the true ML performance for such a large dimension system with $K = 128$ through exhaustive search or sphere decoding is prohibitively complex. Hence, we have used SU performance (i.e., performance with no multiuser interference) as a lower bound on the ML performance in order to assess the nearness of MMSE-LAS performance to ML performance. From Fig. 2, we see that the performance of MMSE-LAS detector is close to SU performance, whereas

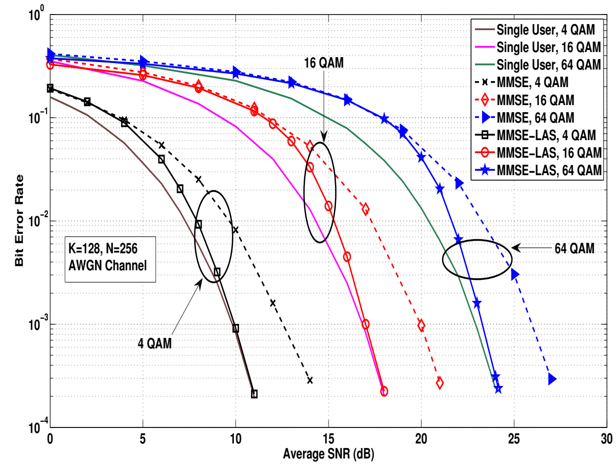

Fig. 2. BER performance of MMSE-LAS detector as a function of average SNR in AWGN channel for 4-QAM/16-QAM/64-QAM with $K = 128$, $N = 256$, $n_t = n_r = n_c = 1$.

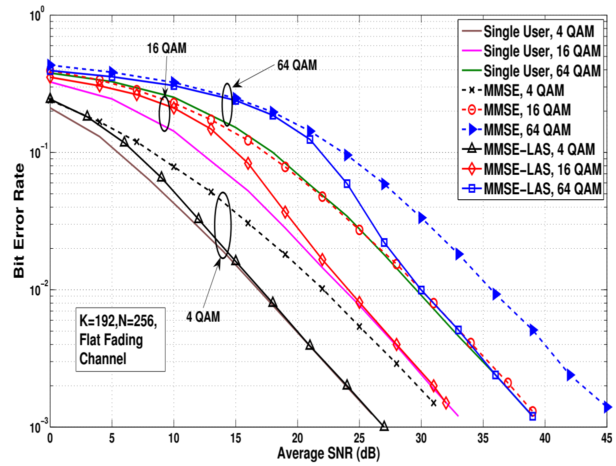

Fig. 3. BER performance of MMSE-LAS detector as a function of average SNR in flat Rayleigh fading for 4-QAM/16-QAM/64-QAM with $K = 192$, $N = 256$, $n_t = n_r = n_c = 1$.

MMSE performance is far from SU performance. This shows the effectiveness of the search operation carried out in MMSE-LAS to improve the MMSE performance towards SU performance.

Performance on flat Rayleigh fading: Figure 3 shows a similar set of BER performance plots for flat Rayleigh fading with 4-QAM, 16-QAM, 64-QAM and $n_c = n_t = n_r = 1$. The number of users and processing gain used are $K = 192$ and $N = 256$. Here again, MMSE-LAS achieves near-SU performance for all the modulations considered, maintaining significant SNR advantage over the MMSE detector (about 5 dB at 10^{-2} BER).

Multicode CDMA in frequency-selective fading: In Fig. 4, we present the MMSE-LAS detector's BER performance for multicode CDMA with 4-QAM, 16-QAM, 64-QAM in frequency-selective fading with $n_c = 4$, $n_t = 1$, $K = 32$, $N = 128$, $L = 3$, $\beta = 0$, and $n_r = 2$. The variance of the channel coefficients are normalized, i.e. $\sum_{l=0}^{L-1} \Omega_l$ is set to 1.

Multicode MIMO-CDMA in frequency-selective fading: In Fig. 5, multicode MIMO-CDMA is considered in frequency-

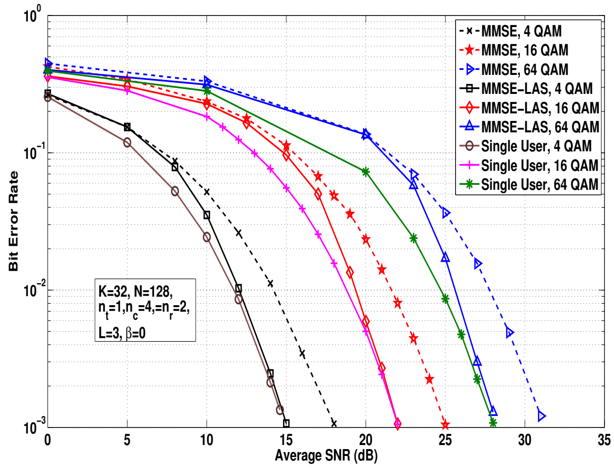

Fig. 4. BER performance of MMSE-LAS detector as a function of average SNR for multicode CDMA in frequency-selective fading with $n_c = 4$, $n_t = 1$, $n_r = 2$, $K = 32$, $N = 128$, $L = 3$. Uniform power delay profile.

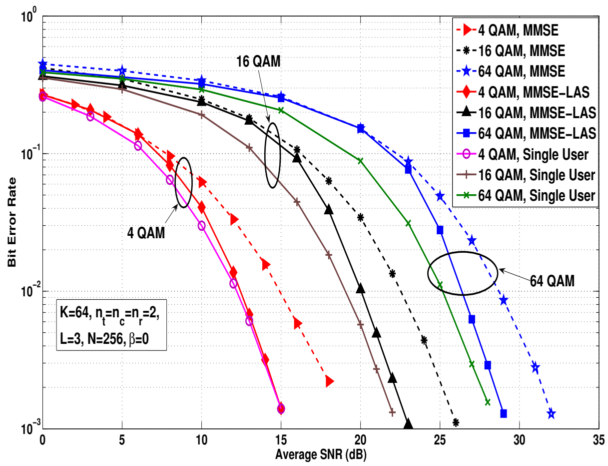

Fig. 5. BER performance of MMSE-LAS detector as a function of average SNR for multicode MIMO-CDMA in frequency-selective fading with $n_c = 2$, $n_t = n_r = 2$, $K = 64$, $N = 256$, $L = 3$. Uniform power delay profile.

selective fading with $n_c = 2$, $n_t = 2$, $K = 64$, $N = 256$, $L = 3$, $\beta = 0$, $n_r = 2$. BER plots are shown for 4-QAM, 16-QAM and 64-QAM. In multicode CDMA (Fig. 4) and multicode MIMO-CDMA (Fig. 5), MMSE-LAS outperforms MMSE detector, and performs close to SU performance.

Comparison with other IC detectors: Figure 6 compares the BER performance of MMSE-LAS detector for single user multicode MIMO with those of the multi-stage SIC receiver proposed in [24], and the group detector proposed in [18]. The number of antennas both at transmitter and receiver side is kept at $n_t = n_r = 4$. The other simulation parameters are taken as in [24], i.e., $N = 32$, $L = 3$, $n_c = 8$, $\beta = 0.5$ and 4-QAM. It can be seen that at 10^{-4} BER, MMSE-LAS BER curve is better than the group detector [18] by about 3 dB. The SIC in [24] exhibits error floors. [24] also proposed a transmit power allocation scheme to better the performance of the SIC. But power allocation requires feedback from receiver and also results in additional processing overhead at the transmitter. The performance achieved by 2-stage SIC with power allocation scheme in [24] could not better the

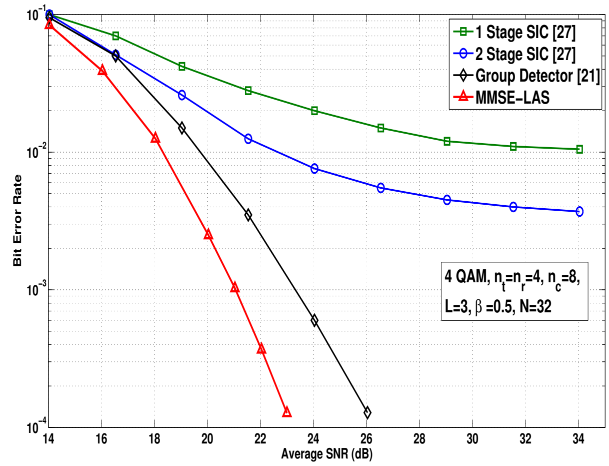

Fig. 6. Comparison of MMSE-LAS detector performance with the performance of interference canceler in [27] and group detector in [21] for 4-QAM in single user multicode MIMO with $n_c = 8$, $n_t = 4$, $n_r = 4$, $K = 1$, $N = 32$, $L = 3$, $\beta = 0.5$.

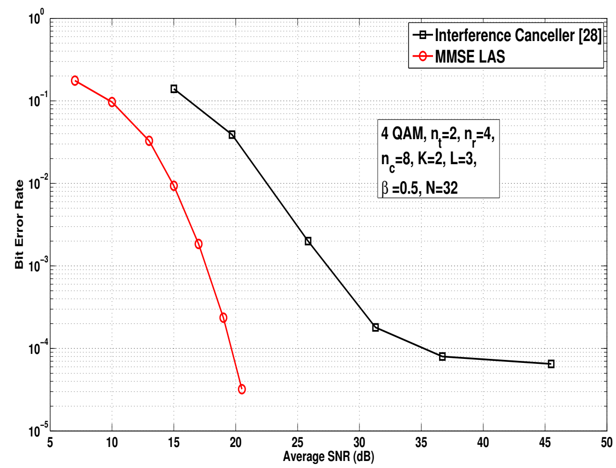

Fig. 7. Comparison of MMSE-LAS detector performance with the performance of the IC in [28] for 4-QAM in multicode MIMO-CDMA with $n_c = 8$, $n_t = 2$, $n_r = 4$, $K = 2$, $N = 32$, $L = 3$, $\beta = 0.5$.

performance of the group detector in [18], whereas MMSE-LAS outperforms even the group detector.

Figure 7 presents the comparison of MMSE-LAS BER performance as a function of average SNR for multiuser multicode MIMO with that of the IC detector proposed in [25]. The simulation parameters, considered as in [25], are $N = 32$, $n_c = 2$, $K = 2$, $n_t = 2$, $n_r = 4$, $n_c = 8$, $L = 3$, $\beta = 0.5$. At 10^{-3} BER MMSE-LAS is nearly 10 dB better than the IC algorithm presented in [25]. The IC receiver in [25] suffers from error floor while MMSE-LAS does not. It is to be noted that [18], [24] and [25] divide the data streams into multiple segments and formulates detection problem individually for each group treating other segments' contribution to received signal power as noise. Whereas we have formulated the problem at the chip matched filter output as joint detection problem over all the data streams. This results in the improved performance observed in Figs. 6, 7.

BER Performance for M-PSK: Finally, in Fig. 8, we present the MMSE-LAS detector's BER performance for 8-PSK with

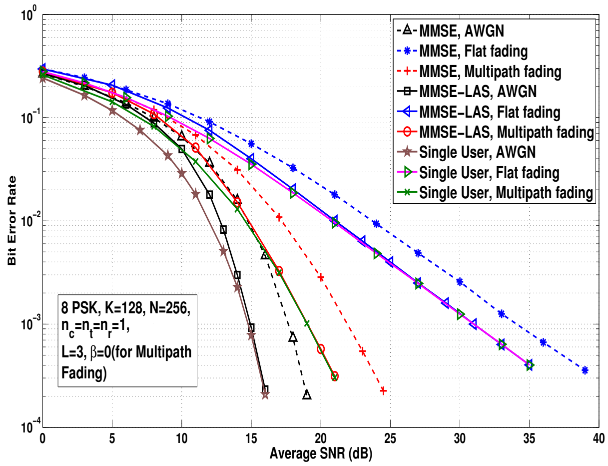

Fig. 8. BER performance of MMSE-LAS detector as a function of average SNR for DS-CDMA with 8-PSK in AWGN, flat fading, and frequency-selective fading channels. $K = 128$, $N = 256$, $n_t = n_r = n_c = 1$.

$K = 128$, $N = 256$, $n_t = n_c = n_r = 1$, in AWGN, flat fading and frequency-selective fading channels. For frequency-selective fading, L is taken to be 3 with uniform power delay profile. For convenience in handling M-PSK symbols, we did not decompose the vector channel model into in-phase and quadrature parts, and applied the detection algorithm directly on the complex system model. The results in Fig. 8 establishes the suitability of LAS algorithm for M-PSK and its ability to achieve near SU performance.

V. Conclusions

We presented a low complexity detection scheme based on local neighborhood search for multicode multi-antenna uplink DS-CDMA systems with higher-order modulation. The detector was shown to scale well for large dimensions and give near SU BER performance. Simulation results showed that the considered MMSE-LAS detector exhibits significant performance advantage over the other detection algorithms reported in the literature for multicode MIMO-CDMA. With its attractive features like low complexity/scalability and near optimal performance, the LAS detector presented in this paper can address the challenge of achieving higher data rates in the next generation MIMO-CDMA wireless systems.

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