Low-Complexity Near-Optimal Signal Detection in Underdetermined Large-MIMO Systems

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Abstract*—In this paper, we consider signal detection in $n_t \times n_r$ underdetermined MIMO (UD-MIMO) systems, where $i)$ $n_t > n_r$ with a overload factor $\alpha = \frac{n_t}{n_r} > 1$, $ii)$ n_t symbols are transmitted per channel use through spatial multiplexing, and $iii)$ n_t, n_r are large (in the range of tens). A low-complexity detection algorithm based on reactive tabu search is considered. A variable threshold based stopping criterion is proposed which offers near-optimal performance in large UD-MIMO systems at low complexities. A lower bound on the maximum likelihood (ML) bit error performance of large UD-MIMO systems is also obtained for comparison. The proposed algorithm is shown to achieve BER performance close to the ML lower bound within 0.6 dB at an uncoded BER of 10^{-2} in 16×8 V-BLAST UD-MIMO system with 4-QAM (32 bps/Hz). Similar near-ML performance results are shown for 32×16, 32×24 V-BLAST UD-MIMO with 4-QAM/16-QAM as well. A performance and complexity comparison between the proposed algorithm and the λ-generalized sphere decoder (λ-GSD) algorithm for UD-MIMO shows that the proposed algorithm achieves almost the same performance of λ-GSD but at a significantly lesser complexity.

Keywords – *Underdetermined MIMO systems, near-optimal performance, low-complexity detection, tabu search, generalized sphere decoder.*

I. Introduction

MIMO wireless systems with large number of antennas are getting increased attention because of their high spectral efficiency advantage [1],[2]. Gigabit transmissions at high spectral efficiencies (tens of bps/Hz) using large number of antennas are being considered in emerging wireless standards; e.g., IEEE 802.11ac (Gigabit WiFi) and LTE-A consider $8 \times n$, $n \leq 8$ and $16 \times m$, $m \leq 16$ MIMO architectures. With spatial multiplexing, such systems will become underdetermined, where the vector of observed statistics lie in a space of dimension smaller than the number of unknowns. Our focus in this paper is on achieving near-optimal detection performance in underdetermined MIMO (UD-MIMO) systems with large number of antennas.

We consider underdetermined $n_t \times n_r$ MIMO systems, where $i)$ $n_t > n_r$ with a overload factor $\alpha = \frac{n_t}{n_r} > 1$, $ii)$ n_t symbols are transmitted per channel use through spatial multiplexing, and $iii)$ n_t, n_r are large (in the range of tens). Achieving near-optimal detection in such underdetermined systems is of interest. Detection in UD-MIMO systems with small number of antennas has been reported in the literature [3]-[7]. In particular, the generalized sphere decoding (GSD), first proposed by Damen *et al* in [3], solved the closet lattice point problem with a complexity exponential in the difference between the number of equations and unknowns. Subsequently, faster versions of GSD were proposed by Dayal and Varanasi in [4], and Yang *et al* in [5]. In [6], Wong and Paulraj have proposed a geometrical approach to achieve near maximum-likelihood (ML) performance in UD-MIMO systems; near-ML BER performance was shown for 2×1, 3×1, 3×2 and 4×3 MIMO. More recently, in [7], Wang and Le-Ngoc proposed an approach (termed as λ-GSD) where the underdetermined problem is transformed into full-column-rank one so that standard SD can be directly applied on the transformed problem; here again, the systems considered are small (up to 6 transmit antennas).

In this paper, we propose a low-complexity detection algorithm that achieves near-ML performance in UD-MIMO systems with tens of antennas. The algorithm is based on reactive tabu search (RTS) [8],[9] in conjunction with a threshold based stopping criterion. In order to compare its BER performance with that of ML, we develop a low complexity lower bound on ML performance. The bound is important for comparison purposes because obtaining exact ML performance through either brute-force exhaustive search or sphere decoding is prohibitively complex for more than 32 real dimensions. Our simulation results show that the proposed algorithm achieves BER performance close to the ML lower bound for 16×8, 16×12, 32×16, 32×24 V-BLAST UD-MIMO with 4-QAM/16-QAM. We also present performance and complexity comparison between the proposed algorithm and the λ-GSD algorithm in [7]. Because of its low complexity, the proposed algorithm, referred to as enhanced RTS (ERTS) algorithm, scales well for large number of antennas whereas the λ-GSD does not scale well due to its high complexity.

II. System Model

Consider an underdetermined V-BLAST MIMO system with n_t transmit and n_r receive antennas, where $n_t > n_r$ and the overload factor $\alpha = \frac{n_t}{n_r} > 1$. The transmitted symbols take values from a modulation alphabet $\mathbb{A}$. Let $\mathbf{x}_c \in \mathbb{A}^{n_t}$ denote the transmitted vector. Let $\mathbf{H}_c \in \mathbb{C}^{n_r \times n_t}$ denote the channel gain matrix, whose entries are assumed to be i.i.d. Gaussian with zero mean and unit variance. The received vector $\mathbf{y}_c$ is

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{x}_c + \mathbf{n}_c, \quad (1)$$

where $\mathbf{n}_c$ is the noise vector whose entries are modeled as i.i.d. $\mathbb{CN}(0, \sigma^2)$. The goal is to obtain an estimate of $\mathbf{x}_c$, given $\mathbf{y}_c$ and $\mathbf{H}_c$. We assume $\mathbf{H}_c$ is known at the receiver. The ML detection rule is given by

$$\widehat{\mathbf{x}}_{ML} = \arg \min_{\mathbf{x}_c \in \mathbb{A}^{n_t}} \|\mathbf{y}_c - \mathbf{H}_c \mathbf{x}_c\|^2 = \arg \min_{\mathbf{x}_c \in \mathbb{A}^{n_t}} \phi(\mathbf{x}_c), \quad (2)$$

where $\phi(\mathbf{x}_c) \stackrel{\triangle}{=} \mathbf{x}_c^H \mathbf{H}_c^H \mathbf{H}_c \mathbf{x}_c - 2\Re\left(\mathbf{y}_c^H \mathbf{H}_c \mathbf{x}_c\right)$ is the ML cost. We will use a real-valued system model corresponding to (1), i.e., use the system model $\mathbf{y} = \mathbf{H}\,\mathbf{x} + \mathbf{n}$, where

$$\mathbf{H} = \begin{bmatrix} \Re(\mathbf{H}_c) & -\Im(\mathbf{H}_c) \\ \Im(\mathbf{H}_c) & \Re(\mathbf{H}_c) \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \Re(\mathbf{y}_c) \\ \Im(\mathbf{y}_c) \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} \Re(\mathbf{x}_c) \\ \Im(\mathbf{x}_c) \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} \Re(\mathbf{n}_c) \\ \Im(\mathbf{n}_c) \end{bmatrix}. \tag{3}$$

III. Proposed Algorithm for UD-MIMO Detection

Tabu search, attributed to F. W. Glover [8],[9], is a mathematical optimization method that can be used to solve combinatorial optimization problems. It is a heuristic method which is found to be very effective when the problem size becomes large to an extent that the computational burden of finding the exact solution becomes prohibitive given its combinatorial complexity. Tabu search methods have yielded impressive successes in a wide range of application domains including multiuser detection in CDMA [10] and MIMO detection [11],[12]. In [11],[12] reactive tabu search (RTS) is shown to achieve good performance in V-BLAST MIMO systems with tens of antennas, but only in fully determined and overdetermined scenarios (i.e., for $n_t \leq n_r$). However, its performance in UD-MIMO systems is far from optimal. Here, we propose an enhanced RTS algorithm which employs a variable threshold based stopping criterion to improve performance in UD-MIMO scenarios.

The RTS algorithm in [11] starts with an initial solution vector, defines a neighborhood around it (i.e., defines a set of neighboring vectors based on a neighborhood criteria), and moves to the best vector among the neighboring vectors (even if the best neighboring vector is worse, in terms of ML cost $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$, than the current solution vector); this allows the algorithm to escape from local minima. This process is continued for a certain number of iterations, after which the algorithm is terminated and the best among the solution vectors in all the iterations is declared as the final solution vector. MMSE solution vector is used as the initial vector. Alternately, running the RTS algorithm multiple times, each time with a different random initial vector, and choosing the best among the resulting solution vectors can be done [12].

A. Motivation for the Proposed Algorithm

The motivation for the proposed algorithm arises from the observed characteristics of the distribution of the ML cost of the output vector from the RTS algorithm in UD-MIMO when the output is incorrect/correct. We observed that the chances of the ML cost of the incorrect RTS output being low are more in underdetermined MIMO than in fully determined MIMO. This observation is illustrated in Figs. 1(a) and 1(b), where histograms of the ML cost of incorrect/correct output from RTS with random initial vector for 8×8 fully determined MIMO and 8×4 underdetermined MIMO are compared at an SNR of 10 dB.

In the algorithm in [12], a fixed threshold of $\Theta = n_r\sigma^2 + 2\sqrt{n_r\sigma^4}$ was used to compare with the RTS output ML cost for limiting the number of restarts. This fixed threshold in the stopping criterion is a good choice for the case of fully determined MIMO since the chances of the ML cost of incorrect RTS output being lower than Θ is very small, as can be seen in Fig. 1(a). However, since the incorrect outputs have lower ML costs than $\Theta = n_r\sigma^2 + 2\sqrt{n_r\sigma^4}$ with high probability in UD-MIMO (as can be seen in Fig. 1(b)), the resulting performance can suffer if this Θ value is used in UD-MIMO (we will see this in the BER plots later).

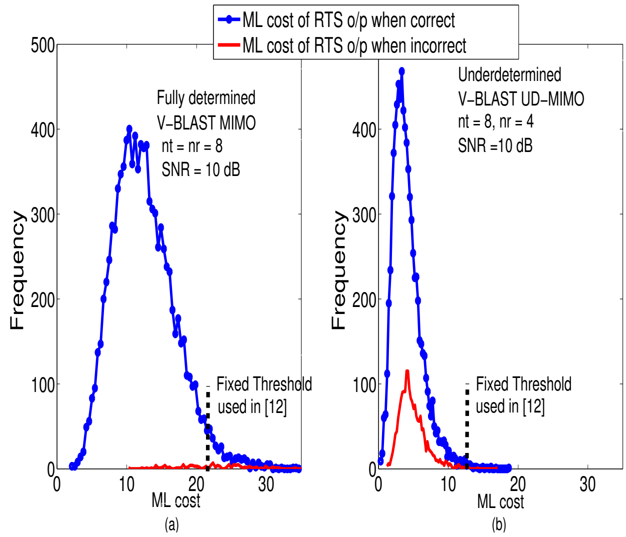

Fig. 1. Histograms of the ML cost of the output vector from RTS with random initial vector when output is incorrect/correct for a) 8×8 fully determined MIMO and b) 8×4 underdetermined MIMO. SNR = 10 dB.

One way to address the above problem is to fix Θ at a much lower value. But this would result in a large number of restarts leading to a significantly increased complexity. Clearly, to improve performance, we need more number of restarts when the RTS output is not trustworthy, i.e., when its ML cost lies in the region where the chances of incorrect decision is not small. So, an intuitive way to improve performance without paying much in complexity would be to start the algorithm with a low value of threshold and increase it in subsequent restarts. The threshold has to be upper-bounded as high values of threshold can cause decision errors. The above ideas are incorporated in the proposed algorithm, referred to as enhanced RTS (ERTS) algorithm, which is presented in the following subsection.

B. Proposed ERTS Algorithm

The following three parameters are defined to limit the number of RTS searches in the ERTS algorithm: MAX, p, $\Theta(n)$. The ERTS algorithm works as follows.

Initialize $n = 0$.

- ***Step 1:*** Increment n by 1. Choose a random initial vector. Run RTS algorithm in [11] using this initial vector. Obtain corresponding solution vector.
- ***Step 2:*** Check if n is less than MAX. If yes, go to Step 3; else go to Step 5.

- ***Step 3:*** If the minimum of the ML costs of the solution vectors obtained so far is less than $\Theta(n)$, then output the solution vector from Step 1 as the final solution vector and stop; else go to Step 4.
- ***Step 4:*** Let L denote the number of distinct solution vectors obtained from Step 1 so far. If $L/n \leq p$, go to Step 5; else go to Step 1.
- ***Step 5:*** Output the best (in terms of ML cost) among the solution vectors obtained so far and stop.

The threshold $\Theta(n)$ is varied in each restart as per the following equations:

$$\Theta(n) \;=\; n_r\sigma^2 + K(n)\sqrt{n_r\sigma^4}, \tag{4}$$

and

$$\begin{aligned} K(n) &= 0, && n \leq T \\ &= 0.5, && T < n \leq 2T \\ &= 1, && 2T < n \leq 3T \\ &= 1.5, && 3T < n \leq 4T \\ &= 2, && n > 4T, \end{aligned} \tag{5}$$

where $T = \lceil (5q(\alpha - 1)) \rceil$, $q = \log_2 M$ for M-QAM alphabet. The threshold comparison in Step 3 reduces the number of searches and hence the complexity. The reason for doing Step 4 is to reduce complexity in realizations where $\|\mathbf{n}\|^2$ happens to be greater than $\Theta(n)$. We have used $p = 0.05$ and MAX=500 in the simulations, which are found to result in good performance. In order to compare the performance achieved by the ERTS algorithm in large dimensions relative to ML performance, we propose to obtain a lower bound on ML performance as outlined in the following section.

IV. A Lower Bound on ML Performance

We obtain a lower bound on the ML bit error performance using the algorithms in [11] and [12] with suitable neighborhood definition and error counting, as follows.

- *Step a):* Run the RTS algorithm in [11] using the transmitted vector $\mathbf{x}$ as the initial vector, and obtain the output vector. Denote this output vector by $\mathbf{x}_A$.
- *Step b):* Run the algorithm in [12] with multiple restarts using random initial vectors and obtain the output vector. Denote this vector by $\mathbf{x}_B$.
- *Step c):* Define $\mathcal{N}_{\mathbf{x}}$ as the m-symbol neighborhood[1] of $\mathbf{x}$. Choose the best vector in $\mathcal{N}_{\mathbf{x}}$ which has the least ML cost. Denote this vector by $\mathbf{x}_N$.
- *Step d):* Choose the best vector among $\mathbf{x}_A$, $\mathbf{x}_B$, and $\mathbf{x}_N$ in terms of ML cost. Denote this vector as $\mathbf{x}_{out}$.

Error Counting for Bounding:

Let e_{out} denote the number of symbol errors in $\mathbf{x}_{out}$ compared to $\mathbf{x}$. For each realization in the simulations, $\mathbf{x}$, $\mathbf{x}_{out}$, and hence e_{out} are known. Also, let $\mathbf{x}_{ML}$ denote the true ML vector, and e_{ML} denote the number of symbol errors in $\mathbf{x}_{ML}$ (which we do not know, and seek to get a lower bound on). Note that the vector $\mathbf{x}_{out}$ may or may not lie in the m-symbol neighborhood of $\mathbf{x}$.

Since $\mathbf{x}_{out}$ has the least ML cost among all the tested vectors, if $\mathbf{x}_{out} \notin \mathcal{N}_{\mathbf{x}}$, then $\mathbf{x}_{ML} \notin \mathcal{N}_{\mathbf{x}}$. Also, by the definition of $\mathcal{N}_{\mathbf{x}}$, the number of errors in $\mathbf{x}_{out}$ and $\mathbf{x}_{ML}$ are lower bounded by $m+1$, i.e., $e_{out}, e_{ML} \geq m+1$. So, in the simulations, if $e_{out} \geq m+1$ in a given realization, then take e_{ML} as $m+1$ as a lower bound on the number symbol errors in the ML vector. On the other hand, if $\mathbf{x}_{out} \in \mathcal{N}_{\mathbf{x}}$, which implies that $e_{out} = k$, $1 \leq k \leq m$, then two cases are possible: 1) $\mathbf{x}_{out}$ is the ML vector, and 2) $\mathbf{x}_{out}$ is not the ML vector. In case 1) $e_{out} = e_{ML} = k$, and in case 2) $e_{out} = k$ and $\mathbf{x}_{ML}$ being outside $\mathcal{N}_{\mathbf{x}}$, $e_{ML} \geq m+1$. So, in the simulations, if $e_{out} = k$, $k \leq m$, then take e_{ML} as k as a lower bound. Lastly, if $e_{out} = 0$, then $\mathbf{x}_{out} = \mathbf{x}$ which may or may not be the ML vector; in such a realization, take $e_{ML} = 0$ as a lower bound. In summary, in the simulations,

- if $e_{out} = k$, $k \leq m$, then take e_{ML} as k, and
- if $e_{out} \geq m+1$, then take e_{ML} as $m+1$,

which results in a lower bound on ML symbol error performance. Since the number of symbol errors is a lower bound on the number of bit errors, it is a bit error bound as well.

A. Results on the ML Lower Bound

To illustrate the tightness of the proposed ML bound, we evaluated the proposed bound (outlined as above) as well as the ML performance (obtained through simulation of λ-GSD in [7]) for 16×8 V-BLAST UD-MIMO with 4-QAM. The results are shown in Fig. 2. Plots for the proposed bound are shown for $m = 1, 2, 3, 4$. In addition to the proposed bounds, we plot the ML lower bound in [11] for comparison. We note that the bound in [11] is obtained using RTS algorithm in [11] alone, whereas the proposed bound improves upon it by using the restart algorithm in [12] (*Step b*) as well as the m-symbol neighborhood of the transmitted vector (*Step c*). It can be seen that the proposed bound gets increasingly tighter as m increases. At 10^{-2} BER, the proposed bound for $m = 4$ is close to within 0.5 dB from the ML performance, whereas the bound in [11] is about 1.7 dB away from ML performance. This illustrates the improved tightness of the proposed bound.

Henceforth, we will use the proposed ML lower bound with $m = 4$ for comparison with the performance of different detection schemes for V-BLAST UD-MIMO with large number of antennas (e.g., $n_t = 16, 32$) reported next.

V. Performance and Complexity Results

We evaluated the BER performance and complexity of the proposed ERTS algorithm through simulations. Perfect channel state information is assumed at the receiver. The following RTS parameters are used in the simulations: $max\_rep = 75$, $max\_iter = 300$, $\beta = 0.1$, $P_0 = 2$ for 4-QAM, and $max\_rep = 250$, $max\_iter = 1000$, $\beta = 0.01$, $P_0 = 2$ for 16-QAM. We compare the performance and complexity of the ERTS algorithm with those of λ-GSD in [7].

[1] A vector is said to be in the m-symbol neighborhood of $\mathbf{x}$, if it differs from $\mathbf{x}$ in i, $i \leq m$ coordinates.

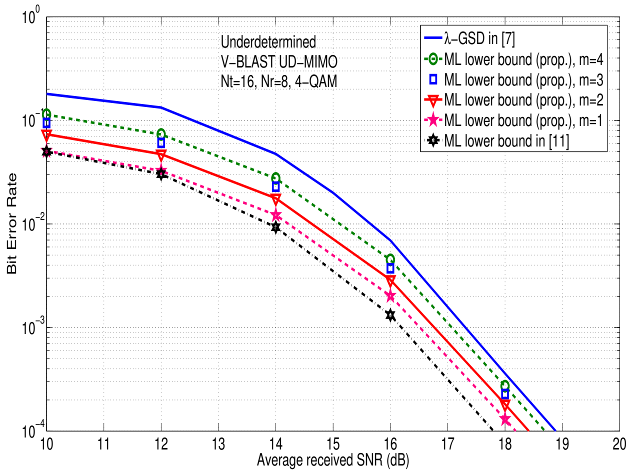

Fig. 2. Proposed ML lower bound for 16×8 V-BLAST UD-MIMO with 4-QAM for $m = 1, 2, 3, 4$. ML performance obtained through λ-GSD in [7] and ML lower bound in [11] are also shown.

In Fig. 3, we present the BER of i) proposed ERTS algorithm, ii) restart RTS algorithm in [12], iii) λ-GSD in [7], and iv) proposed lower bound in a 16×8 V-BLAST UD-MIMO system with 4-QAM. It is observed that ERTS algorithm performs better than the algorithm in [12] by about 1 dB at a BER of 10^{-2}. Also, ERTS performs almost the same as λ-GSD (within 0.2 dB) at 10^{-2} BER. It is interesting to note that the performance of both ERTS and λ-GSD are close to the ML lower bound to within about 0.6 dB, illustrating the near-optimality of the proposed ERTS algorithm. Importantly, the ERTS achieves this near-ML performance at a significantly lesser complexity than λ-GSD. This complexity advantage of ERTS over λ-GSD is illustrated in Figs. 4(a) and 4(b), where the BER and complexity (in number of real operations) of ERTS and λ-GSD are compared for V-BLAST UD-MIMO with $n_t = 16$, 4-QAM, SNR = 14 dB, and n_r varied from 8 to 14 (i.e., α varied from 2 to 1.14).

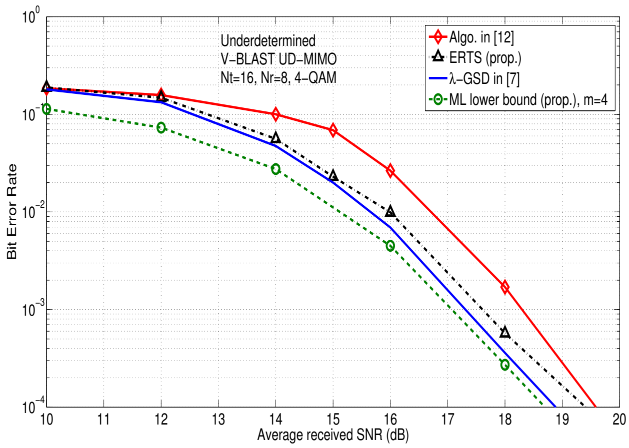

Fig. 3. Comparison of BER performance of the proposed ERTS algorithm with those of the algorithm in [12], the λ-GSD in [7], and the proposed ML lower bound. 16×8 V-BLAST UD-MIMO with 4-QAM.

In Fig. 4(a), we see that the performance of ERTS and λ-GSD are almost the same. However, in Fig. 4(b), we see that the complexity of λ-GSD is about an order higher than that of ERTS, particularly for large values of α. For example, in 16×8 system, the complexity of ERTS is 3×10^6, whereas the complexity of λ-GSD is more than 6×10^7.

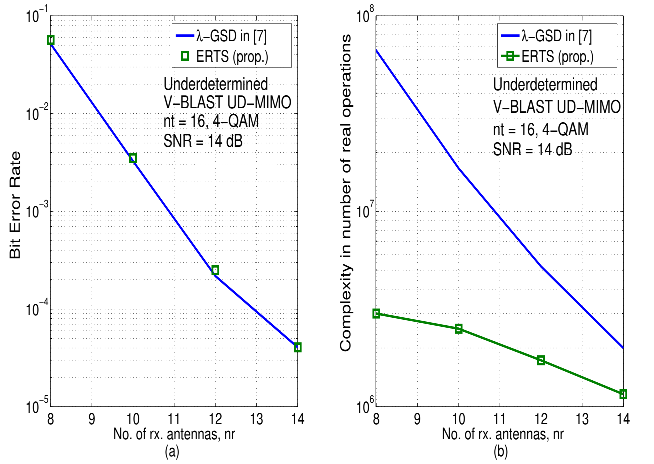

Fig. 4. Comparison of ERTS algorithm performance and complexity with those of λ-GSD. V-BLAST UD-MIMO with $n_t = 16$ and n_r varied from 8 to 14. 4-QAM, SNR = 14 dB.

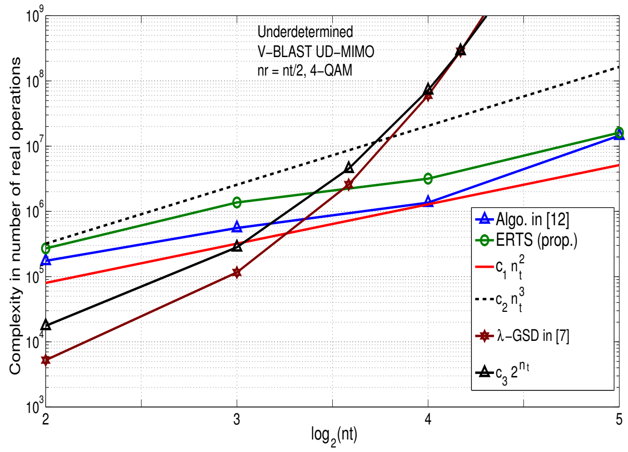

Fig. 5. Complexity (in number of real operations) comparison between ERTS, algorithm in [12], and λ-GSD at 10^{-2} BER in V-BLAST UD-MIMO, $n_r = \frac{n_t}{2}$, 4-QAM.

The complexity attributes of ERTS and λ-GSD are further highlighted in Fig. 5, where the variation of complexity as a function of n_t for a fixed overload factor of $\alpha = 2$ $\left(\text{i.e., } n_r = \frac{n_t}{2}\right)$ and 4-QAM is plotted for different algorithms. $O(n_t^2)$, $O(n_t^3)$, $O(2^{n_t})$ curves are also plotted for comparison. It is seen that the order of complexity is polynomial in n_t for ERTS and the algorithm in [12]. Whereas, the λ-GSD complexity is exponential in n_t. This makes λ-GSD prohibitive for large n_t. In Fig. 6, the performance of ERTS in 32×16 V-BLAST UD-MIMO with 4-QAM is shown along with the performance of the algorithm in [12]. From Figs. 5 and 6, we see that while the complexities of ERTS and the algorithm in [12] are roughly the same for 32×16 UD-MIMO, the performance of ERTS is better by about 1.5 dB at 10^{-2} BER. In addition, the ERTS is found to perform close to the ML lower bound (within 1.5 dB at 10^{-2} BER).

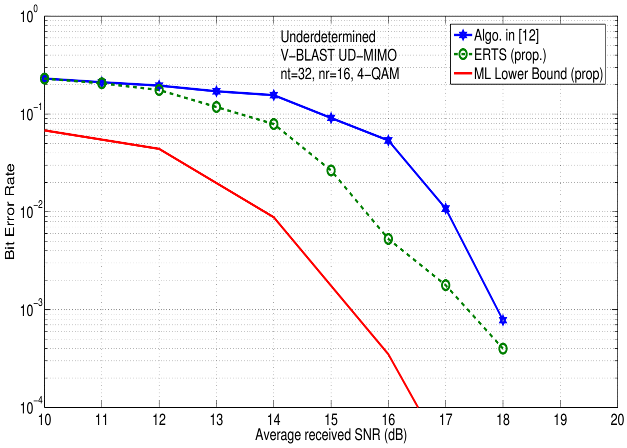

Fig. 6. Performance of ERTS, algorithm in [12], and proposed ML lower bound in 32×16 V-BLAST UD-MIMO with 4-QAM. λ-GSD performance is not shown because of its prohibitive complexity for 32×16 UD-MIMO.

In Fig. 7, we illustrate the BER of ERTS in 16×12, and 32×24 UD MIMO systems ($\alpha = 1.33$) with 4-QAM. For 16×12, we compare ERTS performance with λ-GSD performance; both ERTS and λ-GSD perform very close to the ML lower bound. Exhaustive ML search and λ-GSD simulations for $n_t = 32$ are prohibitively complex. So we do not give λ-GSD performance for comparison in 32×24 UD-MIMO. Instead, we show the comparison with the ML lower bound. It is seen that ERTS performs close to ML performance/lower bound (close to within 0.25 dB at 10^{-2} BER).

In Fig. 8, the ERTS performance for 16-QAM in 16×12 V-BLAST UD-MIMO is shown. The λ-GSD performance and the ML lower bound are also shown. The point to note here again is that the SNR gap between ERTS and λ-GSD is only 0.2 dB at 10^{-2} BER (in favor of λ-GSD), but the complexity gap is about two orders (in favor of the proposed ERTS).

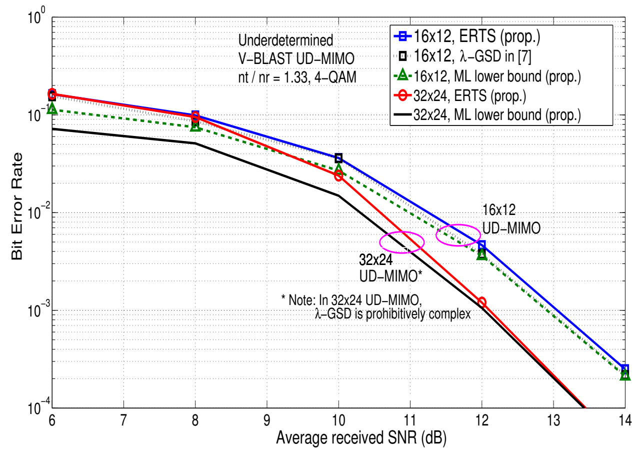

Fig. 7. BER performance of ERTS algorithm and proposed ML lower bound in V-BLAST UD-MIMO systems with $\alpha = 1.33$ and 4-QAM. (n_t, n_r): (16,12), (32,24).

VI. Conclusions

We presented a tabu search based detection algorithm which achieved improved performance in underdetermined MIMO systems by exploiting multiple random restarts and a threshold based stopping criterion. Unlike the generalized sphere decoder, the proposed algorithm scaled well for large number of antennas. In addition, it exhibits near-optimal performance with large number of antennas; e.g., near-ML BER performance was shown for 16×12, 16×8, 32×24, 32×16 UD-MIMO with 4-QAM/16-QAM. A proposed low-complexity ML lower bound aided the assessment of the nearness of the proposed algorithm performance to ML performance.

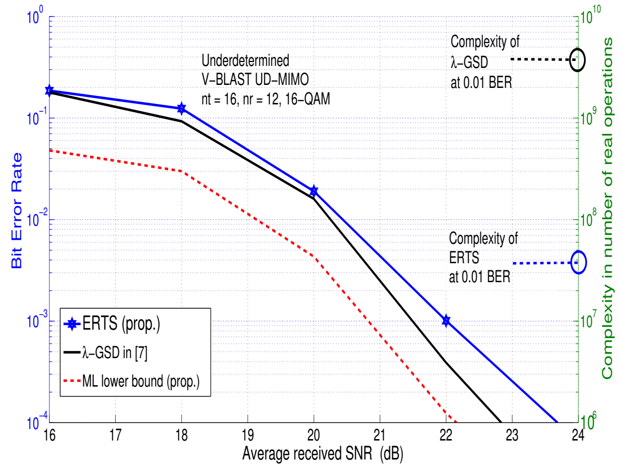

Fig. 8. BER performance and complexity of ERTS and λ-GSD in 16×12 V-BLAST UD-MIMO with 16-QAM.

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