

BER Analysis of Weighted Interference Cancellation in Multicarrier DS-CDMA Systems

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Abstract—In this paper, we present the design and bit error performance analysis of weighted linear parallel interference cancellers (LPIC) for multicarrier (MC) DS-CDMA systems. We propose an LPIC scheme where we estimate (and cancel) the multiple access interference (MAI) based on the soft outputs on individual subcarriers, and the interference cancelled outputs on different subcarriers are combined to form the final decision statistic. We scale the MAI estimate on individual subcarriers by a weight before cancellation; these weights are so chosen to maximize the signal-to-interference ratios at the individual subcarrier outputs. For this weighted LPIC scheme, using an approach involving the characteristic function of the decision variable, we derive exact bit error rate (BER) expressions for different cancellation stages. Using the same approach, we also derive exact BER expressions for the matched filter (MF) and decorrelating detectors for the considered MC DS-CDMA system. We show that the proposed weighted LPIC scheme performs better than the MF detector and the conventional LPIC (where the weights are taken to be unity), and close to the decorrelating detector.

Keywords — Parallel interference cancellation, multicarrier DS-CDMA, signal-to-interference ratio, optimum weights.

I. INTRODUCTION

There has been increased interest in multiuser, multicarrier systems (e.g., multicarrier CDMA) for broadband wireless communications [1]–[3]. Because of their potential to remove multiple access interference (MAI) and increase system capacity, multiuser detection in general [4], and interference cancellation techniques in particular, applied to multicarrier direct-sequence CDMA (MC DS-CDMA) are of interest [5]–[7]. In this paper, we focus on linear parallel interference cancellers (LPIC) for MC DS-CDMA systems.

The conventional way to realize LPIC schemes is to use unscaled values of the soft outputs from different users for MAI estimation. A known problem with this conventional LPIC (CLPIC) approach is that it can perform even worse than the matched filter (MF) detector (where cancellation is not done), particularly at low SNRs [8],[11]. This is because the MAI estimates obtained using unscaled values of soft outputs can become quite inaccurate under poor channel conditions (e.g., low SNRs) to such an extent that it may be better not to do cancellation. This problem can be alleviated by properly weighing (scaling) the MAI estimates before cancellation [8],[11]. A key question in this regard is how to choose these weights (scaling factors) for different stages of the LPIC. For the case of single carrier CDMA systems, the issue of the choice of the weights in LPIC has been addressed

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in [12],[13] for AWGN, and in [11] for Rayleigh fading and diversity channels.

In this paper, we propose a weighted LPIC (WLPIC) scheme for a MC DS-CDMA system, where we scale the MAI estimates on individual subcarriers by weights before cancellation. One way to optimally choose the weights in this scheme is to derive analytical expressions for the average SIR at the output of the IC stages as a function of the weights, and maximize these SIR expressions to obtain the optimum weights for different stages, as done for single carrier CDMA in [11]. However, for the MC DS-CDMA scheme we consider in this paper, the instantaneous SIR expression at the combined output from multiple carriers in the system is such that the unconditioning on the fade variables to obtain the average SIR in closed-form is difficult. Therefore, we adopt an alternate approach where we choose those weights which maximize the average SIR expressions on individual subcarriers (rather than maximizing the average combined output SIR). This approach, though suboptimum relative to maximizing the average SIR at the combined output, has the following advantages: first, closed-form expressions for the weights similar to those derived in [11] for single CDMA can be obtained for the individual subcarriers in the considered MC DS-CDMA system, and second, through an exact bit error rate (BER) analysis in Sec. IV, we show that even these subcarrier-wise optimum weights result in good cancellation performance.

For the proposed WLPIC scheme for MC DS-CDMA, using an approach involving the characteristic function of the decision variable we derive exact BER expressions for different IC stages. Using the same approach, we also derive exact BER expressions for the MF and decorrelating detectors for the considered MC DS-CDMA system. We show that the proposed WLPIC scheme performs better than the MF detector and the conventional LPIC scheme (where the weights are taken to be unity), and close to the decorrelating detector.

II. SYSTEM MODEL

We consider a K -user synchronous multicarrier DS-CDMA system (an asynchronous system can be considered likewise). Figure 1 shows the transmitter of the k th user [1]. M is the number of subcarriers, and $c_{k,i}(t)$ is the spreading waveform of the k th user on the i th subcarrier. The number of chips per bit on each subcarrier is N . The channel coefficients $h_k^{(i)}$, $i = 1, 2, \dots, M$, are assumed to be i.i.d. complex Gaussian r.v.'s (i.e., fade amplitudes are Rayleigh distributed) with zero mean and $E[(h_{kI}^{(i)})^2] = E[(h_{kQ}^{(i)})^2] = 1$, where $h_{kI}^{(i)}$ and $h_{kQ}^{(i)}$ are the real and imaginary parts of $h_k^{(i)}$. It is assumed that the

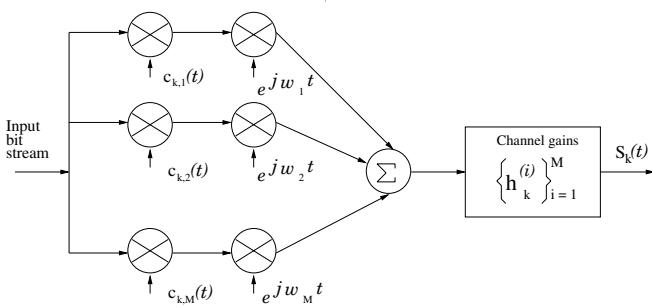


Fig. 1. Multicarrier DS-CDMA transmitter of k th user

channel is frequency non-selective on each subband and fades are independent from one subband to the other. The MC DS-CDMA receiver with the proposed weighted LPIC scheme is shown in Fig. 2.

Let $\mathbf{y}^{(i)} = (y_1^{(i)}, y_2^{(i)}, \dots, y_K^{(i)})^T$, where T denotes the transpose operator, denote the K -length received signal vector on the i th subcarrier; i.e., $y_k^{(i)}$ is the output of the k th user's matched filter on the i th subcarrier. Assuming that the inter-carrier interference is negligible, the K -length received signal vector on the i th subcarrier $\mathbf{y}^{(i)}$ can be written in the form

$$\mathbf{y}^{(i)} = \mathbf{C}^{(i)} \mathbf{H}^{(i)} \mathbf{b} + \mathbf{n}^{(i)}, \quad (1)$$

where $\mathbf{C}^{(i)}$ is the $K \times K$ cross-correlation matrix on the i th subcarrier, given by

$$\mathbf{C}^{(i)} = \begin{bmatrix} 1 & \rho_{12}^{(i)} & \cdots & \rho_{1K}^{(i)} \\ \rho_{21}^{(i)} & 1 & \cdots & \rho_{2K}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1}^{(i)} & \rho_{K2}^{(i)} & \cdots & 1 \end{bmatrix}, \quad (2)$$

where $\rho_{lj}^{(i)}$ is the correlation coefficient between the signature waveforms of the l th and the j th users on the i th subcarrier. $\mathbf{H}^{(i)}$ represents the $K \times K$ channel coefficient matrix, given by

$$\mathbf{H}^{(i)} = \text{diag}\left\{h_1^{(i)}, h_2^{(i)}, \dots, h_K^{(i)}\right\}. \quad (3)$$

The K -length data vector \mathbf{b} is given by

$$\mathbf{b} = \begin{bmatrix} A_1 b_1 & A_2 b_2 & \cdots & A_K b_K \end{bmatrix}^T, \quad (4)$$

where A_k denotes the transmit amplitude and $b_k \in \{+1, -1\}$ denotes the data bit of the k th user, and $[\cdot]^T$ denotes the transpose operator. The K -length noise vector $\mathbf{n}^{(i)}$ is given by

$$\mathbf{n}^{(i)} = \begin{bmatrix} (n_1^{(i)})^* & (n_2^{(i)})^* & \dots & (n_K^{(i)})^* \end{bmatrix}^H, \quad (5)$$

where $n_k^{(i)}$ denotes the additive noise component of the k th user on the i th subcarrier, which is assumed to be complex Gaussian with zero mean with $E[n_k^{(i)}(n_j^{(i)})^*] = 2\sigma^2$ when $j = k$ and $2\sigma^2\rho_{kj}^{(i)}$ when $j \neq k$. Here, $[\cdot]^H$ denotes the Hermitian operator and $(\cdot)^*$ denotes the complex conjugate.

III. WEIGHTED LPIC SCHEME FOR MC DS-CDMA

In the proposed weighted LPIC scheme, we cancel weighted estimates of the MAI on individual subcarriers, and the interference cancelled outputs from all the subcarriers are combined to form the combined decision statistic. The interference cancellation performed on the i th subcarrier in the m th stage is explained as follows.

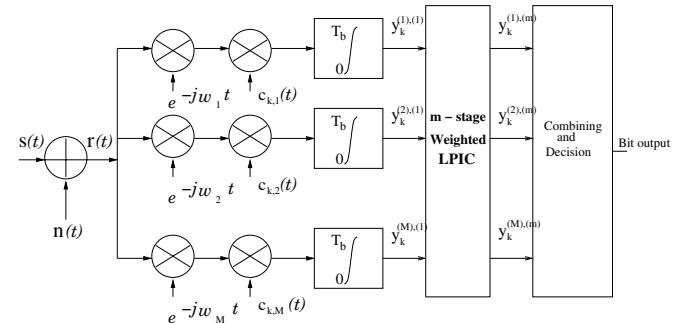


Fig. 2. Multicarrier DS-CDMA receiver with weighted LPIC

A. Interference Cancellation on i th Subcarrier in Stage- m

The estimate of the MAI from the j th interfering user to the desired user k on the i th subcarrier in the m th cancellation stage is scaled by a factor $w_{jk}^{(i),(m)}$ before cancellation. Specifically, the estimate of the MAI from the j th interfering user to the desired user k on the i th subcarrier in stage- m , $m > 1$, is obtained by multiplying $y_j^{(i),(m-1)}$ with $\rho_{jk}^{(i)}$ for all $j \neq k$ and summing them up, where $y_j^{(i),(m-1)}$ is the j th interfering user's soft output at the $(m-1)$ th stage. That is, $\sum_{j \neq k} w_{jk}^{(i),(m)} \rho_{jk}^{(i)} y_j^{(i),(m-1)}$ is the weighted MAI estimate on the i th subcarrier in stage- m for the desired user k . Accordingly, the m th stage interference cancelled output on the i th subcarrier for the desired user k , $y_k^{(i),(m)}$, is given by

$$y_k^{(i),(m)} = y_k^{(i),(1)} - \sum_{j=1, j \neq k}^K w_{jk}^{(i),(m)} \rho_{jk}^{(i)} y_j^{(i),(m-1)}. \quad (6)$$

Note that both the conventional LPIC as well as the MF detector become special cases of the above weighted LPIC for $w_{jk}^{(i),(m)} = 1, \forall i, j, m$ and $w_{jk}^{(i),(m)} = 0, \forall i, j, m$, respectively. All the subcarrier outputs of the desired user are then coherently combined to get the combined output, $y_k^{(m)}$, as

$$y_k^{(m)} = \sum_{i=1}^M \left(h_k^{(i)} \right)^* y_k^{(i), (m)}. \quad (7)$$

The bit decision at the m -th stage output is then obtained as

$$\hat{b}_k^{(m)} = \operatorname{sgn} \left(\operatorname{Re} \left(y_k^{(m)} \right) \right). \quad (8)$$

The optimum choice of the weights $w_{jk}^{(i),(m)}$ can be made based on maximizing the average SIR at the combined output. However, the instantaneous SIR expression at the combined output from the multiple carriers in the system is such that the unconditioning on the fade variables to obtain the average SIR in closed-form is difficult. Therefore, we adopt an alternate approach where we choose those weights which maximize the average SIR expressions on individual subcarriers (rather than maximizing the average combined output SIR).

Average SIR at the 2nd Stage i th Subcarrier Output: Following similar analytical steps in [11], we obtain an exact closed-form expression for the average SIR at the 2nd stage interference cancelled output on the i th subcarrier of the desired user k , $\overline{SIR}_k^{(i), (2)}$, as follows.

From (6), the weighted interference cancelled output of the second stage (i.e., $m = 2$) for the desired user k on the i th subcarrier can be written as

$$y_k^{(i),(2)} = A_k b_k h_k^{(i)} \left(1 - \sum_{j=1, j \neq k}^K w_{jk}^{(i),(2)} (\rho_{jk}^{(i)})^2 \right) + I_{(i,2)} + N_{(i,2)}, \quad (9)$$

where

$$\begin{aligned} I_{(i,2)} &= \sum_{j=1, j \neq k}^K \left(1 - w_{jk}^{(i),(2)} \right) A_j b_j h_j^{(i)} \rho_{jk}^{(i)} \\ &\quad - \sum_{j=1, j \neq k}^K w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \sum_{\substack{l=1 \\ l \neq j, k}}^K \rho_{lj}^{(i)} A_l b_l h_l^{(i)}, \quad (10) \end{aligned}$$

$$N_{(i,2)} = n_k^{(i)} - \sum_{j=1, j \neq k}^K w_{jk}^{(i),(2)} \rho_{jk}^{(i)} n_j^{(i)}. \quad (11)$$

The terms $I_{(i,2)}$ and $N_{(i,2)}$ in (9) represent the interference and noise terms introduced in the 2nd stage i th subcarrier output due to imperfect cancellation in using the soft output values from the first (i.e., MF) stage. Since h 's and n 's are independent complex Gaussian, both $I_{(i,2)}$ and $N_{(i,2)}$ are linear combinations of Gaussian r.v.'s with zero mean. The variances of $I_{(i,2)}$ and $N_{(i,2)}$ can hence be obtained as follows.

$$\begin{aligned} \sigma_{N_{(i,2)}}^2 &= E[N_{(i,2)} N_{(i,2)}^*] = 2\sigma^2 \left(1 - 2 \sum_{j=1, j \neq k}^K w_{jk}^{(i),(2)} (\rho_{jk}^{(i)})^2 \right. \\ &\quad \left. + \sum_{\substack{l=1 \\ l \neq k}}^K w_{lk}^{(i),(2)} \rho_{lk}^{(i)} \sum_{\substack{j=1 \\ j \neq k}}^K w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \rho_{jl}^{(i)} \right), \quad (12) \end{aligned}$$

where we have used $E[n_k^{(i)} (n_j^{(i)})^*] = 2\sigma^2 \rho_{kj}^{(i)}$, for $j \neq k$ and $2\sigma^2$ for $j = k$.

To derive $\sigma_{I_{(i,2)}}^2$, note that $I_{(i,2)}$ in (10) can be rearranged in the form

$$I_{(i,2)} = \sum_{l=1, l \neq k}^K A_l b_l h_l^{(i)} \left(\left(1 - w_{lk}^{(i),(2)} \right) \rho_{lk}^{(i)} - \sum_{\substack{j=1 \\ j \neq k, l}}^K w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \rho_{jl}^{(i)} \right), \quad (13)$$

and hence $\sigma_{I_{(i,2)}}^2 = E[I_{(i,2)} I_{(i,2)}^*]$ can be obtained as

$$\sigma_{I_{(i,2)}}^2 = \sum_{l=1, l \neq k}^K 2A_l^2 \left(\left(1 - w_{lk}^{(i),(2)} \right) \rho_{lk}^{(i)} - \sum_{\substack{j=1 \\ j \neq k, l}}^K w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \rho_{jl}^{(i)} \right)^2. \quad (14)$$

From (6), (12), and (14) the average SIR at the 2nd stage i th subcarrier output can be written as

$$\overline{SIR}_k^{(i),(2)} = \frac{2A_k^2 \left(1 - \sum_{j=1, j \neq k}^K w_{jk}^{(i),(2)} (\rho_{jk}^{(i)})^2 \right)^2}{\sigma_{I_{(i,2)}}^2 + \sigma_{N_{(i,2)}}^2}. \quad (15)$$

Weights for 2nd Stage i th Subcarrier in Closed-form: The optimum values of $w_{jk}^{(i),(2)}$, $j = 1, 2, \dots, K$, $j \neq k$ can be found by numerically maximizing the average SIR expression in (15). However, the time complexity of this numerical optimization is large for large K . A less complex optimization is possible if all other users' interference is weighed equally (yet optimally in terms of maximizing the average SIR) in a given stage, i.e., all other users' interference is weighed by the same weight $w_k^{(i),(2)}$. Indeed, the optimum weights for

this scheme, $w_{k,opt}^{(i),(2)}$, can be obtained in closed-form, by differentiating (15) w.r.t $w_k^{(i),(2)}$ and equating to zero, as

$$w_{k,opt}^{(i),(2)} = \frac{c_1(1 - a_1) + e_1}{-a_1(c_1 + e_1) + c_1 + d_1 + 2e_1 - \sigma^2(a_1^2 - f_1)}, \quad (16)$$

where

$$\begin{aligned} a_1 &= \sum_{\substack{j=1 \\ j \neq k}}^K \left(\rho_{jk}^{(i)} \right)^2, \quad c_1 = \sum_{\substack{l=1 \\ l \neq k}}^K A_l^2 \left(\rho_{lk}^{(i)} \right)^2, \\ d_1 &= \sum_{l=1}^K A_l^2 \left(\sum_{\substack{j=1 \\ j \neq k, l}}^K \rho_{jk}^{(i)} \rho_{lj}^{(i)} \right)^2, \quad f_1 = \sum_{j=1}^K \rho_{jk}^{(i)} \sum_{\substack{l=1 \\ l \neq k}}^K \rho_{lj}^{(i)} \rho_{lk}^{(i)}, \\ e_1 &= \sum_{l=1}^K A_l^2 \rho_{lk}^{(i)} \sum_{\substack{j=1 \\ j \neq k, l}}^K \rho_{jk}^{(i)} \rho_{lj}^{(i)}. \end{aligned}$$

Note: Following similar steps in the above for the 2nd stage, exact closed-form expressions for the average SIR and weights on the i th subcarrier for the 3rd stage (i.e., $m = 3$) can be obtained [11]. Similar derivations can be carried out for stages beyond the third stage ($m > 3$) as well. However, we have restricted our derivation only up to the 3rd stage as most cancellation benefit is found to be realized with $m = 3$, and adding more stages typically results in marginal improvement in performance with added complexity.

Interference cancellation is done on each subcarrier using the optimum weights derived above. The interference cancelled outputs on all the subcarriers are then combined, and the combined signal output vector, $\hat{\mathbf{y}}_{wlpic}$, can be written as

$$\hat{\mathbf{y}}_{wlpic} = \sum_{i=1}^M (\mathbf{H}^{(i)})^H (\mathbf{y}^{(i),(1)} - \mathbf{W}^{(i),(m)} (\mathbf{C}^{(i)} - \mathbf{I}) \mathbf{y}^{(i),(m-1)}), \quad (17)$$

where $\mathbf{y}^{(i),(1)}$ is the output vector of the 1st stage (i.e., MF stage) given by (1), \mathbf{I} denotes the identity matrix of size $K \times K$, and $\mathbf{W}^{(i),(m)}$ denotes the optimum weight matrix on the i th subcarrier for the m th stage, given by

$$\mathbf{W}^{(i),(m)} = \text{diag} \left\{ w_{1,opt}^{(i),(m)}, w_{2,opt}^{(i),(m)}, \dots, w_{K,opt}^{(i),(m)} \right\}. \quad (18)$$

The bit estimate for the k th user at the m th stage output is

$$\hat{b}_k = \text{sgn} \left(\mathbf{e}_k^T \text{Re}(\hat{\mathbf{y}}_{wlpic}) \right), \quad (19)$$

where \mathbf{e}_k is a unit vector with a 1 in the k th position and 0 otherwise.

IV. BER ANALYSIS

In this section, using an approach involving the characteristic function of the decision variable, we derive exact BER expressions for the desired user at the outputs of the different stages of the weighted LPIC scheme for MC DS-CDMA proposed in the previous section. Taking user 1 as the desired user, the bit decision for the desired user is given by

$$\hat{b}_1 = \text{sgn} \left(\mathbf{e}_1^T \text{Re}(\hat{\mathbf{y}}_{wlpic}) \right) = \text{sgn} \left(\text{Re} \left(\mathbf{y}_1^{(m)} \right) \right). \quad (20)$$

$\mathbf{B}^{(i)}(q, j) =$	
$\left(1 - w_1^{(i), (m)} \sum_{l=2}^K (\rho_{l1}^{(i)})^2\right) A_1 b_1$	for $q = j = 1$
$\frac{1}{2} \left[(1 - w_1^{(i), (m)}) \rho_{1j}^{(i)} A_j - w_1^{(i), (m)} A_j \sum_{l=1, l \neq j, 1}^K \rho_{l1}^{(i)} \rho_{lj}^{(i)} \right] b_j$	for $q = 1, j = 2, \dots, K$
$\frac{1}{2} \left[(1 - w_1^{(i), (m)}) \rho_{1q}^{(i)} A_q - w_1^{(i), (m)} A_q \sum_{l=1, l \neq q, 1}^K \rho_{l1}^{(i)} \rho_{lq}^{(i)} \right] b_q$	for $q = 2, \dots, K, j = 1$
$\frac{1}{2}$	for $q = K + 1, j = 1$, and $j = K + 1, q = 1$
$-\frac{1}{2} w_1^{(i), (m)} \rho_{(q-K)1}^{(i)}$	for $q = K + 2, \dots, 2K, j = 1$
$-\frac{1}{2} w_1^{(i), (m)} \rho_{(j-K)1}^{(i)}$	for $j = K + 2, \dots, 2K, q = 1$
0	otherwise

TABLE I

$\mathbf{B}^{(i)}$ MATRIX OF SIZE $2K \times 2K$ FOR THE 2ND STAGE OF THE WLPIC.

We note that the real part of $y_1^{(m)}$ (i.e., the decision variable of the desired user at the m th stage output) can be written in the form [14]

$$\text{Re}(y_1^{(m)}) = \mathbf{V}^H \mathbf{Q} \mathbf{V}, \quad (21)$$

where the vector \mathbf{V} is given by

$$\mathbf{V}_{2MK \times 1} = \begin{bmatrix} (\mathbf{h}_1^{(1)})^* & \dots & (\mathbf{h}_K^{(1)})^* & (\mathbf{n}_1^{(1)})^* & \dots & (\mathbf{n}_K^{(1)})^* \\ \dots & (\mathbf{h}_1^{(M)})^* & \dots & (\mathbf{h}_K^{(M)})^* & (\mathbf{n}_1^{(M)})^* & \dots & (\mathbf{n}_K^{(M)})^* \end{bmatrix}^H,$$

and the \mathbf{Q} matrix is given by

$$\mathbf{Q}_{2MK \times 2MK} = \begin{bmatrix} \mathbf{B}^{(1)} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots & \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{B}^{(M)} \end{bmatrix}. \quad (22)$$

The $\mathbf{B}^{(i)}$ matrix of size $2K \times 2K$ for the 2nd (i.e., $m = 2$) and 3rd (i.e., $m = 3$) stages of the weighted LPIC can be written as shown in Tables I and II, respectively. The correlation matrix of \mathbf{V} is given by

$$\mathbf{L}_{2MK \times 2MK} = \begin{bmatrix} \mathbf{L}^{(1)} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots & \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{L}^{(M)} \end{bmatrix}, \quad (23)$$

where the $\mathbf{L}^{(i)}$ matrix of size $2K \times 2K$ is given by

$$\mathbf{L}_{2K \times 2K}^{(i)} = \begin{bmatrix} 2\mathbf{I}_{K \times K} & \mathbf{0} \\ \mathbf{0} & 2\sigma^2 \mathbf{C}^{(i)} \end{bmatrix}. \quad (24)$$

The characteristic function of (21) can be obtained as [14]

$$\psi(i\omega) = \prod_{j=1}^P \frac{1}{1 - i\omega\lambda_j}, \quad (25)$$

where λ_j 's are the eigen values of the matrix \mathbf{LQ} and P is the number of eigen values of \mathbf{LQ} .

The bit error analysis of the decision rule in (20) can be carried out by conditioning with respect to the transmitted bits (in matrix \mathbf{Q}) and the channel coefficients (in vector \mathbf{V}). Also, the binary coefficients corresponding to the transmitted bits in the above can be dropped since they do not affect the distribution of the decision variable. Hence, from (25), we get the average bit error probability as

$$P_e^{(m)} = \frac{1}{2\pi} \int_{-\infty}^0 \int_{-\infty}^{\infty} \left(\prod_{j=1}^P \frac{1}{1 - i\omega\lambda_j} \right) e^{-i\omega x} d\omega dx. \quad (26)$$

Ignoring the positions where $\lambda_j = 0$ since the product term is unaltered, the above integral can be evaluated by splitting the product term in (26) into partial fractions. Let the number of distinct eigen values be Z . Let the multiplicity of eigen value λ_l be \mathcal{K}_l . Splitting the product term into partial fractions, we get

$$P_e = \frac{1}{2\pi} \int_{-\infty}^0 \int_{-\infty}^{\infty} \sum_{l=1}^Z \sum_{\substack{j=1 \\ \lambda_l \neq 0}}^{\mathcal{K}_l} \frac{A_l^{(j)}}{(1 - i\omega\lambda_l)^j} e^{-i\omega x} d\omega dx. \quad (27)$$

Using Eqns. 3.382 ET 1 118(3) and 118(4) in [15], it can be shown that

$$P_e = \sum_{l=1}^Z \sum_{\substack{j=1 \\ \lambda_l < 0}}^{\mathcal{K}_l} A_l^{(j)}. \quad (28)$$

For the case of distinct eigen values, $A_i^{(1)}$'s in the above equation can be calculated as

$$A_i^{(1)} = \prod_{\substack{j=1 \\ j \neq i \\ \lambda_j \neq 0}}^P \frac{\frac{1}{\lambda_j}}{\frac{1}{\lambda_j} - \frac{1}{\lambda_i}}. \quad (29)$$

We point out that computing the BER in (28) requires the computation of the eigen values of the matrix \mathbf{LQ} .

A. BER Expressions for MF and Decorrelating Detectors

We obtain exact BER expressions for the MF detector and the decorrelating detector for the considered MC DS-CDMA system, again using the $\text{Re}(y_1^{(m)}) = \mathbf{V}^H \mathbf{Q} \mathbf{V}$ formulation.

1) *MF Detector:* In the case of the MF detector (i.e., $m = 1$), the combined signal output vector is given by

$$\hat{\mathbf{y}}_{mf} = \sum_{i=1}^M (\mathbf{H}^{(i)})^H \mathbf{y}^{(i)}, \quad (30)$$

where $\mathbf{y}^{(i)}$ is given by (1). For this detector, the $\mathbf{B}^{(i)}$ matrix in (22) is given by

$$\mathbf{B}_{mf}^{(i)} = \begin{bmatrix} A_1 b_1 & \frac{A_2 \rho_{12}^{(i)}}{2} & \dots & \frac{A_K \rho_{1K}^{(i)}}{2} & \frac{1}{2} & 0 & \dots & 0 \\ \frac{A_2 \rho_{12}^{(i)}}{2} & 0 & \dots & & & & & \\ 0 & \ddots & & 0 & & & & \\ \frac{A_K \rho_{1K}^{(i)}}{2} & 0 & \dots & \dots & 0 & & 0 & \\ \frac{1}{2} & 0 & \dots & \dots & 0 & & 0 & \\ \vdots & \ddots & & & & & & \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \end{bmatrix}. \quad (31)$$

$\mathbf{B}^{(i)}(q, j) =$	$\begin{aligned} & \left[1 - w_1^{(i), (m)} \left(\sum_{l=2}^K (\rho_{l1}^{(i)})^2 (1 - w_l^{(i), (m-1)}) \right. \right. \\ & \left. \left. - \sum_{l=2}^K \sum_{\substack{r=1, \\ r \neq l, 1}}^K \rho_{l1}^{(i)} w_l^{(i), (m)} \rho_{rl}^{(i)} \rho_{r1}^{(i)} \right) \right] A_1 b_1 \end{aligned}$	for $q = j = 1$
	$\begin{aligned} & \frac{1}{2} A_q b_q \left[w_1^{(i), (m-1)} \sum_{\substack{l=1, \\ l \neq q, 1}}^K \sum_{\substack{r=1, \\ r \neq q, l}}^K w_l^{(i), (m-1)} \rho_{l1}^{(i)} \rho_{lr}^{(i)} \rho_{rq}^{(i)} \right. \\ & \left. - w_1^{(i), (m)} \sum_{\substack{l=2, \\ l \neq q}}^K \rho_{l1}^{(i)} \rho_{lq}^{(i)} (1 - w_l^{(i), (m-1)}) \right. \\ & \left. + \rho_{q1}^{(i)} (1 - w_1^{(i), (m)} (1 - w_q^{(i), (m-1)} \sum_{\substack{l=1, \\ l \neq q}} (\rho_{lq}^{(i)})^2)) \right] \end{aligned}$	for $q = 2, \dots, K, j = 1$
	$\begin{aligned} & \frac{1}{2} A_j b_j \left[w_1^{(i), (m-1)} \sum_{\substack{l=1, \\ l \neq j, 1}}^K \sum_{\substack{r=1, \\ r \neq j, l}}^K w_l^{(i), (m-1)} \rho_{l1}^{(i)} \rho_{lr}^{(i)} \rho_{rj}^{(i)} \right. \\ & \left. - w_1^{(i), (m)} \sum_{\substack{l=2, \\ l \neq j}}^K \rho_{l1}^{(i)} \rho_{lj}^{(i)} (1 - w_l^{(i), (m-1)}) \right. \\ & \left. + \rho_{j1}^{(i)} (1 - w_1^{(i), (m)} (1 - w_q^{(i), (m-1)} \sum_{\substack{l=1, \\ l \neq j}} (\rho_{lj}^{(i)})^2)) \right] \end{aligned}$	for $q = 2, \dots, K, j = 1$
	$\frac{1}{2} \left[1 + w_1^{(i), (m)} \sum_{l=2}^K (\rho_{l1}^{(i)})^2 w_l^{(i), (m-1)} \right]$	for $q = K + 1, j = 1$, and $j = K + 1, q = 1$
	$\frac{1}{2} w_1^{(i), (m)} \left[- \rho_{(q-K)1}^{(i)} + \sum_{\substack{l=1, \\ l \neq (q-K), 1}}^K \rho_{l1}^{(i)} \rho_{l(q-K)}^{(i)} w_l^{(i), (m-1)} \right]$	for $q = K + 2, \dots, 2K$, $j = 1$
	$\frac{1}{2} w_1^{(i), (m)} \left[- \rho_{(j-K)1}^{(i)} + \sum_{\substack{l=1, \\ l \neq (j-K), 1}}^K \rho_{l1}^{(i)} \rho_{l(m-K)}^{(i)} w_l^{(i), (m-1)} \right]$	for $j = K + 2, \dots, 2K$, $q = 1$
	0	otherwise

TABLE II
 $\mathbf{B}^{(i)}$ MATRIX OF SIZE $2K \times 2K$ FOR THE 3RD STAGE OF THE WLPIC.

Using the $\mathbf{B}^{(i)}$ matrices, $i = 1, 2, \dots, M$, in the above, the corresponding eigen values and the probability of bit error in (28) can be computed for the MF detector.

2) *Decorrelating Detector*: In the case of the decorrelating detector, the decorrelated and combined signal output vector is given by

$$\hat{\mathbf{y}}_{dc} = \sum_{i=1}^M (\mathbf{H}^{(i)})^H (\mathbf{C}^{(i)})^{-1} \mathbf{y}^{(i)}. \quad (32)$$

For this decorrelating detector, the $\mathbf{B}^{(i)}$ matrix is given by

$$\mathbf{B}^{(i)}(q, j) = \begin{cases} A_1 b_1, & q = j = 1 \\ \frac{(\mathbf{C}^{(i)})_{(q-K, 1)}^{-1}}{2}, & K + 1 \leq q \leq 2K, j = 1 \\ \frac{(\mathbf{C}^{(i)})_{(j-K, 1)}^{-1}}{2}, & q = 1, K + 1 \leq j \leq 2K \\ 0, & \text{otherwise,} \end{cases}$$

using which the BER in (28) can be computed.

V. RESULTS AND DISCUSSION

In this section, we present numerical results of the BER performance of the proposed WLPIC scheme for MC DS-CDMA. We computed the analytical BER performance for the 2nd and 3rd stages of the weighted LPIC for different number of subcarriers, M , and number of users K . We used random binary sequences of length N as the spreading sequences on

each subcarrier. In all the performance plots NM is taken to be 256 (i.e., the number of chips per bit on each subcarrier is chosen such that the total system bandwidth is fixed regardless of the number of subcarriers used). We take the number of subcarriers M to be 1, 2, and 4. We also keep the total transmit power to be the same irrespective of the number of subcarriers used. BER performance is computed in near-far scenarios where some users transmit with higher powers than the desired user. We take user 1 as the desired user.

In Fig. 3, we plot the bit error performance of the desired user at the 2nd and 3rd stage outputs of the WLPIC for $K = 16$ users, $M = 2$ subcarriers, and $N = 128$ chips per bit on each subcarrier, in a near-far scenario where users 2, 4, and 5 are received with 10 times more power than the desired user 1 (i.e., $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$). The performance of the 2nd and 3rd stages of the conventional LPIC (CLPIC), the MF detector, and the decorrelating (DC) detector are also shown for comparison. It is noted that in CLPIC, all weights are unity (i.e., weights are not optimized to maximize the subcarrier SIR). From Fig. 3, it can be observed that the proposed WLPIC clearly performs better than the MF detector as well as the CLPIC; this is expected since in the MF detector there is no cancellation, whereas, in the CLPIC there is cancellation but the weights are not optimum. Also, the proposed WLPIC is found to perform close to the DC detector. We have also evaluated the BER performance through simulations and compared with the analytical results. The analytical and simulation results matched as there are no approximations involved in the analysis.

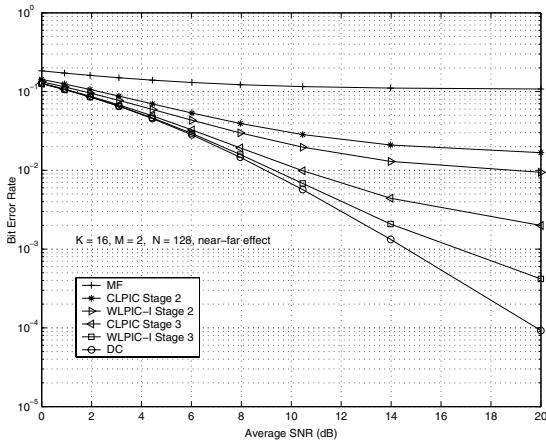


Fig. 3. BER vs average SNR performance at the 2nd and 3rd stage outputs of the weighted LPIC scheme. $K = 16, M = 2, N = 128$. Near-far effect: $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. Random spreading sequences.

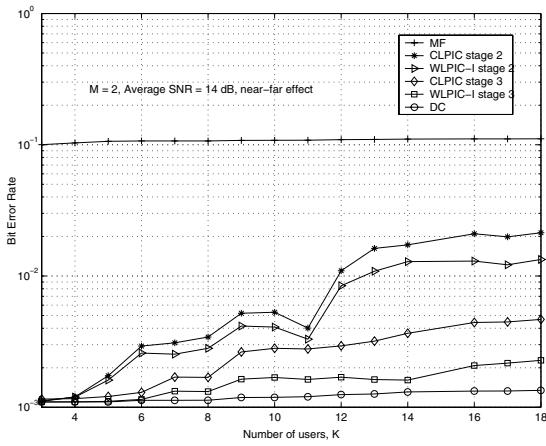


Fig. 4. BER vs number of users, K , performance of the 2nd and 3rd stage outputs of the weighted LPIC scheme. $M = 2, N = 128$, average SNR = 14 dB. Near-far effect: $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. Random spreading sequences.

In Fig. 4 we present the performance comparison of various detectors as a function of number of users, K , for $M = 2, N = 128$, average SNR = 14 dB with near-far effect such that $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. Here again, the WLPIC scheme clearly performs better than the MF detector as well as the CLPIC and quite close to the DC detector. Figure 5 shows the performance of the WLPIC scheme and the DC detector for different number of subcarriers, $M = 1, 2, 4$ for $NM = 256$, average SNR=10 dB with near-far effect. The performance of $M = 4$ is better than $M = 2$ and $M = 1$ because of frequency diversity effect.

VI. CONCLUSIONS

We presented the design and BER analysis of a weighted LPIC scheme for multicarrier DS-CDMA systems. In the proposed WLPIC scheme, the MAI on the individual subcarriers are estimated based on soft outputs, scaled and cancelled, and the interference cancelled outputs on different subcarriers are combined to form the final decision statistic. We derived exact closed-form expressions for the subcarrier output average SIRs and the weights that maximize these subcarrier SIRs. Using the characteristic function of the deci-

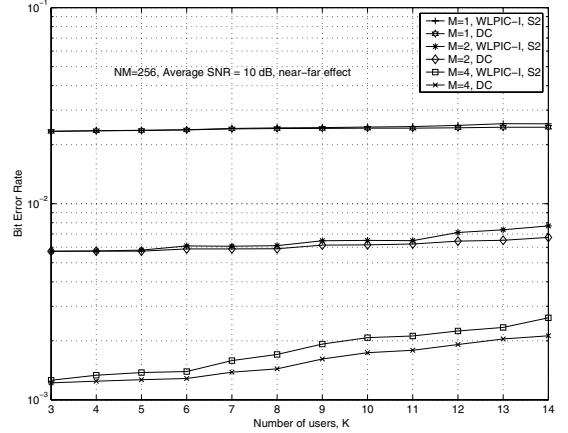


Fig. 5. BER vs number of users, K , performance at the 2nd stage output of the weighted LPIC scheme for different number of subcarriers, $M = 1, 2, 4$, $NM = 256$, average SNR = 10 dB. Near-far effect: $A_2/A_1 = A_4/A_1 = A_5/A_1 = 3$. Random spreading sequences.

sion variable, we derived exact analytical expressions for the BER at the output of the different stages of the weighted LPIC scheme for MC DS-CDMA. We also derived exact BER expressions for the matched filter and decorrelating detectors for the considered MC DS-CDMA system. We showed that the proposed weighted LPIC scheme performs better than the MF detector and the conventional LPIC scheme (where the weights are unity), and close to the decorrelating detector.

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